한국해양공학회지 제22권 5호, pp 1-6, 2008년 10월 (ISSN 1225-0767)

Kelvin Ship Wake Modification due to Wind Waves

KWI-JOO LEE*, I.V. SHUGAN * AND JUNG-SUN AN*
*Department of Naval Architecture and Ocean Engineering, Chosun University, Gwangju , Korea

KEY WORDS: Kinematics model, Ship wake, Surface waves, Angle of the wake, Kelvin model

ABSTRACT: *A kinematics model of a ship wake in the presence of surface waves generated by wind is presented. It was found that a stationary wave structure behind a ship covered a wedge region with the angle at the top of the wake and that only divergent waves were present in a ship wake instead of both the longitudinal and cross-waves, which are known as the Kelvin model. Ship motion at some angle to wind waves can cause an essential asymmetry of the wake, compressing its windward half.*

1.Introduction

A moving ship (or any other object moving at or near the water surface) generates a kind of trace on the water surface, which is called a wake. Around and directly behind the ship, the wake is rather complex, with so-called bow and stern waves, eddies and currents, and foam. It depends on the actual shape of the ship, the ship's screws, and the ship speed, among other factors. From about 3 ship lengths behind the ship, the main features of the ship wake are rather universal and do not depend much on the shape of the ship or the screws. Here, the ship wake is a combination of two different phenomena:

1. the turbulent wake, i.e., foam, turbulent water and sometimes surface films in the ship's track.
2. the Kelvin wake , i.e. a characteristic wave pattern behind the ship (named after Lord Kelvin who first explained the physics of ship wakes in 1887 by Kelvin, 1910).

The wave pattern of the Kelvin wake consists of two kinds of waves: transverse waves (crests across the ship's track) and divergent waves (crests roughly parallel to the ship's track, moving outward). They are confined to a wedge-shaped region behind the ship, and the half angle of that wedge is 19.5°, independent of the ship's speed, as long as the deep water condition is satisfied. For a typical ship speed of 10(20knots), "deep" as defined above means a depth of more than 5.6. Since the water depth along shipping lanes in the sea is usually 10m or more, the water can usually be considered deep for most ships except speedboats. The length of the longest waves in the Kelvin wake is between about 15 and 60 for typical ship speeds between 5 and 10/m/s The wave's structure behind the ship is presented in Fig. 1.

교신저자 안정선: 광주광역시 동구 서석동 371번지
062-230-7882 running79@naver.com

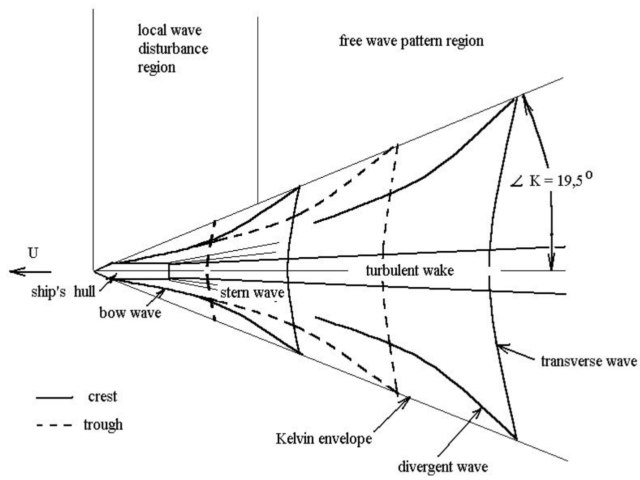

Fig. 1 Scheme of ship wake

Remote sensing diagnostics of the sea surface nevertheless evidently show an essential difference of measuring parameters of the ship wake from the existing theoretical values by Shemdin (1990), Reed and Milgram (2002), Melsheimer et al. (1999). In the first hand it is the more narrow or sometimes more wide envelope angle than Kelvin one.

Another particularity is connected with asymmetry of observed form of ship wake several additional mechanisms for forming of the ship waves were investigated recently, such as redistribution of the surface organic film inside the wake, nonlinear wave's interactions and Bragg wave's modulation by Pelinovsky and Talipova (1990), Brown et al. (1989). One more of the possible reasons for such variability in a ship wave's structure are that classical solution (Kelvin, 1910) considered the structure of ship wake in the calm water. But the ship wave's structure in the presence of external sea surface wave field can be strongly different from the classical one and possess of some conceptually new properties.

