Simulation Models for Investigation of Multiuser Scheduling in MIMO Broadcast Channels

Seung Hwan Lee and John S. Thompson

Spatial correlation is a result of insufficient antenna spacing among multiple antenna elements, while temporal correlation is caused by Doppler spread. This paper compares the effect of spatial and temporal correlation in order to investigate the performance of multiuser scheduling algorithms in multiple-input multiple-output (MIMO) broadcast channels. This comparison includes the effect on the ergodic capacity, on fairness among users, and on the sum-rate capacity of a multiuser scheduling algorithm utilizing statistical channel state information in spatio-temporally correlated MIMO broadcast channels. Numerical results demonstrate that temporal correlation is more meaningful than spatial correlation in view of the multiuser scheduling algorithm in MIMO broadcast channels. Indeed, the multiuser scheduling algorithm can reduce the effect of the Doppler spread if it exploits the information of temporal correlation appropriately. However, the effect of spatial correlation can be minimized if the antenna spacing is sufficient in rich scattering MIMO channels regardless of the multiuser scheduling algorithm used.

Keywords: Multiple-input multiple-output (MIMO), broadcast channel (BC), multiuser scheduling, channel model, spatial correlation, temporal correlation.

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I. Introduction

In the original multiple-input multiple-output (MIMO) capacity analyses undertaken in [1] and [2], the entries of MIMO channel matrices were assumed to be independent and identically distributed (*i.i.d.*) complex Gaussian random variables. Subsequently, various MIMO channel models were proposed to describe real channel environments. A survey of channel and radio propagation models for wireless MIMO systems was presented in [3]. The Kronecker model is the best known simplified spatial model for MIMO channels. This concept was validated in several papers by comparison with measured results [4]. However, the Kronecker structure of the channel covariance matrix is suitable for arrays with a moderate number of antenna elements [5]. The Kronecker model fails in correlated environments where the spatial correlations of transmitter and receiver are not separable [6]. However, the elements of real MIMO channels are correlated both in time, due to Doppler spread effects, and space, due to insufficient spacing between multiple antenna elements. Joint spatial temporal models were presented in [7], where spatio-temporal correlations are modeled based on angle of departure (AOD), Doppler spread, shadowing and path loss, in addition to conventional spatial model parameters, such as antenna spacing, beamwidth, and so on. As the number of parameters considered increases, the complexity of the model also grows, making it difficult to implement and run the MIMO channel efficiently despite its improved accuracy. It might be too complex to include all these factors in a realistic simulation model. Some parameters may have little effect despite the complexity of modeling them accurately.

From the viewpoint of a multiuser scheduler in a MIMO broadcast channel (BC, or downlink, that is, channels from the base station to mobile terminals), temporal correlation is one of the most important factors. The reason is that if a MIMO precoding technique like zero-forcing dirty-paper coding (ZF-DPC) or Tomlinson-Harashima precoding (THP) is used in the MIMO BC, channel state information (CSI) at the transmitter is essential for transmit processing. It is likely that there is a time difference between obtaining CSI and using that CSI at the transmitter in a temporally correlated channel. Because the channel is time-varying due to the Doppler spread, there are always mismatches of the channel estimates caused by that difference. This degrades the performance of the MIMO precoding technique. However, if the multiuser scheduler exploits the statistical CSI (SCSI) such as mean and covariance from the past instantaneous CSI (ICSI), it can allocate users to different time slots more efficiently by considering temporal correlation [8]. This can minimize the performance degradation due to imperfect CSI caused by temporal variations. Unlike temporal correlation, the effect of spatial correlation caused by insufficient antenna spacing might not be overcome by the multiuser scheduler with ease because it is determined by the physical antenna configuration and its corresponding MIMO channel environments. In this case, performance degradation can be minimized if the antenna spacing is sufficient to guarantee very small spatial correlation values or rich scattering, so that the MIMO channel matrix has *i.i.d.* elements. This means that parameters related to temporal correlation are more important than those related to spatial correlation in making a realistic simulation model to investigate the performance of the multiuser scheduler in the MIMO broadcast channel.

The objective of this paper is to show that the temporal correlation of MIMO broadcast channels has a greater impact on the performance of multiuser scheduling than spatial correlation in terms of system performance factors, such as the sum-rate capacity and fairness among users. Indeed, we will show that a multiuser scheduler considering the effect of the temporal correlation when allocating multiple users in two dimensional spatio-temporal slots can mitigate the effect of the temporal correlation. By doing this, it justifies the fact that a temporally correlated MIMO broadcast channel that does not consider spatial correlation is not only simple but realistic enough to design a multiuser scheduling algorithm in the MIMO broadcast channel and analyze its performance.

