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SENSITIVITY ANALYSIS FOR A SYSTEM OF GENERALIZED NONLINEAR MIXED QUASI-VARIATIONAL INCLUSIONS WITH (A, η)-ACCRETIVE MAPPINGS IN BANACH SPACES

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Abstract. In this paper, we study the behavior and sensitivity analysis of the solution set for a new system of parametric generalized nonlinear mixed quasi-variational inclusions with (A, η)-accretive mappings in q-uniformly smooth Banach spaces. The present results improve and extend many known results in the literature.

1. Introduction

Sensitivity analysis of solutions of variational inequalities with single-valued mappings has been studied by many authors via quite different techniques.

By using the projection method, Dafermos [2], Yen [9], Mukherjee and Verma [6], Noor [7], and Pan [8] studied the sensitivity analysis of solutions of some variational inequalities with single-valued mappings in finite-dimensional spaces or Hilbert spaces.

In 2004, using the concept and technique of resolvent operators, Agawal et. al [1] introduced and studied the behavior and sensitivity analysis of the solution set for a system of parametric variational inclusions in a Hilbert space H, which is called the system of parametric generalized nonlinear mixed quasi-variational inclusion problem:

For a given two nonempty open subsets Ω and Λ of H in which the parameters ω and λ take values, two maximal monotone mappings $M : H \times \Omega \to 2^H$ and $N : H \times \Lambda \to 2^H$, nonlinear single-valued mappings $A, S : H \times \Omega \to H$ and $B, T : H \times \Lambda \to H$, find $(x, y) \in H \times H$ such that

$$0 \in x - y + \rho(A(y, \omega) + S(y, \omega)) + \rho M(x, \omega),$$
$$0 \in y - x + \gamma(B(x, \lambda) + T(x, \lambda)) + \gamma N(y, \lambda),$$

where $\rho, \lambda > 0$ are two constants.

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In this paper, we study the behavior and sensitivity analysis of the solution set for a system of parametric generalized nonlinear mixed quasi-variational inclusions with (A,η)-accretive mappings in q-uniformly smooth Banach spaces. The present results improve and extend many known results in the literature.

2. Preliminaries

Let E be a real Banach space with dual space E^*, $\langle\cdot,\cdot\rangle$ be the dual pair between E and E^* and 2^E denote the family of all the nonempty subsets of E. The generalized duality mapping $J_q : E \to 2^{E^*}$ is defined by

$$J_q(x) = \{f \in E^* | \langle x, f^*\rangle = \|x\|^q \quad \text{and} \quad \|f^*\| = \|x\|^{q-1}\}, \forall x \in E,$$

where $q > 1$ is a constant. In particular, J_2 is the usual normalized duality mapping. It is known that, in general, $J_q(x) = \|x\|^{q-2}J_2(x)$ for all $x \neq 0$ and J_q is single-valued if E^* is strictly convex. If $E = H$ is a Hilbert space, then J_2 becomes the identity mapping of H.

The modulus of smoothness of E is the function $\rho_E : [0,\infty) \to [0,\infty)$ defined by

$$\rho_E(t) = \sup\{\frac{1}{2}(\|x+y\| + \|x-y\|) - 1 : \|x\| \le 1, \|y\| \le t\}.$$

A Banach space E is called uniformly smooth if $\lim_{t\to 0} \frac{\rho_E(t)}{t} = 0$. E is called q-uniformly smooth if there exists a constant $c > 0$ such that $\rho_E(t) \le ct^q$, $q > 1$. Note that J_q is single-valued if E is uniformly smooth.

We consider now a system of parametric generalized nonlinear mixed quasi-variational inclusions with (A,η)-accretive mappings in q-uniformly smooth Banach spaces. To this end, let Ω and Λ be two nonempty open subsets of E in which the parameters ω and λ take values, $S, T : E \times \Omega \to E$ and $U, V : E \times \Lambda \to E$ be nonlinear single-valued mappings. Let $M : E \times \Omega \to 2^E$ and $N : E \times \Lambda \to 2^E$ be set-valued mappings such that for each given $(\omega,\lambda) \in \Omega\times\Lambda$, $M(\cdot,\omega)$ and $N(\cdot,\lambda) : E \to 2^E$ are (A,η)-accretive mappings. For each fixed $(\omega,\lambda) \in \Omega \times \Lambda$, the system of parametric generalized nonlinear mixed quasi-variational inclusions with (A,η)-accretive mappings in q-uniformly smooth Banach spaces consist of finding $(x,y) \in E \times E$ such that

$$\begin{aligned} &0 \in A(x) - y + \rho(S(y,\omega) + T(y,\omega)) + \rho M(x,\omega),\\ &0 \in A(y) - x + \gamma(U(x,\lambda) + V(x,\lambda)) + \gamma N(y,\lambda), \end{aligned} \tag{2.1}$$

where $\rho > 0$ and $\gamma > 0$ are two constants.

We now discuss some special cases.

Case I. Let $E = H$ be a Hilbert space, $A = I$, the identity mappings, Ω and Λ be two nonempty open subsets of H in which the parameters ω and λ takes values. Let $\phi_1 : H\times\Omega \to R\cup\{+\infty\}$ and $\phi_2 : H\times\Lambda \to R\cup\{+\infty\}$ be functionals such that for $(x,\omega) \in H \times \Omega$ and $(y,\lambda) \in H \times \Lambda$, $\partial\phi_1(\cdot,\omega)$ and $\partial\phi_2(\cdot,\lambda)$ denote the subdifferential of proper convex lower semicontinuous functions ϕ_1 and ϕ_2, respectively. Let $M(\cdot,\omega) = \partial\phi_1(\cdot,\omega)$ and $N(\cdot,\lambda) = \partial\phi_2(\cdot,\lambda)$ for all

$(\omega, \lambda) \in \Omega \times \Lambda$. Then problem (2.1) is equivalent to finding $(x^*, y^*) \in H \times H$ such that

$$\begin{aligned} &\langle \rho(S(y^*, \omega) + T(y^*, \omega)) + x^* - y^*, x - x^* \rangle \geq \rho\phi_1(x^*, \omega) - \rho\phi_1(x, \omega), \\ &\langle \gamma(U(x^*, \lambda) + V(x^*, \lambda)) + y^* - x^*, x - y^* \rangle \geq \gamma\phi_2(y^*, \lambda) - \gamma\phi_2(x, \lambda) \end{aligned} \tag{2.2}$$

for all $x \in H$, which is called the system of parametric generalized nonlinear mixed variational inequalities in Hilbert spaces [1].

Case II. Let K be a nonempty closed convex subset of H and $\phi_i = I_K$ $(i = 1, 2)$ are the indicator functions of K. Then problem (2.2) reduces to the problem of finding $(x^*, y^*) \in K \times K$ such that

$$\begin{aligned} &\langle \rho(S(y^*, \omega) + T(y^*, \omega)) + x^* - y^*, x - x^* \rangle \geq 0, \\ &\langle \gamma(U(x^*, \lambda) + V(x^*, \lambda)) + y^* - x^*, x - y^* \rangle \geq 0 \end{aligned}$$

for all $x \in K$, which is called the system of parametric generalized nonlinear quasi-variational inequalities in Hilbert spaces [1].

