

# Compensation Algorithm of Arrival Time Mismatch in the Space-Time Coded Systems

Minhyuk Kim, Suksoon Choi, Jiwon Jung, Seongro Lee, Han Na Cho, and Myeongsoo Choi, *Member, KIMICS*

**Abstract**— One objective in developing the next generation of wireless communication systems is to increase data rates and reliability. A promising way to achieve this is to combine multiple-input and multiple-output signal processing with a space-time coding scheme, which offers higher coding and diversity gains and improves the spectrum efficiency and reliability of a wireless communication system. It is noted, however, that time delay differences and phase differences among different channels increase symbol interference and degrade system performance. In this letter, we investigate phase differences and their effects on multiple-input and multiple-output systems, and propose a compensation algorithm for the Rayleigh fading model to minimize their effects.

**Index Terms**— Multiple-input and multiple-output, space-time code, Rayleigh fading, phase compensation algorithm.

## I. INTRODUCTION

One of objective in developing next generation wireless communication systems is to increase the data rates and thus improve reliability to satisfy the rapidly growing demand for high quality, multi-media services. However, such systems suffer from multi-path propagation effects, which exhibit rapidly time-varying channel characteristics. Space-time coding techniques have been widely proposed to combat these adverse effects. In [1]-[3], the authors proposed several space-time coding schemes, which offer higher coding and diversity gains and are suitable to improve the spectrum efficiency and reliability of broadband applications. In their studies, it is assumed that there are no delay or phase errors among the multiple transmitter and receiver chains. This assumption is difficult to warrant in practice. Since transmit chains inevitably have different characteristics due to component mismatches, the transmitted signals usually have different phase shifts and time delay. Therefore, phase differences increase symbol interference and degrade system

performance. In this letter, we attempt to model phase differences, and investigate their effects on MIMO systems. We also propose a compensation algorithm for the Rayleigh fading model to minimize the effects of phase differences.

## II. System Model

Fig. 1 shows a block diagram of the transmitter and receiver equipped with the two transmit and receive antennas. The information bits are encoded by a space-time encoder and the output of the space-time encoder is split into two symbol streams. Each symbol is then pulse-shaped modulated and transmitted through the corresponding antenna. The transmitted signals are received by two antennas, pass through matched filters, and then phase tracking algorithms. The outputs are then processed by a space-time decoder to produce the decoded bit stream.

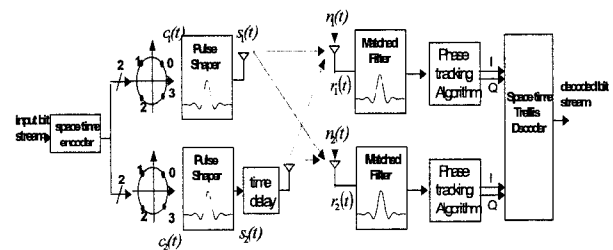


Fig.1 Simulation model of space-time coded system in presence of time delay

The transmitted signal can be expressed as

$$s_i(t) = \sqrt{E_s} \cdot \sum_l c_i(l) p(t - lT_s), i = 1, 2 \quad (1)$$

where  $c_i(l)$  is modulation symbols at  $l$ -th time in the  $i$ -th transmit antenna,  $T_s = 1/R_s$  is symbol period, and  $p(t)$  is the pulse shaping function. Without loss of generality, we will assume that  $p(t)$  is a square root raised cosine pulse shape filter given by [6]

$$p(t) = \frac{4\varepsilon}{\pi\sqrt{T_s}} \cdot \frac{\cos((1+\varepsilon)\pi t/T_s) + \frac{\sin((1+\varepsilon)\pi t/T_s)}{4\varepsilon t/T_s}}{(4\varepsilon t/T_s)^2 - 1} \quad (2)$$

Where,  $\varepsilon$  is bandwidth expansion or roll-off factor. Fig. 2 shows the unit sample response of the square raised cosine filter. In this report, in order to investigate

