GLFP 모형하에서의 가속수명시험 데이터 분석

김종만*†

* 명지대학교 산업경영공학과

Analyses of Accelerated Life Tests Data from General Limited Failure Population

Chong Man Kim*†

* Department of Industrial and Management Engineering, Myongji University

Key Words : Accelerated Life Tests, Expectation and Maximization Algorithm, Wear-out and Infant-mortality Failures, General Limited Failure Population.

Abstract

This paper proposes a method of estimating the lifetime distribution at use condition for constant stress accelerated life tests when an infant-mortality failure mode as well as wear-out one exists. General limited failure population model is introduced to describe these failure modes. It is assumed that the log lifetime of each failure mode follows a location-scale distribution and a linear relation exists between the location parameter and the stress. An estimation procedure using the expectation and maximization algorithm is proposed. Specific formulas for Weibull distribution are obtained. An illustrative example and the simulation results are given.

1. 서 론

Accelerated life tests (ALTs) are used to obtain information on life distributions of products or parts quickly and economically. Test items are run at higher-than-usual levels of stress to induce early failures. Test data are then extrapolated to estimate the lifetime distribution at design stress in terms of a model to relate life to stress. The stress can be applied in various ways; the most common method is to test units at constant stress until all units fail or censoring time is reached.

The analyses of ALT data usually assume that the lifetime distribution at each stress comes from a prespecified parametric family of distributions such as exponential, Weibull, lognormal etc; See, for instance, Nelson and Meeker(1978) for Weibull distribution and Nelson and Kielpinski(1976) for lognormal distribution. See Nelson(1990) for detailed treatments of ALTs.

Most of previous works assume that the lifetime distribution has only one failure mode. However, some electronic devices or other system components are subject to not only wear-out but also infant-mortality failures which are attributed to the presence of randomly occurring defects in the manufacturing process. For example, failures due to pinholes, particulates and contaminants in the capacitors are infant-mortality failures. Environmental stress screening and burn-in reduce the defect-related failures. However, they can not eliminate all of

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† 교신저자 chongman@mju.ac.kr

them as Sichart and Vollertsen (1991) pointed out. Consequently, infant-mortality failures have an important effect upon the lifetime distribution at use condition. Therefore, it is necessary to consider infant-mortality failure mode as well as wear-out one. ; see, for instance, Mori et al. (1991), Prendergast et al.(1997), Martin et al. (1997), and Croes et al.(1998) for infant- mortality failure mode.

When more than one failure mode exist, a mixture of distributions has been widely used in describing the lifetimes of units. Kim and Bai (2002) used the mixed distribution in order to describe two failure modes and considered the problem of estimating the lifetime distribution at use condition for constant stress ALTs. Kim (2006) also used the mixed distribution and considered the optimum design of ALT under two failure modes.

Another model for representing the situation where two failure modes coexist is general limited failure population(GLFP) model. Chan and Meeker(1999) proposed GLFP model in which the defective units will usually lead to an infant-mortality failure early in their lifetimes and the nondefective units will eventually fail from wearout.

This paper proposes a method of estimating the lifetime distribution at use condition for constant stress ALTs when an infant-mortality failure mode as well as wear-out one exists. The GLFP model is used to describe the two failure modes. Assuming that the log lifetime of each failure mode follows a location-scale distribution and its location parameter is a linear function of stress, the maximum likelihood estimates (MLEs) of the distribution parameters and the proportion of infant-mortality failure are obtained by expectation and maximization(EM) algorithm. Section 2 describes an ALT model with wear-out and infant-mortality failure modes. EM algorithm and estimators of the lifetime distribution are presented and a numerical example is given in Section 3. Simulation results on the properties of the estimators are given in Section 4. The following notations will be used in this paper.