Definition 2.1. Let $A : E \to E$, $\eta : E \times E \to E$ be two single-valued mappings. Then a multivalued mapping $M : E \to 2^E$ is said to be

(i) accretive if

$$\langle u - v, J_q(x - y) \rangle \geq 0, \quad \forall x, y \in E, u \in M(x), v \in M(y);$$

(ii) m-relaxed η-accretive if there exists a constant $m > 0$ such that

$$\langle u - v, J_q(\eta(x, y)) \rangle \geq -m\|x - y\|^q, \quad \forall x, y \in E, u \in M(x), v \in M(y);$$

(iii) (A, η)-accretive if M is m-relaxed η-accretive and $(A + \rho M)(E) = E$ for every $\rho > 0$.

Definition 2.2. A mapping $S : E \times \Omega \to E$ is said to be

(i) δ-strongly accretive with respect to the first argument, $\delta \in (0, 1)$, if

$$\langle S(x, \omega) - S(y, \omega), J_q(x - y) \rangle \geq \delta\|x - y\|^q, \quad \forall x, y \in E;$$

(ii) λ_S-Lipschitz continuous with respect to the first argument if there exists a constant $\lambda_S > 0$ such that

$$\|S(x, \omega) - S(y, \omega)\| \leq \lambda_S\|x - y\|, \quad \forall (x, y, \omega) \in E \times E \times \Omega.$$

Definition 2.3. A single-valued mapping $A : E \to E$ is said to be

(i) η-accretive if

$$\langle A(x) - A(y), J_q(\eta(x, y)) \rangle \geq 0, \quad \forall x, y \in E;$$

(ii) strictly η-accretive if A is η-accretive and equality holds if and only if $x = y$.

(iii) γ-strongly η-accretive if there exists a constant $\gamma > 0$ such that

$$\langle A(x) - A(y), J_q(\eta(x, y)) \rangle \geq \gamma\|x - y\|^q, \quad \forall x, y \in E.$$

If $A : E \to E$ is a strictly η-accretive mapping and $M : E \to 2^E$ is an (A, η)-accretive mapping, then for a constant $\rho > 0$, the resolvent operator associated with A and M is defined by

$$R_{M,\rho}^{A,\eta}(u) = (A + \rho M)^{-1}(u), \quad \forall u \in E.$$

It is well known that $R_{M,\rho}^{A,\eta}$ is a single-valued mapping [5].

Remark 2.1. Since M is an (A, η)-accretive mapping with respect to the first argument, for any fixed $\omega \in \Omega$, we define

$$R_{M(\cdot,\omega),\rho}^{A,\eta}(u) = (A + \rho M(\cdot, \omega))^{-1}(u), \quad \forall u \in D(M),$$

which is called the parametric resolvent operator associated with A and $M(\cdot, \omega)$.

Remark 2.2. Resolvent operators associated with (A, η)-accretive mappings include as special cases the corresponding resolvent operators associated with (H, η)-accretive operators [8], (H, η)-monotone operators [10], H-accretive operators [4], H-monotone operators [3], A-monotone operators [11], the classical m-accretive and maximal monotone operators [14].

Now we need some lemmas which will be used in the proofs for the main results in the next section.

Lemma 2.1 ([12]). *Let E be a real uniformly smooth Banach space. Then E is q-uniformly smooth if and only if there exists a constant $c_q > 0$ such that for all $x, y \in E$*

$$\|x + y\|^q \le \|x\|^q + q\langle y, J_q(x)\rangle + c_q\|y\|^q.$$

Lemma 2.2 ([5]). *Let E be a q-uniformly smooth Banach space, $\eta : E \times E \to E$ be τ-Lipschitz continuous, $A : E \to E$ be r-strongly η-accretive mapping and $M : E \to 2^E$ be an (A, η)-accretive mapping. Then the resolvent operator $R_{M(\cdot,\omega),\rho}^{A,\eta} : E \to E$ is $\frac{\tau^{q-1}}{r-\rho m}$-Lipschitz continuous, i.e.,*

$$\|R_{M(\cdot,\omega),\rho}^{A,\eta}(u) - R_{M(\cdot,\omega),\rho}^{A,\eta}(v)\| \le \frac{\tau^{q-1}}{r - \rho m}\|u - v\|, \quad \forall u, v \in E,$$

where $\rho \in (0, \frac{r}{m})$ is a constant.

3. Sensitivity analysis of solution set

Throughout the rest of this paper, we always assume that E is a real q-uniformly smooth Banach space. First of all, we prove the following lemma.

Lemma 3.1. *For all fixed $(\omega, \lambda) \in \Omega \times \Lambda$, $(\bar{x}(\omega, \lambda), \bar{y}(\omega, \lambda))$ is a solution of the system of parametric generalized nonlinear quasi-variational inclusions with (A, η)-accretive mapping in q-uniformly smooth Banach space* (2.1) *if and only if for some given $\rho, \gamma > 0$, the mapping $F : E \times \Omega \times \Lambda \to E$ defined by*

$$
\begin{aligned}
F(x,\omega,y) = R^{A,\eta}_{M(\cdot,\omega),\rho}[&R^{A,\eta}_{N(\cdot,\lambda),\gamma}(x-\gamma(U+V)(x,\lambda)) \\
&- \rho(S+T)(R^{A,\eta}_{N(\cdot,\lambda),\gamma}(x-\gamma(U+V)(x,\lambda)),\omega)]
\end{aligned} \tag{3.1}
$$

has a fixed point $\bar{x}$.

Proof. For each fixed $(\omega,\lambda) \in \Omega \times \Lambda$, let $(\bar{x}(\omega,\lambda), \bar{y}(\omega,\lambda))$ be a solution of problem (2.1). Then for given $\rho, \gamma > 0$,

$$
\begin{aligned}
0 &\in A(\bar{x}) - \bar{y} + \rho(S(\bar{y},\omega) + T(\bar{y},\omega)) + \rho M(\bar{x},\omega), \\
0 &\in A(\bar{y}) - \bar{x} + \gamma(U(\bar{x},\lambda) + V(\bar{x},\lambda)) + \gamma N(\bar{y},\lambda),
\end{aligned}
$$

which implies that

$$
\begin{aligned}
\bar{y} - \rho(S+T)(\bar{y},\omega) &\in [A + \rho M(\cdot,\omega)](\bar{x}), \\
\bar{x} - \gamma(U+V)(\bar{x},\lambda) &\in [A + \gamma N(\cdot,\lambda)](\bar{y}).
\end{aligned}
$$

Thus, we obtain

$$
\begin{aligned}
(A + \rho M(\cdot,\omega))^{-1}[\bar{y} - \rho(S+T)(\bar{y},\omega)] &= \bar{x}, \\
(A + \gamma N(\cdot,\lambda))^{-1}[\bar{x} - \gamma(U+V)(\bar{x},\lambda)] &= \bar{y},
\end{aligned}
$$

i.e.,

$$
\begin{aligned}
R^{A,\eta}_{M(\cdot,\omega),\rho}[\bar{y} - \rho(S+T)(\bar{y},\omega)] &= \bar{x}, \\
R^{A,\eta}_{N(\cdot,\lambda),\gamma}[\bar{x} - \gamma(U+V)(\bar{x},\lambda)] &= \bar{y}.
\end{aligned}
$$

Hence, we have

$$
\begin{aligned}
\bar{x} &= R^{A,\eta}_{M(\cdot,\omega),\rho}[R^{A,\eta}_{N(\cdot,\lambda),\gamma}(\bar{x}-\gamma(U+V)(\bar{x},\lambda)) \\
&\quad - \rho(S+T)(R^{A,\eta}_{N(\cdot,\lambda),\gamma}(\bar{x}-\gamma(U+V)(\bar{x},\lambda)),\omega)] \\
&= F(\bar{x},\omega,\lambda).
\end{aligned}
$$

This means that $\bar{x}$ is a fixed point of $F(x,\omega,\lambda)$.

