One-to-All Broadcasting of Odd Networks for One-Port and All-Port Models

Jong-Seok Kim and Hyeong-Ok Lee

ABSTRACT—Odd networks were introduced in the context of graph theory. However, their potential as fault-tolerant multiprocessor networks has been shown. Broadcasting is one of the most important communication primitives used in multiprocessor networks. In this letter, we introduce efficient one-to-all broadcasting schemes of odd networks for one-port and all-port models. We show the broadcasting time of the former is 2d–2 and that of the latter is d–1. The total time steps taken by the proposed algorithms are optimal.

Keywords—Odd network, one-to-all broadcasting, spanning tree, one-port, all-port.

I. Introduction

The class of odd networks was introduced by [1] in the context of graph theory. However, [2] pointed out their potential as fault-tolerant multiprocessor networks. Their efficiency was analyzed in terms of routing, diagnosability, combinatorial structure, maximal fault tolerance [2], symmetry [1], fault diameter [2], [3], and embedding [4], [5]. Odd networks are competitive with mesh and hypercube variants. For the same number of nodes, odd networks are superior to comparable mesh and hypercube variants when the network cost (degree×diameter) is used as a measure.

Broadcasting is the transfer of a piece of information, owned by a certain node called the originator, to all other nodes. This is one of the primitives of communication in parallel processing. Therefore, inefficient broadcasting can cause bottlenecks in multiprocessor networks. In broadcasting, a series of calls are placed along the communication lines of the network. At any time, the informed nodes contribute to the information transfer process by informing one of their uninformed neighbors. The most common method to find a broadcasting algorithm is to utilize a spanning tree. To implement a broadcasting algorithm, a broadcasting tree that is a spanning tree is embedded with the source node as the root [6]. Broadcasting algorithms can be implemented in either a one-port or an all-port model. In a one-port model, a node can transmit information along only one incident edge and can simultaneously receive information along only one incident edge. In an all-port model, all incident edges of a node can be used simultaneously for information transmission and reception.

In this letter, we present efficient one-to-all broadcasting schemes of odd networks for one-port and all-port models. We show the broadcasting time (BT) of the former is $2d$–2 and that of the latter is d–1. The total time steps taken by the proposed algorithms are optimal.

II. Odd Networks

Odd network O_d with $d\geq 2$ has a set of binary bitstrings of length $2d$–1 with exactly d 1s as the node set. The number of nodes in O_d is ${}_{(2d-1)}C_d$, the degree of O_d is d, and its diameter is d–1. Two nodes are adjacent if and only if their Hamming distance is $2d$–2. An edge connecting two nodes $u=u_1u_2\cdots u_i\cdots u_{2d-1}$ and $v=v_1v_2\cdots v_i\cdots v_{2d-1}$ with $u_i=v_i=1$ is called an i-edge, where the Hamming distance is $2d$–2. In other words, nodes u and v are connected when v is obtained from the operation $\sigma_i(u)$. The Hamming distance between u and v is the number of positions of bitstrings at which they differ. In this paper, we write a node 0…01…1 with d–1 0s and d 1s as $0^{d-1}1^d$.

A layered network consists of nodes in t+1 layers numbered

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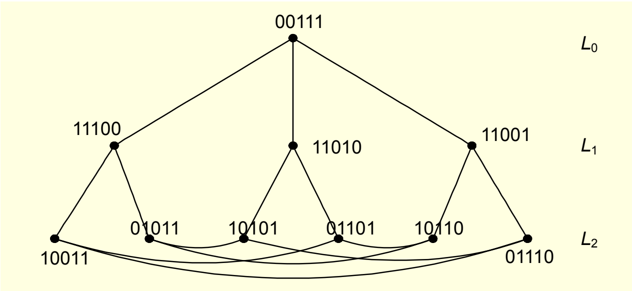

Fig. 1. O_3 network as a form similar to a layered network.

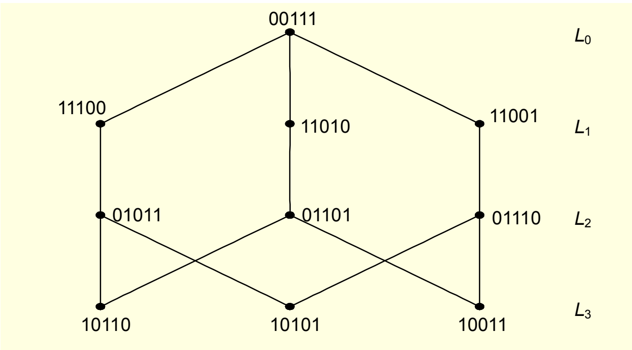

Fig. 2. O_3^{-1} network as a layered network.