1.1 Notation

h	Number of stress levels.
k	Failure mode index; 1(wear-out), 2(infant-mortality).
s_j	j th stress level, $j=1,\cdots,h$.
n	Total number of test units.
n_j	Number of test units at stress s_j, $j=1,\cdots,h$.
r_j	Number of units that observed failure at stress s_j, $j=1,\cdots,h$.
ξ_j	Standardized stress, $\xi_j=\frac{s_j-s_d}{s_h-s_d}$, $j=1,\cdots,h$.
α_{0k},α_{1k}	Parameters of linear relation.
β_{0k},β_{1k}	Parameters of standardized linear relation.
μ_{jk}	Location parameter at stress s_j, $j=1,\cdots,h$.
σ_k	Scale parameter.
$F_k(\cdot)$	Location-scale cdf(cumulative distribution function).
$f_k(\cdot)$	Location-scale pdf (probability density function).
π	Proportion of population subject to infant-mortality failure mode.
Θ_k	$\{\beta_{0k},\beta_{1k},\sigma_k\}$.
Θ	$\{\phi,\Theta_1,\Theta_2\}$.
Y_{ij}	Log-lifetime of unit under stress s_j, $i=1,\cdots,n_j$, $j=1,\cdots,h$.

2. Model

2.1 Assumptions

(1) At any stress s_j, the log-lifetime of a test unit follows the GLFP model with location and scale parameters, μ_{jk} and σ_k, $k=1$

(wear-out), 2(infant-mortality).

(2) μ_{jk} is a linear function of a(possibly transformed) stress s_j; that is, $\mu_{jk}=\alpha_{0k}+\alpha_{1k}s_j$.

(3) σ_k is constant and is independent of the stress.

(4) The lifetimes of test units are independent and identically distributed.

(5) The cause of failure is not observed.

2.2 LEP and GLFP model

For some electronic components such as integrated circuits(ICs), it is known that lifetimes of non-defective units are very long and the probability of failure during the technical life of the unit is essentially 0. Such population is called a limited failure population (LFP).

The time-to-failure cdf of a unit selected at random form the LFP is

$$P(T\leqq t)=pF_T(t;\theta) \qquad (1)$$

This is a mixture distribution with a probability point mass $(1-p)$ at infinity (or at some other arbitrarily large time beyond the period of observation and interest). This model has been used in other applications with $F_T(t;\theta)$ assumed to be exponential, lognormal, or Weibull distribution by several authors. See, Meeker(1987) for more detailed treatments of LFP.

The GLFP model proposed by Chan and Meeker[1] is an extension of the LFP model. It combines components from the LFP model for infant mortality with a competing risk model for long-term wearout.

$$P(T\leqq t)=1-\overline{F_1}(t;\theta_1)\left[1-pF_2(t;\theta_2)\right] \qquad (2)$$

Therefore, the GLFP model can be a good alternative to represent the lifetime of units with two failure modes. See Chan and Meeker(1999) for more detailed treatment of GLFP model.

2.3 Lifetime distribution

From the assumption of the GLFP model, the cdf and pdf of at higher-than-usual condition are:

$$F(y_{ij};\Theta)=1-\overline{F_1}(y_{ij};\Theta_1)\left[1-\pi F_2(y_{ij};\Theta_2)\right] \qquad (3)$$

$$f(y_{ij};\Theta)=f_1(y_{ij};\Theta_1)\left[1-\pi F_2(y_{ij};\Theta_2)\right] +\pi f_2(y_{ij};\Theta_2)\overline{F_1}(y_{ij};\Theta_1) \qquad (4)$$

where $\overline{F_1}(y_{ij};\Theta_1)=1-F_1(y_{ij};\Theta_1)$.

3. Estimation with EM Algorithm

From assumption (5), the only information available is the time to failure. One reason justifying this assumption is that despite the fact that in some circumstances the cause of failure could be classified as wear-out or as infant-mortality, it would require costs and time to classify all the failures. Another justification is that in many cases it is technically difficult to find out the real source of failure. Thus the cause of failure can be regarded as a missing variable and the EM algorithm can be utilized.

The EM algorithm can obtain iterative solutions to the maximum likelihood equations in a wide class of missing data problems. On each iteration of the EM algorithm there are two steps; the expectation step (E-step) and the maximization step (M-step). In the E-step, log-likelihood including missing data is replaced by its conditional expectation given the observed data. In the M-step, MLEs of the parameters are computed which maximize the conditional expectation of the log-likelihood calculated in the expectation step. See Dempster et al.(1977) and Kim and Bai(2002) for more details of the EM algorithm.