Now, for any fixed point $(\omega,\lambda) \in (\Omega \times \Lambda)$, let $\bar{x}$ be a fixed point of $F(x,\omega,\lambda)$. By the definition of F,

$$
\begin{aligned}
\bar{x} &= F(\bar{x},\omega,\lambda) \\
&= R^{A,\eta}_{M(\cdot,\omega),\rho}[R^{A,\eta}_{N(\cdot,\lambda),\gamma}(\bar{x}-\gamma(U+V)(\bar{x},\lambda)) \\
&\quad - \rho(S+T)(R^{A,\eta}_{N(\cdot,\lambda),\gamma}(\bar{x}-\gamma(U+V)(\bar{x},\lambda)),\omega)].
\end{aligned}
$$

Let

$$
\bar{y} = R^{A,\eta}_{N(\cdot,\lambda),\gamma}(\bar{x} - \gamma(U+V)(\bar{x},\lambda)).
$$

Then we have

$$
\bar{x} = R^{A,\eta}_{M(\cdot,\omega),\rho}[\bar{y} - \rho(S+T)(\bar{y},w)].
$$

By the definitions of $R^{A,\eta}_{M(\cdot,\omega),\rho}$ and $R^{A,\eta}_{N(\cdot,\lambda),\gamma}$, we get

$$
\bar{x} = (A + \rho M(\cdot,\omega))^{-1}[\bar{y} - \rho(S+T)(\bar{y},\omega)],
$$

$$\bar{y} = (A + \gamma N(\cdot, \lambda))^{-1}[\bar{x} - \gamma(U + T)(\bar{x}, \lambda)],$$

which implies that

$$\bar{y} - \rho(S + T)(\bar{y}, \omega) \in [A + \rho M(\cdot, \omega)](\bar{x}),$$
$$\bar{x} - \gamma(U + V)(\bar{x}, \lambda) \in [A + \gamma N(\cdot, \lambda)](\bar{y}).$$

Hence

$$0 \in A(\bar{x}) - \bar{y} + \rho(S(\bar{y}, \omega) + T(\bar{y}, \omega)) + \rho M(\bar{x}, \omega),$$
$$0 \in A(\bar{y}) - \bar{x} + \gamma(U(\bar{x}, \lambda) + V(\bar{x}, \lambda)) + \gamma N(\bar{y}, \lambda).$$

This completes the proof. □

Theorem 3.1. *Let $A : E \to E$, $S, T : E \times \Omega \to E$, $U, V : E \times \Lambda \to E$ be five mappings and $M : E \times \Omega \to 2^E$, $N : E \times \Lambda \to 2^E$ be two set-valued mappings satisfying the following conditions*:

(i) *A is r-strongly η-accertive mapping,*
(ii) *S is λ_S-Lipschitz continuous with respect to the first argument,*
(iii) *T is δ-strongly accretive and λ_T-Lipschitz continuous with respect to the first argument,*
(iv) *U is λ_U-Lipschitz continuous with respect to the first argument,*
(v) *V is α-strongly accretive and λ_V-Lipschitz continuous with respect to the first argument,*
(vi) *M and N are (A, η)-accretive with respect to the first argument.*

Suppose that there exist $\rho > 0$ and $\gamma > 0$ such that

$$1 - \alpha q\gamma + c_q\gamma^q\lambda_V^q < (\frac{\gamma - \rho m}{\tau^{q-1}} - \gamma\lambda_U)^q,$$

$$1 - q\rho\delta + c_q\rho^q\lambda_T^q < (\frac{\gamma - \lambda m}{\tau^{q-1}} - \rho\lambda_S)^q. \tag{3.2}$$

Then

(1) *the mapping $F : E\times\Omega\times\Lambda \to E$ defined by* (3.1) *is a uniform θ-contractive mapping with respect to $(\omega, \lambda) \in \Omega \times \Lambda$.*

(2) *for each $(\omega, \lambda) \in \Omega \times \Lambda$, the system of parametric generalized nonlinear mixed quasi-variational inclusions with (A, η)-accretive mappings in q-uniformly smooth Banach spaces* (2.1) *has a nonempty solution set $S(\omega, \lambda)$ and $S(\omega\lambda)$ is a closed subset of E.*

Proof. (1) By the definition of F, for any $x, y \in E$, we have

(3.3)

$$\begin{aligned}
&\|F(x, \omega, \lambda) - F(y, \omega, \lambda)\| \\
&= \|R^{A,\eta}_{M(\cdot,\omega),\rho}[R^{A,\eta}_{N(\cdot,\lambda),\gamma}(x - \gamma(U + V)(x, \lambda)) \\
&\qquad - \rho(S + T)(R^{A,\eta}_{N(\cdot,\lambda),\gamma}(x - \gamma(U + V)(x, \lambda)), \omega)] \\
&\quad - R^{A,\eta}_{M(\cdot,\omega),\rho}[R^{A,\eta}_{N(\cdot,\lambda),\gamma}(y - \gamma(U + V)(y, \lambda))
\end{aligned}$$

$$
\begin{aligned}
&\qquad - \rho(S+T)(R^{A,\eta}_{N(\cdot,\lambda),\gamma}(y-\gamma(U+V)(y,\lambda)),\omega)]\| \\
&\le \frac{\tau^{q-1}}{r-\rho m}\|R^{A,\eta}_{N(\cdot,\lambda),\gamma}(x-\gamma(U+V)(x,\lambda)) \\
&\qquad - \rho(S+T)(R^{A,\eta}_{N(\cdot,\lambda),\gamma}(x-\gamma(U+V)(x,\lambda)),\omega) \\
&\quad - (R^{A,\eta}_{N(\cdot,\lambda),\gamma}(y-\gamma(U+V)(y,\lambda)) \\
&\qquad - \rho(S+T)(R^{A,\eta}_{N(\cdot,\lambda),\gamma}(y-\gamma(U+V)(y,\lambda)),\omega))\| \\
&\le \frac{\tau^{q-1}}{r-\rho m}\|R^{A,\eta}_{N(\cdot,\lambda),\gamma}(x-\gamma(U+V)(x,\lambda)) - R^{A,\eta}_{N(\cdot,\lambda),\gamma}(y-\gamma(U+V)(y,\lambda)) \\
&\quad - \rho[T(R^{A,\eta}_{N(\cdot,\lambda),\gamma}(x-\gamma(U+V)(x,\lambda)),\omega) \\
&\qquad - T(R^{A,\eta}_{N(\cdot,\lambda),\gamma}(y-\gamma(U+V)(y,\lambda)),\omega)]\| \\
&\quad + \frac{\tau^{q-1}}{r-\rho m}\rho\|S(R^{A,\eta}_{N(\cdot,\lambda),\gamma}(x-\gamma(U+V)(x,\lambda)),\omega) \\
&\qquad - S(R^{A,\eta}_{N(\cdot,\lambda),\gamma}(y-\gamma(U+V)(y,\lambda)),\omega)\|.
\end{aligned}
$$