L_0 to L_t, such that each node is in one layer, and each edge connects nodes in consecutive layers. The network O_d can be shown in a form similar to a layered network with layers L_0 to L_{d-1}, although it is not layered network. The nodes of O_d included in L_0 to L_{d-2} are connected to those in consecutive layers. The nodes included in L_{d-1} are connected to those included in the same layer and those included in the upper layer. Figure 1 shows O_3 as a form similar to a layered network. Assuming that a network in which 1-edges are entirely removed from O_d is O_d^{-1}, O_d^{-1} is a layered network with layers from L_0 to L_{2d-3}. Figure 2 shows O_3^{-1} as a layered network.

III. One-to-All Broadcasting of Odd Networks

The one-to-all broadcasting scheme can be easily implemented in the all-port model because O_d is node-symmetric [1] and similar to a layered network. In the one-to-all broadcasting scheme in the all-port model, let node u in L_t hold the message M, t=0. Then, all nodes in L_t send M to all nodes in L_{t+1}, t=t+1. This operation is performed continuously until t+1=d–1. This scheme takes d–1 time, which is optimal, since the diameter of O_d is d–1.

In the broadcasting scheme in the one-port model using a spanning tree, since O_d is node-symmetric, we define the spanning tree with node $u=0^{d-1}1^d$ as the root node. Let Pa(v) be a function that represents the parent of v, and Ch(v) be a function that represents the child of v. Then, for node v and its

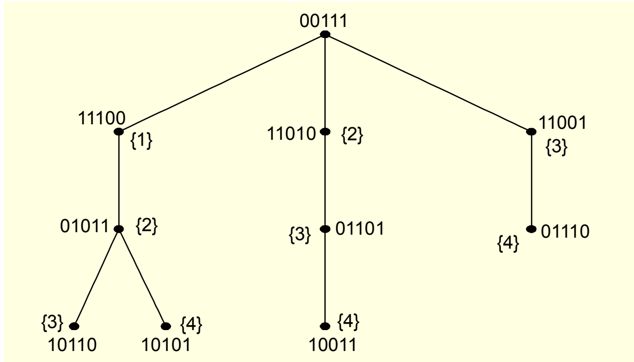

Fig. 3. ST(00111) of O_3^{-1}.

grandparent $g=g_1g_2\cdots g_i\cdots g_{2d-1}$, let Æ={$i$|$r_i$=$g_i\oplus v_i$=1}. Define τ∈Æ with 1≤τ≤d–1, define ρ∈Æ with d≤ρ≤2d–1 and Γ={ρ+1, ρ+2,⋯, Þ}, Þ<2d, when v is in an even layer. Define τ∈Æ with d≤τ≤2d–1, ρ∈Æ with 1≤ρ≤d–1, and Γ={ρ+1, ρ+2, ⋯, Þ}, Þ<d, when v is in an odd layer.

Definition 1. Let source node u be $0^{d-1}1^d$, and let spanning tree ST(u) include all nodes of O_d^{-1}. Then, ST(u) rooted at u is defined by the functions Pa(v) and Ch(v) as follows:

$$\mathrm{Ch}(v) = \sigma_h(v), \text{ for all } h \text{ in } \Gamma,$$

$$\mathrm{Pa}(v) = \sigma_\tau(v).$$

In particular, when v=u, Pa(v) does not exist and Ch(v)=$\sigma_c(v)$, d≤c≤2d–1. When v is in L_1, Pa(v)=u and Ch(v)=$\sigma_j(v)$, 2≤j≤d–1. When v is in L_{2d-3}, Ch(v) does not exist.

By definition 1, ST(00111) of O_3^{-1} is as follows:

u=00111. Ch(u)=11100 (by $\sigma_3(u)$), 11010 (by $\sigma_4(u)$), 11001 (by $\sigma_5(u)$).

Since 11100, 11010, and 11001 are in L_1, Pa(11100), Pa(11010), and Pa(11001) are the same, 00111. Ch(11100)=01011 (by σ_2(11100)). Ch(11010)=01101 (by σ_2(11010)). Ch(11001)=01110 (by σ_2(11001)).

Since 01011 is in L_2 and g=00111, Æ={2,3}, τ={2}, ρ={3}, and Γ={4,5}. Ch(01011)=10110 (by σ_4(01011)), 10101 (by σ_5(01011)). Pa(01011)=11100 (by σ_2(01011)).

Since 01101 is in L_2 and g=00111, Æ={2,4}, τ={2}, ρ={4}, and Γ={5}. Ch(01101)=10011 (by σ_5(01101)). Pa(01101)= 11010 (by σ_2(01101)).

Since 01110 is in L_2 and g=00111, Æ={2,5}, τ={2}, ρ={5}, and Γ={}. Ch(01110)={}. Pa(01110)=11001 (by σ_2(01110)).

