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구간 시변 지연이 존재하는 불확실 확률적 뉴럴 네트웍의 지연의존 안정성 판별법

Delay-dependent Stability Criteria for Uncertain Stochastic Neural Networks with Interval Time-varying Delays

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Abstract – In this paper, the problem of global asymptotic stability of uncertain stochastic neural networks with delay is considered. The delay is assumed to be time-varying and belong to a given interval. Based on the Lyapunov stability theory, new delay-dependent stability criteria for the system is derived in terms of LMI(linear matrix inequality). Three numerical examples are given to show the effectiveness of proposed method.

Key Words : Interval time-varying delays, Linear matrix inequalities, Lyapunov method, Stochastic neural networks.

1. Introduction

During the last two decades, considerable efforts have been done to the stability analysis of cellular neural networks since cellular neural networks have been widely applied to pattern recognition, associative memory, signal processing, and fixed-point computation, For reference, see [1-3] and reference therein. These applications rely on the dynamic behaviors of the equilibrium point of the related network.

In the processing of storage and transmission of information, time-delays often occur due to the finite switching speed of amplifiers in electronic networks or finite speed for signal propagation in biological networks. The important factor is that the delays may cause instability and oscillation of neural networks. Therefore, many researchers have focused on the stability analysis of delayed cellular neural networks in recnet years [4-20].

Recently, some attentions for the stability analysis of the stochastic neural networks have been paid by some researchers[21-22] since the stochastic perturbations are unavoidable when one models the neural networks. In this regard, Zhang et al. [22] investigated the stability problem for uncertain stochastic Hop-field neural networks with time-varying delays. The time-delays treated in the work [22] was restricted to be differentiable and its derivative was less than one. More recently, the delay-dependent stability criteria for uncertain stochastic neural networks with no requirement of the bounds of delay-derivative terms was proposed [23-24].

On the other hands, the stability analysis of dynamic systems with interval time-varying delay has been a focused topic of theoretical and practical impotance [25-28]. The system with interval time-varying delays means that the lower bound of time-delay which guarantees the stability of system is not restricted to zero. A typical example of dynamic systems with interval time-varying delays is networked control systems [26]. Michiels et al. [27] proposed a frequency-domain method for the stability analysis of time-delay system with periodic time-varying delay functions. Yu and Lien [28] investigated the stability of neutral systems with interval time-varying delays and two classes of uncertainties.

Unfortunately, to the best of authors' knowledge, the problem of stability analysis for uncertain stochastic neural networks with interval time-delay has not been investigated

With this motivation, in the paper, we propose a new stability criterion for uncertain stochastic neural networks with interval time-varying delays for the first time. By constructing a suitable Lyapunov functional, a delay-dependent criterion, which is less conservative than delay-independent one when the size of delays is small, is established in terms of LMI which can be solved efficiently by using the interior-point algorithm [29]. Three numerical example are given to show the effectiveness of the proposed method.

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Notation : R^n is the n-dimensional Euclidean space, $R^{m\times n}$ denotes the set of $m\times n$ real matrix. $\|\cdot\|$ refers to the Euclidean vector norm and the induced matrix norm. For symmetric matrices X and Y , the notation $X>Y$ is positive definite, (respectively, nonnegative). $diag\{\cdots\}$ denotes the block diagonal matrix. $\star$ represents the elements below the main diagonal of a symmetric matrix. I is the identity matrix with appropriate dimension. A^T means the transpose of the matrix A. For $h>0$, $C([-h,\ 0],R^n)$ means the family of continuous functions ϕ from $[-h,\ 0]$ to R^n with the norm $\|\phi\|=\sup_{-h\le s\le 0}|\phi(s)|$. Let $(\Omega,F,\{F_t\}_{t\ge 0},P)$ be a complete probability space with a filtration $\{F_t\}_{t>0}$ satisfying the usual conditions (i.e. it is right continuous and F_0 and contains all P-pull sets). $L^p_{F_0}([-h,\ 0],R^n)$ the family of all F_0-measurable $C([-h,\ 0],R^n)$-valued random variables$\xi=\{\xi(\theta)\ :\ -h\le\theta\le 0\}$ such that $\sup_{-h\le s\le 0}\mathrm{E}|\xi(\theta)|^p<\infty$ where $\mathrm{E}\{\cdot\}$ stands for the mathematical expectation operator with respect to the given probability measure P . Denote by $C^{2,1}(R^n\times R^+,R^+)$ the family of all nonnegative functions $V(x,t)$ on $R^n\times R^+$ which are continuously twice differentiable in x and differentiable in t .