From Lemma 2.1, the δ-strong accretivity and λ_T-Lipschitz continuity of T it follows that

(3.4)

$$
\begin{aligned}
&\|R^{A,\eta}_{N(\cdot,\lambda),\gamma}(x-\gamma(U+V)(x,\lambda)) - R^{A,\eta}_{N(\cdot,\lambda),\gamma}(y-\gamma(U+V)(y,\lambda)) \\
&\quad - \rho\{T(R^{A,\eta}_{N(\cdot,\lambda),\gamma}(x-\gamma(U+V)(x,\lambda)),\omega) \\
&\qquad - T(R^{A,\eta}_{N(\cdot,\lambda),\gamma}(y-\gamma(U+V)(y,\lambda)),\omega)\}\|^q \\
&\le \|R^{A,\eta}_{N(\cdot,\lambda),\gamma}(x-\gamma(U+V)(x,\lambda)) - R^{A,\eta}_{N(\cdot,\lambda),\gamma}(y-\gamma(U+V)(y,\lambda))\|^q \\
&\quad - q\rho\langle T(R^{A,\eta}_{N(\cdot,\lambda),\gamma}(x-\gamma(U+V)(x,\lambda)),\omega) \\
&\qquad - T(R^{A,\eta}_{N(\cdot,\lambda),\gamma}(y-\gamma(U+V)(y,\lambda)),\omega), J_q(R^{A,\eta}_{N(\cdot,\lambda),\gamma}(x-\gamma(U+V)(x,\lambda)) \\
&\qquad - R^{A,\eta}_{N(\cdot,\lambda),\gamma}(y-\gamma(U+V)(y,\lambda)))\rangle \\
&\quad + c_q\rho^q\|T(R^{A,\eta}_{N(\cdot,\lambda),\gamma}(x-\gamma(U+V)(x,\lambda)),\omega) \\
&\qquad - T(R^{A,\eta}_{N(\cdot,\lambda),\gamma}(y-\gamma(U+V)(y,\lambda)),\omega)\|^q \\
&\le (1-q\rho\delta+c_q\rho^q\lambda_T^q)\|R^{A,\eta}_{N(\cdot,\lambda),\gamma}(x-\gamma(U+V)(x,\lambda)) \\
&\quad - R^{A,\eta}_{N(\cdot,\lambda),\gamma}(y-\gamma(U+V)(y,\lambda))\|^q \\
&\le (1-q\rho\delta+c_q\rho^q\lambda_T^q)(\frac{\tau^{q-1}}{r-\lambda m})^q(\|x-y-\gamma(V(x,\lambda)-V(y,\lambda))\| \\
&\quad + \gamma\|U(x,\lambda)-U(y,\lambda)\|)^q.
\end{aligned}
$$

Since V is α-strongly accretive and λ_V-Lipschitz continuous with respect to the first argument,

$$\begin{aligned} &\|x-y-\gamma(V(x,\lambda)-V(y,\lambda))\|^q \\ &\le \|x-y\|^q - q\gamma\langle V(x,\lambda)-V(y,\lambda), J_q(x-y)\rangle \\ &\quad + c_q\gamma^q\|V(x,\lambda)-V(y,\lambda)\|^q \\ &\le (1-\alpha q\gamma + c_q\gamma^q\lambda_V^q)\|x-y\|^q. \end{aligned} \tag{3.5}$$

By the λ_U-Lipschitz continuity of U, we have

$$\|U(x,\lambda)-U(y,\lambda)\| \le \lambda_U\|x-y\|. \tag{3.6}$$

By the λ_S-Lipschitz continuity of S, (3.5) and (3.6), we obtain

(3.7)

$$\begin{aligned} &\|S(R^{A,\eta}_{N(\cdot,\lambda),\gamma}(x-\gamma(U+V)(x,\lambda)),\omega) - S(R^{A,\eta}_{N(\cdot,\lambda),\gamma}(y-\gamma(U+V)(y,\lambda)),\omega)\| \\ &\le \lambda_S\|R^{A,\eta}_{N(\cdot,\lambda),\gamma}(x-\gamma(U+V)(x,\lambda)),\omega) - R^{A,\eta}_{N(\cdot,\lambda),\gamma}(y-\gamma(U+V)(y,\lambda)),\omega)\| \\ &\le \lambda_S\frac{\tau^{q-1}}{r-\lambda m}(\|x-y-\gamma(V(x,\lambda)-V(y,\lambda))\| + \gamma\|U(x,\lambda)-U(y,\lambda)\|) \\ &\le \frac{\lambda_S\tau^{q-1}}{r-\lambda m}[(1-\alpha q\gamma+c_q\gamma^q\lambda_V^q)^{\frac{1}{q}}+\gamma\lambda_U]\|x-y\|. \end{aligned}$$

By (3.3)-(3.7), we have

$$\begin{aligned} &\|F(x,\omega,\lambda)-F(y,\omega,\lambda)\| \\ &\le (\frac{\tau^{q-1}}{r-\rho m})(\frac{\tau^{q-1}}{r-\lambda m})[(1-\alpha q\gamma+c_q\gamma^q\lambda_V^q)^{\frac{1}{q}}+\gamma\lambda_U] \\ &\qquad [(1-q\rho\delta+c_q\rho^q\lambda_T^q)^{\frac{1}{q}}+\rho\lambda_S]\|x-y\| \\ &= (\frac{\tau^{q-1}}{r-\rho m})(\frac{\tau^{q-1}}{r-\lambda m})\theta_1\theta_2\|x-y\| \\ &= \theta\|x-y\|, \end{aligned} \tag{3.8}$$

where $\theta_1 = (1-\alpha q\gamma+c_q\gamma^q\lambda_V^q)^{\frac{1}{q}}+\gamma\lambda_U$, $\theta_2 = (1-q\rho\delta+c_q\rho^q\lambda_T^q)^{\frac{1}{q}}+\rho\lambda_S$ and $\theta = (\frac{\tau^{q-1}}{r-\rho m})(\frac{\tau^{q-1}}{r-\lambda m})\theta_1\theta_2$. It follows from condition (3.2) that $\theta < 1$. Thus, (3.8) implies that F is a contractive mapping which is uniform with respect to $(\omega,\lambda)\in\Omega\times\Lambda$.