Manuscript received February 12, 2008; revised July 20, 2008. Minhyuk Kim is with the Department of Radio Communication Engineering, Korea Maritime University, Busan, 606-791, Korea (Tel: +82-51-410-4920, Email: squaru@hotmail.com)

the effect of time delay between two transmitters, roll-off factor  $\epsilon$  of 0.35, number of sample  $L_s$  of 10, number of symbol  $L_d$  of 8 and number of coefficient  $L_c$  of 81 are assigned. At the receiver, the received signal at each antennas is filtered using a receive filter with impulse response  $\bar{p}(t)$  that is matched to the transmit pulse shape  $p(t)$ ,  $p(t) = \bar{p}(t)$ . Since the transmit chains inevitably have different characteristics due to component mismatch, the two transmitted of signals have usually

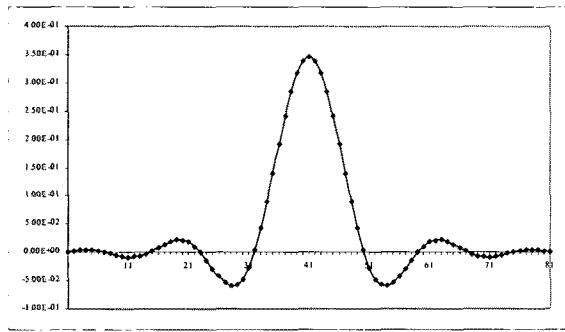


Fig. 2 Unit sample response of square root raised cosine filter ( $\epsilon = 0.35, L_s = 10, L_d = 8, L_c = 81$ )

different delay. Therefore, a parameter,  $\nabla$ , is introduced to represent the time delay of the 2<sup>nd</sup> transmit antenna relative to the first one. A sample point  $\nabla_s$  due to  $\nabla$  may be expressed by

$$\Delta_s = \Delta \cdot L_s \text{ samples} \tag{3}$$

If  $\nabla = 0.1$ , it means that the 2<sup>nd</sup> transmit antenna transmits one sample ( $\Delta_s = 1$ ) delayed relative to the first one. We can write the received signal at the  $k$ th antenna,  $r_k(t), k = 1, 2$ , as

$$r_k(t) = s_1(t) + \rho(\Delta)s_2(t + \Delta) + \eta_k(t) \tag{4}$$

$\rho(\Delta)$  denotes the amplitude of sample point at time  $t + \Delta$ . We assume that if there is no time delay between two transmit antennas, that is  $\Delta = 0$ , then  $\rho(\Delta)$  is equal to 1. The channel noise,  $\eta_j(t)$ , is modeled as a zero-mean complex Gaussian random variable with variance  $N_0/2$  per dimension. After some manipulation, Equation (4) can be written as

$$r_k(t) = \sqrt{2} e^{j\theta_1(t)} + \sqrt{2} \rho(\Delta) e^{j\theta_2(t)} e^{j\phi} + \eta_k(t) \tag{5}$$

where  $\theta_1(t)$  and  $\theta_2(t)$  are phase information of the two transmitted signals at time  $t$  and  $\phi$  is the phase error due to  $\nabla$ . After some manipulations, Equation (5) can also expression as

$$r_k(t) = \tilde{a}(t) e^{j\theta_o(t)} e^{j\frac{\phi}{2}} + \eta_k(t) \tag{6}$$

where  $\theta_o(n) = \frac{\theta_1(t) + \theta_2(t)}{2}$  is a function of the transmitted information, and  $\tilde{a}(n) = 2\sqrt{1 + \rho(\Delta)^2} \cos\left(\frac{\theta_1(t) - \theta_2(t) - \phi}{2}\right)$  is the amplitude of the received signal. Fig. 3 shows three received constellations in the noise-free channel: (a)  $\Delta = 0$  (b)  $\Delta = 0.1$  and (c)  $\Delta = 0.3$ . We notice that the time delay rotates constellation around its position and disperses some points.