3.1 Estimation procedure for ALT data

When the data at stress s_j consist of r_j failure times and $(n-r_j)$ censoring times out of n_j units tested, the log-likelihood becomes

$$
\begin{aligned}
\log L &= \sum_{j=1}^{h}\left[\sum_{i=1}^{r_j}\log f(y_{ij};\Theta)+(n_j-r_j)\log S(\eta_j;\Theta)\right] \\
&= \sum_{j=1}^{h}\Bigg[\sum_{j=1}^{r_j}\log\left[f_1(y_{ij};\Theta_1)\{1-\pi F_2(y_{ij};\Theta_2)\}\right. \\
&\quad \left.+\pi f_2(y_{ij};\Theta_2)\overline{F_1}(y_{ij};\Theta_1)\right] \\
&\quad +(n_j-r_j)\{\log\overline{F_1}(\eta_j;\Theta_1) \\
&\quad +\log(1-\pi F_2(\eta_j;\Theta_2))\}\Bigg]
\end{aligned} \tag{5}
$$

Let I_{ij1} and $I_{ij2}(=1-I_{ij1})$ be the indicator variables denoting whether unit i at stress s_j follows wear-out or infant-mortality failure mode, respectively. If these I_{ijk}'s were observable, then the log-likelihood of a complete data set would become

$$
\begin{aligned}
\log L_c &= \sum_{j=1}^{h}\Bigg[\sum_{i=1}^{r_j}I_{ij1}\{\log f_1(y_{ij};\Theta_1) \\
&\quad +\log(1-\pi F_2(y_{ij};\Theta_2))\}] \\
&\quad +I_{ij2}\{\log\pi+\log f_2(y_{ij};\Theta_2)+\log\overline{F_1}(y_{ij};\Theta_1)\} \\
&\quad +(n_j-r_j)\{\log\overline{F_1}(\eta_j;\Theta_1) \\
&\quad +\log(1-\pi F_2(\eta_j;\Theta_2))\}\Bigg]
\end{aligned} \tag{6}
$$

Here, I_{ijk}'s are the missing variables.

E-step : As the log-likelihood of a complete data set is linear in I_{ijk}'s, the expectation step simply requires the calculation of the conditional expectation of I_{ijk} given the observed data y_{ij}. We have

$$
\begin{aligned}
\Pr\{I_{ij1}=1|y_{ij}\} &= \frac{f_1(y_{ij};\Theta_1)\left[1-\pi F_2(y_{ij};\Theta_2)\right]\}}{f(y_{ij};\Theta)} \\
&= \tau_1(y_{ij};\Theta)
\end{aligned} \tag{7}
$$

for $i=1,\cdots,r_j$ and $j=1,\cdots,h$. The quantity $\tau_1(y_{ij};\Theta)$ $=-\tau_2(y_{ij};\Theta)$ is the posterior probability that the failure is wear-out given y_{ij}. Thus the expectation of I_{ijk} given y_{ij} on the pth iteration is

$$
E_{\Theta^{(p-1)}}(I_{ijk}|y_{ij})=\tau_k(y_{ij};\Theta^{(p-1)}) \tag{8}
$$

where $\Theta^{(p-1)}$ is the parameter set obtained on the $(p-1)$th iteration. Thus the conditional expectation of log-likelihood is

$$
\begin{aligned}
Q &= \sum_{j=1}^{h}\Bigg[\sum_{i=1}^{r_j}\tau_1\{\log f_1(y_{ij};\Theta_1) \\
&\quad +\log(1-\pi F_2(y_{ij};\Theta_2))\}] \\
&\quad +\tau_2\{\log\pi+\log f_2(y_{ij};\Theta_2)+\log\overline{F_1}(y_{ij};\Theta_1)\} \\
&\quad +(n_j-r_j)\{\log\overline{F_1}(\eta_j;\Theta_1) \\
&\quad +\log(1-\pi F_2(\eta_j;\Theta_2))\}\Bigg]
\end{aligned} \tag{9}
$$

where $Q=Q(\Theta;\Theta^{(p-1)})$ and $\tau_k=\tau_k(y_{ij};\Theta^{(p-1)})$

M-step : At the M-step of the pth iteration, the intent is to maximize $Q(\Theta;\Theta^{(p-1)})$ with respect to Θ to produce a new estimate $\widehat{\Theta^{(p)}}$ of Θ. The values $\widehat{\Theta^{(p)}}$ of Θ can be obtained by simultaneously solving maximum likelihood equations obtained from Eq. (9).