Since 10110, 10101, and 10011 are in L_{2d-3}, Ch(10110), Ch(10101), and Ch(10011) do not exist. Pa(10110)=01011 (by σ_4(10110)). Pa(10101)=01011 (by σ_5(10101)). Pa(10011)= 01101 (by σ_5(10011)).

Figure 3 shows ST(00111) of O_3^{-1}. In Fig. 3, {x}, 1≤x≤2d–2, denotes BT.

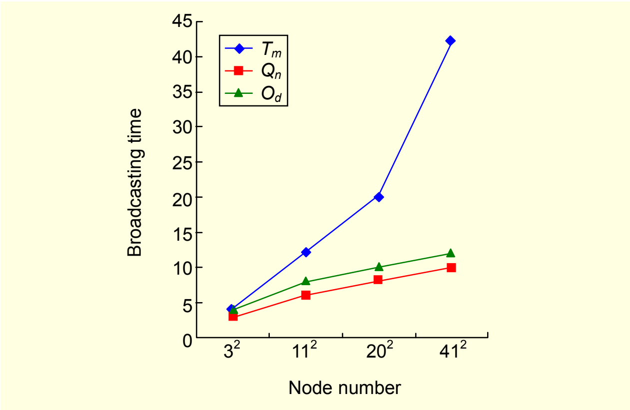

Fig. 4. Comparison of BT for T_m, Q_n, and O_d with one-port model.

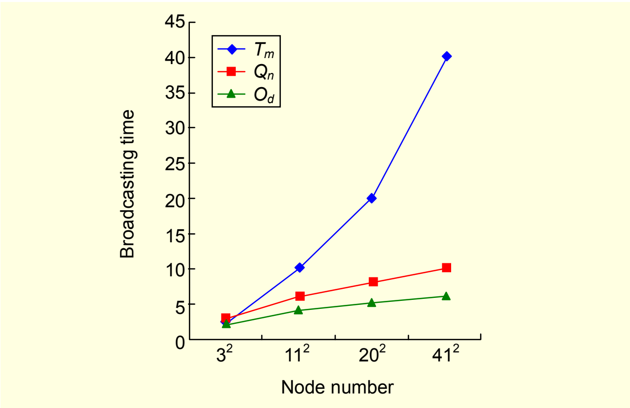

Fig. 5. Comparison of BT for T_m, Q_n, and O_d with all-port model.

Lemma 1. Tree ST(u) is the spanning tree of O_d^{-1}.

Proof. For the proof, we show that there is no cycle in ST(u). Since g is the grandparent of v, the number of elements of Æ is 2 at all times. One element is τ, and the other is ρ. Since only one τ exists, all nodes in ST(u) have just one parent. Therefore, there is no cycle in ST(u). □

In the one-to-all broadcasting scheme in the one-port model using ST(u), find all nodes v's with the message M. Search child nodes Ch(v)'s without M, and send M to the left-most Ch(v). This operation is performed continuously until all nodes in $L_{2d\text{-}3}$ receive M.

Theorem 1. The time taken to perform the proposed broadcasting in the one-port model using a spanning tree is $2d$–2. This is optimal.

Proof. Broadcasting in a one-port model, a node can send the message M along only one incident edge and can simultaneously receive M along only one incident edge. In a one-port model, the first node with M sends M to a neighbor node. All nodes with M send M to neighbor nodes. This operation is performed continuously until all nodes in the network receive M.

The number of nodes for broadcasting in the one-port model is 2^n, and n denotes BT. The height of ST(u) is $2d$–3, implying that the optimal BT must be $2d$–3. However, the optimal BT is not $2d$–3, since the number of nodes in O_d^{-1} is greater than $2^{2d\text{-}3}$. Therefore, the optimal BT is $2d$–3+1=$2d$–2. □

Figures 4 and 5 compare BT among m×m torus T_m and hypercubes Q_n and O_d which include similar nodes. The BT of T_m for the all-port model is m–1, and the BT of T_m for the one-port model is m. The BT of Q_n for the all-port model is n, and the BT of Q_n for the one-port model is n.

As the results in Fig. 4 demonstrate, the BT of Q_n is slightly better than that of O_d, and the BT of O_d is much better than that of T_m. The BT of O_d which is introduced in this letter is optimal. As seen in Fig. 5, the BT of O_d is better than that of Q_n and of T_m.

IV. Conclusion

In this letter, we proposed efficient one-to-all odd network broadcasting schemes for one-port and all-port models. We proved the broadcasting time of the former is $2d$–2 and that of the latter is d–1. We showed that the total time steps taken by the proposed algorithms are optimal. The result will be used to analyze other properties, such as the edge-disjoint spanning tree or all-to-all broadcasting for one-port and all-port models of odd networks.

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