

# Characteristic-Function-Based Analysis of MIMO Systems Applying Macroscopic Selection Diversity in Mobile Communications

Wun-Cheol Jeong, Jong-Moon Chung, and Dongfang Liu

**Multiple-input multiple-output (MIMO) systems can provide significant increments in capacity; however, the capacity of MIMO systems degrades severely when spatial correlation among multipath channels is present. This paper demonstrates that the influence of shadowing on the channel capacity is more substantial than that of multipath fading; therefore, the shadowing effect is actually the dominant impairment. To overcome the composite fading effects, we propose combining macroscopic selection diversity (MSD) schemes with MIMO technology. To analyze the system performance, the capacity outage expression of MIMO-based MSD (MSD-MIMO) systems using a characteristic function is applied. The analytic results show that there are significant improvements when MSD schemes are applied, even for the two-base-station diversity case. It is also observed that the effect of spatial correlation due to multipath fading is almost negligible when multiple base stations cooperatively participate in the mobile communication topology.**

**Keywords:** MIMO systems, macroscopic selection diversity, capacity outage, characteristic function.

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Wun-Cheol Jeong (email: wjeong@etri.re.kr) is with IT Convergence Technology Research Laboratory, ETRI, Daejeon, Rep. of Korea.

Jong-Moon Chung (phone: +82 2 2123 5863, email: jmc@yonsei.ac.kr) is with the School of Electrical and Electronic Engineering, Yonsei University, Seoul, Rep. of Korea.

Dongfang Liu (email: dfliu@qualcomm.com) is with the Systems R&D, Qualcomm Incorporated, San Diego, USA.

## I. Introduction

Future mobile communication systems will be required to support broadband networking applications, where the reliability and robustness of the wireless link quality will be of prime concern. This paper investigates the performance of multiple-input multiple-output MIMO systems in outdoor environments under the presence of shadowing, where most research has been focused on the effect of small scale fading on channel capacity. The capacity outage expression of MIMO systems using a characteristic function approach was originally introduced in [1]. In this paper, an extension to this approach is applied to demonstrate the capacity gain that can be obtained through the application of macroscopic selection diversity technologies.

Since its original conception, numerous papers have investigated the relation between channel capacity and the number of deployed multiple antennas at the transmitter and receiver for MIMO systems [1]-[8]. As the number of antenna elements at the transmitter and receiver increases the information channel capacity of MIMO systems can linearly increase, provided that a rich scattering environment ensures an uncorrelated relation among the transmitter and receiver antenna elements. However, this increment in channel capacity cannot be obtained if correlation is present between the antennas. This can result from numerous causes, including insufficient antenna spacing or lack of local scattering objects. In [5], [7], and [9]-[11], different correlation models are introduced to simulate different propagation environments and to investigate the effects of antenna correlation. In [12] and [13], the asymptotic capacities of correlated MIMO systems are

investigated. All of these studies show how the MIMO capacity decreases as the channel correlation increases.

To evaluate the capacity of spatially correlated MIMO systems, a physical model of the channel correlation is required. One approach is to use a ray tracing model. Another approach is to construct a scatter model that can provide a reasonable description of the propagation environment [14]. In this paper, the scatter model approach is applied. An overview of multipath correlation models can be found in [7], [10], and [15].

In addition to multipath correlation, the received signal is degraded by large scale physical obstacles in the propagation path, resulting in a phenomenon called shadowing. Usually, the transmitted directional signals from a base station (BS) are under a common shadowing influence even though the multipath signal scattering profile may be uncorrelated. In such a system, there are significant reductions in obtainable channel capacity despite an ample scattering profile. In fact, as we will show in this paper, shadowing has a more dominant influence on the channel capacity than multipath correlation.

Various studies have analyzed space diversity techniques in order to combat multipath Rayleigh fading and shadowing effects in mobile communication. A majority of the early space diversity techniques were introduced in [9]. The term macrodiversity was first used in [16]. The author of [17] and [18] provided a theoretical analysis of composite microscopic plus macroscopic selection diversity (MSD) combining within continuous phase modulation (CPM) systems. In [19], the multiply-detected macrodiversity (MDM) scheme was introduced. It utilizes the maximum-likelihood bit-by-bit decision criterion on the combined information. In [20], the channel capacity and its upper and lower bounds are derived for MIMO-based macroscopic diversity combining systems applying selection diversity and stochastic water filling in composite fading environments. In [21], a performance analysis based on computer simulation was conducted for MIMO distributed antenna systems.

As we demonstrate in this study, the capacity of a MIMO system is severely degraded when shadowing is present on the propagation channels, even when the multipath channel components are statistically uncorrelated. The applied MSD topology presented in this paper enables the mobile terminal and base stations to maximize the spatial multiplexing gain while combating the shadowing phenomena.

In this paper, two aspects of the capacity are investigated: channel capacity and capacity outage. In terms of MSD, two BS selection schemes are presented in this paper. The first MSD scheme is based on maximizing the channel capacity. The second MSD scheme is based on minimizing shadowing by selecting the maximum signal to noise ratio (SNR) link. In the following sections, the complexity and the capacity outage

performance of the two schemes are compared.

The subsequent sections of this paper are organized as follows. In section II, the capacity of MIMO-only and MSD-MIMO systems under composite fading is analyzed. In section III, the capacity outage probability for MIMO systems based on two MSD schemes is derived using a characteristic function approach [1]. In section IV, numerical results are provided. The paper concludes with a summary and analysis of the observations in section V.

## II. Capacity of MSD-MIMO System in Composite Fading Channels

Figure 1 illustrates a typical outdoor wireless environment. For downlink transmission, the received signal vector  $\mathbf{Y}$  of an  $(N_T, N_R)$ -MIMO system under composite fading channel can be written as

$$\mathbf{Y} = \mathbf{H}_C \mathbf{X} + \mathbf{N}, \quad (1)$$

where  $N_T$  is the number of transmitter antennas,  $N_R$  is the number of receiver antennas,  $\mathbf{H}_C$  is the composite fading channel gain matrix with an  $N_R \times N_T$  dimension,  $\mathbf{X}$  is the  $N_T \times 1$  signal vector transmitted with total transmit power  $E[\mathbf{X}^H \mathbf{X}] = P_T$ , and  $\mathbf{N}$  is the  $N_R \times 1$  additive white Gaussian noise (AWGN) vector with covariance of  $\mathbf{Q} = E[\mathbf{N} \cdot \mathbf{N}^H] = \sigma_N^2 \mathbf{I}$ .