2. Problem Statements

Consider the following uncertain neural networks with discrete time-varying delays:

$$\dot{v}(t)=(-A+\Delta A(t))v(t)+(W_0+\Delta W_0(t))f(v(t)) + (W_1+\Delta W_1(t))f(v(t-h(t)))+b \quad (1)$$

where $v(t)=[v_1(t),\cdots,v_n(t)]^T\in R^n$ is the neuron state vector, n denotes the number of neurons in a neural network, $f(v(t))=[f_1(v_1(t)),\cdots,f_n(v_n(t))]^T\in R^n$ denotes the neuron activation function, $f(v(t-h(t)))=[f_1(v_1(t-h(t))),\cdots,f_n(v_n(t-h(t)))]^T\in R^n$, $A=diag\{a_i\}$ is a positive diagonal matrix, $W_0=(w^0_{ij})_{n\times n}$, and $W_1=(w^1_{ij})_{n\times n}$ are the interconnection matrices representing the weight coefficients of the neurons, $b=[b_1,b_2,\cdots,b_n]^T$ means a constant input vector, and $\Delta A(t)$, $\Delta W_0(t)$, and $\Delta W_1(t)$, are the uncertainties of system matrices of the form

$$[\ \Delta A(t)\ \ \Delta W_0(t)\ \ \Delta W_1(t)\ ]=DF(t)[\ E\ \ E_0\ \ E_1\ ], \quad (2)$$

where the time-varying nonlinear function $F(t)$ satisfies

$$F^T(t)F(t)\le I,\quad \forall t\ge 0. \quad (3)$$

The delays, $h(t)$, are time-varying continuous function that satisfies

$$0\le h_L\le h(t)\le h_U,\ \dot{h}(t)\le h_D, \quad (4)$$

where h_L, and h_U are positive constants and h_D is any constant one.

The activation functions, $f_i(v_i(t)), i=1,\cdots,n$, are assumed to posses the following properties :

(A1) f_i is bounded on R^n, $i=1,2,\cdots,n$.

(A2) There exist real numbers $l_i>0$ such that

$$|f_i(\xi_i)-f_j(\xi_j)|\le|l_i(\xi_i-\xi_j)|,\ \xi_i,\ \xi_j\in R,\ \xi_i\ne\xi_j,\ i,j=1,\cdots,n. \quad (5)$$

Note that using the Brouwer's fixed-point theorem [4], it can be easily proven that there exists at least one equilibrium point Eq. (1).

For simplicity, in stability analysis of the system (1), the equilibrium point $v^*=[v_1^*,\cdots,v_n^*]^T$ is shifted to the origin by utilizing the transformation $x(\cdot)=x(\cdot)-v^*$, which leads the system (1) to the following form:

$$\dot{x}(t)=(-A+\Delta A(t))x(t)+(W_0+\Delta W_0(t))g(x(t)) + (W_1+\Delta W_1(t))g(x(t-h(t))) \quad (6)$$

where $x(t)=[x_1(t),\cdots,x_n(t)]^T\in R^n$ is the state vector of the transformed system, $g(x(t))=[g_1(x(t)),\cdots,g_n(t))]^T$ and $g_i(x_i(t))=f_i(x_i(t)+v_i^*)-f_i(v_i^*)$.

Here, the activation functions g_i satisfy the following properties:

(H1) g_i is bounded on R^n, $i=1,2,\cdots,n$.

(H2) There exist real numbers $l_i>0$ such that

$$|g_i(\xi_i)-g_j(\xi_j)|\le|l_i(\xi_i-\xi_j)|,\ \xi_i,\ \xi_j\in R,\ \xi_i\ne\xi_j,\ i,j=1,\cdots,n. \quad (7)$$

(H3) $g_i(0)=0\ (i=1,\cdots,n)$.

In this paper, we consider the following uncertain stochastic neural networks with interval time-varying delays

$$\begin{aligned} dx(t)=&[(-A+\Delta A(t))x(t)+W_0+\Delta W_0(t))g(t)) \\ &+(W_1+\Delta W_1(t))g(x(t-h(t)))]dt \\ &+[H_0x(t)+H_1x(t-h(t))]dw(t) \end{aligned} \quad (8)$$

where H_0 and H_1 are known constant matrices with appropriate dimensions, $w(t)$ is a scalar Wiener Process (Brownian Motion) on $(\Omega,F,\{F_t\}_{t\ge 0},P)$ which satisfies $\mathrm{E}\{dw(t)\}=0$ and $\mathrm{E}\{dw^2(t)\}=dt$.

Now, system (8) can be written as:

$$\begin{aligned} dx(t)&=[-Ax(t)+W_0g(x(t))+W_1g(x(t-h(t)))+Dp(t)]dt \\ &\quad+[H_0x(t)+H_1(x-h(t))]dw(t), \\ p(t)&=F(t)q(t), \\ q(t)&=Ex(t)+E_0g(x(t))+E_1g(x(t-h(t))). \end{aligned} \quad (9)$$

Before deriving our results, we state the following facts, lemma and definition.

Fact 1. (Schur complement) Given constant symmetric matrices Σ_1, Σ_2, Σ_3 where $\Sigma_1=\Sigma_1^T$ and $0<\Sigma_2=\Sigma_2^T$, then $\Sigma_1+\Sigma_3^T\Sigma_2^{-1}\Sigma_3<0$ if and only if

$$\begin{bmatrix}\Sigma_1 & \Sigma_3^T\\ \Sigma_3 & -\Sigma_2\end{bmatrix}<0,\ \text{or}\ \begin{bmatrix}-\Sigma_2 & \Sigma_3\\ \Sigma_3^T & \Sigma_1\end{bmatrix}<0.$$

Fact 2. For any real vectors a, b and any matrix $Q>0$ with appropriate dimensions, it follow that:

$$2a^Tb\le a^TQa+b^T+b^TQ^{-1}b.$$

Lemma 1.[30] For any constant matrix $M\in R^{n\times n}$,

$M=M^T>0$, scalar $\gamma>0$, vector function $x:[0,\ \gamma]\to R^n$ such that the integrations concerned are well defined, then

$$\left(\int_0^\gamma x(s)ds\right)^T M\left(\int_0^\gamma x(s)ds\right)\le \int_0^\gamma x^T(s)Mx(s)ds. \qquad (10)$$

Definition 1. For the uncertain stochastic neural networks (9) and every $\phi\in L^2_{F_0}([-h,\ 0]],\ R^n)$, the trivial solution is globally asymptotically stable in the mean square if, for all admissible uncertainties,

$$\lim_{t\to\infty}\mathrm{E}|x(t,\phi)|^2=0 \qquad (11)$$

In deriving our main results, Itô's formula plays a key role in stability analysis of stochastic systems (See [31-32] for details).