The rest of this paper is organized as follows. In section II, the system model is introduced. This includes a channel model, a brief overview of the MIMO precoding technique used, and its sum-rate capacity. It also deals with the weighted sum-rate maximization rule briefly. Section III compares the effect of spatial and temporal correlations in terms of system performance factors, such as the ergodic capacity, fairness among users and the sum-rate capacity. Conclusions are drawn in section IV.

II. System Model

We use boldface to denote matrices and vectors. For any general matrix $\mathbf{A}$, $\mathbf{A}^T$ denotes the transpose, $\mathbf{A}^H$ denotes the conjugate transpose, Tr($\mathbf{A}$) denotes the trace, diag$\{\lambda_i\}$ denotes a diagonal matrix with the (i, i) entry equal to λ_i, and $\mathbf{I}$ denotes the identity matrix. Also, $E[\ ]$ denotes expectation, and MOD[ ] denotes modulo operation. The field $\mathbb{C}$ denotes complex numbers. For any complex scalar value a, a^* denotes the complex conjugate.

1. Channel Model

Consider a narrowband MIMO broadcast channel with M_T transmit antennas at a base station and K ($K \geq M_T$) users each with a single receive antenna. Let $\mathbf{h}_k(t) \in \mathbb{C}^{M_T \times 1}$ denote the channel at time t between the transmit antenna array and the receive antenna for user k. Then, the MIMO BC at time t can be represented as

$$y_k(t) = \mathbf{h}_k^T(t)\mathbf{s}(t) + z_k(t), \qquad k = 1, \cdots, K, \tag{1}$$

where $\mathbf{s}(t) \in \mathbb{C}^{M_T \times 1}$ is the transmit signal vector with a power constraint $\mathbf{R}_{ss} = \mathrm{Tr}\left(E\left[\mathbf{s}(t)\mathbf{s}(t)^H\right]\right) \leq P$, $y_k(t)$ is the received signal for user k, and $z_k(t)$ is the complex additive white Gaussian noise (AWGN) with zero mean and unit variance for user k at time instant t. The covariance matrix $\mathbf{R}_{ss} = \sigma_s^2\mathbf{I}$, where σ_s^2 is the average power of the transmit signal.

2. Tomlinson-Harashima Precoding and Its Sum-Rate Capacity

In [9], Costa presented the principle of *dirty-paper coding* (DPC), in which the capacity of a system with known interference at the transmitter is the same as if there were no interference present. In MIMO broadcast channels, DPC can achieve the sum-rate capacity [10]. However, because DPC is difficult to implement in real systems, practical precoding techniques have been developed using the result of DPC. ZF-DPC exploits the DPC principle [11], which is a nonlinear suboptimal implementation of DPC. It decomposes a MIMO channel $\mathbf{H}$ into a transmit beamforming matrix $\mathbf{F}$ and a lower triangular matrix $\mathbf{B}$ by using the QR decomposition as $\mathbf{H} = \mathbf{BF}$. Any interference caused by data stream $j > i$ on each data stream i is forced to zero by pre-subtraction at the transmitter. Due to the pre-subtraction of interference, the transmit power of ZF-DPC increases, which may not be feasible in practical implementation.

THP employs a modulo operation to prevent a possibly large increase in the transmit power of ZF-DPC. Although THP prevents the large power increase of ZF-DPC, it still suffers from three kinds of losses: receive modulo loss, transmit power loss, and shaping loss [12]. Receive modulo loss is due to the existence of more neighbors at the edges of the original constellations at the receiver. Transmit power loss is due to the extension of the original constellations up to the modulo boundary at the transmitter. Shaping loss is due to the cubic shape of the M-QAM constellation, which generates a capacity loss of 1.53 dB from the Shannon capacity for spherical Gaussian distributed signals. If we assume that the transmitted symbols of M-QAM THP are uniformly distributed over the boundary region of $2\sqrt{M}$, the power increase is calculated as in [13] as

$$\Gamma_{\text{THP}} = \frac{M}{M-1}. \quad (2)$$

For more details on the operation of THP, see [14].

For the THP technique in multiuser MIMO BCs, a channel matrix $\mathbf{H}(S)$ is formed with a user set S_0 ($|S_0| \le M_T$) and is decomposed into a unitary transmit beamforming matrix $\mathbf{F}$ and lower triangular matrix $\mathbf{B}$ by taking the QR decomposition. Denoting the transmit power allocated to user k as P_k and b_{kk} as the k-th diagonal element of the matrix $\mathbf{B}$, the achievable sum-rate capacity of THP is given by

$$\begin{gathered} C(S_0) = \max \sum_{k \in S_0} \log_2\left(1 + \frac{b_{kk}^2 P_k}{\Gamma_{\text{THP}}^k}\right) \\ \text{subject to } P_k \ge 0, \sum_{k \in S_0} P_k \le P, \end{gathered} \quad (3)$$