The received signal under a composite fading channel can be expressed as the product of the multipath fading and shadowed fading [22]. When the covariance of the channel gain matrix is not available at the transmitter, the capacity of a  $(N_T, N_R)$ -MIMO system employing uniform transmit power allocation with composite fading can be expressed as

$$C = \log_2 \left[ \det \left( \mathbf{I} + \frac{\rho}{N_T} \mathbf{H}_C \cdot \mathbf{H}_C^H \right) \right], \quad (2)$$

where the average SNR is  $\rho = P_T / \sigma_N^2$ . In (2), the channel gain matrix is

$$\mathbf{H}_C = \begin{bmatrix} h_{11} \omega_{11}^{1/2} & h_{12} \omega_{12}^{1/2} & \cdots & h_{1N_T} \omega_{1N_T}^{1/2} \\ h_{21} \omega_{21}^{1/2} & h_{22} \omega_{22}^{1/2} & \cdots & h_{2N_T} \omega_{2N_T}^{1/2} \\ \vdots & \vdots & \ddots & \vdots \\ h_{N_R 1} \omega_{N_R 1}^{1/2} & h_{N_R 2} \omega_{N_R 2}^{1/2} & \cdots & h_{N_R N_T} \omega_{N_R N_T}^{1/2} \end{bmatrix}, \quad (3)$$

where  $N_T$  and  $N_R$  are, respectively, the number of transmitter antennas and receiver antennas; and  $h_{ij}$  and  $\omega_{ij}^{1/2}$  are, respectively, the channel gain between the  $j$ -th transmitter antenna and the  $i$ -th receiver antenna for multipath fading and shadowed fading.

In outdoor wireless environments, the height of the antenna

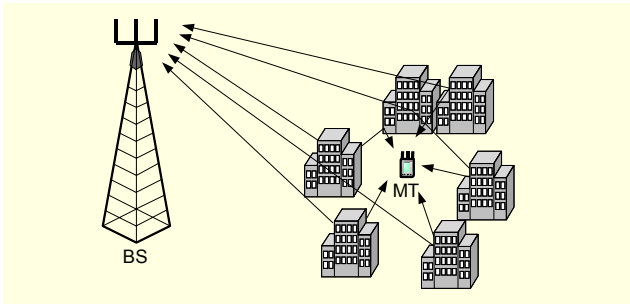


Fig. 1. Illustration of outdoor wireless propagation environment.

at the mobile terminal (MT) is typically lower than that of the antenna at the BS site. In addition, mobile communication BSs are normally located in open areas with a wide range of line of sight, while an MT is typically surrounded by ample local scatterers (see Fig. 1). Thus, the multipath signals at the MT antennas are more likely to be uncorrelated, while those at the BS antennas have a higher probability of being correlated due to lack of local scatterers. Meanwhile, the large-scale shadowing term  $\omega_{ij}$  requires much wider antenna spacing to be uncorrelated. Thus, all shadowing terms at the receiver (that is,  $\{\omega_{ij}^{1/2} : \forall i\}$ ) from the  $j$ -th transmitter can be considered completely correlated because of the physical limitation of antenna spacing at the MT (that is,  $\omega_{ij}^{1/2} = \omega_j^{1/2}$ ). Moreover, regardless of the multipath signal scattering profile, the shadowing components of the composite faded signals result in a common factor among the scattered channels (that is,  $\omega_{ij}^{1/2} = \omega^{1/2}$ ). From this fact, the capacity can be simplified to the form of

$$\begin{aligned}
 C &= \log_2 \left[ \det \left( \mathbf{I} + \frac{\rho}{N_T} \mathbf{H}_C \cdot \mathbf{H}_C^H \right) \right] \\
 &= \log_2 \left[ \det \left( \mathbf{I} + \frac{\rho}{N_T} \omega \mathbf{H}_S \cdot \mathbf{H}_S^H \right) \right] \\
 &= \sum_{i=1}^N \log_2 \left[ 1 + \frac{\rho}{N_T} \omega \lambda_i \right] = \sum_{i=1}^N \log_2 \left[ 1 + \frac{\rho}{N_T} x_i \right], \quad (4)
 \end{aligned}$$

where  $N = \min(N_T, N_R)$ ,  $\mathbf{H}_S$  is a channel gain matrix of multipath fading channels whose elements are zero mean complex Gaussian random variables with unit variance ( $CN(0,1)$ ),  $x_i = \omega \lambda_i$ , and  $\lambda_i$  is the  $i$ -th largest eigenvalue of the matrix  $\mathbf{W} = \mathbf{H}_S \cdot \mathbf{H}_S^H$ . In our analysis, we assume  $N_R = N_T = N$  for simplicity. However, simple manipulation will yield the result for imbalanced antenna deployment (that is,  $N_T \neq N_R$ ). The large scale shadowing statistics are known to be log-normally distributed and can be represented by the probability density function (pdf) of [9]:

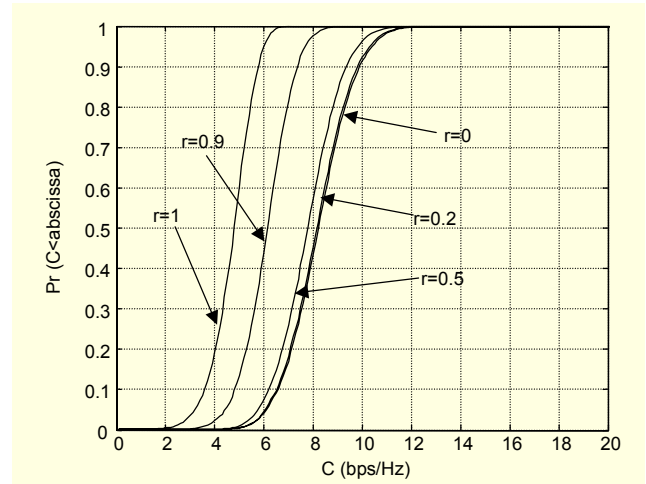


Fig. 2. Capacity outage probability of a MIMO system for multipath fading channel. Note that exponential correlation model is used to illustrate the effect of spatial correlation.

$$f_{\omega}(\omega) = \frac{1}{\omega \sigma_{\omega}^{\xi} \sqrt{2\pi}} \exp \left[ -\frac{(10 \log_{10} \omega - \mu_{\omega})^2}{2\sigma_{\omega}^2} \right], \quad (5)$$

where  $\mu_{\omega}$  and  $\sigma_{\omega}^2$  are the local mean power (LMP) in dBm and variance of the log-normal random variable  $\omega$ , respectively, and  $\xi = \ln 10 / 10$ .