As the iteration of the expectation and the maximization steps progresses, $\hat{\Theta}$ converges to the stationary solution. If the likelihood function is unimodal, the stationary solution of the algorithm is the unique MLE 「Wu, 1983」. Even if the likelihood function is not unimodal, MLE could still be obtained by choosing the solution with the largest maximum among the local maximums 「Jiang and Kececioglu, 1992」.

3.2 Confidence intervals for Θ and t_q

The Fisher information matrix is obtained by taking expectations of negative of the second partial derivatives of the log likelihood function. Asymptotic variance-covariance matrix is the inverse of Fisher information matrix and the confidence intervals for parameters can be obtained from it.

The qth quantile of the GLFP model satisfies the following equation

$$F(t_q)=1-\overline{F_1}(t_q)(1-\pi F_2(t_q))=q \tag{10}$$

The construction of confidence interval for t_q using asymptotic variance-covariance matrix is computationally difficult and impractical since t_q of the GLFP model cannot be obtained in a closed form.

One way of solving this problem is the random walk approximation of confidence interval suggested by Murdoch(1998). The basic idea is to approximate the target confidence region by generating many uniformly distributed points within confidence region. Thus we use random work algorithm to compute the confidence interval of t_q numerically. See Murduch (1998) for more details.

3.3 A numerical example

If the lifetime T follows Weibull distribution with scale parameter λ and shape parameter δ, then the log lifetime $Y=\log T$ has an extreme value distribution with location parameter $\mu=\log\lambda$ and scale parameter $\sigma=1/\delta$.

We illustrate the estimation method in previous section with the ALT data generated from GLFP for two extreme value distributions with parameters;

$$\pi=0.2;$$
$$\beta_{01}=16,\ \beta_{11}=-6,\ \sigma_1=0.8;$$
$$\beta_{02}=12,\ \beta_{12}=-8,\ \sigma_2=0.5;$$
$$\eta_1=13,\ \eta_2=10;$$

In this example, the two-stress ALT is considered. Given $\xi_1=0.5$, $\xi_2=1.0$, $n_1=40$ and $n_2=20$. <Table 1> contains the failure and censoring times in minutes under each stress level.

<Table 1> Failure times with censoring
: Weibull case

Low Stress				High stress	
8.06	12.86	10.44	12.71	3.51	3.06
*13.00	10.56	11.89	8.03	6.84	8.97
*13.00	*13.00	11.18	11.07	9.77	9.45
12.71	*13.00	*13.00	12.99	8.54	*10.00
11.43	*13.00	12.35	12.15	3.56	9.33
10.13	12.39	9.33	10.17	3.89	1.97
11.77	*13.00	12.66	13.89	5.38	9.82
12.01	12.59	8.35	6.16	8.10	*10.00
*13.00	12.09	*13.00	12.57	9.19	*10.00
12.61	11.69	7.31	12.10	3.67	9.08

'*' denotes censored observation

Initial step : Initial estimates $\pi^{(0)}=0.5$, $\beta_{01}^{(0)}=20$, $\beta^{(0)_{11}}=-4$, $\sigma_1^{(0)}=0.9$, $\beta_{02}^{(0)}=11$, $\beta_{12}^{(0)}=-9$ and $\sigma_2^{(0)}=0.6$ are chosen.

E-step : With the initial estimates, the $\tau_k(y_{ij};\Theta^{(0)})$ can be computed for $i=1,\cdots,n_j$, $j=1,\cdots,h$ and $k=1,2$. For example,

$$\tau_1(y_{11};\Theta^{(0)})=\frac{f_1(y_{11};\Theta_1^{(0)})\left[1-\pi F_2(y_{11};\Theta_2^{(0)})\right]\}}{f(y_{11};\Theta^{(0)})}$$
$$=0.6379$$

M-step : The first partial derivatives of Eq. (9) for the extreme value distribution are given in the Appendix. With $\tau_1(y_{ij};\Theta^{(0)})$ and $\tau_2(y_{ij};\Theta^{(0)})$, $\pi^{(1)}$, $\Theta_1^{(1)}$ and $\Theta_2^{(1)}$ can be obtained by simultaneously solving maximum likelihood equations, using a numerical method such as Newton-Rapson algorithm.