where Γ_{THP}^k denotes the modulo loss of user k [12]. For simplicity, several assumptions are made. First, although the maximum sum-rate capacity of THP can be achieved by the optimal transmit power allocation [15], an equal power allocation over spatial channels is assumed. Second, the number of elements in a user set S is assumed to be the same as that of transmit antennas ($S = M_T$). Finally, if the target bit-error rate (BER) of the system is very small (BER $\le 10^{-6}$) and a high SNR is assumed for all users, the modulo loss can be ignored except for the shaping loss of 1.53 dB, which can be achieved by using multidimensional lattice codes rather than M-QAM modulation. Then, the sum-rate capacity of THP at time t is approximated as

$$C_{\text{THP}}(S,t) = \sum_{k=1}^{M_T} \log_2\left(1 + \frac{b_{kk}^2(t) P}{M_T}\right). \quad (4)$$

3. Weighted Sum-Rate Maximization Rule

In general, the number of users in real MIMO BC scenarios is likely to be greater than that of transmit antennas at the base station. This generates a multiuser selection problem which selects a user set for transmission according to certain performance criteria. Because all users in a wireless channel are likely to have independent fading with rich scattering, any user selection algorithm should utilize this characteristic well. Obviously, if one user should be selected at a time to maximize the capacity, the optimal selection would be to select the user with the best channel condition. This exploits the multiuser diversity gain [16]. However, if the base station serves multiple users simultaneously by transmitting different data through different spatial channels as in spatial multiplexing in a single user MIMO channel, it can increase the multiuser capacity compared to the case of using all transmit antennas for one user at a time [17]. If all users are orthogonal to each other, the optimal multiuser selection technique may be to select users with the largest individual spatial channel gains up to the number of transmit antennas. In reality, unfortunately, it is not likely that all users are orthogonal. This necessitates a joint consideration of multiple users in obtaining a user set for transmission. When nonlinear MIMO precoding techniques exploiting the DPC principle such as ZF-DPC and THP are used, any interference from other users can be successively canceled by a certain encoding order if perfect CSI is available at the transmitter. However, linear MIMO precoding techniques like zero-forcing beamforming (ZFBF) cannot cancel the interference. They only use orthogonal channel directions by suppressing other users' interference. Therefore, the capacity of nonlinear MIMO precoding techniques is usually better than that of linear MIMO precoding techniques. However, as the number of users increases to infinity, the performance of linear MIMO precoding techniques approaches that of DPC [18]. This is because the probability of finding orthogonal users among all users may increase as the number of users becomes very large [19].

When the number of users is K, the total user set $U=\{u_1, u_2, \cdots, u_K\}$. If K is larger than the number of transmit antennas M_T in MIMO broadcast channels, a multiuser selection algorithm finds a user set $S\left(S \subset U, |S| = M_T\right)$, which satisfies performance criteria by considering all possible choices of user set S. If the performance criterion is to maximize the sum-rate capacity, the maximum sum-rate capacity can be obtained by

$$C_{\text{MAX}} = \max_S C(S), \quad (5)$$

where $C(S)$ is the capacity determined by user set S and the type of MIMO precoding technique used. When there is an additional performance constraint and the sum-rate capacity is maximized, the sum-rate maximization includes a weight vector $\boldsymbol{\mu}(t) = \left[\mu_1(t), \cdots, \mu_K(t)\right]$ to form a weighted sum-rate maximization rule.

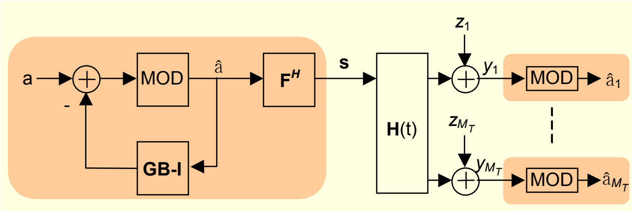

Fig. 1. Block diagram of THP in the MIMO BC.

From (4), the weighted sum-rate maximization rule finds a user set $S_{\max}$ according to

$$S_{\max}(t) = \arg \max_{S\subset U, |S|=M_T} \sum_{k=1}^{M_T} \mu_k(t) \log_2 \left(1 + \frac{b_{kk}^2(t)P}{M_T}\right), \quad (6)$$

where $\mu_k(t)$ denotes the weight for user k at time t.

Figure 1 shows the block diagram of THP with the selected user set $S_{\max}$ in the MIMO BC. The vector $\mathbf{a} \in \mathbb{C}^{M_T \times 1}$ denotes the input data vector with the covariance matrix $\mathbf{R}_{aa} = \sigma_a^2 \mathbf{I}$. The matrix $\mathbf{G} = \text{diag}\{b_1^{-1}, \cdots, b_{M_T}^{-1}\}$ determines the effective channel gain of all users at the receiver.