Equation (4) shows that the capacity of a MIMO system depends on the distribution of the multipath channel gain matrix and shadow fading. Extensive research has been conducted on MIMO systems in multipath fading channels [1]-[6]. In such studies, the capacity was investigated for multipath fading conditions to analyze the multiplexing gain of MIMO systems. It has been observed that the mean capacity of MIMO systems under independent multipath fading channels increases linearly as the number of antennas at both ends increases. However, capacity improvement degrades when the multipath channels are correlated. Figure 2 illustrates the degradation of capacity outage probability along with spatial correlation considering multipath fading only. In the simulation, a simple exponential model was used, which is explained in section IV, to model the spatial correlation among multipath channels. As shown in Fig. 2, the capacity outage decreases as the correlation between the multipath fading channels increases.

However, when shadowing is considered, the capacity outage probability degrades severely, as shown in Fig. 3, for  $\sigma_{\omega} = 8$  dB. In particular, the capacity barely increases as the multipath fading correlation decreases over the low outage probability region of interest (below the 10% outage region). This is because the strength of the received signal suffering severe shadowing is not enough to take advantage of spatial multiplexing. This implies that shadowing, rather than

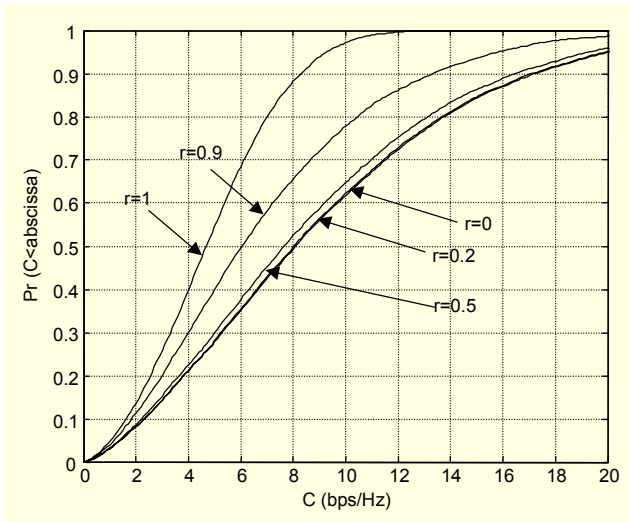


Fig. 3. Capacity outage probability of a MIMO system for composite fading channel ( $\sigma_\omega = 8$  dB). Note that exponential correlation model is used to illustrate the effect of spatial correlation.

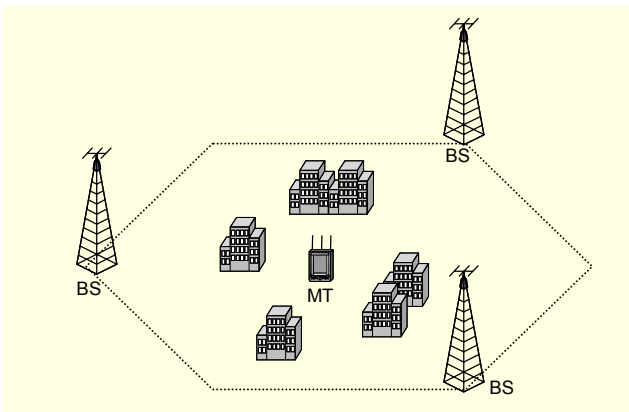


Fig. 4. Illustration of the  $(M, N_T, N_R)$  MSD-MIMO system, where  $M=3$  and  $N_T = N_R = 3$  is deployed for illustrative purpose.

multipath fading correlation, is responsible for the capacity outage degradation.

MSD is known to be an efficient scheme to overcome the shadowing phenomenon in cellular communications [9]. In MSD schemes, a BS is selected among a number of BSs. Figure 4 illustrates the configuration of an  $(M, N_T, N_R)$  MSD-MIMO system, where  $M$  BSs cover a cellular area,  $N_T$  represents the number of transmitter antennas at the BS, and  $N_R$  represents the number of receiver antennas at the MT. In this system, the optimal selection scheme is to select the BS that provides largest capacity, that is,

$$C = \max[C_1 \ C_2 \ \dots \ C_M], \quad (6)$$

where  $C_l$ , as given in (4), is the capacity of the communication link between the MT and the  $l$ -th BS. The capacity outage

probability of an  $(M, N_T, N_R)$  MSD-MIMO system is

$$\begin{aligned} \Pr\{C \leq C_{Th}\} &= F_C(C_{Th}) \\ &= \Pr\{\max(C_1, C_2, \dots, C_M) \leq C_{Th}\} \\ &= F_C(C_{Th}, C_{Th}, \dots, C_{Th}), \end{aligned} \quad (7)$$

where the joint cumulative distribution function (cdf) of the random vector  $\mathbf{C}=[C_1 \ C_2 \ \dots \ C_M]$  is given by

$$\begin{aligned} F_C(C_{Th}, C_{Th}, \dots, C_{Th}) \\ = \Pr\{C_1 \leq C_{Th}, C_2 \leq C_{Th}, \dots, C_M \leq C_{Th}\}. \end{aligned}$$

As shown in (4), the capacity of the communication link between the MT and the  $l$ -th BS,  $C_l$ , is determined by the distribution of the shadowing term ( $\omega_l$ ) and multipath gain matrix  $\mathbf{H}_{S,l}$ . We assume that the configuration of the local scatterers at the MT site is symmetric. In other words, the geographical configurations of local scatterers seen at any BSs are *statistically* identical. Thus,  $\omega_l$  and  $\mathbf{H}_{S,l}$  for all  $l$  are assumed to be identical and independently distributed (IID), thereby,  $C_l$  is also IID. With this assumption, the capacity outage probability can be simplified to

$$\Pr\{C \leq C_{Th}\} = F_C(C_{Th}, C_{Th}, \dots, C_{Th}) = [F_{C_l}(C_{Th})]^M, \quad (8)$$

where  $F_{C_l}(C_{Th})$  is the outage probability of the  $l$ -th link given as  $F_{C_l}(C_{Th}) = \Pr\{C_l \leq C_{Th}\}$  [16]. In wireless communication configurations where the local scatterers at the mobile terminal are symmetric and the shadow fading terms from the base stations are IID, a macroscopic selection diversity system with two or more BSs will always have a lower capacity outage probability compared to a single BS communication topology. Moreover, the difference between  $F_{C_l}(C_{Th})$  and  $F_C(C_{Th})$  increases as  $F_{C_l}(x)$  decreases.