3. Main results

In this section, we propose a new stability criterion for uncertain stochastic neural networks with interval time-varying delays described by (6).

For each $V\in C^{2,1}([-h_U,\infty]\times R^+,\ R^+)$, define a notation $\mathcal{L}$ which means the weak infinitesimal operator [33] associated with stochastic neural networks (9) from $R^n\times R^+$ to R by

$$\begin{aligned}\mathcal{L}V(x(t),t)&=V_t(x(t),t)+V_x(x(t),t)[-Ax(t)+W_0g(x(t))\\&\quad+W_1g(x(t-h(t))+Dp(t)]\\&\quad+\frac{1}{2}trace\left[[H_0x(t)+H_1x(t-h(t))]^T\right.\\&\qquad\left.V_{xx}(x(t),t)[H_0x(t)+H_1x(t-h(t))]\right]\end{aligned}$$

where

$$V_t(x(t),t)=\frac{\partial V(x(t),t)}{\partial t},$$

$$V_x(x(t),t)=\left(\frac{\partial V(x(t),t)}{\partial x_1},\cdots,\frac{\partial V(x(t),t)}{\partial x_n}\right)$$

$$V_{xx}(x(t),t)=\left(\frac{\partial^2 V(x(t),t)}{\partial x_i\partial x_j}\right),\ i,j=1,\cdots,n.$$

Before stating our main results, the notations of several matrices are defined for simplicity in Appendix I. Now, we have the following theorem.

Theorem 1 For given h_L, h_U, h_D, and $L=diag\{l_1,\ l_2,\cdots,l_n\}$, the equilibrium point of (9) is globally asymptotically stable in the mean square if there exist positive diagonal matrices $M_i(i=1,2)$, positive definite matrices N, P_1, $Q_i(i=1,2)$, $R_i(i=1,\cdots,4)$, $S_i(i=1,2)$ and any matrices $P_i,(i=2,3,\cdots,45)$ satisfying the following LMI:

$$\begin{bmatrix}\Sigma & P^TU_1 & P^TU_2 & P^TU_3 & \Xi_1^TP_1 & \sqrt{h_L}\,\Xi_1^TS_1 & (h_U-h_L)\Xi_1^TS_2 & \Xi_2^TN\\ \star & -S_1 & 0 & 0 & 0 & 0 & 0 & 0\\ \star & \star & -S_2 & 0 & 0 & 0 & 0 & 0\\ \star & \star & \star & -S_2 & 0 & 0 & 0 & 0\\ \star & \star & \star & \star & -P_1 & 0 & 0 & 0\\ \star & \star & \star & \star & \star & -S_1 & 0 & 0\\ \star & \star & \star & \star & \star & \star & -(h_U-h_L)S_2 & 0\\ \star & \star & \star & \star & \star & \star & \star & -N\end{bmatrix}<0 \quad (12)$$

where Σ is defined in Appendix I, and

$$P=\begin{bmatrix}P_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\ P_2 & P_3 & P_4 & P_5 & P_6 & P_7 & P_8 & P_9 & P_{10} & P_{11} & P_{12}\\ P_{13} & P_{14} & P_{15} & P_{16} & P_{17} & P_{18} & P_{19} & P_{20} & P_{21} & P_{22} & P_{23}\\ P_{24} & P_{25} & P_{26} & P_{27} & P_{28} & P_{29} & P_{30} & P_{31} & P_{32} & P_{33} & P_{34}\\ P_{35} & P_{36} & P_{37} & P_{38} & P_{39} & P_{40} & P_{41} & P_{42} & P_{43} & P_{44} & P_{45}\end{bmatrix},$$

$$U_1^T=[0\ \ 0\ \ -I\ \ 0\ \ 0],$$

$$U_2^T=[0\ \ 0\ \ 0\ \ -I\ \ 0],$$

$$U_3^T=[0\ \ 0\ \ 0\ \ 0\ \ -I],$$

$$\Xi_1=[H_0\ \ 0\ \ H_1\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0],$$

$$\Xi_2=[E\ \ 0\ \ H_1\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ E_1\ \ E_2\ \ 0].$$

Proof. First of all, let us define

$$\begin{aligned}y(t)&=-Ax(t)+W_0g(x(t))+W_1g(x(t-h(t)))+Dp(t),\\ m(t)&=H_0x(t)+H_1x(t-h(t)).\end{aligned} \qquad (13)$$

From [31], the following three equations

$$\begin{aligned}&z_1(t):x(t)-x(t-h(t))-\int_{t-h_L}^{t}y(s)ds-\int_{t-h_L}^{t}m(s)dw(s)=0,\\ &z_2(t):x(t-h_L)-x(t-h(t))\\ &\qquad-\int_{t-h(t)}^{t-h_L}y(s)ds-\int_{t-h(t)}^{t-h_L}m(s)dw(s)=0,\\ &z_3(t):x(t-h(t))-x(t-h_U)\\ &\qquad-\int_{t-h_U}^{t-h(t)}y(s)ds-\int_{t-h_U}^{t-h(t)}m(s)dw(s)=0.\end{aligned} \qquad (14)$$

hold.