III. Comparison of Spatial and Temporal Correlations

1. Effect of the Spatial Correlation on the Ergodic Capacity

With the Kronecker spatial correlation model, the channel matrix **H** can be given as in [5] and [20] by

$$\mathbf{H} = \mathbf{R}_r^{1/2} \mathbf{H}_w \mathbf{R}_t^{1/2}, \quad (7)$$

where $\mathbf{R}_r$ is the $M_T \times M_T$ spatial correlation matrix at the receiver of the selected user set S, $\mathbf{R}_t$ is the $M_T \times M_T$ spatial correlation matrix at the base station, and $\mathbf{H}_w$ is an $M_T \times M_T$ matrix of which elements are *i.i.d.* complex Gaussian random variables with zero mean and unit variance. In this section, time index t will be omitted to simplify the equations because we assume that spatial correlation is not affected by time. For the spatial correlation matrices, $\mathbf{R}_r$ can be assumed to be an identity matrix if users are separated enough to ignore the spatial correlation between them. Assuming a linear array of equally-spaced M_T parallel antenna elements at the transmitter, the spatial correlation matrix can be given as in [21] and [22] by the following Toeplitz matrix:

$$\mathbf{R}_t = \begin{bmatrix} 1 & \rho_s & \rho_s^4 & \cdots & \rho_s^{(M_T-1)^2} \\ \rho_s & 1 & \rho_s & \ddots & \vdots \\ \rho_s^4 & \rho_s & 1 & \ddots & \rho_s^4 \\ \vdots & \ddots & \ddots & \ddots & \rho_s \\ \rho_s^{(M_T-1)^2} & \cdots & \rho_s^4 & \rho_s & 1 \end{bmatrix}, \quad (8)$$

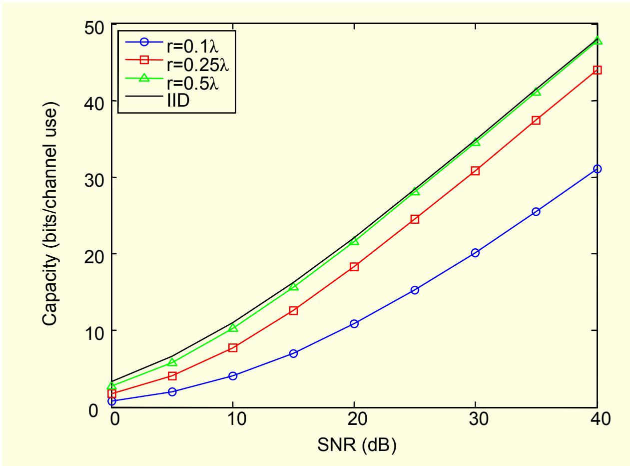

Fig. 2. Ergodic capacity of a spatially-correlated 4×4 MIMO channel with the angular spread $\Lambda = \pi/4$.

where ρ_s represents the spatial correlation between two adjacent antenna elements, which can be approximated as in [23] as

$$\rho_s \approx \exp\left[-23\Lambda^2 \left(\frac{d}{\lambda}\right)^2\right], \quad (9)$$

where Λ denotes the angular spread, d denotes the distance between two adjacent transmit antenna elements, and λ denotes the wavelength of the carrier frequency.

Figure 2 shows the ergodic capacity of a spatially-correlated 4 × 4 MIMO channel with the angular spread $\Lambda = \pi/4$ for different antenna spacing values. The ergodic capacity decreases as the antenna spacing decreases. However, as the distance between antenna elements increases, the effect of spatial correlation on the ergodic capacity becomes negligible. When $d = 0.5\lambda$, the capacity is almost the same as in the case of *i.i.d.* fading channels.

2. Effect of the Temporal Correlation on the Ergodic Capacity

Assuming the Jakes power density spectrum, if the MIMO broadcast channel has temporally correlated fading, a temporal correlation function for user k can be given as in [24] and [25] as

$$E\left[h_{i,k}(t) h_{j,k}^*(t-\tau_k)\right] = J_0\left(2\pi f_d^k \tau_k\right), \quad (10)$$

where $h_{i,k}$ denotes the i-th element of the channel vector $\mathbf{h}_k$ for user k, τ_k denotes the time delay for user k, J_0 is the 0th order Bessel function of the first kind, and f_d^k is the Doppler spread for user k which is determined by the carrier frequency f_c and the velocity v_k as $f_d^k = v_k f / C$, where C is the speed of light.