### III. Outage Probability of MSD-MIMO Systems

In this section, we provide exact expressions of capacity outage probability for the MSD-MIMO system. First, we provide an expression of the capacity outage probability for the  $l$ -th link,  $F_{C_l}(C_{Th})$ , using the characteristic function (CF). Then, the capacity outage probability of an MSD-MIMO system is obtained as the product of  $F_{C_l}(C_{Th})$  as shown in (8).

In this paper, two BS selection algorithms are presented and analyzed.

#### Scheme 1. Maximum Channel Capacity MSD Scheme

Based on the criteria of achieving maximum channel capacity, an optimal MSD BS selection algorithm for a MIMO system is one that will result in the selection of the base station

that can provide the largest channel capacity.

### Scheme 2. Minimum Shadowing MSD Scheme

Based on the criteria of achieving minimum shadowing, an optimal MSD BS selection algorithm for a MIMO system is one that will result in the selection of the base station that has the highest SNR.

Scheme 2 may result in a different selection than scheme 1. The two schemes are compared in this section.

For scheme 1 we consider an MT that detects signals from  $M$  BSs *statistically* independent. Suppose that the MIMO system channel capacity at time  $t$  of the  $i$ -th BS to the MT is denoted as  $C_{BSi}(t)$  (for  $i = 1, \dots, M$ ). At time  $t$ , an optimal MSD base station selection algorithm will be one that selects the BS that maximizes its capacity; therefore, the BS selection criteria will be  $\max[C(t)] = \sup_{t \geq 0} (C_{BS1}(t), C_{BS2}(t), \dots, C_{BSM}(t))$ .

For scheme 2, we consider an MT that detects signals from  $M$  BSs statistically independent. Correspondingly, the MIMO system average SNR at time  $t$  of the  $i$ -th BS to the MT is denoted as  $\rho_{BSi}(t)$  (for  $i = 1, \dots, M$ ). An MSD algorithm that selects the BS that suffers the least shadowing will make its decision based on the BS that satisfies  $\sup_{t \geq 0} (\rho_{BS1}(t), \rho_{BS2}(t), \dots, \rho_{BSM}(t))$ , where it is assumed that the BS that is selected from this maximum SNR criteria is denoted as  $C_{\max(\rho_{BSi})}(t)$ . From the channel capacity point of view, if we compare the two, we obtain  $\max[C(t)] \geq C_{\max(\rho_{BSi})}(t)$ , which indicates that the two schemes may result in different selections and channel capacity performance.

In terms of complexity, scheme 1 requires an estimation of multipath channel gains and shadowing terms for all links involved in the communications. An alternative MSD method is scheme 2, which selects the BS that suffers the least shadowing. Although scheme 2 does not guarantee maximum channel capacity, it is much simpler to implement, and in section IV it is shown that scheme 2 has an impressive capacity outage performance that is very close to the performance of scheme 1. Scheme 2 is less complex because it does not require computationally massive channel estimation for all links.

### 1. MSD-MIMO System with Maximum Channel Capacity BS Selection Scheme

The CF of capacity for the  $l$ -th link is given as

$$\begin{aligned} \phi_{C_l}(z) &= E_C [e^{j2\pi Cz}] = E_{\mathbf{X}} \left[ \prod_{i=1}^N \left( 1 + \frac{\rho}{N} x_i \right)^{\frac{j2\pi z}{\ln 2}} \right] \\ &= \int_{\mathbf{X}} f_{\mathbf{X}}(x_1, \dots, x_N) \prod_{i=1}^N \left( 1 + \frac{\rho}{N} x_i \right)^{\frac{j2\pi z}{\ln 2}} d\mathbf{X} \\ &= \int_0^{\infty} \frac{1}{\omega^N} f_{\omega}(\omega) \cdot \Theta(\omega, z) d\omega, \end{aligned} \quad (9)$$

where  $E_{\alpha}[\cdot]$  is an expectation taken with respect to the random variable  $\alpha$ ,  $\mathbf{X}$  is a vector given by  $\mathbf{X} = [x_1, x_2, \dots, x_N]$ , and  $f_{\lambda}(\lambda_1, \lambda_2, \dots, \lambda_N)$  is the joint pdf of the ordered eigenvalues of  $\mathbf{W}$ . In (9), the integral is taken over the ordered statistics,  $\{x_1 > x_2 > \dots > x_N > 0\}$ , and

$$\Theta(\omega, z) = \int_{\mathbf{X}} \prod_{i=1}^N \left( 1 + \frac{\rho}{N} x_i \right)^{\frac{j2\pi z}{\ln 2}} \cdot f_{\lambda} \left( \frac{x_1}{\omega}, \frac{x_2}{\omega}, \dots, \frac{x_N}{\omega} \right) d\mathbf{X}. \quad (10)$$

Extending equation (32) of [1], the capacity outage probability of the  $l$ -th link can be obtained from its CF as

$$\begin{aligned} \Pr[C_l \leq C_{Th}] &= F_{C_l}(C_{Th}) = \int_0^{C_{Th}} \int_{-\infty}^{\infty} \phi_{C_l}(z) e^{-j2\pi z c} dz dc \\ &= \int_{-\infty}^{\infty} \phi_{C_l}(z) \cdot \left[ \frac{1 - e^{-j2\pi z C_{Th}}}{j2\pi z} \right] dz. \end{aligned} \quad (11)$$

By substituting (11) into (8), the capacity outage probability of the  $(M, N_T, N_R)$  MSD-MIMO system can be expressed as

$$\begin{aligned} \Pr\{C \leq C_{Th}\} &= [F_{C_l}(C_{Th})]^M \\ &= \left\{ \int_{-\infty}^{\infty} \phi_{C_l}(z) \left[ \frac{1 - e^{-j2\pi z C_{Th}}}{j2\pi z} \right] dz \right\}^M. \end{aligned} \quad (12)$$

Numerical evaluation techniques such as the inverse fast Fourier transform can be applied to evaluate (11).