For positive definite matrices P_1, $Q_i(i=1,2)$, $R_i(i=1,\cdots,4)$, $S_i(i=1,2)$,and any matrices $P_i(i=1,2,3,\cdots,45)$, let us consider the Lyapunov-krasovskii functional candidate:

$$V(x(t),t)=\sum_{i=1}^{5}V_i(x(t),t) \qquad (15)$$

where

$$V_1(x(t),t)=\zeta^T(t)\Gamma P\zeta(t),$$

$$\begin{aligned}V_2(x(t),t)&=\int_{t-h_L}^{t}x^T(s)R_1x(s)ds+\int_{t-h(t)}^{t}x^T(t)R_2(s)ds\\&\quad+\int_{t-h_U}^{t}x^T(s)R_3(s)x(s)ds\\&\quad+\int_{t-h(t)}^{t}g^T(x(s))R_4g(x(s))ds,\end{aligned}$$

$$V_3(x(t),t)=h_L\int_{t-h_L}^{t}\int_s^t y^T(u)Q_1y(u)duds,$$

$$V_4(x(t),t)=\int_{t-h_U}^{t-h_L}\int_s^t y^T(u)Q_2y(u)duds,$$

$$\begin{aligned}V_5(x(t),t)&=\int_{t}^{t-h_L}\int_s^t m^T(u)S_1m(u)duds\\&\quad+\int_{t-h_U}^{t-h_L}\int_s^t m^T(u)S_2m(u)duds.\end{aligned}$$

Here, E and $\zeta(t)$ in V_1 are defined as

$$\Gamma^T=\begin{bmatrix}I & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\end{bmatrix},$$

$$\zeta^T(t)=\Big[x^T(t)\ \ x^T(t-h_L)\ \ x^T(t-h(t))\ \ y^T(t-h_U)\ \ y^T(t)\ \ \left(\int_{t-h(t)}^{t-h_L}y(s)ds\right)^T$$

$$\left(\int_{t-h_U}^{t-h(t)} y(s)ds\right)^T \; g^T(x(t)) \; g^T(x(t-h(t))) \; p^T(t)\Big]. \tag{16}$$

Then, $\mathcal{L}V_1(x(t),t)$ can obtained as

$$\mathcal{L}V_1(x(t),t)$$

$$= 2\zeta^T(t)P^T\begin{bmatrix} y(t) \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} + m^T(t)P_1m(t)$$

$$= 2\zeta^T(t)P^T\begin{bmatrix} y(t) \\ -y(t)-Ax(t)+W_0g(x(t))+W_1g(x(t-h(t)))+Dp(t) \\ z_1(t) \\ z_2(t) \\ z_3(t) \end{bmatrix}$$

$$+m^T(t)P_1m(t)$$

$$= \zeta^T(t)(P^TG+G^TP)\zeta(t)+m^T(t)P_1m(t)$$

$$+2\zeta^T(T)P^TU_1\int_{t-h_L}^{t} m(s)dw(s)$$

$$+2\zeta^T(t)P^TU_2\int_{t-h(t)}^{t-h_L} m(s)dw(s)$$

$$+2\zeta^T(t)P^TU_3\int_{t-h_U}^{t-h(t)} m(s)dw(s)$$

$$\leq \zeta^T(t)(P^TG+G^TP+\Xi_1^TP_1\Xi_1)\zeta(t)$$

$$+\zeta^T(t)(P^TU_1S_1^{-1}U_1^TP+P^TU_2S_2^{-1}U_2^TP+P^TU_3S_2^{-1}U_3^TP)\zeta(t)$$

$$+\left(\int_{t-h_L}^{t} m(s)dw(s)\right)^T S_1\left(\int_{t-h_L}^{t} m(s)dw(s)\right)$$

$$+\left(\int_{t-h(t)}^{t-h_L} m(s)dw(s)\right)^T S_2\left(\int_{t-h(t)}^{t-h_L} m(s)dw(s)\right)$$

$$+\left(\int_{t-h_U}^{t-h(t)} m(s)dw(s)\right)^T S_2\left(\int_{t-h_U}^{t-h(t)} m(s)dw(s)\right), \tag{17}$$

where

$$G = \begin{bmatrix} 0 & 0 & 0 & 0 & I & 0 & 0 & 0 & 0 & 0 & 0 \\ -A & 0 & 0 & 0 & -I & 0 & 0 & 0 & W_0 & W_1 & D \\ I & -I & 0 & 0 & 0 & -I & 0 & 0 & 0 & 0 & 0 \\ 0 & I & -I & 0 & 0 & 0 & I & 0 & 0 & 0 & 0 \\ 0 & 0 & I & -I & 0 & 0 & 0 & I & 0 & 0 & 0 \end{bmatrix}, \tag{18}$$

and Fact 2 was utilized in obtaining an upper bound of $\mathcal{L}V_1(x(t),t)$.