In investigating the effect of temporal correlation, it is assumed that one downlink frame of the MIMO BC with time period T_f

consists of $N_s\,(N_s > 1)$ time slots each with time period T_s. Then, the ICSI is updated once per frame at the base station. Denoting discrete slot index as n, the temporal correlation function $\rho_t(nT_s)$ when t=nT_s after the initial accurate channel measurement $t = 0$ can be written as

$$\rho_t(nT_s) = J_0\left(2\pi f_d^k nT_s\right). \tag{11}$$

ICSI can be obtained by using either a feedback channel or the reciprocity principle for time-division duplex (TDD) systems. There are several sources for imperfect ICSI but it is assumed that the ICSI is perfect at the beginning of every frame [25] for simplicity. In this case, the only imperfection is the mismatch of the channel estimate due to scheduling delay. This mismatch results in performance degradation, which generally increases through the time slots of each frame. The amount of degradation for user k is determined by temporal correlation, which is a function of the Doppler spread f_d^k and multiples of the slot period nT_s, which is given by (11).

Based on the results in [25], an equivalent time-varying channel matrix for SCSI reflecting scheduling delay within the frame at slot index n can be written as

$$\mathbf{H}(n) = \mathbf{P}(n)\mathbf{H}_0 + \sqrt{\mathbf{I} - \mathbf{P}(n)\mathbf{P}^H(n)}\,\mathbf{H}_m,\quad n = 1,\cdots N_s, \tag{12}$$

where $\mathbf{P}(n) = \mathrm{diag}\{\rho_{t,1}(nT_s),\cdots,\rho_{t,M_T}(nT_s)\}$ represents the autocorrelation matrix of the user set $S_{\max}$, $\sqrt{\mathbf{I} - \mathbf{P}(n)\mathbf{P}^H(n)}$ represents the amplitude increase in the channel estimation error due to scheduling delay, $\mathbf{H}_0$ is the perfectly estimated channel matrix at the beginning of each frame, and $\mathbf{H}_m$ is an uncorrelated estimation error matrix which has the same statistical characteristics as the estimated channel $\mathbf{H}_0$. Equation (12) implies that any decrease in ICSI becomes a delay-induced channel estimation error due to scheduling delay.

From (1), the received signal via the equivalent MIMO BC becomes

$$\mathbf{y}(n) = \mathbf{P}(n)\mathbf{H}_0\mathbf{s} + \sqrt{\mathbf{I} - \mathbf{P}(n)\mathbf{P}^H(n)}\,\mathbf{H}_m\mathbf{s} + \mathbf{z}(n), \tag{13}$$

where $\mathbf{y}(n) \in \mathbb{C}^{M_T\times 1}$ and $\mathbf{z}(n) \in \mathbb{C}^{M_T\times 1}$ are vector forms of the received signal and complex AWGN, respectively. In (13), the first term denotes the desired signal, and the second term denotes the additive measurement noise due to the delay-induced estimation error. After the modulo operation at the receiver, the output data vector becomes

$$\hat{\mathbf{a}} = \mathrm{MOD}\left[\mathbf{P}(n)\mathbf{H}_0\mathbf{s}\right] = \mathbf{P}(n)\mathbf{G}^{-1}\mathbf{a}\,. \tag{14}$$

The variance of the estimate noise, $\sqrt{\mathbf{I} - \mathbf{P}(n)\mathbf{P}^H(n)}\,\mathbf{H}_m\mathbf{s}$, is given by

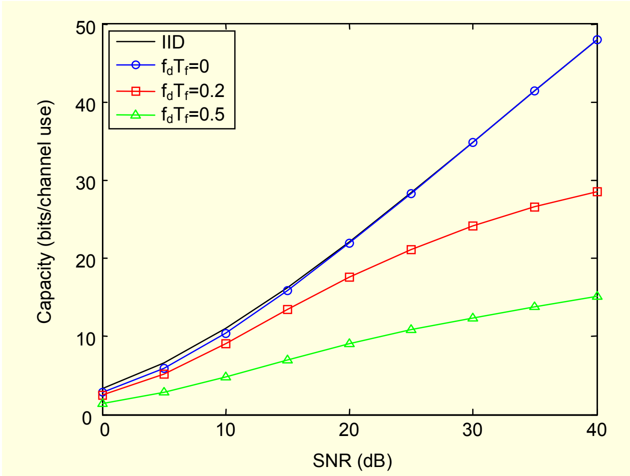

Fig. 3. Ergodic capacity of a temporally-correlated 4×4 MIMO channel.

$$\begin{aligned}&\left(\mathbf{I} - \mathbf{P}(n)\mathbf{P}^H(n)\right)E\left[(\mathbf{H}_m\mathbf{s})(\mathbf{H}_m\mathbf{s})^H\right]\\ &= M_T\left(\mathbf{I} - \mathbf{P}(n)\mathbf{P}^H(n)\right)\mathbf{R}_{ss},\end{aligned} \tag{15}$$

which follows from the fact that $E[\mathbf{H}_m\mathbf{H}_m^H] = M_T\mathbf{I}$. Since $\sigma_s^2 \simeq \sigma_a^2$ with THP [26] and $\sigma_a^2 = P/M_T$, the sum-rate capacity considering the temporal variation is written as

$$C_{\mathrm{THP}}(S,n) = \sum_{k=1}^{M_T}\log_2\left(1 + \frac{b_{kk}^2\rho_{t,k}^2(n)P}{M_T\left\{\left(1-\rho_{t,k}^2(n)\right)P+1\right\}}\right). \tag{16}$$

Figure 3 shows the ergodic capacity of a 4 × 4 MIMO channel with temporal correlation for various relative Doppler spread values.