#### A. Uncorrelated Multipath Fading Case

Evaluation of (9) involves an  $(N+1)$ th order integral, which usually increases the computational complexity. Chiani and others provide a simpler form to evaluate the inner integral of  $\Theta(\omega, z)$ . Using corollary 2 in the appendix of [1], the CF of the  $l$ -th link can be simplified as

$$\begin{aligned} \phi_{C_l}(z) &= \int_0^{\infty} \frac{1}{\omega^N} f_{\omega}(\omega) \cdot \Theta(\omega, z) d\omega \\ &= K \int_0^{\infty} \frac{1}{\omega^N} f_{\omega}(\omega) \cdot \det[\mathbf{U}] d\omega, \end{aligned} \quad (13)$$

where  $\mathbf{U}$  is a Hankel matrix with the  $ij$ -th element given by

$$u_{i,j} = \int_0^{\infty} \left(\frac{x}{\omega}\right)^{i+j-2} \cdot e^{-\frac{x}{\omega}} \cdot \left(1 + \frac{\rho}{N}x\right)^{\frac{j2\pi z}{\ln 2}} dx \quad (14)$$

and  $K = \frac{\pi^{N(N-1)}}{[\tilde{\Gamma}_N(N)]^2}$  with

$$\tilde{\Gamma}_\alpha(\beta) = \pi^{\alpha(\alpha-1)/2} \prod_{i=1}^{\alpha} (\beta-i)!$$

### B. Correlated Multipath Fading Case

When the small-scale channel gains are correlated, the inner integral of the CF function shown in (10) can be obtained using corollary 2 in the appendix of [1]:

$$\Theta(\omega, z) = K_\Sigma \det[\mathbf{G}], \quad (15)$$

where the  $ij$ -th elements of the matrix  $\mathbf{G}$  is given by

$$g_{i,j} = \int_0^{\infty} \left(\frac{x}{\omega}\right)^{j-1} \cdot e^{-\frac{x}{\sigma_i \omega}} \cdot \left(1 + \frac{\rho}{N}x\right)^{\frac{j2\pi z}{\ln 2}} dx, \quad (16)$$

and the normalizing constant is given by

$$K_\Sigma = K \cdot \prod_{j=1}^N (j-1)! \cdot \frac{|\boldsymbol{\Sigma}|^N}{|\mathbf{V}_2(\boldsymbol{\sigma})|},$$

where  $\boldsymbol{\Sigma}$  is the correlation matrix of small-scale fading with ordered eigenvalues of  $\boldsymbol{\sigma} = [\sigma_1, \sigma_2, \dots, \sigma_N]$ , and  $\mathbf{V}_2(\boldsymbol{\sigma})$  is a Vandermonde matrix given by

$$\mathbf{V}_2(\boldsymbol{\sigma}) = \begin{bmatrix} 1 & 1 & \dots & 1 \\ -\sigma_1^{-1} & -\sigma_2^{-1} & \dots & -\sigma_N^{-1} \\ \vdots & \vdots & \ddots & \vdots \\ -\sigma_1^{1-N} & -\sigma_2^{1-N} & \dots & -\sigma_N^{1-N} \end{bmatrix}.$$

Note that (14) and (16) can be evaluated in a compact form using the identity

$$\begin{aligned} & \int_0^{\infty} x^n (1+bx)^y e^{-\frac{x}{a}} dx \\ &= \frac{n! \Gamma(-1-n-y) {}_1F_1(1+n, n+y+2, 1/ab)}{b^{n+1} \Gamma(-y)} \\ &+ a^{n+1+y} b^y \Gamma(n+1+y) {}_1F_1(-y, -n-y, 1/ab), \end{aligned} \quad (17)$$

which is valid for  $R\{a\} > 0$ ,  $n \geq 0$ ,  $\arg\{b\} \neq \pi$ , where  $\Gamma(\cdot)$  is the Gamma function, and  ${}_1F_1(\cdot, \cdot, \cdot)$  is the hypergeometric function [23].

## 2. MSD-MIMO System with Minimum Shadowing BS Selection Scheme

If the distributions of  $\mathbf{H}_s$  for all BSs are *statistically* identical, that is, if the configuration of the local scatters is symmetrical, the selection of the BS to maximize the capacity might be comparable to the capacity of the BS which suffers the least amount of shadowing. Thus, the capacity is given as

$$C = \sum_{i=1}^N \log_2 \left[ 1 + \frac{\rho}{N} \lambda_i \omega_{\max} \right] = \sum_{i=1}^N \log_2 \left[ 1 + \frac{\rho}{N} x_i \right], \quad (18)$$

where  $\omega_{\max} = \max[\omega_1 \ \omega_2 \ \dots \ \omega_M]$ , and  $x_i = \lambda_i \omega_{\max}$ .

The distribution function of the shadow fading component is given as in [17] by

$$F_{\omega_{\max}}(\omega) = \prod_{k=1}^M Q\left(\frac{10 \log_{10} \omega - \mu_\omega}{\sigma_\omega}\right), \quad (19)$$

where  $Q(x) = \int_x^{\infty} \frac{1}{\sqrt{2\pi}} e^{-t^2/2} dt$ . By differentiating (19), the pdf of the shadowing component can be simply obtained as

$$\begin{aligned} f_{\omega_{\max}}(\omega) &= \frac{M}{\omega \sigma_\omega^2 \sqrt{2\pi}} \exp\left[-\frac{(10 \log_{10} \omega - \mu_\omega)^2}{2\sigma_\omega^2}\right] \\ &\times \left[ \operatorname{erf}\left(\frac{10 \log_{10} \omega - \mu_\omega}{\sigma_\omega}\right) \right]^{M-1}. \end{aligned} \quad (20)$$