Therefore one of the main problems to be considered here is a ship wake and subsurface currents behind the ship in the presence of external surface waves field. Directional distribution of surface waves inside the wake will be considered on the basis of general theory of wave's propagation in the moving non-stationary media. Detailed analysis of ship waves will permit to determine the structure of subsurface currents behind the ship, its influence on bubbles motion and cavitations dynamic characteristics.

2. Statement of the Problem

We'll consider the ship waves dynamics in the moving frame of reference with the ship source at its origin (P), and uniform liquid flow with constant velocity U in the right direction along the X axes (see Fig. 2).

Main purpose of the present investigation is to analyze the stationary waves structure behind the ship in the presence of external free surface wind wave's pattern. Kinematical approach for a ship wave's motion in calm water first was suggested by Lighthill et al. (1955).

Dispersion relation for surface gravity waves includes a Doppler shift in this case and has the form:

$$\omega = \vec{U}\cdot\vec{k} + \sqrt{g|\vec{k}|} \tag{1}$$

Stationary picture of surface waves takes place for zero value of frequency $\omega = 0$. One can easily show from (1) that ship velocity projection on the direction of the wave propagation has to be equal to phase velocity of surface waves:

$$U\cos\psi = \sqrt{g/|\vec{k}|} \tag{2}$$

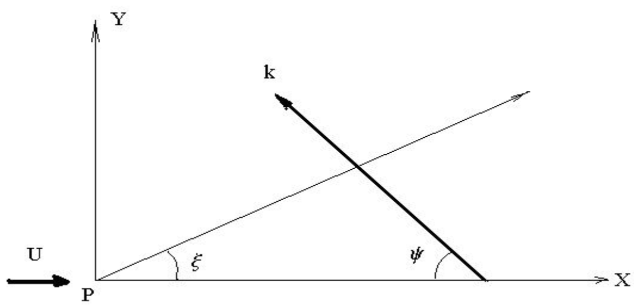

Fig. 2 Geometry of Waves Behind the Ship.

where (k,ψ)- polar coordinates of wave number vector, see Fig. 2.

Kinematics of wave propagation is governed by conservation equation for wave number by Lighthill et al. (1955):

$$\frac{\partial\vec{k}}{\partial t} + \nabla\omega = 0 \tag{3}$$

and consistency relation for phase function:

$$\frac{\partial k_1}{\partial Y} = \frac{\partial k_2}{\partial X} \tag{4}$$

Equation (3) is identically valid for the stationary problem and zero frequency of surface wave field.

So, finally we have to solve the pair of equations (2),(4) for two unknown functions (k_1,k_2). Dependence $k_1 = f(k_2)$ is known from the relation (2) and so the quasi-linear equation (4) can be analyzed by the method of characteristics. Let us rewrite it as equation for k_2 function:

$$\frac{\partial k_2}{\partial X} = f'(k_2)\frac{\partial k_2}{\partial Y} \tag{5}$$

Characteristics system for this equation has the form:

$$\frac{dY}{dX} = -f'(k_2) \qquad \frac{dk_2}{dX} = 0 \tag{6}$$

For the point source P we have the central wave, characteristics are straight rays with the constant wave number along the ray:

$$\frac{dY}{dX} = -f'(k_2) \qquad \frac{dk_2}{dX} = 0 \tag{7}$$

By using relation (1) function $f'(k_2)$ can be easily calculated:

$$\frac{df}{dk_2} = -\frac{\partial\omega}{\partial k_2}\Big/\frac{\partial\omega}{\partial k_1} \tag{8}$$

Substituting (2) and (8) into first of the expressions (6) gives following equation for the ray directions:

$$tg\xi = \frac{Y}{X} = \frac{-k_2/k_1}{1+2(k_2/k_1)^2} \tag{9}$$

where ξ is the angle between the ray and the X-axis (see Fig. 2). Phase synchronism condition (2) and equation for rays direction (9) defines the signs of wave-number components $k_1 < 0, k_2 > 0$ for $X > 0, Y > 0$. Picture of waves is symmetric with respect to X axes, so we'll consider only the region $X > 0, Y > 0$. Function $\zeta = \zeta(-k_2/k_1)$, defined by (9) has maximum, equal to $\xi_m = 19.5^\circ$. Every crest line has a cusp point at this critical ray, dividing wave system into divergent $(-k_2/k_1 < 1/\sqrt{2})$ and transverse $(-k_2/k_1 > 1/\sqrt{2})$ parts. Typical crest curve is presented at Fig. 3, and common wave's scheme in Fig. 1.