We notice that as the relative Doppler spread f_dT_f increases, the capacity becomes saturated due to the increase in mismatches of the channel estimate. This trend corresponds to the result in [27], where the additive channel estimation error without the temporal correlation reduces the effective SNR, which causes saturation in the capacity as well. If f_dT_f =0, which means there is no scheduling delay, the ergodic capacity is almost the same as in the case of *i.i.d.* fading channels.

3. Fairness among Users with Spatial and Temporal Correlations

For the analysis of the effect of spatial and temporal correlations on a multiuser scheduling algorithm, fairness among users is considered as a performance constraint in addition to maximizing the sum-rate capacity. Proportional fair (PF) scheduling [28] is applied for the weighted sum-rate maximization rule described in section II. The PF scheduling rule is designed to satisfy long-term throughput fairness among users by considering the channel conditions and the amount of past throughput while simultaneously exploiting multiuser diversity. The PF scheduler selects a user with index k at time t according to

$$k(t) = \arg \max_{i \in \{1,\cdots,K\}} \frac{r_i(t)}{\overline{r}_i(t)}, \tag{17}$$

where $r_i(t)$ is the data rate of user i at time t, and $\overline{r}_i(t)$ is the exponential moving average of the past throughput of user i, which is updated as

$$\overline{r}_i(t+1) = \left(1 - \frac{1}{\alpha_t}\right)\overline{r}_i(t) + \frac{1}{\alpha_t} r_i(t), \tag{18}$$

where α_t is the smoothing factor, which determines the fairness window size of the scheduler. Large values of α_t indicate a large time window, which results in long-term throughput fairness among users. From (6), (17), and (18), the weighted sum-rate maximization rule considering throughput fairness with the PF algorithm finds a user set according to

$$S_{\max}(t) = \arg \max_{S \subset U, |S| = M_T} \sum_{k=1}^{M_T} \frac{1}{\overline{r}_k(t)} \log_2 \left(1 + \frac{b_{kk}^2(t) P}{M_T}\right). \tag{19}$$

Several fairness indices have been suggested for the measurement of fairness for different resource allocation schemes. The Jain's fairness index [29], [30] is one quantitative measure of fairness. Denoting the number of users as K, the Jain's fairness index is given by

$$I_{Jain} = \frac{\left(\sum_{k=1}^{K} \gamma_k\right)^2}{K \sum_{k=1}^{K} \gamma_k^2}, \tag{20}$$

where γ_k is the fraction of transmission resource allocated to user k.

Figures 4 and 5 compare the Jain's fairness index and the sum-rate capacity against the fairness window, respectively. For these plots, it is assumed that the number of users K = 8, and the number of transmit antennas M_T = 4. For spatial correlation, it is assumed that $\Lambda = \pi/8$, and $d = 0.5\lambda$. For temporal correlation, it is assumed that $f_d T_f$ = 0.001. The labels "SCH," "TCH," and "STCH" mean spatially correlated channel, temporally correlated channel, and spatio-temporally correlated channel, respectively. First of all, when there is only spatial correlation (SCH), the difference in the fairness index with and without the PF algorithm is small compared to when there is temporal correlation only (TCH). This is because the channel realizations with spatial correlation are *i.i.d.* random variables with no temporal correlation in terms of time; thus, statistical fairness among users can be achieved without any fairness consideration. In contrast to the case of the spatial correlation, the existence of

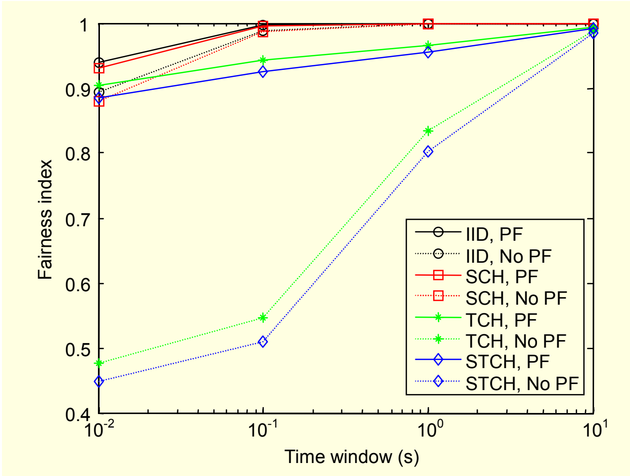

Fig. 4. Jain's fairness index with the weighted sum-rate maximization rule, the number of users K = 8, and the number of transmit antennas M_T = 4.