Computations are iterated until the differences between $(p-1)$th and pth value of parameters are smaller than 10^{-5}. The stationary solutions

$$\hat{\pi}=0.1750,$$
$$\widehat{\beta_{01}}=15.9831,\ \widehat{\beta_{11}}=-6.3975,\ \sigma_1=1.0017,$$
$$\widehat{\beta_{02}}=12.4098,\ \widehat{\beta_{12}}=-8.8776,\ \sigma_2=0.4496,$$

are obtained after 90 iterations and the estimates of qth quantile of the lifetime distribution at use condition are

$$\widehat{t_{0.05}}=11.7489,\ \widehat{t_{0.1}}=12.1841$$

By taking expectations of negative of the second partial derivatives of the log likelihood function, we obtain the following Fisher information matrix with $n_1=40$ and $n_2=20$ and the asymptotic variance and confidence interval of each parameter.

$$I(\Theta)=\begin{pmatrix} 387.1 & -4.1 & -2.3 & 16.9 & -7.1 & -4.3 & -11.8 \\ & 35.6 & 24.2 & 0.1 & -0.9 & -0.5 & -1.8 \\ & & 18.5 & 0.4 & -0.5 & -0.4 & -1.1 \\ & & & 36.9 & 2.3 & 1.5 & 7.7 \\ & & & & 39.8 & 27.3 & 8.6 \\ & & & & & 21.1 & 6.4 \\ & & & & & & 65.3 \end{pmatrix}$$

Table 2 shows the asymptotic variances and 95% confidence intervals of the parameters.

To obtain confidence intervals for t_q with the random walk approximation, many uniformly distributed points are generated. Murduch(1998) lists the required number of points depending on the number of unknown parameters. Our model is a 7-parameter model and the required number of simulated trials is 2×10^6. The confidence intervals of t_q calculated by the random walk algorithm are

$$10.4920\leq\widehat{t_{0.05}}\leq12.7837,$$
$$11.0916\leq\widehat{t_{0.1}}\leq13.2618.$$

<Table 2> Estimation results

parameter	Asymptotic variance	Confidence Intervals	
		95% lower limit	95% upper limit
π	0.0027	0.0732	0.2768
β_{01}	0.2540	14.9953	16.9709
β_{11}	0.4899	-7.7694	-5.0256
σ_1	0.0285	0.6708	1.3326
β_{02}	0.2284	11.4731	13.3465
β_{12}	0.4309	-10.1642	-7.5910
σ_2	0.0163	0.1994	0.6998

4. SIMULATION STUDY

In this section, finite sample properties of the estimators of parameters for the GLFP model are investigated by Monte Carlo simulation. The GLFP data for Weibull distribution are obtained through the inverse transformation method. Two-stress ALTs with $\xi_1=0.5$ and $\xi_2=1.0$ were considered. 2,000 repetitions of the simulation were performed and the deviations and squared deviations of the estimates from the true value were averaged to obtain biases and MSEs for parameters.

For each test, a sample of size $n_1=400$ and $n_2=200$ from GLFP with following parameters was generated and the observations were censored at $\eta_1=13$ and $\eta_2=10$.

$$\beta_{01}=16,\ \beta_{11}=-6,\ \sigma_1=0.8,\ \beta_{12}=-8;$$
$$\pi=0.1,0.2,0.3,\ D=2,3,4,\ \sigma_2=0.5,1.5,2.0$$

where $D=\beta_{01}-\beta_{02}$.

For each set of data, the iteration began from 4 different initial points since the iterative equations in Section 3 may result in multiple solutions. The results of the simulations are shown in Table 3. One can see from the table that:

- The MSEs of parameters for infant-mortality (wear-out) failure distribution tend to decrease as π increases (decreases).

- The increase in MSEs of infant-mortality failure distribution is much larger than that of wear-out one as $\sigma_2(D)$ increases(decreases). This indicates that the estimators of infant-mortality failure distribution are more affected by the discriminations between two