Calculating $\mathcal{L}V_2(x(t),t)$ gives that

$$\mathcal{L}V_2(x(t),t) \leq x^T(t)R_1x(t)-x^T(t-h_L)R_1x(t-h_L)$$

$$+x^T(t)R_2x(t)-(1-h_D)x^T(t-h(t))R_2x(t-h(t))$$

$$+x^T(t)R_3x(t)-x^T(t-h_U)R_3x(t-h_U)$$

$$+g^T(x(t))R_4g(s(t))$$

$$-(1-h_D)g^T(x(t-h(t)))R_2x(t-h(t))). \tag{19}$$

By using Lemma 1, we can obtain an upper bounds of $\mathcal{L}V_3(x(t),t)$ and $\mathcal{L}V_4(x(t),t)$ as follows:

$$\mathcal{L}V_3(x(t),t) = h_L^2y^T(t)Q_1y(t)-h_L\int_{t-h_L}^{t} y^T(s)Q_1y(s)ds$$

$$\leq h_L^2y^T(t)Q_1y(t)-\left(\int_{t-h_L}^{t} y(s)ds\right)^T Q_1\left(\int_{t-h_L}^{t} y(s)ds\right),$$

$$\mathcal{L}V_4(x(t),t) = \int_{t-h_L}^{t} y^T(s)Q_2y(s)ds - \int_{t-h_U}^{t} y^T(s)Q_2y(s)ds$$

$$+(h_U-h_L)y^T(t)Q_2y(t)$$

$$= (h_U-h_L)y^T(t)Q_2y(t)-\int_{t-h_U}^{t-h_L} y^T(s)Q_2y(s)ds$$

$$= (h_U-h_L)y^T(t)Q_2y(t)-\int_{t-h(t)}^{t-h_L} y^T(s)Q_2y(s)ds$$

$$-\int_{t-h_U}^{t-h_L} y^T(s)Q_2y(t)ds$$

$$\leq (h_U-h_L)y^T(t)Q_2y(t)$$

$$-(h_U-h_L)^{-1}\left(\int_{t-h(t)}^{t-h_L} y(s)ds\right)^T Q_2\left(\int_{t-h(t)}^{t-h_L} y(s)ds\right)$$

$$-(h_U-h_L)^{-1}\left(\int_{t-h_U}^{t-h_L} y(s)ds\right)^T Q_2\left(\int_{t-h_U}^{t-h(t)} y(s)ds\right). \tag{20}$$

By calculating $\mathcal{L}V_5(x(t),t)$, we have

$$\mathcal{L}V_5(x(t),t) \leq h_L\zeta^T(t)\Xi_1^TS_1\Xi_1\zeta(t)-\int_{t-h_L}^{t} m^T(s)S_1m(s)ds$$

$$+(h_U-h_L)\zeta^T(t)\Xi_1^TS_2\Xi_1\zeta(t)$$

$$-\int_{t-h(t)}^{t-h_L} m^T(s)S_2m(s)ds$$

$$-\int_{t-h_U}^{t-h(t)} m^T(s)S_2m(s)ds \tag{21}$$

Here note that Eq. (7) means that

$$g_j^2(x_j(t))-l_j^2x_j^2(t) \leq 0 \;\; (j=1,\cdots,n). \tag{22}$$

and

$$g_j^2(x_j(t-h(t))-l_j^2x_j^2(t-h(t)) \leq 0 \;\; (j=1,\cdots,n). \tag{23}$$

From two inequalities (22) and (23) above, for any diagonal positive matrices $M_1 = diag\{m_{11},\cdots,m_{1n}\}$ and $M_2 = diag\{m_{21},\cdots,m_{2n}\}$, the following inequalities hold

$$0 \leq -\sum_{j=1}^{n} m_{1j}\left[g_j^2(x_j(t))-l_j^2x_j^2(t)\right]$$

$$-\sum_{j=1}^{n} m_{2j}\left[g_j^2(x_j(t-h(t)))-l_j^2x_j^2(t-h(t))\right]$$

$$= x^T(t)L^TM_1Lx(t)-g^T(x(t))M_1g(x(t))$$

$$+x^T(t-h(t))L^TM_2Lx(t-h(t))$$

$$-g^T(x(t-h(t)))M_2g(x(t-h(t))). \tag{24}$$

Since the following inequality holds from (3) and (9),

$$p^T(t)p(t) \leq q^T(t)q(t) = \zeta^T(t)\Xi_2^T\Xi_2\zeta(t), \tag{25}$$

there exist a positive matrix N satisfying the following inequlity

$$\zeta^T(t)\Xi_2^TN\Xi_2\zeta(t)-p^T(t)NP(t) \geq 0, \tag{26}$$

where Ξ_2 is defined in Theorem 1.