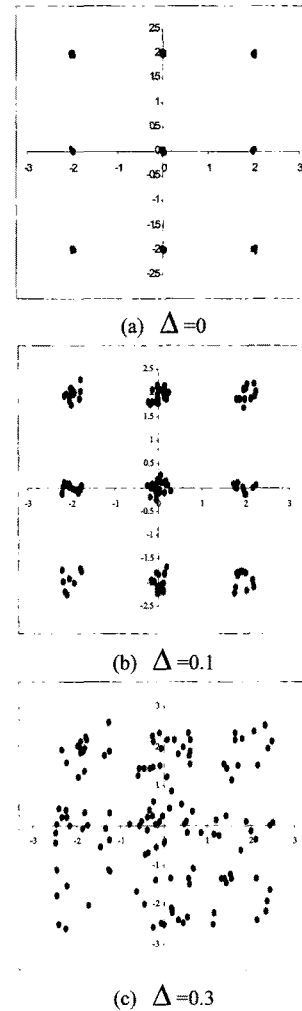


Fig. 3 The received constellations for time delay  $\Delta$

In the next section, we describe a decision-directed phase error estimation and compensation algorithm, and use it to solve the phase error problem in a space-time

coded communication system. Based on the received signal model of Equation (2), a decision-directed approach can be used to estimate and track the total phase error in each receive channel

### III. Phase Estimation and Compensation

The covariance approach [9] can only estimate the phase offset, and it does not correct it. We will now describe a decision-directed phase error estimation and compensation algorithm, and use this algorithm in a space-time coded communication system. Given the received signal model of Equation (2), a decision-directed approach can be used to estimate and track the total phase error in each receive channel, as shown in Fig. 4.

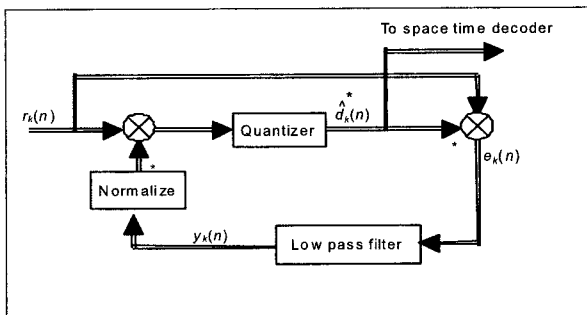


Fig. 4 Block diagram of the decision-directed phase estimation and compensation

Where “\*” denotes the complex-conjugate operator,  $\hat{d}_k^*(n)$  is the quantizer output after the phase error is compensated. Ideally, we have

$$\hat{d}_k^*(n) = e^{j\theta_0(n)}. \tag{7}$$

In the figure, the phase error is given by

$$e_k(n) = r_k(n) \cdot \hat{d}_k^*(n). \tag{8}$$

Correspondingly, the phase error estimate can be obtained by taking the expectation of (8) as

$$e_k(n) = E[e^{j\theta_0(n)} e^{j\phi_k} e^{-j\theta_0(n)} + \eta_k(n) e^{-j\theta_0(n)}] = e^{j\phi_k}. \tag{9}$$

The expectation operation is implemented as the low-pass filtering in Fig. 4. Both infinite impulse response (IIR) and finite impulse response (FIR) filters can be used for the low-pass filter [5]. In this letter, a first order IIR filter is used. It is defined as follows:

$$y_k(n) = \beta y_k(n-1) + (1-\beta)e_k(n), \tag{10}$$

where  $\beta(0 < \beta < 1)$  means the loop bandwidth

parameter of a low pass filter. Its value controls the estimation accuracy and how fast the phase estimate reaches its steady-state value. The larger  $\beta$  is, the more accurate the estimate is, but the slower it is to converge. In the noiseless case, we can show that

$$y_k(n) = \beta y_k(n-1) + (1-\beta)e^{-j\phi_k} = e^{-j\phi_k}(1-\beta^n) \tag{11}$$

As the time  $n$  increases,  $\beta^n$  decreases. Eventually,  $y_k(n)$  will approach its steady-state value, that is,

$$y_k(n) \rightarrow e^{-j\phi_k}, \text{ as } n \rightarrow \infty. \tag{12}$$

Therefore, when  $y_k(n)$  reaches its steady-state value, we can remove the effect of the phase error by multiplying its complex-conjugate to the input signal  $r_k(n)$ .