<Table 3> Performance of the estimators

(1) $\sigma_2 = 0.5$

		Biases							MSEs						
D	π	π	β_{01}	β_{11}	σ_1	β_{02}	β_{12}	σ_2	π	β_{01}	β_{11}	σ_1	β_{02}	β_{12}	σ_2
2	0.1	-0.0009	-0.0029	0.0032	0.0022	-0.0148	0.0142	0.0209	0.0004	0.0178	0.0346	0.0035	0.1146	0.1912	0.0091
	0.2	0.0001	-0.0040	0.0066	0.0016	-0.0075	0.0098	0.0079	0.0004	0.0203	0.0395	0.0036	0.0359	0.0627	0.0026
	0.3	-0.0003	-0.0059	0.0087	0.0027	-0.0078	0.0103	0.0048	0.0005	0.0241	0.0470	0.0045	0.0194	0.0362	0.0015
3	0.1	0.0000	-0.0023	0.0024	0.0005	-0.0085	0.0160	0.0144	0.0002	0.0171	0.0337	0.0022	0.0740	0.1339	0.0048
	0.2	-0.0003	-0.0050	0.0078	0.0021	-0.0070	0.0105	0.0046	0.0003	0.0180	0.0355	0.0022	0.0258	0.0472	0.0017
	0.3	0.0001	-0.0049	0.0067	0.0008	-0.0051	0.0100	0.0040	0.0004	0.0226	0.0450	0.0029	0.0151	0.0294	0.0011
4	0.1	0.0002	-0.0027	0.0037	0.0002	-0.0055	0.0135	0.0128	0.0002	0.0165	0.0325	0.0018	0.0538	0.1048	0.0035
	0.2	-0.0004	-0.0049	0.0083	0.0020	-0.0035	0.0067	0.0047	0.0002	0.0180	0.0350	0.0018	0.0218	0.0415	0.0014
	0.3	0.0003	-0.0040	0.0055	0.0005	-0.0035	0.0092	0.0039	0.0003	0.0225	0.0450	0.0024	0.0132	0.0265	0.0010

(2) $\sigma_2 = 1.5$

		Biases							MSEs						
D	π	π	β_{01}	β_{11}	σ_1	β_{02}	β_{12}	σ_2	π	β_{01}	β_{11}	σ_1	β_{02}	β_{12}	σ_2
2	0.1	-0.0450	-0.0153	-0.0146	0.0189	-0.3588	0.0040	0.0542	0.0293	0.0247	0.0455	0.0122	3.8070	3.3090	0.1786
	0.2	-0.0142	-0.0101	-0.0006	0.0124	-0.0910	0.0344	0.0134	0.0054	0.0273	0.0469	0.0143	2.4295	1.1926	0.0977
	0.3	-0.0064	-0.0064	0.0040	0.0042	-0.0713	0.0457	0.0142	0.0038	0.0345	0.0555	0.0168	0.5561	0.9098	0.0243
3	0.1	-0.0127	-0.0099	0.0025	0.0116	-0.2318	0.0355	0.0221	0.0051	0.0188	0.0356	0.0055	2.3169	2.3712	0.1040
	0.2	-0.0050	-0.0081	0.0066	0.0093	-0.0793	0.0354	0.0100	0.0012	0.0228	0.0427	0.0058	0.6409	0.8552	0.0338
	0.3	-0.0016	-0.0062	0.0076	0.0040	-0.0369	0.0407	0.0100	0.0008	0.0270	0.0510	0.0069	0.3049	0.4645	0.0161
4	0.1	-0.0027	-0.0049	0.0040	0.0057	-0.1076	0.0374	0.0194	0.0007	0.0174	0.0339	0.0029	1.4397	1.8354	0.0752
	0.2	-0.0012	-0.0039	0.0050	0.0049	-0.0321	0.0269	0.0111	0.0004	0.0200	0.0396	0.0031	0.3585	0.6011	0.0218
	0.3	-0.0004	-0.0048	0.0063	0.0029	-0.0228	0.0302	0.0081	0.0004	0.0240	0.0477	0.0039	0.1953	0.3472	0.0124