So, main results of Kelvin solution for the ship wake are the following:

- Wave number vector function is constant at every ray starting from the source point P (Fig. 2);
- Two system of waves are presented inside the ship wake: transverse waves (crests across the ship's track) and divergent waves (crests roughly parallel to the ship's track, moving outward). They are confined to a wedge-shaped region behind the ship, and the half angle of that wedge is 19.5°. Angle geometry of wake does not depend from the ship speed and determined only by dispersive properties of the surface gravity waves. Waves picture is symmetric with respect to the direction of ship motion.

We'll begin consideration with the simplest, but very important case of co propagation of the ship and surface wind waves along the X-axis in the phase synchronism conditions:

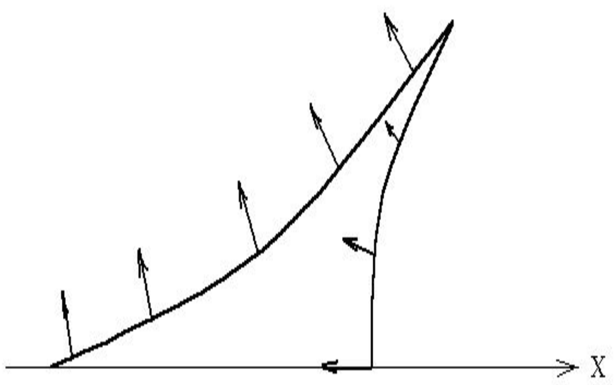

Fig. 3 Crest lines of waves in the ship trace

3. One-Dimensional Interaction of Ship Wake and External Surface Wave's Field

$$\vec{U} = \sqrt{\frac{g}{|k_{10}|}}\frac{\overrightarrow{k_{10}}}{|k_{10}|} \tag{10}$$

,where $\overrightarrow{k_{10}}$ - wave-number of the external wave field. Constant wave-number field $(k_{10}, k_{20} = 0)$ satisfied both kinematics equations (2), (5) with equal to zero wave frequency in the chosen moving frame of reference:

$$Uk_{10} + \sqrt{g|k_{10}|} = 0 \tag{11}$$

But continuous solution for wave-number function, including both ship wake region and external field is impossible due to different values of wave-number vector along the critical rays $\zeta = \zeta_m$. Weak solution of the problem, including breaks of functions will be constructed here. Usual structure for such a solutions has two or more regions of continuous solution, joined by break lines; functions have a jump across these lines, nevertheless the integral physical conservation laws are still valid there and give the connection between values before and after the break line. It is well known, that for quasi-linear differential equation of the first order like (5) exist an infinite number of conservation laws and special consideration has to be given for making a choice between them. In our case equation (5) has a clear physical sense: it is the consequence condition, necessary for the phase of wave motion to exist the phase of wave motion. Keeping in mind the common definition of wave-number function:

$$\vec{k} = \nabla\theta,$$

where θ is the phase of wave motion, we'll assume the integral conservation law, expressing the continuity of phase function along the break line with direction D:

$$[k_1] + D[k_2] = 0, \tag{12}$$

where square brackets [ ] mean the jump of function before and after the break: $[k_i] = k_i - k_{i0}, i = 1,2$. Direction of break line D can be expressed from (2), (11), (12) as a pure function of wave-number components ratio $(-k_2/k_1)$ immediately after the jump:

$$D = -\frac{k_1}{k_2}\left(1 - \frac{1}{(1+(k_2/k_1)^2)^{1/2}}\right) \tag{13}$$

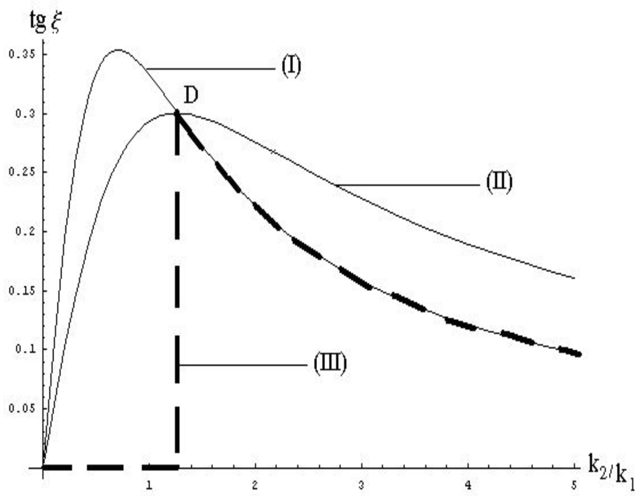

Fig. 4 Graphical solution of the equation (14). Curve (I) represents solution for ship wake in calm water; curve (II) possible directions of the break line; curve (III) combined weak solution. Intersection point D defines the unique position of the break line.