### IV. Simulation Results

Computer simulations were used to study the effect of phase errors on system performance. A space-time coded system with two transmit and two receive antennas was simulated. The source symbols were transmitted in frames of length 130. The code generator matrix for the QPSK space-time code with 32 states proposed in [1] was used. The additive white Gaussian noise was introduced as the channel noise in the simulation.

Fig. 5 shows the bit error rate for different time delay  $\Delta$  without any compensation. The dotted line represents the BER of an uncoded QPSK system with one transmit and one receive antenna. As  $\Delta$  increases, the BER performance deteriorates rapidly. If  $\Delta > 0.3$ , the performance is worse than that of the uncoded system. The results in Fig 5 clearly show that phase error compensation is necessary to preserve both the coding and diversity gains of the space-time coded system. In the following, we examine the performance of the proposed phase estimation and compensation algorithm and the BER improvement it can achieve.

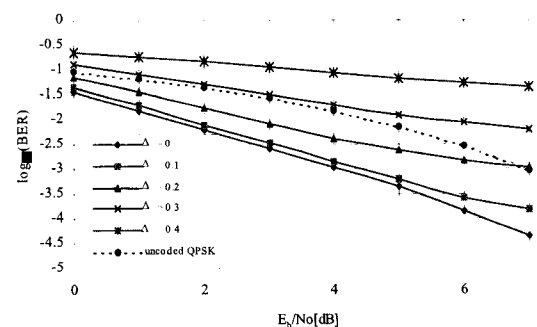


Fig. 5 The bit error rate for various time delay  $\Delta$

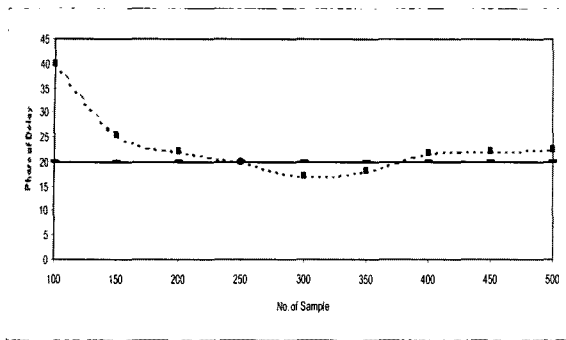


Fig. 6 Phase error tracking performance

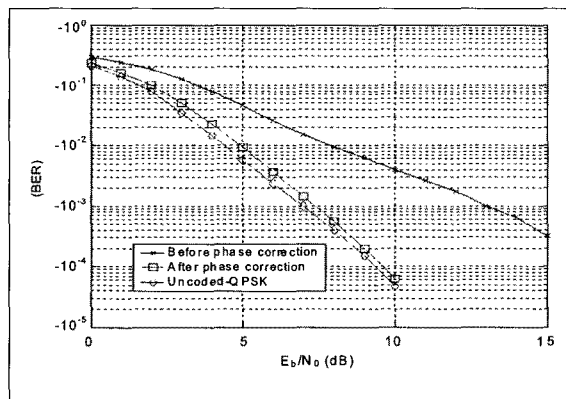


Fig. 7 Bit error rate comparison

The following phase errors  $\phi = 20^\circ$  is used. Fig. 6 shows the phase error estimate in two cases: a noise free case which is represented straight line, and another case with an  $E_b/N_0 = 7$  dB and  $\beta = 0.9$  which is represented by noisy curves. We notice that the proposed algorithm estimates the phase errors correctly and they converge within about 200 samples.