(3) $\sigma_2 = 2.0$

		Biases							MSEs						
D	π	π	β_{01}	β_{11}	σ_1	β_{02}	β_{12}	σ_2	π	β_{01}	β_{11}	σ_1	β_{02}	β_{12}	σ_2
2	0.1	-0.0738	-0.0109	-0.0214	0.0131	-0.7537	0.0026	0.0763	0.0593	0.0255	0.0462	0.0097	9.5522	5.499	0.2622
	0.2	-0.0323	-0.0062	-0.0220	0.0134	-0.3647	0.0111	0.0281	0.0187	0.0281	0.0521	0.0156	3.4201	2.6068	0.1106
	0.3	-0.0205	-0.0006	-0.0162	0.0060	-0.1554	0.0395	0.0201	0.0125	0.0355	0.0581	0.0210	1.6024	1.4166	0.0640
3	0.1	-0.0329	-0.0136	-0.0034	0.0147	-0.5721	-0.0159	0.0299	0.0179	0.0208	0.0376	0.0070	6.3566	4.7194	0.1894
	0.2	-0.0160	-0.0112	0.0019	0.0140	-0.2171	0.0290	0.0071	0.0055	0.0246	0.0441	0.0086	1.8193	1.9583	0.0773
	0.3	-0.0062	-0.0078	0.0054	0.0055	-0.0874	0.0547	0.0124	0.0023	0.0302	0.0534	0.0108	0.8757	1.1162	0.0402
4	0.1	-0.0088	-0.0083	0.0032	0.0100	-0.2848	0.0360	0.0187	0.0023	0.018	0.0346	0.0040	3.6674	4.1136	0.1444
	0.2	-0.0038	-0.0064	0.0056	0.0080	-0.0888	0.0384	0.0099	0.0007	0.0211	0.0410	0.0044	0.9933	1.4478	0.0525
	0.3	-0.0015	-0.0061	0.0070	0.0041	-0.0467	0.0492	0.0117	0.0006	0.0255	0.0496	0.0056	0.4901	0.7802	0.0271

failure modes than those of wear-out one.

5. CONCLUSIONS

We have proposed a method of estimating the lifetime distribution from the data obtained at higher-than-use condition when an infant-mortality failure mode as well as wear-out one exists. The GLFP model is introduced to describe two failure modes and EM algorithm is used to estimate the parameters of the lifetime distributions and the mixing proportion simultaneously. Although we used Weibull distributions to demonstrate the method, it can be used for any location-scale distributions. Monte Carlo simulations show that proportion of population subject to infant-mortality failure and discriminations between wear-out and infant-mortality failure modes have an important effect upon the estimators.

The accuracy of estimates greatly depends on stress levels and proportion of units tested at each stress level. Therefore, the problem of optimally designing ALTs when the lifetime follows GLFP model can be considered. The analyses of step-stress ALT data under GLFP model can be also considered.

6. APPENDIX

When the log lifetime of each failure mode follows an extreme value distribution, the first partial derivatives of $Q(\Theta;\Theta^{(p-1)})$ with respect to Θ are

$$\frac{\partial Q}{\partial \pi}=\sum_{j=1}^{h}\left\{\sum_{i=1}^{r_j}\tau_1\left(\frac{-F_2(y_{ij})}{1-\pi F_2(y_{ij})}\right)+\frac{\tau_2}{\pi}\right.$$
$$\left.-(n_j-r_j)\frac{f_2(\eta_j)}{1-\pi F_2(\eta_j)}\right\} \tag{A.1}$$

$$\frac{\partial Q}{\partial \beta_{lk}}=\sum_{j=1}^{h}\left\{\sum_{i=1}^{r_j}\tau_k\frac{\left(-1+e^{z_k(y_{ij})}\right)}{\sigma_k}\right.$$
$$+(1-\tau_k)\frac{\pi^{k-1}f_k(y_{ij})}{1-\pi^{k-1}F_k(y_{ij})}$$
$$\left.+(n_j-r_j)\frac{\pi^{k-1}f_k(\eta_j)}{1-\pi^{k-1}F_k(\eta_j)}\right\}\xi_j^l \tag{A.2}$$

$$\frac{\partial Q}{\partial \sigma_k}=\sum_{j=1}^{h}\left\{\sum_{i=1}^{r_j}\tau_k\frac{\left(-1-z_k(y_{ij})+z_k(y_{ij})e^{z_k(y_{ij})}\right)}{\sigma_k}\right.$$
$$+(1-\tau_k)\frac{\pi^{k-1}z_k(y_{ij})f_k(y_{ij})}{1-\pi^{k-1}F_k(y_{ij})}$$
$$\left.+(n_j-r_j)\frac{\pi^{k-1}z_k(\eta_j)f_k(\eta_j)}{1-\pi^{k-1}F_k(\eta_j)}\right\} \tag{A.3}$$

where $Q=Q(\Theta;\Theta^{(p-1)})$, $\tau_k=\tau_k(y_{ij};\Theta^{(p-1)})$ $z_k(y_{ij})=\frac{y_{ij}-(\beta_{0k}+\beta_{1k}\xi_j)}{\sigma_k}$, $l=1,2$, $k=1,2$.

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