(2) Since $F(x,\omega,\lambda)$ is a uniform θ-contractive mapping with respect to $(\omega,\lambda)\in\Omega\times\Lambda$, by the Banach fixed point theorem, $F(x,\omega,\lambda)$ has a fixed point $\bar{x}(\omega,\lambda)$ for each $(\omega,\lambda)\in\Omega\times\Lambda$. By Lemma 3.1, $S(\omega,\lambda)\ne\phi$. For each $(\omega,\lambda)\in\Omega\times\Lambda$, let $(x_n,y_n)\in S(\omega,\lambda)$ and $x_n\to x_0$, $y_n\to y_0$ as $n\to\infty$. Then we have

$$x_n \in F(x_n,\omega,\lambda),\quad n=1,2,\ldots.$$

By (1), we have

$$\|F(x_n,\omega,\lambda) - F(x,\omega,\lambda)\| \le \theta\|x_n - x\|.$$

It follows that

$$\begin{aligned}\|x_0 - F(x_0,\omega,\lambda)\| &\le \|x_0 - x_n\| + \|x_n - F(x_n,\omega,\lambda)\| \\ &\quad + \|F(x_n,\omega,\lambda) - F(x_0,\omega,\lambda)\| \\ &\le (1+\theta)\|x_n - x_0\| \\ &\to 0 \quad \text{as} \quad n \to \infty.\end{aligned}$$

Hence we have $x_0 \in F(x_0,\omega,\lambda)$. From Lemma 3.1 we have $(x_0,y_0) \in S(\omega,\lambda)$. Therefore $S(\omega,\lambda)$ is a nonempty closed subset of E. □

Theorem 3.2. *Under the hypotheses of Theorem 3.1, further assume that for any $x,y \in E$, the mappings $\omega \mapsto S(x,\omega)$, $\omega \mapsto T(x,\omega)$, $\lambda \mapsto U(y,\lambda)$ and $\lambda \mapsto V(y,\lambda)$ are Lipschitz continuous with constants l_S, l_T, l_U, l_V, respectively. Suppose that for any $(t,\omega.\bar{\omega}) \in E \times \Omega \times \Omega$ and $(z,\lambda,\bar{\lambda}) \in E \times \Lambda \times \Lambda$,*

$$\|R^{A,\eta}_{M(\cdot,\omega),\rho}(t) - R^{A,\eta}_{M(\cdot,\bar{\omega}),\rho}(t)\| \le \mu\|\omega - \bar{\omega}\|,$$

$$\|R^{A,\eta}_{N(\cdot,\lambda),\gamma}(z) - R^{A,\eta}_{M(\cdot,\bar{\lambda}),\gamma}(z)\| \le \tau\|\lambda - \bar{\lambda}\|, \tag{3.9}$$

where $\mu > 0$ and $\tau > 0$ are two constants.

Then the solution $(x(\omega,\lambda), y(\omega,\lambda))$ for the system of parametric generalized nonlinear mixed quasi-variational inclusions with (A,η)-accretive mappings in q-uniformly smooth Banach spaces (2.1) *is Lipschitz continuous.*

Proof. For each $(\omega,\lambda),(\bar{\omega},\bar{\lambda}) \in \Omega\times\Lambda$, by Theorem 3.1, $S(\omega,\lambda)$ and $(S(\bar{\omega},\bar{\lambda})$ are both nonempty closed subsets. Also, $F(x,\omega,\lambda)$ and $F(x,\bar{\omega},\bar{\lambda})$ are contractive mappings with same constant $\theta \in (0,1)$ and have fixed points $x(\omega,\lambda)$ and $x(\bar{\omega},\bar{\lambda})$, respectively. For any fixed $(\omega,\lambda),(\bar{\omega},\bar{\lambda}) \in \Omega \times \Lambda$, we have

$$\begin{aligned}&\|x(\omega,\lambda) - x(\bar{\omega},\bar{\lambda})\| \\ &= \|F(x(\omega,\lambda),\omega,\lambda) - F(x(\bar{\omega},\bar{\lambda}),\bar{\omega},\bar{\lambda})\| \\ &\le \|F(x(\omega,\lambda),\omega,\lambda) - F(x(\bar{\omega},\bar{\lambda}),\omega,\lambda)\| \\ &\quad + \|F(x(\bar{\omega},\bar{\lambda}),\omega,\lambda) - F(x(\bar{\omega},\bar{\lambda}),\bar{\omega},\bar{\lambda})\| \\ &\le \theta\|x(\omega,\lambda) - x(\bar{\omega},\bar{\lambda})\| + \|F(x(\bar{\omega},\bar{\lambda}),\omega,\lambda) - F(x(\bar{\omega},\bar{\lambda}),\bar{\omega},\bar{\lambda})\|,\end{aligned}$$

which implies that

$$\|x(\omega,\lambda) - x(\bar{\omega},\bar{\lambda})\| \le \frac{1}{1-\theta}\|F(x(\bar{\omega},\bar{\lambda}),\omega,\lambda) - F(x(\bar{\omega},\bar{\lambda}),\bar{\omega},\bar{\lambda})\|. \tag{3.10}$$

By condition (3.9), we have

$$\begin{aligned}&\|F(x(\bar{\omega},\bar{\lambda}),\omega,\lambda) - F(x(\bar{\omega},\bar{\lambda}),\bar{\omega},\bar{\lambda})\| \\ &\le \|R^{A,\eta}_{M(\cdot,\omega),\rho}[R^{A,\eta}_{N(\cdot,\lambda),\gamma}(x(\bar{\omega},\bar{\lambda}) - \gamma(U+V)(x(\bar{\omega},\bar{\lambda}),\lambda))\end{aligned} \tag{3.11}$$