In a similar fashion as in the optimal selection scheme, the capacity outage probability is represented as

$$\Pr\{C \leq C_{Th}\} = \int_{-\infty}^{\infty} \phi_C(z) \left[ \frac{1 - e^{-j2\pi z C_{Th}}}{j2\pi z} \right] dz. \quad (21)$$

The CF in (21) is

$$\begin{aligned} \phi_C(z) &= E_C [e^{j2\pi C z}] = E_{\mathbf{X}} \left[ \prod_{i=1}^N \left( 1 + \frac{\rho}{N} x_i \right)^{\frac{j2\pi z}{\ln 2}} \right] \\ &= \int_{\mathbf{X}} f_{\mathbf{X}}(x_1, \dots, x_N) \prod_{i=1}^N \left( 1 + \frac{\rho}{N} x_i \right)^{\frac{j2\pi z}{\ln 2}} d\mathbf{X} \\ &= \int_0^{\infty} \frac{1}{\omega^N} f_{\omega_{\max}}(\omega) \cdot \Theta(\omega, z) d\omega, \end{aligned} \quad (22)$$

where  $\Theta(\omega, z)$  is given in (10).

## IV. Numerical Results

In this section, we provide numerical results of the capacity

outage probability. As shown in (4), the capacity of MIMO systems is determined by large-scale shadowing and multipath fading. In wireless environments, the signals can be correlated to each other for many reasons, including insufficient antenna spacing or the lack of local scatters. The capacity of MIMO systems can be significantly lower in correlated fading environments than in independent fading environments.

For downlink transmission, the distribution of a multipath channel gain matrix  $\mathbf{H}_S$  can be approximated as given in [7] as

$$\mathbf{H}_S \sim \mathbf{H}_W \mathbf{\Sigma}^{1/2}, \quad (23)$$

where  $\mathbf{H}_W$  is a complex Gaussian matrix whose elements are IID. Complex Gaussian  $CN(0,1)$ , and  $\mathbf{\Sigma}$  is the channel correlation matrix whose elements represent the spatial correlation of the multipath profile among the antennas. To model the channel correlation matrix  $\mathbf{\Sigma}$ , we consider two correlation models: the exponential model and the one-ring model.

In our evaluation model, we assume  $N_T=N_R=3$ ,  $\rho = 10$  dB, and  $\sigma_\omega = 8$  dB.

### 1. Exponential Correlation Model

For a linear antenna array, the correlation among antennas decreases as the distance between antenna elements increases. In [11], exponential correlation was used to model the spatial correlation between any two individual fading channels for a uniform linear antenna array. The correlation matrix using this model is given as  $\mathbf{\Sigma} = \{r^{|i-j|}\}_{i,j=1,\dots,N}$  with  $r \in [0,1)$ .

Figure 5 shows the capacity outage probability of MSD-MIMO systems for various numbers of BSs. The results show that the mean capacity can be substantially improved by employing multiple BSs. The largest improvement occurs between  $M=1$  and  $M=2$ . At the 10% outage level, an approximate 120% capacity gain is obtained, and at the 5% outage level an approximate 180% capacity gain is obtained. Comparing Fig. 5 to the uncorrelated case ( $r = 0$ ) of Fig. 2, the mean capacity is not greatly degraded even for  $r = 0.5$ . This means that the comparable spatial correlation in small scale fading does not degrade the mean capacity. Rather, the capacity is greatly affected by the shadowing effect. Regarding the difference between the suboptimal to optimal selection methods, despite  $M$  increasing, the capacity difference is less than 4%. The performance of scheme 2 (minimum shadowing BS selection) is very close to that of scheme 1 (maximum channel capacity BS selection) but can be obtained at a fraction of the computation complexity. Thus, from the point of view of implementation, scheme 2 would be a natural preference. Its performance is further illustrated in the following figures.

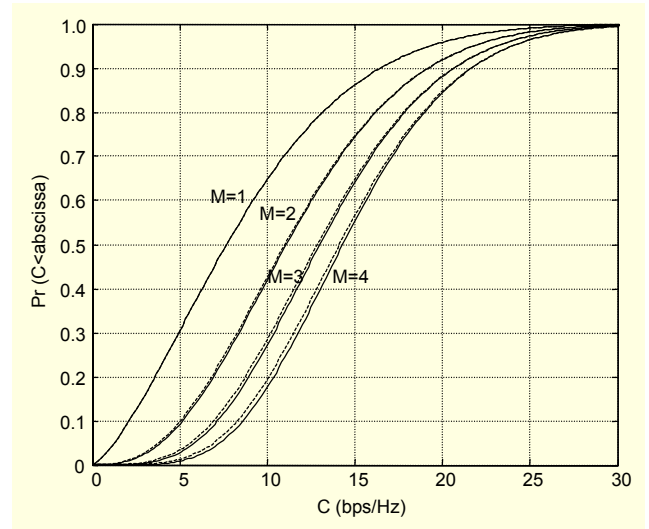


Fig. 5. Outage capacity of  $(M, 3, 3)$  MSD-MIMO system for different number of BSs ( $r = 0.5$ ). Note that the solid curves are the capacity outage probabilities for the maximum capacity MSD scheme, while dashed curves are those for minimum shadowing MSD scheme.

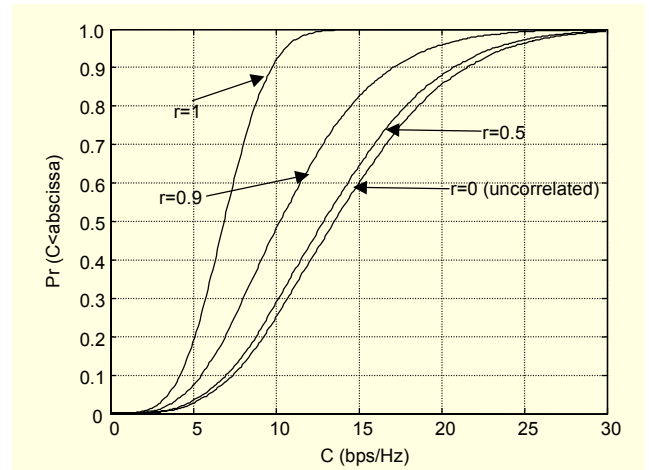


Fig. 6. Outage capacity of  $(3, 3, 3)$  MSD-MIMO system for different multipath correlation values (the minimum shadowing MSD scheme is applied).