Displacement of the break line D is found by solving the consequence equation, following from (9) and (13):

$$-\frac{k_1}{k_2}\left(1-\frac{1}{(1+(k_2/k_1)^2)^{1/2}}\right) =-\frac{k_2}{k_1}\frac{1}{1+2(k_2/k_1)^2} \tag{14}$$

Graphical solution of equation (14) is presented in Fig. 4. Most significant result here is that the break line, defined by direction D:

$$\xi_0=\mathrm{arc}tgD\approx 16.9^\circ \tag{15}$$

reduces the maximal angle of ship wave's structure ($\xi_m=19.5^\circ$). Another most important particularity of weak solution is, that longitudinal part of ship waves is eliminated eliminates longitudinal part of ship waves only cross waves behind the ship are presented by high frequency part of the solution (see curve (III), Fig. 4) to

the rewritten of the intersection point D.

Final combined solution (dashed line (III) at Fig. 4) includes the external surface wave's field of zero frequency (11), breaking line, defined by direction ξ_0 and relatively high frequency cross-waves inside the ship wake.

4. Two-Dimensional Interaction of Ship Wake and External Surface Wave's Field

Let us consider next the stationary surface waves, which have some nonlinear angle of propagation to the axis of ship movement. Observer on the ship will see stationary surface picture of constant wave-number field $\vec{k}_{10}=(k_{10},k_{20})$ for waves of zero frequency:

$$Uk_{10}+\sqrt{g|\vec{k}_{10}|}=0 \tag{16}$$

It can be emphasized, that the wave's picture can not be symmetrical in this case due to nonzero value of vertical wave number component k_{20}.

Continuous solution for the wave-number function, including both ship wake angle region and external field is impossible due to different values of wave-number vector along the critical rays $\xi=\xi_m$ at the boundaries of ship wake. Weak solution of the problem, including breaks of functions will be constructed here. We'll assume the validity of first integral conservation law, expressing the continuity of phase function along the break line with direction D:

$$[k_1]+D[k_2]=0 \tag{17}$$

where square brackets [ ] mean the jump of function before and after the break: $[k_i]+k_i-k_{i0}, i=1,2$. Direction of break line D can be expressed from (16)-(17) as a function of external wave-number angle $tg\psi_0=k_{20}/k_{10}$ and wave number angle immediately after the jump $tg\psi=k_2/k_1$:

$$D=-\frac{(1+tg\psi^2)^{1/2}-(1+tg\psi_0{}^2)^{1/2}}{tg\psi(1+tg\psi^2)^{1/2}-tg\psi_0(1+tg\psi_0{}^2)^{1/2}} \tag{18}$$

Displacement of the break line D is found by solving the consequence equation, expressed the fact that the value of wave-number function at the shock ray (18) must be equal to the corresponding value of continuous solution of (7), (9) in the same direction:

$$-\frac{(1+tg\psi^2)^{1/2}-(1+tg\psi_0{}^2)^{1/2}}{tg\psi(1+tg\psi^2)^{1/2}-tg\psi_0(1+tg\psi_0{}^2)^{1/2}} =-\frac{tg\psi}{1+2tg\psi^2} \tag{19}$$

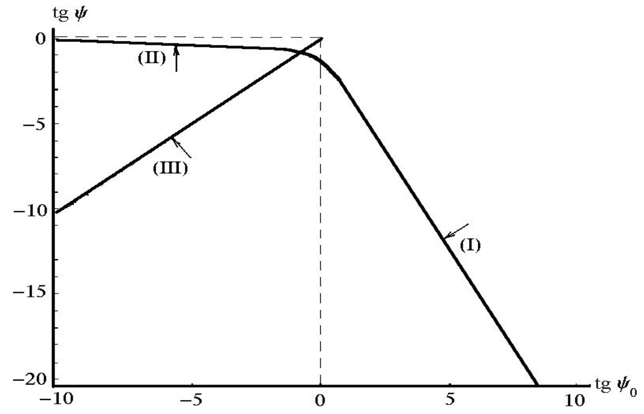

Fig. 5 Graphical representation of function $tg\psi = f(tg\psi_0)$

For every direction of external waves field $tg\psi_0$ the relation (19) implicitly determines the direction of wave number immediately after the jump $tg\psi$ and correspondingly the possible directions of break line D. Graphical representation of solution of the equation (19) is presented in Fig. 5.

Curves (I) and (II) in Fig. 5 represent the possible directions of the strong break lines of wave number function for downward $k_{20} < 0$ and upward $k_{20} > 0$ directions of external wave field, correspondingly. Curve (III) expresses the solution of (19): $tg\psi = tg\psi_0$ which evidently indicates the possibility of smooth transition from internal to external wave's field.