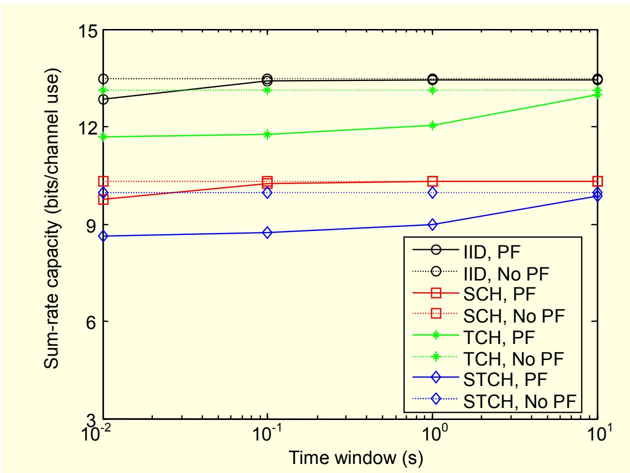

Fig. 5. Sum-rate capacity with the weighted sum-rate maximization rule, the number of users K = 8, and the number of transmit antennas M_T = 4.

the PF algorithm significantly improves fairness among users in the temporally correlated channel.

If the MIMO channel is temporally correlated, the coherence time due to the Doppler spread and the size of the fairness window are closely related and affect the performance. When the coherence time is much smaller than the fairness window, the effect of temporal correlation can be negligible, and the performance difference with and without the PF algorithm becomes small. As the size of the fairness window grows, the temporally correlated channel can be considered an *i.i.d.* channel in view of the multiuser scheduling algorithm. We also notice that the difference in the fairness index with and without the PF algorithm in the spatio-temporally correlated channel is similar to that in the temporally correlated channel. This implies that temporal correlation can have a much bigger impact on resource fairness and that PF scheduling is an important technique to improve fairness if temporal correlation exists in the MIMO channel.

When there is no fairness constraint, the weighted sum-rate maximization rule finds a user set by maximizing the sum-rate capacity, which is not affected by the size of fairness window regardless of the type of MIMO channel correlation considered. Due to spatial correlation, the sum-rate capacity of the spatially correlated channel is smaller than that of the temporally correlated channel. If the PF algorithm is applied, the sum-rate capacity decreases in order to maintain fairness among users. This is true for all cases if we ignore the amount of decrease. However, as the difference in the Jain's fairness index of the temporally correlated channel is much larger than that of the spatially correlated channel at a certain fairness window, the decrease in the sum-rate capacity of the temporally correlated channel is also much larger than that of the spatially correlated channel due to the fairness constraint. For example, when the time window is 0.1 s, the decrease in the sum-rate capacity with temporal correlation is about 2 bits per channel used in order to increase the Jain's fairness index from 0.54 to 0.91. For comparison, the decrease of the spatially correlated channel is only about 0.3 bits per channel used in order to increase the Jain's fairness index from 0.98 to 0.99. The effect of spatial correlation on the sum-rate capacity corresponds to the trend of the results shown in Fig. 4. That is, the spatial correlation decreases the sum-rate capacity regardless of PF scheduling.

4. Multiuser Scheduling in Spatio-temporally Correlated MIMO Broadcast Channels

In this section, we compare the performance of the SCSI-assisted multiuser scheduling algorithm [8] in spatio-temporally correlated MIMO broadcast channels. The SCSI-assisted multiuser scheduling algorithm utilizes SCSI for the selection of a user set which can minimize the mismatch of channel estimates in order to improve the sum-rate capacity. Because a frame consists of multiple slots, with the SCSI-assisted multiuser scheduling algorithm, a user set for transmission is selected on a slot-by-slot basis using SCSI, so that the sum-rate capacity can be improved compared to a frame-by-frame multiuser scheduler without SCSI. Although the base station obtains ICSI once per frame, it can utilize the previous ICSI to obtain SCSI, such as mean and covariance due to channel stationarity [25]. This SCSI can be used to estimate the temporal correlation of the MIMO BC when precise ICSI is not available. The SCSI-assisted sum-rate maximization rule finds a user set by using the effective MIMO channel reflecting SCSI as in (12) at every slot index n according to

