

# Outage Probability of Opportunistic Amplify-and-Forward Relaying in Nakagami-m Fading Channels

Qinghai Yang, Yingji Zhong, Kyung Sup Kwak, and Fenglin Fu

**ABSTRACT**—We address the outage performance for the opportunistic amplify-and-forward relaying strategies under Nakagami- $m$  fading channels. A closed-form expression for the outage probability is derived. Simulation results verify our theoretical solutions.

**Keywords**—Outage probability, opportunistic AF, Nakagami- $m$  fading.

## I. Introduction

Recently, cooperative communication systems that use various relay strategies have attracted much interest from researchers [1] aiming to achieve spatial diversity, to enlarge coverage, and/or to potentially increase capacity. In [2], opportunistic relaying with the amplify-and-forward (AF) strategy is presented, and the outage probability is investigated under Rayleigh fading channels. However, [2] merely provides an exact expression for the outage probability but does not reach a closed form.

In this letter, we extend the result of [2] in Nakagami- $m$  fading channels and derive a closed-form expression for the outage probability of opportunistic AF relay strategies. The Nakagami- $m$  model represents a wide variety of realistic line-of-sight/non-line-of-site fading channels encountered in practice. By setting  $m$  to 1, we can directly reach a closed-form for the outage probability expression presented in [2], in which the scenario of a Rayleigh fading channel is considered.

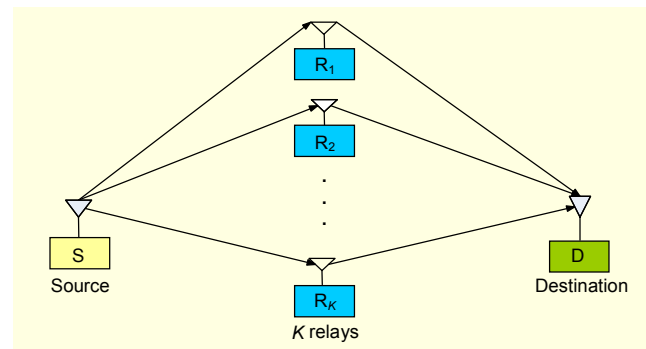


Fig. 1.  $K$ -relay channel model.

Computer simulations demonstrate our analytical results.

## II. Signal Model

The half-duplex dual-hop wireless network under consideration is shown in Fig. 1. The source node transmits its information to the destination node with the aid of a set of  $K$  relay nodes  $\{R_1, \dots, R_K\}$ . During the first hop, the source-node transmits information, and the relays listen. For the second hop, a selected relay-node amplifies the received signal and then forwards it to the destination node. The opportunistic relay-node selection can be performed proactively before the source transmission or reactively after the source transmission [2].

Let us denote the channel gains corresponding to the links of the source to  $k$ -th relay and the  $k$ -th relay to destination as  $h_k$  and  $\bar{h}_k$ , respectively. Considering Nakagami- $m$  fading channels,  $h_k$  and  $\bar{h}_k$  are Nakagami distributed with parameters  $(m_k, \lambda_k)$  and  $(\bar{m}_k, \bar{\lambda}_k)$ , respectively. Thus, the variables  $X_k = |h_k|^2$  and  $Y_k = |\bar{h}_k|^2$  will respectively follow the gamma distribution with the probability density functions

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Qinghai Yang (phone: 86 15091330019, email: qhyang@xidian.edu.cn) and Fenglin Fu (email: flfu@xidian.edu.cn) are with the State Key Laboratory on ISN, School of Telecommunications Engineering, Xidian University, Shaanxi, China.

Yingji Zhong (email: zhongyingji32@sdu.edu.cn) and Kyung Sup Kwak (email: kskwak@inha.ac.kr) are with the Graduate School of Information Technology & Telecommunications, Inha University, Incheon, Rep. of Korea.

(PDF) as

$$f_{X_k}(x) = \frac{(\theta_k)^{m_k} x^{m_k-1} \exp(-x\theta_k)}{\Gamma(m_k)}, \quad (1)$$

and

$$f_{Y_k}(x) = \frac{(\bar{\theta}_k)^{\bar{m}_k} x^{\bar{m}_k-1} \exp(-x\bar{\theta}_k)}{\Gamma(\bar{m}_k)}, \quad (2)$$

where  $\theta_k = m_k / \lambda_k$ , and  $\bar{\theta}_k = \bar{m}_k / \bar{\lambda}_k$ . We assume that all channels are independent, and the receivers at the relays and destination have exact channel state information (CSI), but the transmitters at the source or relays have no CSI.