From (17)-(26) and by applying S-procedure [29], the $\mathcal{L}V(x(t),t) = \sum_{i=1}^{5}\mathcal{L}V_i(x(t),t)$ has a new upper bound as

$$\mathcal{L}V(x(t),t) \leq \zeta^T(t)\Phi\zeta(t)$$

$$+\left(\int_{t-h_L}^{t} m(s)dw(s)\right)^T S_1\left(\int_{t-h_L}^{t} m(s)dw(s)\right)$$

$$+\left(\int_{t-h(t)}^{t-h_L} m(s)dw(s)\right)^T S_1\left(\int_{t-h(t)}^{t-h_L} m(s)dw(s)\right)$$

$$+\left(\int_{t-h_U}^{t-h(t)} m(s)dw(s)\right)^T S_1\left(\int_{t-h_U}^{t-h(t)} m(s)dw(s)\right)$$

$$-\int_{t-h_L}^{t} m^T(s)S_1m(s)ds-\int_{t-h(t)}^{t-h_L} m^T(s)S_2m(s)ds$$
$$-\int_{t-h_U}^{t-h(t)} m^T(s)S_2m(s)ds. \tag{27}$$

where

$$\Phi=\Sigma+P^TU_1S_1^{-1}U_1^TP+P^TU_2S_2^{-1}U_2^TP+P^TU_3S_2^{-1}U_3^TP$$
$$+\Xi_1^T(P_1+h_LS_1+(h_U-h_L)S_2)\Xi_1+\Xi_2^TN\Xi_2.$$

Using Fact 1. the inequality $\Phi<0$, is equivalent to the LMI (29). Then, there exist a positive scalar γ such that

$$\Phi+diag\{\gamma I,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0\}<0.$$

Since

$$\mathrm{E}\left\{\left(\int_{t-h_L}^{t} m(s)dw(s)\right)^T S_1\left(\int_{t-h_L}^{t} m(s)dw(s)\right)\right\}$$
$$=\mathrm{E}\left\{\int_{t-h_L}^{t} m^T(s)S_1m(s)ds\right\},$$
$$\mathrm{E}\left\{\left(\int_{t-h(t)}^{t-h_L} m(s)dw(s)\right)^T S_2\left(\int_{t-h(t)}^{t-h_L} m(s)dw(s)\right)\right\}$$
$$=\mathrm{E}\left\{\int_{t-h(t)}^{t-h_L} m^T(s)S_2m(s)ds\right\},$$
$$\mathrm{E}\left\{\left(\int_{t-h_U}^{t-h(t)} m(s)dw(s)\right)^T S_2\left(\int_{t-h_U}^{t-h(t)} m(s)dw(s)\right)\right\}$$
$$=\mathrm{E}\left\{\int_{t-h_U}^{t-h(t)} m^T(s)S_2m(s)ds\right\},$$

by taking the mathematical expectation on both side of (27), we have

$$\mathrm{E}\{\mathcal{L}V(x(t),t)\}\le \mathrm{E}\{\zeta^T(t)\Phi\zeta(t)\}\le-\gamma\mathrm{E}|x(t)|^2. \tag{28}$$

The obtained inequality (28) indicates that the system (9) is globally asymptotically stable in the mean square. This complete our proof. ■

Remark 1. The solutions of Theorem 1 can be obtained by solving the eigenvalue problem with respect to solution variables, which is a convex optimization problem. In this paper, we utilize Matlab's LMI Control Toolbox [34] which implements the interior-point algorithm. This algorithm is faster than classical convex optimization algorithms [29].

Remark 2. By iteratively solving the LMI given in Theorem 1 with respect to h_U for fixed h_L, and h_D, one can find the maximum upper bound of time delay h_U for guaranteeing asymptotic stability of system (6).

Remark 3. If $h_D=1$, the restriction that time delay is to be differentiable and its time-derivative is less than one is not required, which is considered in [23].

Corollary 1 For given h_L, h_U, h_D, and $L=diag\{l_1,\ l_2,\cdots,\ l_n\}$, the equilibrium of (6) is globally asymptotically stable in the mean square if there exist positive diagonal matrices $M_i(i=1,2)$, positive definite matrices N, P_1, $Q_i(i=1,2)$, $R_i(i=1,\cdots,4)$, and any matrices $P_i,(i=2,3,\cdots,45)$ satisfying the following LMI:

$$\begin{bmatrix}\Sigma & \Xi_2^TN\\ \star & -N\end{bmatrix}<0 \tag{29}$$

where Σ, P, Ξ_2 are the same ones in Theorem 1.

4. Numerical Example

Example 1. Consider the following two-neuron neural networks

$$\dot{x}_{1(t)}=-0.8x_1(t)+0.1f_1(x_1(t-h(t))+0.3f_2(x_2(t-h(t))+5,$$
$$\dot{x}_{2(t)}=-5.3x_1(t)+0.9f_1(x_1(t-h(t))+0.1f_2(x_2(t-h(t))-3,$$
$$f_1(\alpha)=f_2(\alpha)=sin(\alpha). \tag{30}$$

From system (30), we can obtain the following system matrices:

$$A=\begin{bmatrix}0.8 & 0\\ 0 & 5.3\end{bmatrix},\ W_0=\begin{bmatrix}0 & 0\\ 0 & 0\end{bmatrix},\ W_1=\begin{bmatrix}0.1 & 0.3\\ 0.9 & 0.1\end{bmatrix},\ L=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$$
$$D=E=E_0=E_1=\begin{bmatrix}0 & 0\\ 0 & 0\end{bmatrix} \tag{31}$$

By applying Corollary 1 to the system (30), one can obtain the delay bounds for guaranteeing the asymptotic stability of (30) for different values of h_D as listed in Table 2. From Table 2, Corollary 1 provides much larger delay bounds than those of [13].