Figure 6 shows the outage capacity of a  $(3, 3, 3)$  MSD-MIMO system applying scheme 2 for different spatial correlation values. The outage capacity degradation of the MSD-MIMO system at  $r = 0.5$  is approximately 10%. In addition, Fig. 7 shows the effect of SNR on the 10% capacity outage performance for a  $(3, 3, 3)$  MSD-MIMO communication topology. As the SNR level (in dB) increases, the capacity outage increment factor is larger than 0.27 bps/Hz/dB for the uncorrelated and correlated cases. In comparison of the uncorrelated case ( $r = 0$ ) to the  $r = 0.5$  correlated case, a less than 5% loss in outage capacity occurs

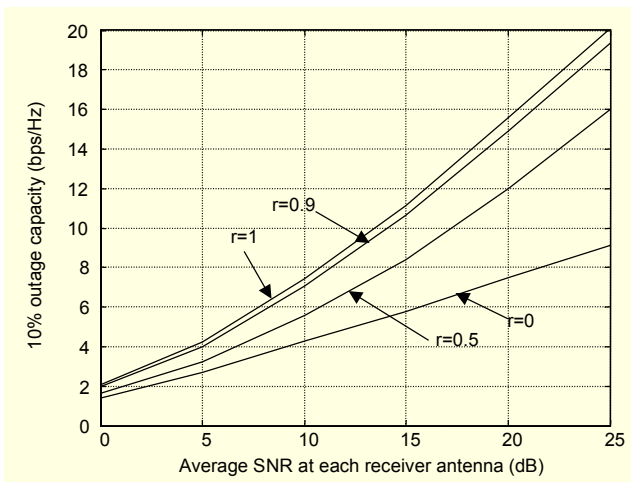


Fig. 7. 10% capacity outage of (3, 3, 3) MSD-MIMO system for different multipath correlation values (the minimum shadowing MSD scheme is applied).

for a wide range of SNR levels. This demonstrates the level of robustness of the MSD-MIMO scheme against composite fading environments. For a fixed value of correlation, this improvement is due to base station diversity. Thus, the benefit of diversity increases logarithmically as the diversity order increases.

It is important to note that the results also apply if the channel model is reversed (that is, if the transmitter and receiver roles are reversed in the communication model) due to the channel reciprocal properties.

## 2. One-Ring Model

Figure 8 illustrates a one-ring model applied as an alternative method to analyze the effects of correlation on the capacity outage in composite fading environments. The model assumes that the local scatterers are placed on a continuous ring surrounding the MT. Since the MT is surrounded by local scatterers, the correlation introduced by the MT antennas is negligible if the antenna spacing is greater than half a wavelength. Thus, the  $ij$ -th element of the multipath correlation matrix at the transmitter site is given by

$$\Sigma_{ij} \approx J_0(2\pi \cdot \Theta \cdot d(i, j)), \quad (24)$$

where  $J_0(x)$  is the Bessel function of the first kind of the 0-th order,  $\Theta$  is the angle spread at the transmitter site as shown in Fig. 8, and  $d(i, j)$  is the distance in wavelength between the  $i$ -th and the  $j$ -th transmitter antennas.

Figures 9, 10, and 11 respectively show the capacity outage performance for the angle spread ( $\Theta$ ) of  $10^\circ$ ,  $30^\circ$ , and  $50^\circ$ . In each graph, the  $(M, 3, 3)$  MSD-MIMO topology performance is illustrated for the cases of  $M$  equaling 1, 2, 3, and 4. Comparing

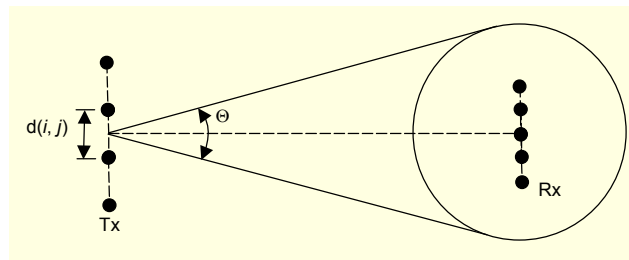


Fig. 8. Continuous scatter one-ring model.

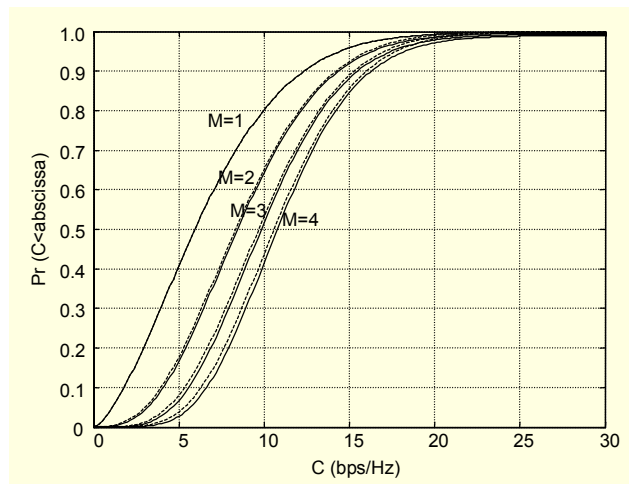


Fig. 9. Capacity outage probability of  $(M, 3, 3)$  MSD-MIMO system for different number of BSs. Angle spread of 10 degree is used. Note that the solid curves are the capacity outage probabilities for maximum capacity MSD scheme, while dashed curves are those for minimum shadowing MSD scheme.

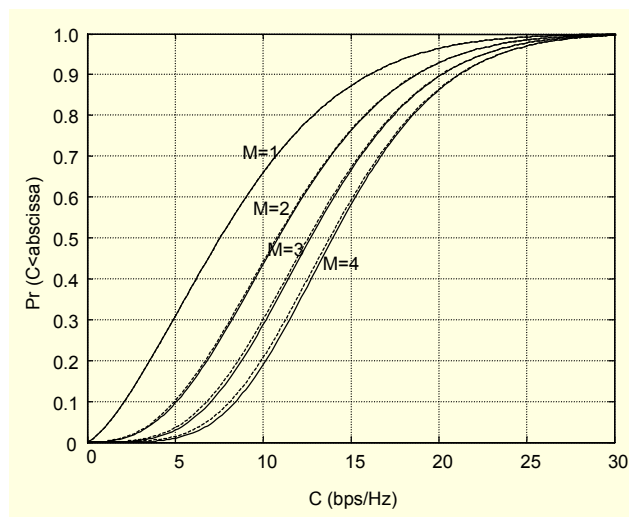


Fig. 10. Capacity outage probability of  $(M, 3, 3)$  MSD-MIMO system for different number of BSs. Angle spread of 30 degree is used. Note that the solid curves are the capacity outage probabilities for maximum capacity MSD scheme, while dashed curves are those for minimum shadowing MSD scheme.