### III. Outage Probability

The mutual information of the opportunistic AF relaying scheme is given as in [2] by

$$I = \frac{1}{2} \log_2 \left( 1 + \gamma_S \max_{k \in [1, K]} \frac{X_k Y_k}{\varpi_k + Y_k} \right), \quad (3)$$

where  $\gamma_S = E_S / \sigma_n^2$ ,  $\varpi_k = (\lambda_k \gamma_S + 1) / \gamma_R$ , and  $\gamma_R = E_R / \sigma_n^2$  with  $E_S$ ,  $E_R$  and  $\sigma_n^2$  being the transmit energy at the source node, the transmit energy at the relay node, and the power of additive white Gaussian noise (AWGN), respectively, and  $\varpi_k$  corresponds to the power constraint factor [2]. The outage probability is mathematically defined as the probability that  $I$  is less than a target rate  $R$ , denoted as  $P_{out} = \Pr(I < R)$ . Equivalently, we write

$$P_{out} = \Pr \left( \max_{k \in [1, K]} \frac{X_k Y_k}{\varpi_k + Y_k} < \eta_0 \right), \quad (4)$$

where  $\eta_0 = (2^{2R} - 1) / \gamma_S$ . Due to the independent channel assumption, it is given by

$$P_{out} = \prod_{k=1}^K P_{out}^k \quad (5)$$

with

$$P_{out}^k = \int_0^\infty \int_0^{\eta_0(\varpi_k + y)^{1/y}} f_{X_k}(x) f_{Y_k}(y) dx dy. \quad (6)$$

Inserting (1), we can obtain (3.381.1) in [3], which is

$$P_{out}^k = \frac{1}{\Gamma(m_k)} \int_0^\infty \Gamma_{inc}(m_k, \eta_0 \theta_k (1 + \frac{\varpi_k}{y})) f_{Y_k}(y) dy, \quad (7)$$

where  $\Gamma_{inc}(a, x)$  is the Pearson's incomplete gamma function expressed as  $\Gamma_{inc}(a, x) = \int_0^x e^{-t} t^{a-1} dt$ .

Subsequently, we assume that all coefficients  $\{m_k\}$  are natural numbers. This does not represent a strong limitation for

the following reasons [4]. First, the channel may sometimes merely be characterized or measured to an accuracy corresponding to whole integer arithmetic. Second, if channels are known more accurately, the upper and lower bounds of natural numbers can be employed to offer bounds for the outage probabilities, and a linear approximation between these results may be used to obtain an accurate approximations. Under this assumption, we can adopt the expansion (8.352.1) in [3]:

$$\Gamma_{inc}(m, x) = \Gamma(m) \left( 1 - e^{-x} \sum_{n=0}^{m-1} \frac{x^n}{n!} \right). \quad (8)$$

Then,

$$P_{out}^k = 1 - e^{-\eta_0 \theta_k} \int_0^\infty e^{-\frac{\eta_0 \theta_k \varpi_k}{y}} \sum_{n=0}^{m_k-1} \frac{1}{n!} (\eta_0 \theta_k)^n \left( 1 + \frac{\varpi_k}{y} \right)^n f_{Y_k}(y) dy. \quad (9)$$

In (9), rewriting  $(1 + \varpi_k / y)^n$  by virtue of the power series (1.111) in [3], which is

$$(a + x)^n = \sum_{j=0}^n C_n^j x^j a^{n-j}, \quad (10)$$

we reach

$$P_{out}^k = 1 - e^{-\eta_0 \theta_k} \frac{(\bar{\theta}_k)^{\bar{m}_k}}{\Gamma(\bar{m}_k)} \cdot \sum_{n=0}^{m_k-1} \frac{1}{n!} (\eta_0 \theta_k)^n \cdot \sum_{j=0}^n C_n^j (\varpi_k)^j \int_0^\infty y^{\bar{m}_k-j-1} e^{-\left(\frac{\eta_0 \theta_k \varpi_k}{y} + \bar{\theta}_k y\right)} dy, \quad (11)$$

where  $C_n^k = \frac{n!}{k!(n-k)!}$ . It finally yields (3.471.9) in [3]:

$$P_{out}^k = 1 - \frac{2e^{-\eta_0 \theta_k} (\bar{\theta}_k)^{\bar{m}_k}}{\Gamma(\bar{m}_k)} \cdot \sum_{n=0}^{m_k-1} \frac{1}{n!} (\eta_0 \theta_k)^n \cdot \sum_{j=0}^n C_n^j (\varpi_k)^j \left( \frac{\eta_0 \theta_k \varpi_k}{\bar{\theta}_k} \right)^{\frac{\bar{m}_k-j}{2}} K_{\bar{m}_k-j} \left( 2\sqrt{\eta_0 \varpi_k \theta_k \bar{\theta}_k} \right), \quad (12)$$