$$
\begin{aligned}
&\quad - \rho(S+T)(R^{A,\eta}_{N(\cdot,\lambda),\gamma}(x(\bar\omega,\bar\lambda) - \gamma(U+V)(x(\bar\omega,\bar\lambda),\lambda)),\omega)] \\
&- R^{A,\eta}_{M(\cdot,\omega),\rho}[R^{A,\eta}_{N(\cdot,\bar\lambda),\gamma}(x(\bar\omega,\bar\lambda) - \gamma(U+V)(x(\bar\omega,\bar\lambda),\bar\lambda)) \\
&\quad - \rho(S+T)(R^{A,\eta}_{N(\cdot,\bar\lambda),\gamma}(x(\bar\omega,\bar\lambda) - \gamma(U+V)(x(\bar\omega,\bar\lambda),\bar\lambda)),\bar\omega)]\| \\
&+ \|R^{A,\eta}_{M(\cdot,\omega),\rho}[R^{A,\eta}_{N(\cdot,\bar\lambda),\gamma}(x(\bar\omega,\bar\lambda) - \gamma(U+V)(x(\bar\omega,\bar\lambda),\bar\lambda)), \\
&\quad - \rho(S+T)(R^{A,\eta}_{N(\cdot,\bar\lambda),\gamma}(x(\bar\omega,\bar\lambda) - \gamma(U+V)(x(\bar\omega,\bar\lambda),\bar\lambda)),\bar\omega)] \\
&- R^{A,\eta}_{M(\cdot,\bar\omega),\rho}[R^{A,\eta}_{N(\cdot,\bar\lambda),\gamma}(x(\bar\omega,\bar\lambda) - \gamma(U+V)(x(\bar\omega,\bar\lambda),\bar\lambda)) \\
&\quad - \rho(S+T)(R^{A,\eta}_{N(\cdot,\bar\lambda),\gamma}(x(\bar\omega,\bar\gamma) - \gamma(U+V)(x(\bar\omega,\bar\lambda),\bar\lambda)),\bar\omega)]\| \\
\le\ & \frac{\tau^{q-1}}{r-\rho m}[\|R^{A,\eta}_{N(\cdot,\lambda),\gamma}(x(\bar\omega,\bar\lambda) - \gamma(U+V)(x(\bar\omega,\bar\lambda),\lambda)) \\
&- R^{A,\eta}_{N(\cdot,\bar\lambda),\gamma}(x(\bar\omega,\bar\lambda) - \gamma(U+V)(x(\bar\omega,\bar\lambda),\bar\lambda)) \\
&- \rho\{T(R^{A,\eta}_{N(\cdot,\lambda),\gamma}(x(\bar\omega,\bar\lambda) - \gamma(U+V)(x(\bar\omega,\bar\lambda),\lambda)),\omega) \\
&\quad - T(R^{A,\eta}_{N(\cdot,\bar\lambda),\gamma}(x(\bar\omega,\bar\lambda) - \gamma(U+V)(x(\bar\omega,\bar\lambda),\bar\lambda)),\bar\omega)\}\| \\
&+ \rho\|S(R^{A,\eta}_{N(\cdot,\lambda),\gamma}(x(\bar\omega,\bar\lambda) - \gamma(U+V)(x(\bar\omega,\bar\lambda),\lambda)),\omega) \\
&\quad - S(R^{A,\eta}_{N(\cdot,\bar\lambda),\gamma}(x(\bar\omega,\bar\lambda) - \gamma(U+V)(x(\bar\omega,\bar\lambda),\bar\lambda)),\bar\omega)\|] \\
&+ \mu\|\omega - \bar\omega\| \\
\le\ & \frac{\tau^{q-1}}{r-\rho m}[\|R^{A,\eta}_{N(\cdot,\lambda),\gamma}(x(\bar\omega,\bar\lambda) - \gamma(U+V)(x(\bar\omega,\bar\lambda),\lambda)) \\
&- R^{A,\eta}_{N(\cdot,\lambda),\gamma}(x(\bar\omega,\bar\lambda) - \gamma(U+V)(x(\bar\omega,\bar\lambda),\bar\lambda)) \\
&- \rho\{T(R^{A,\eta}_{N(\cdot,\lambda),\gamma}(x(\bar\omega,\bar\lambda) - \gamma(U+V)(x(\bar\omega,\bar\lambda),\lambda)),\omega) \\
&\quad - T(R^{A,\eta}_{N(\cdot,\lambda),\gamma}(x(\bar\omega,\bar\lambda) - \gamma(U+V)(x(\bar\omega,\bar\lambda),\bar\lambda)),\omega)\}\| \\
&+ \|R^{A,\eta}_{N(\cdot,\lambda),\gamma}(x(\bar\omega,\bar\lambda) - \gamma(U+V)(x(\bar\omega,\bar\lambda),\bar\lambda)) \\
&- R(_{N(\cdot,\bar\lambda),\gamma}(x(\bar\omega,\bar\lambda) - \gamma(U+V)(x(\bar\omega,\bar\lambda),\bar\lambda))\| \\
&+ \rho\|T(R^{A,\eta}_{N(\cdot,\lambda),\gamma}(x(\bar\omega,\bar\lambda) - \gamma(U+V)(x(\bar\omega,\bar\lambda),\bar\lambda)),\omega)\| \\
&\quad - T(R^{A,\eta}_{N(\cdot,\lambda),\gamma}(x(\bar\omega,\bar\lambda) - \gamma(U+V)(x(\bar\omega,\bar\lambda),\bar\lambda)),\bar\omega)\| \\
&+ \rho\|T(R^{A,\eta}_{N(\cdot,\lambda),\gamma}(x(\bar\omega,\bar\lambda) - \gamma(U+V)(x(\bar\omega,\bar\lambda),\bar\lambda)),\bar\omega) \\
&\quad - T(R^{A,\eta}_{N(\cdot,\bar\lambda),\gamma}(x(\bar\omega,\bar\lambda) - \gamma(U+V)(x(\bar\omega,\bar\lambda),\bar\lambda)),\bar\omega)\| \\
&+ \rho\|S(R^{A,\eta}_{N(\cdot,\lambda),\gamma}(x(\bar\omega,\bar\lambda) - \gamma(U+V)(x(\bar\omega,\bar\lambda),\lambda)),\omega) \\
&\quad - S(R^{A,\eta}_{N(\cdot,\lambda),\gamma}(x(\bar\omega,\bar\lambda) - \gamma(U+V)(x(\bar\omega,\bar\lambda),\bar\lambda)),\bar\omega)\| \\
&+ \rho\|S(R^{A,\eta}_{N(\cdot,\lambda),\gamma}(x(\bar\omega,\bar\lambda) - \gamma(U+V)(x(\bar\omega,\bar\lambda),\bar\lambda)),\bar\omega)\|
\end{aligned}
$$

$$- S(R^{A,\eta}_{N(\cdot,\bar{\lambda}),\gamma}(x(\bar{\omega},\bar{\lambda}) - \gamma(U+V)(x(\bar{\omega},\bar{\lambda}),\bar{\lambda})),\bar{\omega})\|$$
$$+ \mu\|\omega - \bar{\omega}\|.$$