Now we note that if $tg\psi, tg\psi_0$, D satisfied equations (16)-(19), then $-tg\psi_0, -tg\psi_0$, -D will also be the solution of these equations. So, for modeling the common schematic of waves behind the ship it is sufficient to consider only the upper semi plate $\xi > 0$ and various directions of external surface waves field $tg\psi_0$. For downward external waves $k_{20} < 0$ strong break line (corresponding to points of the curve (I), Fig. 5) will be placed inside the wake, cut the transverse part of Kelvin waves and provide the transition from divergent waves inside the wake to external field. Upward external wave field can be smoothly joined with Kelvin solution by using the solution represented at curve (III), Fig. 5 and both transverse and divergent waves will be presented in the ship wake for such a case. On the basis of described solution one can easily construct the common schematic of the ship wake in the presence external wave field. A principal schematic of the common solution is presented in Fig. 6.

Interaction of the ship wake with free surface waves under phase synchronism conditions (16) will cut some part of ship waves directed oppositely to the wave's field and correspondingly the angle of this region of ship wake will be narrower than the classical one. "Shadow" half of ship wake roughly will keep its initial structure and still consist of divergent and transverse systems of waves.

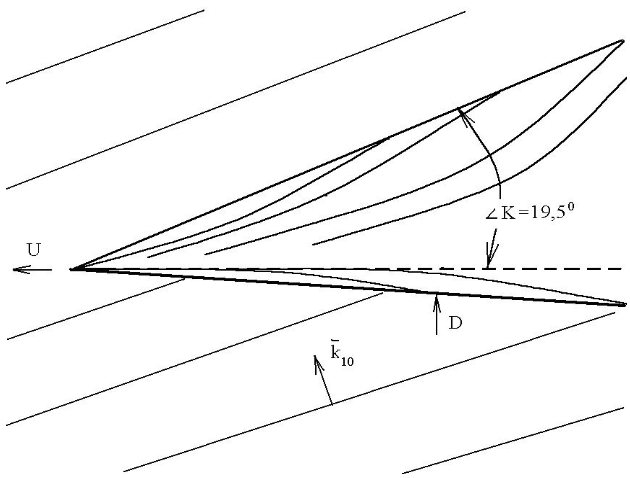

Fig. 6 Structure of ship wake under external surface waves field

5. Conclusions

Shows kinematical model of the ship wake in presence of external wind surface waves clearly shows the possibility of changing the structure of classical Kelvin solution. Ship wake can be narrower and loses its symmetric property. Analysis is carried out for linear or weakly nonlinear surface wave field. The above mentioned effects are visible in the far-field of ship wake for essentially developed wind waves of the same order as divergent ship waves.

More precisely speaking, the region of applicability for the present model can be estimated by a series of the corresponding experiments in laboratory or under real conditions.

References

Brown, E.D., Buchsbaum, S.B., Hall, R.E., Penhune, J.P., Schmitt, K.F., Watson, K.M. & Wyatt, D.C. (1989). "Observation of a Nonlinear Solitary Wave Packet in the Kelvin Wake of a Ship". J. Fluid Mech., Vol 204, pp 263- 293.

Melsheimer C., Lim H. and Shen. C. (1999). "Observation and Analysis of Ship Wakes in ERS-SAR and SPOT

Images", in: Proceedings of the 20th Asian Conference on Remote Sensing, Hong Kong, China, pp 554-559.

Kelvin, W.T. (1910). "Deep Sea Ship-waves", Mathematical and Physical Papers, Cambridge, Vol 4, pp 303-306.

Lighthill M.J. and Whitham G.B (1955). On Kinematic Waves: I. Flood Movement in Long Rivers; II. Theory of Traffic Flow on Long Crowded Roads, Proc. Roy. Soc. A., Vol 229, pp 281-345.

Pelinovsky E.H. and Talipova T.G. (1990). Films of Surface Active Materials on the Sea Surface. Preprint 219. Institute of Heat, Ac. Sc. USSR, (Siberian Department) (Russian).

Reed A. and Milgram J. (2002). "Ship Wakes and Their Radar Images". Annu. Rev. Fluid Mech. Vol 34, pp 469-502.

Shemdin O.H. (1990). "Synthetic Aperture Radar Imaging of Ship Wakes in the Gulf of Alaska. Journal of Geophysical Research", Vol 95, C, pp 16319-16338.

2008년 5월 25일 원고 접수

2008년 6월 23일 최종 수정본 채택