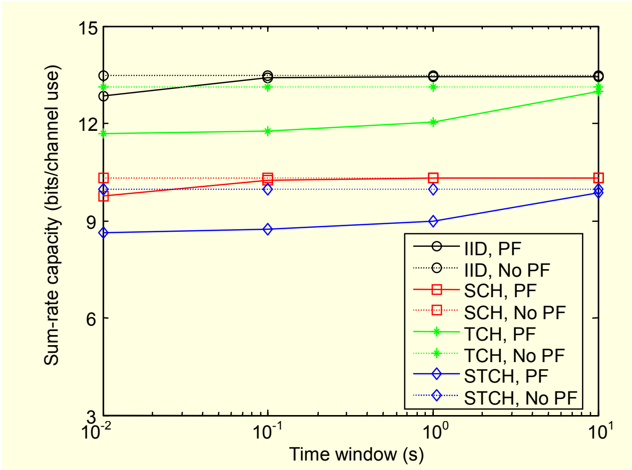

Fig. 6. Performance of the SCSI-assisted multiuser scheduling algorithm in spatio-temporally correlated MIMO broadcast channels (frame period T_f = 2 ms, number of transmit antennas M_T = 4, and number of users K=8).

$$S_{\max}(n) = \arg \max_{S \subset U, |S| = M_T} C_{\text{THP}}(S, \mathbf{H}(n)) \ . \tag{21}$$

According to this selection rule, any user with a rapid decrease in temporal correlation may be assigned to the first few slots in the frame. This may improve the sum-rate capacity with a trade-off in increased complexity. This is because the multiuser selection procedure has to be performed on a slot-by-slot basis if the SCSI-assisted algorithm is applied. When the temporal variations of the MIMO BC are small compared to the coherence time, the SCSI-assisted algorithm shows little improvement over the normal sum-rate maximization algorithm, which is performed on a frame-by-frame basis. This would give the same choice in the multiuser selection procedure for all slots.

Figure 6 shows the performance of the SCSI-assisted multiuser scheduling algorithm for different spatio-temporal channel parameters. We assume the frame period T_f= 2 ms, the number of transmit antennas M_T=4, the number of users K=8, and the number of slots in one frame Ns=5. The relative Doppler spread values of all users ($f_d T_f$) are assumed to be *i.i.d.* uniform random variables over [0.1, 0.5) or fixed at the same values. The antenna spacing is assumed to be d = 0.5λ. When the relative Doppler spread values are the same, there is no difference between the normal multiuser scheduling algorithm ("No SCSI" in the figure) and the proposed SCSI-assisted multiuser scheduling algorithm ("SCSI" in the figure). This is because the number of the channel estimation errors due to scheduling delay is identical for all users. However, the sum-rate capacity decreases as the relative Doppler spread increases due to an increase in channel estimation error. When the relative Doppler spreads of all users have a uniform distribution over [0.1, 0.5), the SCSI-assisted multiuser scheduling algorithm performs better than the normal multiuser scheduling algorithm, which does not use SCSI and applies a frame-basis multiuser selection.

The performance difference between these two schemes is evident as the SNR increases because the delay-induced channel estimation error produces an irreducible error floor in the high SNR region. Performance improvement with the SCSI-assisted multiuser scheduling algorithm can be achieved regardless of the spatial correlation parameter, namely, the angular spread Λ. When the SNR is 20 dB, the capacity increase is about 2 bits per channel used regardless of the spatial channel parameters. A smaller angular spread means higher spatial correlation between antennas, which reduces the sum-rate capacity. This plot shows that the effect of temporal correlation can be minimized by using an appropriate multiuser scheduling algorithm.

It is useful to note that the receive spatial correlation caused by the geographic location of multiple users with a single receive antenna can be minimized by the sum-rate maximization rule indirectly, which might exclude a pair of highly correlated users from the selected user set to maximize the sum-rate capacity. However, transmit spatial correlation is not overcome completely by the multiuser scheduling algorithm presented in this paper.

IV. Conclusion

In this paper, we compared the effect of spatial and temporal correlation on system performance. We demonstrated that temporal correlation has a greater effect than spatial correlation on the fairness and capacity performance of multiuser scheduling algorithms in MIMO broadcast channels. With a narrowband channel assumption, the base station can consider space and time for its multiuser scheduling in the MIMO broadcast channels. In this case, the sum-rate maximization rule finds a set of the most orthogonal users to maximize the sum-rate capacity. However, this does not mean that it can reduce the effect of spatial correlation at the transmitter. Unlike the effect of spatial correlation, the effect of temporal correlation can be minimized if the multiuser scheduler exploits information of the time-varying channel appropriately. The coherence time, which can be approximated as the inverse of the Doppler spread, affects the degree of fairness among users. When the coherence time is very short compared to the fairness window, the channel can be considered an *i.i.d.* channel, so statistical fairness can be achieved automatically without any fairness consideration in the multiuser scheduling metric. However, when the coherence time is similar to the time window, temporal correlation affects fairness among users, and the multiuser scheduler should consider fairness among users in its scheduling metric. Numerical results showed that the SCSI-assisted scheduling algorithm can mitigate the decrease in the sum-rate capacity because it allocates users with high temporal correlation to the front part of a frame; thus, it can improve the performance in spatio-temporally correlated MIMO broadcast channels regardless of the spatial correlation parameters.

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