The δ-strongly accretivity of T and the Lipschitz continuity of T, U and V imply

$$\begin{aligned}
&\|R^{A,\eta}_{N(\cdot,\lambda),\gamma}(x(\bar{\omega},\bar{\lambda}) - \gamma(U+V)(x(\bar{\omega},\bar{\lambda}),\lambda)) \\
&\quad - R^{A,\eta}_{N(\cdot,\lambda),\gamma}(x(\bar{\omega},\bar{\lambda}) - \gamma(U+V)(x(\bar{\omega},\bar{\lambda}),\bar{\lambda})) \\
&\quad - \rho\{T(R^{A,\eta}_{N(\cdot,\lambda),\gamma}(x(\bar{\omega},\bar{\lambda}) - \gamma(U+V)(x(\bar{\omega},\bar{\lambda}),\lambda)),\omega) \\
&\qquad - T(R^{A,\eta}_{N(\cdot,\lambda),\gamma}(x(\bar{\omega},\bar{\lambda}) - \gamma(U+V)(x(\bar{\omega},\bar{\lambda}),\bar{\lambda})),\omega)\}\|^q \\
&\le \|R^{A,\eta}_{N(\cdot,\lambda),\gamma}(x(\bar{\omega},\bar{\lambda}) - \gamma(U+V)(x(\bar{\omega},\bar{\lambda}),\lambda)) \\
&\quad - R^{A,\eta}_{N(\cdot,\lambda),\gamma}(x(\bar{\omega},\bar{\lambda}) - \gamma(U+V)(x(\bar{\omega},\bar{\lambda}),\bar{\lambda}))\|^q \\
&\quad - q\rho\langle T(R^{A,\eta}_{N(\cdot,\lambda),\gamma}(x(\bar{\omega},\bar{\lambda}) - \gamma(U+V)(x(\bar{\omega},\bar{\lambda}),\lambda)),\omega) \\
&\qquad - T(R^{A,\eta}_{N(\cdot,\lambda),\gamma}(x(\bar{\omega},\bar{\lambda}) - \gamma(U+V)(x(\bar{\omega},\bar{\lambda}),\bar{\lambda})),\omega), \\
&\quad J_q(R^{A,\eta}_{N(\cdot,\lambda),\gamma}(x(\bar{\omega},\bar{\lambda}) - \gamma(U+V)(x(\bar{\omega},\bar{\lambda}),\lambda)) \\
&\qquad - R^{A,\eta}_{N(\cdot,\lambda),\gamma}(x(\bar{\omega},\bar{\lambda}) - \gamma(U+V)(x(\bar{\omega},\bar{\lambda}),\bar{\lambda})))\rangle \\
&\quad + c_q\rho^q\|T(R^{A,\eta}_{N(\cdot,\lambda),\gamma}(x(\bar{\omega},\bar{\lambda}) - \gamma(C+D)(x(\bar{\omega},\bar{\lambda}),\lambda)),\omega) \\
&\qquad - T(R^{A,\eta}_{N(\cdot,\lambda),\gamma}(x(\bar{\omega},\bar{\lambda}) - \gamma(C+D)(x(\bar{\omega},\bar{\lambda}),\bar{\lambda})),\omega)\|^q \\
&\le (1 - q\rho\delta + c_q\rho^q\lambda_T^q)\|R^{A,\eta}_{N(\cdot,\lambda),\gamma}(x(\bar{\omega},\bar{\lambda}) - \gamma(U+V)(x(\bar{\omega},\bar{\lambda}),\lambda)) \\
&\quad - R^{A,\eta}_{N(\cdot,\lambda),\gamma}(x(\bar{\omega},\bar{\lambda}) - \gamma(U+V)(x(\bar{\omega},\bar{\lambda}),\bar{\lambda}))\|^q \\
&\le (1 - q\rho\delta + c_q\rho^q\lambda_T^q)\frac{\tau^{q(q-1)}\gamma^q}{(r-\lambda m)^q}(\|U(x(\bar{\omega},\bar{\lambda}),\lambda) - U(x(\bar{\omega},\bar{\lambda})\bar{\lambda})\| \\
&\quad + \|V(x(\bar{\omega},\bar{\lambda}),\lambda) - V(x(\bar{\omega},\bar{\lambda}),\bar{\lambda})\|)^q \\
&\le (1 - q\rho\delta + c_q\rho^q\lambda_T^q)\frac{\tau^{q(q-1)}\gamma^q}{(r-\lambda m)^q}(l_U + l_V)^q\|\lambda - \bar{\lambda}\|^q.
\end{aligned} \tag{3.12}$$

By condition (3.9), we have

$$\begin{aligned}
&\|R^{A,\eta}_{N(\cdot,\lambda),\gamma}(x(\bar{\omega},\bar{\lambda}) - \gamma(U+V)(x(\bar{\omega},\bar{\lambda}),\bar{\lambda})) \\
&\quad - R^{A,\eta}_{N(\cdot,\bar{\lambda}),\gamma}(x(\bar{\omega},\bar{\lambda}) - \gamma(U+V)(x(\bar{\omega},\bar{\lambda}),\bar{\lambda}))\| \\
&\le \tau\|\lambda - \bar{\lambda}\|.
\end{aligned} \tag{3.13}$$

By the l_T-Lipschitz continuity of T, we have

$$\begin{aligned}
&\|T(R^{A,\eta}_{N(\cdot,\lambda),\gamma}(x(\bar{\omega},\bar{\lambda}) - \gamma(U+V)(x(\bar{\omega},\bar{\lambda}),\bar{\lambda}),\omega) \\
&\quad - T(R^{A,\eta}_{N(\cdot,\lambda),\gamma}(x(\bar{\omega},\bar{\lambda}) - \gamma(U+V)(x(\bar{\omega},\bar{\lambda}),\bar{\lambda})),\bar{\omega})\|
\end{aligned} \tag{3.14}$$

$$\leq l_T\|\omega-\bar{\omega}\|.$$

By using same argument, we can prove

$$\begin{aligned}(3.15)\qquad &\|T(R^{A,\eta}_{N(\cdot,\lambda),\gamma}(x(\bar{\omega},\bar{\lambda})-\gamma(U+V)(x(\bar{\omega},\bar{\lambda}),\bar{\lambda})),\bar{\omega})\\ &\quad -T(R^{A,\eta}_{N(\cdot,\bar{\lambda}),\gamma}(x(\bar{\omega},\bar{\lambda})-\gamma(U+V)(x(\bar{\omega},\bar{\lambda}),\bar{\lambda})),\bar{\omega})\|\\ &\leq \lambda_T\tau\|\lambda-\bar{\lambda}\|,\end{aligned}$$

$$\begin{aligned}(3.16)\qquad &\|S(R^{A,\eta}_{N(\cdot,\lambda),\gamma}(x(\bar{\omega},\bar{\lambda})-\gamma(U+V)(x(\bar{\omega},\bar{\lambda}),\lambda)),\omega)\\ &\quad -S(R^{A,\eta}_{N(\cdot,\lambda),\gamma}(x(\bar{\omega},\bar{\lambda})-\gamma(U+V)(x(\bar{\omega},\bar{\lambda}),\bar{\lambda})),\bar{\omega})\|\\ &\leq \|S(R^{A,\eta}_{N(\cdot,\lambda),\gamma}(x(\bar{\omega},\bar{\lambda})-\gamma(U+V)(x(\bar{\omega},\bar{\lambda}),\lambda)),\omega)\\ &\quad -S(R^{A,\eta}_{N(\cdot,\lambda),\gamma}(x(\bar{\omega},\bar{\lambda})-\gamma(U+V)(x(\bar{\omega},\bar{\lambda}),\lambda)),\bar{\omega})\|\\ &\quad +\|S(R^{A,\eta}_{N(\cdot,\lambda),\gamma}(x(\bar{\omega},\bar{\lambda})-\gamma(U+V)(x(\bar{\omega},\bar{\lambda}),\lambda)),\bar{\omega})\\ &\quad\quad -S(R^{A,\eta}_{N(\cdot,\lambda),\gamma}(x(\bar{\omega},\bar{\lambda})-\gamma(U+V)(x(\bar{\omega},\bar{\lambda}),\bar{\lambda})),\bar{\omega})\|\\ &\leq l_S\|\omega-\bar{\omega}\|+\frac{\lambda_S\tau^{q-1}\gamma}{r-\lambda m}[\|U(x(\bar{\omega},\bar{\lambda}),\lambda)-U(x(\bar{\omega},\bar{\lambda}),\bar{\lambda})\|\\ &\quad +\|V(x(\bar{\omega},\bar{\lambda}),\lambda)-V(x(\bar{\omega},\bar{\lambda}),\bar{\lambda})\|]\\ &\leq l_S\|\omega-\bar{\omega}\|+\frac{\lambda_S\tau^{q-1}\gamma}{r-\lambda m}(l_U+l_V)\|\lambda-\bar{\lambda}\|,\end{aligned}$$