Example 2. Consider the uncertain stochastic neural networks (9) with

$$A=\begin{bmatrix}4 & 0\\ 0 & 5\end{bmatrix},\ W_0=\begin{bmatrix}0.4 & -0.7\\ 0.1 & 0\end{bmatrix},\ W_1=\begin{bmatrix}-0.2 & 0.6\\ 0.5 & -0.1\end{bmatrix},$$
$$H_0=\begin{bmatrix}0.5 & 0\\ 0 & 0.5\end{bmatrix},\ H_1=\begin{bmatrix}0.4 & -0.7\\ 0.1 & 0\end{bmatrix},$$
$$L=0.5I,\ D=[0.1\ \ -0.1]^T,$$
$$E=[0.2\ \ 0.3],\ E_0=[0.2\ \ -0.3],\ E_1=[-0.2\ \ -0.3],$$

By applying Theorem 1 to the above system, the maximum delay bounds of $h(t)$ with the condition $h_D=1$ for different h_L are shown in Table 1. From Table 2, one can ses that h_U increases as h_L does.

Example 3. Consider the following uncertain stochastic neural networks in [23]:

$$dx(t)=[(-A+\Delta A(t)x(t)+(W_0+\Delta W_0(t))g(x(t))$$
$$+(W_1+\Delta W_1(t))g(x(t-h(t)))]dt$$
$$+[H_0x(t)+H_1x(t-h(t))]dw(t)$$

where

$$A=\begin{bmatrix}4.5 & 0 & 0\\ 0 & 5.2 & 0\\ 0 & 0 & 3.6\end{bmatrix},\ W_0=\begin{bmatrix}-1 & 0.4 & -0.5\\ 0 & -0.7 & 0.7\\ 0.2 & 0.6 & 0.8\end{bmatrix},\ W_1=\begin{bmatrix}0.5 & 0.7 & 1.1\\ -0.1 & 0.4 & 0\\ 0 & -0.2 & -0.8\end{bmatrix},$$
$$H_0=\begin{bmatrix}1.2 & 0.4 & -0.8\\ -1.5 & -1.8 & 0.9\\ 0.5 & 1.1 & 2.1\end{bmatrix},\ H_1=\begin{bmatrix}0.2 & 0.1 & -0.4\\ 0 & 0.2 & 0.5\\ 0.6 & 0 & 0\end{bmatrix},$$
$$L=0.4I,\ D=[0.1\ \ 0\ \ 0.2]^T,$$
$$E=[0.4\ 0.1\ 0.2],\ E_0=[-0.2\ 0.2\ 0.1],\ E_1=[0.2\ -0.2\ 0.1],$$

In [24], the obtained maximum allowable delay bounds with $h_D=1$ was 0.0664. However, by applying Theorem 1

표 1 예제 1에서 $h_L=0$일 때 다양한 h_D값에 따른 안정성을 보장하는 h_U의 상한 값

Table 1 Delay bounds h_U with $h_L=0$ and different values of h_D (Example 1).

	$h_D=0$	$h_D \le 0.8$	$h_D=0.9$	$h_D \ge 1$
Chen et al [13].	0.058	not applicable	not applicable	not applicable
Corollary 1	∞	∞	9.9483	2.0128

표 2 예제 2에서 일 때 다양한 $h_L=0$값에 따른 안정성을 보장하는 h_U의 상한 값

Table 2 Delay bounds h_U with different h_L (Example 2).

h_L	0	0.1	0.3	0.5
Huang and Feng [24].	0.4109	not applicable	not applicable	not applicable
Theorem 1	0.6519	0.7512	0.9475	1.1475

to the above system, one can obtain the maximum allowable delay bounds with $h_D=1$ is 0.0833. This shows our criterion gives a larger delay bound than the result in [24].

5. Conclusion

In this paper, the problem of the global asymptotic stability for uncertain stochastic neural networks with interval time varying delays is considered. To obtain a less conservative result, a Lyapounv-krasovskii functional of descriptor form, which includes three zero equations, is utilized by combining with the LMI framework for obtaining the stability criterion of the neural network (1). Through three numerical example, the effectiveness of the proposed stability criterion is successfully verified.