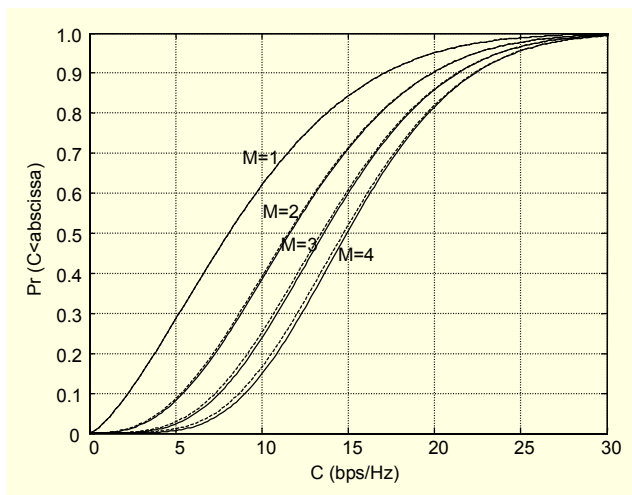


Fig. 11. Capacity outage probability of  $(M, 3, 3)$  MSD-MIMO system for different number of BSs. Angle spread of 50 degree is used. Note that the solid curves are the capacity outage probabilities for maximum capacity MSD scheme, while dashed curves are those for minimum shadowing MSD scheme.

Fig. 9 to Fig. 10, for each corresponding case of  $M = 2, 3$ , and 4, an approximate 27% gain in capacity is obtained at the 10% outage level. Next, comparing Fig. 10 to 11, for the corresponding cases of  $M = 2, 3$ , and 4, an approximate 5% to 10% gain in capacity is obtained at the 10% outage level. These results in Figs. 9 to 11 demonstrate the effects of correlation on the capacity outage performance based on the one-ring model. As the angle spread increases from  $10^\circ$ , to  $30^\circ$ , to  $50^\circ$ , the correlation decreases, resulting in an increment in the obtainable capacity. As the angle spread reaches  $50^\circ$ , the signals become almost uncorrelated resulting in a capacity outage performance near the saturated level.

## V. Conclusion

MIMO systems can provide significant increments in capacity, which make them a favorable candidate for futuristic mobile communication applications. However, the inherent structure of MIMO technology makes it sensitive to the wireless channel environment. Namely, the signal correlation factor of the multipath signal profile is one aspect that has been heavily investigated in past studies [1]-[7]. This paper adds to the findings of previous studies by investigating the effects of shadowing to the MIMO system capacity. With this intent, the capacity outage expression of MSD-MIMO systems using a characteristic function approach has been developed. From the results, we conclude that the shadowing effect is a stronger factor in decreasing the capacity than multipath correlation. In relation to MSD BS selection schemes, we demonstrated that

the maximum channel capacity selection scheme, which is more complex, has a negligible advantage in outage capacity performance compared to the simpler minimum shadowing selection scheme. Based on the analysis of MSD systems in correlated composite fading environments, we conclude that the MSD-MIMO scheme can provide a capacity outage performance beyond the performance range of MIMO systems. The exact capacity outage equations and simulation results provide an accurate method to quantitatively analyze the performance gain of MSD technology in MIMO systems.

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**Wun-Cheol Jeong** received his BS degree in electrical engineering from Konkuk University, Seoul, Rep. of Korea in 1996, and his MS and PhD degrees in electrical engineering from the Pennsylvania State University, University Park, in 1999 and 2002, respectively. From 2002 to 2005, Dr. Jeong worked in the Advanced Communication Systems Engineering Laboratory (ACSEL), Oklahoma Communication Laboratory for Networking and Bioengineering (OCLNB), and the School of Electrical and Computer Engineering of the Oklahoma State University as a research assistant professor. Since 2005, he has been a senior researcher at the Electronics and Telecommunications Research Institute (ETRI), Daejeon, Rep. of Korea. His research interests include wireless communications, information theory, ad-hoc networks, and signal processing.



**Jong-Moon Chung** is an associate professor in the School of Electrical and Electronic Engineering at Yonsei University, Seoul, Rep. of Korea, since Sept. 2005. Dr. Chung received his PhD degree in electrical engineering from the Pennsylvania State University in 1999, and his MS and BS degrees in electronic engineering from Yonsei University, Seoul, Rep. of Korea, in 1994 and 1992, respectively. From 1997 to 1999, he was an instructor and assistant professor in the Department of Electrical Engineering at the Pennsylvania State University. From 2000 to 2005, he served as Director of the Oklahoma Communication Laboratory for Networking and Bioengineering (OCLNB) and associate professor of Electrical and Computer Engineering at the Oklahoma State University. Dr. Chung's research is in the area of mobile communication and networking. In 2008 and 2007, respectively, Dr. Chung received the Outstanding Professor Award and the Outstanding Teaching Award from Yonsei University. In 2005 October, Dr. Chung was awarded the Regents Distinguished Research Award (USA) and in the same year September he received the Halliburton Outstanding Young Faculty Award (USA). In 2004 and 2003 respectively, he received the Technology Innovator Award and the Distinguished Faculty Award both from the Oklahoma State University (USA). In addition, in 2003 Dr. Chung received the Top Gun Award (USA) and in 2000, Dr. Chung received the 'The First Place' Outstanding Paper Award at the IEEE EIT conference.



**Dongfang Liu** received his Bachelor's and Master's degrees in electrical engineering from the University of Science and Technology Beijing, China, in 1993 and 1996 respectively. He received the PhD degree in electrical engineering from the Oklahoma State University, USA, in 2005. After graduation, he has been with Qualcomm Incorporated at San Diego, USA. His research interests include RF circuit design/testing, OFDM/MIMO technologies, and cellular communications.