where  $K_\nu(z)$  denotes the Bessel function of the imaginary argument (8.432) in [3]. The function  $K_\nu(z)$  is not available directly in popular symbolic software such as MATLAB. Whereas its zero and first orders ( $K_0(z)$  and  $K_1(z)$ ) have a concise and closed-form expression as in (8.447.3) and (8.446) in [3], respectively, the higher orders can be calculated via the formula (8.486.10) in [3], which is  $zK_{\nu+1}(z) = zK_{\nu-1}(z) + 2\nu K_\nu(z)$ . We did not constrain  $\bar{m}_k$  to

be a natural number during the derivation of (12).

As is well known, the Nakagami fading channel with parameter  $m_k=1$  will become the Rayleigh fading channel. For this special case, (12) can be written as

$$P_{out}^k = 1 - 2e^{-\eta_0 \theta_k} (\eta_0 \theta_k \bar{\sigma}_k)^{\frac{\bar{m}_k}{2}} K_{\frac{\bar{m}_k}{2}}(2\sqrt{\eta_0 \theta_k \bar{\sigma}_k}), \quad (13)$$

where  $\theta_k = 1/\lambda_k$ . If we further assume that  $\bar{m}_k = 1$ , that is, the links of relay-destination under Rayleigh fading, we have

$$P_{out}^k = 1 - 2e^{-\frac{\eta_0}{\lambda_k}} \sqrt{\frac{\eta_0 \bar{\sigma}_k}{\lambda_k \bar{\lambda}_k}} K_1(2\sqrt{\frac{\eta_0 \bar{\sigma}_k}{\lambda_k \bar{\lambda}_k}}). \quad (14)$$

It is noted that (14) signifies the closed-form expression for (28) in [2].

In summary, we first derive  $P_{out}^k$  based on the PDFs of  $X_k$  and  $Y_k$ . Then, a closed form expression for the outage probability  $P_{out}$  can be obtained via a product of  $P_{out}^k \forall k$ .

#### IV. Simulations

In this section, we present simulations to illustrate our theoretical results. In simulations, identical Nakagami- $m$  fading channels are assumed for all links of source-relay and relay-destination. That is, we let  $m_k = \bar{m}_k = m$  and  $\lambda_k = \bar{\lambda}_k = 1 \forall k \in [1, K]$ . The transmit SNRs at the source and at the selected relay are equal, that is,  $\gamma_S = \gamma_R$ . The target rate threshold  $R$  is selected as 0.5.

Figure 2 depicts the outage probability against the averaged transmit SNR for the cases in which  $m=1$  and  $K=3$  and in which  $m=2$  and  $K=5$ . It is shown that the theoretical results match the simulation results perfectly. As expected, with the

adoption of more relay nodes (larger  $K$ ), better outage performance is achieved. Also, with larger  $m$ , superior outage performance is achieved. Note that the Nakagami fading model with  $m=1$  corresponds to the Rayleigh fading scenario.

#### V. Conclusion

We have analyzed the outage performance of the opportunistic AF relaying strategy over Nakagami- $m$  fading channels. A closed-form expression for the outage probability was derived, and it was demonstrated by simulations.

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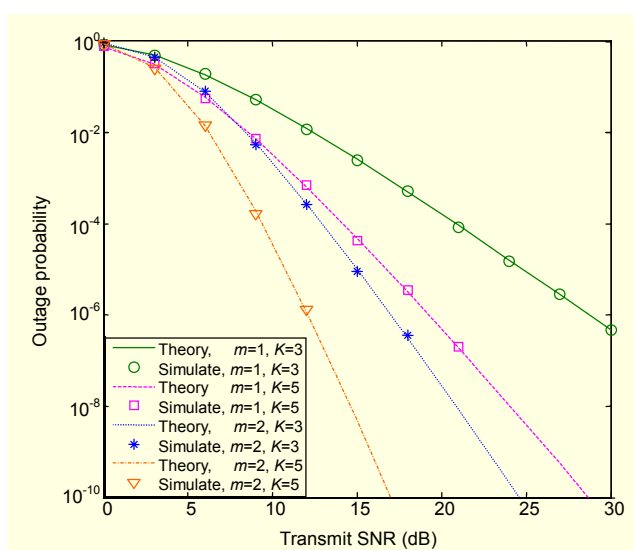


Fig. 2. Outage probability against averaged transmit SNR.