$$\begin{aligned}(3.17)\qquad &\|S(R^{A,\eta}_{N(\cdot,\lambda),\gamma}(x(\bar{\omega},\bar{\lambda})-\gamma(U+V)(x(\bar{\omega},\bar{\lambda}),\bar{\lambda})),\bar{\omega})\\ &\quad -S(R^{A,\eta}_{N(\cdot,\bar{\lambda}),\gamma}(x(\bar{\omega},\bar{\lambda})-\gamma(U+V)(x(\bar{\omega},\bar{\lambda}),\bar{\lambda})),\bar{\omega})\|\\ &\leq \lambda_S\tau\|\lambda-\bar{\lambda}\|.\end{aligned}$$

It follows from (3.10)-(3.17) that

$$\begin{aligned}&\|x(\omega,\lambda)-x(\bar{\omega},\bar{\lambda})\|\\ &\leq \frac{1}{1-\theta}[\frac{\tau^{q-1}}{r-\rho m}\{(1-q\rho\delta+c_q\rho^q\lambda_T^q)^{\frac{1}{q}}\frac{\tau^{q-1}\gamma}{r-\rho m}(l_U+l_V)\|\lambda-\bar{\lambda}\|\\ &\quad +\tau\|\lambda-\bar{\lambda}\|+\rho l_T\|\omega-\bar{\omega}\|+\rho\lambda_T\tau\|\lambda-\bar{\lambda}\|+\rho l_S\|\omega-\bar{\omega}\|\\ &\quad +\rho\lambda_S\frac{\tau^{q-1}\gamma}{r-\lambda m}(l_u+l_V)\|\lambda-\bar{\lambda}\|+\rho\lambda_S\tau\|\lambda-\bar{\lambda}\|\}+\mu\|\omega-\bar{\omega}\|]\\ &= \frac{1}{1-\theta}[\frac{\tau^{q-1}\rho}{r-\rho m}(l_S+l_T)+\mu]\|\omega-\bar{\omega}\|\\ &\quad +(\frac{1}{1-\theta})(\frac{\tau^{q-1}}{r-\rho m})[\frac{\tau^{q-1}}{r-\rho m}(1-q\rho\delta+c_q\rho^q\lambda_T^q)^{\frac{1}{q}}\gamma(l_U+l_V)\\ &\quad +\tau+\rho\lambda_T\tau+\frac{\tau^{q-1}}{r-\lambda m}\rho\lambda_S\gamma(l_U+l_V)+\rho\lambda_S\tau]\|\lambda-\bar{\lambda}\|,\end{aligned}$$

where θ is the constant of (3.8). This proves that $x(\omega,\lambda)$ is Lipschitz continuous in $(\omega,\lambda)\in\Omega\times\Lambda$.

On the other hand,

$$y(\omega,\lambda)=R^{A,\eta}_{N(\cdot,\lambda),\gamma}(x(\omega,\lambda)-\gamma(U+V)(x(\omega,\lambda),\lambda)),$$

$$y(\bar\omega,\bar\lambda)=R^{A,\eta}_{N(\cdot,\bar\lambda),\gamma}(x(\bar\omega,\bar\lambda)-\gamma(U+V)(x(\bar\omega,\bar\lambda),\bar\lambda)).$$

Hence, we have

$$\begin{aligned}
&\|y(\omega,\lambda)-y(\bar\omega,\bar\lambda)\|\\
&\le \|R^{A,\eta}_{N(\cdot,\lambda),\gamma}(x(\omega,\lambda)-\gamma(U+V)(x(\omega,\lambda),\lambda))\\
&\qquad -R^{A,\eta}_{N(\cdot,\lambda),\gamma}(x(\bar\omega,\bar\lambda)-\gamma(U+V)(x(\bar\omega,\bar\lambda),\bar\lambda))\|\\
&\quad +\|R^{A,\eta}_{N(\cdot,\lambda),\gamma}(x(\bar\omega,\bar\lambda)-\gamma(U+V)(x(\bar\omega,\bar\lambda),\bar\lambda))\\
&\qquad -R^{A,\eta}_{N(\cdot,\bar\lambda),\gamma}(x(\bar\omega,\bar\lambda)-\gamma(C+D)(x(\bar\omega,\bar\lambda),\bar\lambda))\|\\
&\le \frac{\tau^{q-1}}{r-\lambda m}[\|x(\omega,\lambda)-x(\bar\omega,\bar\lambda)-\gamma(V(x(\omega,\lambda),\lambda)-V(x(\bar\omega,\bar\lambda),\lambda))\|\\
&\quad +\gamma\|V(x(\bar\omega,\bar\lambda),\lambda)-V(x(\bar\omega,\bar\lambda),\bar\lambda)\|+\gamma\|U(x(\omega,\lambda),\lambda)-U(x(\bar\omega,\bar\lambda),\lambda)\|\\
&\quad +\gamma\|U(x(\bar\omega,\bar\lambda),\lambda)-U(x(\bar\omega,\bar\lambda),\bar\lambda)\|]+\tau\|\lambda-\bar\lambda\|\\
&\le \frac{\tau^{q-1}}{r-\lambda m}[(1-\alpha q\gamma+c_q\gamma^q\lambda_V^q)^{\frac{1}{q}}\|x(\omega,\lambda)-x(\bar\omega,\bar\lambda)\|+\gamma l_V\|\lambda-\bar\lambda\|\\
&\quad +\gamma\lambda_U\|x(\omega,\lambda)-x(\bar\omega,\bar\lambda)\|+\gamma l_U\|\lambda-\bar\lambda\|]+\tau\|\lambda-\bar\lambda\|\\
&= \frac{\tau^{q-1}}{r-\lambda m}[(1-\alpha q\gamma+c_q\gamma^q\lambda_V^q)^{\frac{1}{q}}+\gamma\lambda_U]\|x(\omega,\lambda)-x(\bar\omega,\bar\lambda)\|\\
&\quad +[\frac{\tau^{q-1}}{r-\lambda m}\gamma(l_U+l_V)+\tau]\|\lambda-\bar\lambda\|.
\end{aligned}$$

It follows from the Lipschitz continuity of $x(\omega,\lambda)$ that $y(\omega,\lambda)$ is Lipschitz continuous. This completes the proof of Theorem 3.2. □

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