Fig. 7 shows the BER performance of the space-time coded system with and without the phase estimation and compensation algorithm for the same set of phase error values used in Fig. 6. It is noticed that the proposed algorithm significantly improves the BER performance. For example, at a bit error rate of  $10^{-3}$ , about 5.5 dB improvement is achieved.

## V. Conclusion

In a space-time coded system, it is often assumed that there are no phase errors among the multiple transmitter and receiver chains. This assumption is difficult to warrant in practice. The phase differences between the propagation paths increase the symbol interference and degrade system performance. In this letter, we have studied the effect of the phase errors between different transmit antennas and different propagation paths in a space-time coded communication system, and have shown through computer simulations of BER performance that the BER performance can be severely degraded. A decision-directed estimation and compensation algorithm has been

proposed to minimize the effects on system performance. The described decision-directed approach works well under the assumption of constant phase over a frame. If phase is variable over a frame, the circuit needs a long transmit packet size to estimate the phase or we choose low pass filter parameter trade-offs between the estimate time and performance. Therefore, this circuit may be useful as a phase estimator before transmitting data.

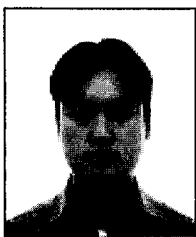
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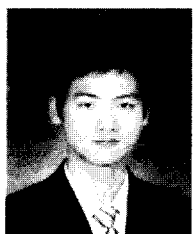
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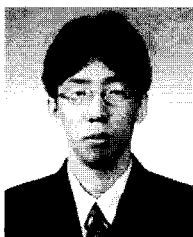
#### **Min-Hyuk Kim**

received his BS, MS degrees in radio sciences and engineering from Korea Maritime University, Pusan, Korea, in 2006 and 2008. He is currently working toward the PhD degree at Korea Maritime University. His research interests are channel coding, digital modem, field-programmable gate-array (FPGA) design technology, and digital broadcasting system.



#### **Seok-Soon Choi**

received the BS degree in radio sciences and engineering from Korea Maritime University, Pusan, Korea, in 2007. He is currently working toward the MS degree at Korea Maritime University. His research interests are channel coding, digital modem, field-programmable gate-array (FPGA) design technology, and digital broadcasting system.



#### **Ji-Won Jung**

received his BS in 1989, MS in 1991, and PhD in 1995 from Sungkyunkwan University, Seoul, Korea, all in electronics engineering. From Nov. 1990 to Feb. 1992, he was with LG Research Center, Korea. From Sept. 1995 to Aug. 1996, he was with Korea Telecom (KT). From Aug. 2001 to July 2002, he was an Invited Researcher at the Communication Research Center Canada supported by NSERC. Since 1996, he joined the Department of Radio Science and Engineering, Korea Maritime University, Busan, Korea. His research interests are channel coding, digital modem, FPGA design technology, and digital broadcasting system.



#### **Seong-ro Lee**

received his BS degree in electronics engineering from Korea University, Seoul, Korea, in 1987, the MS and PhD degrees from Korea Advanced Institute of Science and Technology, Daejeon, Korea, in 1990 and 1996. Since 2005, he joined the Department of Electronic Engineering, Mokpo National University, Mokpo, Korea. His research interests are digital communication system, mobile and satellite communication system, USN/ telematics application, embedded system, biometrics system.



#### **Choi-Han Na**

received the BS degree in department of early childhood education from Gwangju University, Gwangju, Korea, in 2006. She is currently working toward the MS degree at the Department of Electronic Engineering, Mokpo National University, Mokpo, Korea. Her research interests are digital communication system, USN/ telematics application, embedded system, biometrics system.



#### **Myeong-soo Choi**

received his BS, MS degrees in Department of Electronic Engineering, Mokpo National University, Mokpo, Korea, in 2000 and 2002. He is currently working toward the PhD degree at Mokpo National University. His research interests are digital communication system, USN/ telematics application, embedded system, biometrics system.