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Appendix

$\Sigma=(\Sigma_{(i,j)}),\ i,\ j=1,\cdots,11,$

$\Sigma_{(1,1)}=-P_2^TA-A^TP_2+P_{13}^T+P_{13}+R_1+R_2+R_3+L^TM_1L,$

$\Sigma_{(1,2)}=-P_{13}^T+P_{24}^T-A^TP_3+P_{14},$

$\Sigma_{(1,3)}=-P_{24}^T+P_{35}^T-A^TP_4+P_{15},$

$\Sigma_{(1,4)}=-P_{35}^T-A^TP_5+P_{16},$

$\Sigma_{(1,5)}=P_1-P_2^T-A^TP_6+P_{17},$

$\Sigma_{(1,6)}=-P_{13}^T-A^TP_7+P_{18},$

$\Sigma_{(1,7)}=-P_{24}^T-A^TP_8+P_{19},$

$\Sigma_{(1,8)}=-P_{35}^T-A^TP_9+P_{20},$

$\Sigma_{(1,9)}=P_2^TW_0-A^TP_{10}+P_{21},$

$\Sigma_{(1,10)}=P_2^TW_1-A^TP_{11}+P_{22},$

$\Sigma_{(1,11)}=P_2^TD-A^TP_{12}+P_{23},$

$\Sigma_{(2,2)}=-P_{14}^T-P_{14}+P_{25}^T+P_{25}-R_1,$

$\Sigma_{(2,3)}=-P_{25}^T+P_{36}^T-P_{15}+P_{26},$

$\Sigma_{(2,4)}=-P_{36}^T-P_{16}+P_{27},$

$\Sigma_{(2,5)}=-P_3^T-P_{17}+P_{28},$

$\Sigma_{(2,6)}=-P_{14}^T-P_{18}+P_{29},$

$\Sigma_{(2,7)}=-P_{25}^T-P_{19}+P_{30},$

$\Sigma_{(2,8)}=-P_{36}^T-P_{20}+P_{31},$

$\Sigma_{(2,9)}=P_3^TW_0-P_{21}+P_{32},$

$\Sigma_{(2,10)}=P_3^TW_1-P_{22}+P_{33},$

$\Sigma_{(2,11)}=P_3^TD-P_{23}+P_{34},$

$\Sigma_{(3,3)}=-P_{26}-P_{26}^T+P_{37}+P_{37}^T-(1-h_D)R_2+L^TM_2L,$

$\Sigma_{(3,4)}=-P_{37}^T-P_{27}+P_{38},$

$\Sigma_{(3,5)}=-P_4^T-P_{28}+P_{39},$

$\Sigma_{(3,6)}=-P_{15}^T-P_{29}+P_{40},$

$\Sigma_{(3,7)}=-P_{26}^T-P_{30}+P_{41},$

$\Sigma_{(3,8)}=-P_{37}^T-P_{31}+P_{42},$

$\Sigma_{(3,9)}=P_4^TW_0-P_{32}+P_{43},$

$\Sigma_{(3,10)}=P_4^TW_1-P_{33}+P_{44},$

$\Sigma_{(3,11)}=P_4^TD-P_{34}+P_{45},$

$\Sigma_{(4,4)}=-P_{38}-P_{38}^T-R_3,$

$\Sigma_{(4,5)}=-P_5^T-P_{39},$

$\Sigma_{(4,6)}=-P_{16}^T-P_{40},$

$\Sigma_{(4,7)}=-P_{27}^T-P_{41},$

$\Sigma_{(4,8)}=-P_{38}^T-P_{42},$

$\Sigma_{(4,9)}=P_5^TW_0-P_{43},$

$\Sigma_{(4,10)}=P_5^T-P_{44},$

$\Sigma_{(4,11)}=P_5^TD-P_{45},$

$\Sigma_{(5,5)}=-P_6-P_6^T+h_L^2Q_1+(h_U-h_L)Q_2,$

$\Sigma_{(5,6)}=-P_{17}^T-P_7,$

$\Sigma_{(5,7)}=-P_{28}^T-P_8,$

$\Sigma_{(5,8)}=-P_{39}^T-P_9,$

$\Sigma_{(5,9)}=P_6^TW_0-P_{10},$

$\Sigma_{(5,10)}=P_6^TW_1-P_{11},$

$\Sigma_{(5,11)}=P_6^TD-P_{12},$

$\Sigma_{(6,6)}=-P-18-P_{18}^T-Q_1,$

$\Sigma_{(6,7)}=-P_{29}^T-P_{19},$

$\Sigma_{(6,8)}=-P_{40}^T-P_{20},$

$\Sigma_{(6,9)}=P_7^TW_0-P_{21},$

$\Sigma_{(6,10)}=P_7^TW_1-P_{22},$

$$\Sigma_{(6,11)} = P_7^T D - P_{23},$$
$$\Sigma_{(7,7)} = -P_{30} - P_{30}^T - (h_U - h_L)^{-1} Q_2,$$
$$\Sigma_{(7,8)} = -P_{41}^T - P_{31},$$
$$\Sigma_{(7,9)} = P_8^T W_0 - P_{23},$$
$$\Sigma_{(7,10)} = P_8^T W_1 - P_{23},$$
$$\Sigma_{(7,11)} = P_8^T D - P_{34},$$
$$\Sigma_{(8,8)} = -P_{42} - P_{42}^T - (h_U - h_L)^{-1} W_2,$$
$$\Sigma_{(8,9)} = P_9^T W_0 - P_{43},$$
$$\Sigma_{(8,10)} = P_9^T W_1 - P_{44},$$
$$\Sigma_{(8,11)} = P_9^T D - P_{45},$$
$$\Sigma_{(9,9)} = P_{10}^T W_0 + W_0^T P_{10} + R_4 - M_1,$$
$$\Sigma_{(9,10)} = P_{10}^T W_1 + W_0^T P_{11},$$
$$\Sigma_{(9,11)} = P_{10}^T D + W_0^T P_{12},$$
$$\Sigma_{(10,10)} = P_{11}^T W_1 + W_1^T P_{11} - (1 - h_D) R_4 - M_2,$$
$$\Sigma_{(10,11)} = P_{11}^T D + W_1^T P_{12},$$
$$\Sigma_{(11,11)} = P_{12}^T D + D^T P_{12} - N. \qquad (32)$$

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