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Hybrid Adaptive Volterra Filter Robust to Nonlinear Distortion

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Abstract

In this paper, the new hybrid adaptive Volterra filter was proposed to be applied for compensating the nonlinear distortion of memoryless nonlinear systems with saturation characteristics. Through computer simulations as well as the analytical analysis, it could be shown that it is possible for both conventional Volterra filter and proposed hybrid Volterra filter, to be applied for linearizing the memoryless nonlinear system with nonlinear distortion. Also, the simulations results demonstrated that the proposed hybrid filter may have faster convergence speed and better capability of compensating the nonlinear distortion than the conventional Volterra filter.

Keywords: *Hybrid adaptive, Volterra filter, Nonlinear distortion, Convergence*

I. Introduction

The linear filters have played a very popular in the development of various signal processing techniques. The obvious advantage of linear filters is their inherent simplicity. Design, analysis, and implementation of such filters are relatively straightforward tasks in many applications. However, for systems with high performances, the distortion problem exists due to nonlinearities of the system and decreases the performance of conventional signal processing system. For example, there are several situations in which the performance of linear filters is unacceptable. A simple but highly pervasive type of nonlinearity is the saturation-type nonlinearity. Trying to identify these types of systems using linear models can often give misleading results. Another situation where nonlinear models will do well when linear models will fail miserably is that of trying to relate two signals with non-overlapping spectral components. Therefore, a variety of workers have recognized the need for the nonlinear control system and in practicality, it may be studied in many applications [1-2]. Fortunately, with development of high-speed processor in recent, it is possible to implement the signal processing algorithms for the nonlinear system, which it have not been impossible due to its complexities and computation problem.

In general, the nonlinear filter to represent the nonlinear system, may be based on the functional series. And the characteristics of the nonlinear filter is similar to that of linear systems because the filter output is composed of linear combination of filter coefficients. Also, the adaptive algorithms in linear systems to adapt the filter coefficients, can be applied to the nonlinear system directly, and the analysis is similar to that of the linear system. As the adaptive algorithms to adapt the filter coefficients in the nonlinear system, the least mean square (LMS) and least square (LS) algorithms *etc.* can be used. However, the nonlinear filter may have more computation complexities and slower convergence speed than those of the linear

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system because it may use more coefficients than the nonlinear system. Therefore, a variety of workers have recognized the need for the nonlinear control system with less computation complexities and faster convergence speed and it may be developed in many applications [3-7].

In this paper, the new hybrid adaptive Volterra filter was proposed to be applied for compensating the nonlinear distortion of memoryless nonlinear systems with saturation characteristics. Through computer simulations as well as the analytical analysis, it could be shown that it is possible for the proposed hybrid Volterra filter to be applied for linearizing the memory-less nonlinear system with nonlinear distortion. Also, the simulations results demonstrated that the proposed hybrid filter may have faster convergence speed and better capability of compensating the nonlinear distortion than the conventional Volterra filter.

II. Analysis of Proposed Algorithm

2.1. Conventional Algorithm

In the case of active noise control, the propagating of an acoustic wave, with an amplitude up to that corresponding to an extremely loud noise, is a very nearly linear process. However, the overall active control systems may be nonlinear due to some com-ponents of the systems. The most significant cause of nonlinearity present in an active noise control system is usually due to the loudspeaker acting as the secondary source, including amplifiers, converters, and microphones acting as the sensor, *etc.*, although with good design this nonlinearity, too, can be made small.

The linearization schemes are divided into three schemes as follows : linearization by cancellation at the output, linearization with a post-processor, and linearization with a pre-processor. The first and second schemes are not suitable for a loudspeaker application because these schemes require processing of sound signals after sound waves leave the loudspeaker. The processing of sound is not easy and not simple. Thus, the last schemes, linearization with a pre-processor can generally be used in application areas. The block diagram of the adaptive compensator of nonlinearities using a pre-processor is shown in Fig. 1. The memoryless adaptive compensator, or pre-processor $\boldsymbol{W}$ is located in front of a nonlinear system H. Here, H is assumed to be the linear combination of the nonlinear functions representing overall nonlinear system such as loudspeakers, amplifiers, microphones, converters, and *etc.* The adaptive compensator is composed of the linear combination of the nonlinear functions and the relations between inputs and outputs are as follows [8]

$$y = \sum_{i=1}^{P} w_i f_i[x] \tag{1}$$

where P is the number of the used functions and f_i, $i=1,2\cdots,P$ represents the nonlinear functions series and the Taylor series, the Fourier series *etc.* can be used for it. When the outputs of the nonlinear functions and the coefficients of the adaptive compensator can be defined as follows,

$$\boldsymbol{B}(n) = [b_1(n)\ ,\ b_2(n)\ ,\ \cdots\ ,b_P(n)]^T$$
$$\text{with } b_m(n) = f_i[x(n)], \qquad i=1,2,\cdots,P$$
$$\boldsymbol{W}(n) = [w_1(n)\ ,\ w_2(n)\ ,\ \cdots\ ,w_P(n)]^T,$$

the outputs of the adaptive compensator and the outputs of the nonlinear function H can be represented as follows, respectively

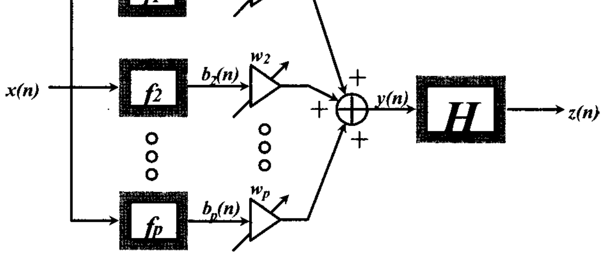

Fig. 1. The linearization scheme for a nonlinear function using the pre-processor.

$$y(n) = W^T(n)B(n) \quad (2)$$

$$z(n) = H[W^T(n)B(n)] \quad (3)$$

Also, the distortion, $e(n)$, included in $z(n)$ can be defined as the difference between the output $z(n)$ and the input $x(n)$

$$e(n) = x(n) - z(n). \quad (4)$$

The optimum coefficient value of the compensator is given by minimizing the variance of the distortion

$$\frac{\partial E[e^2(n)]}{\partial B(n)} = 0, \quad (5)$$

$$E[H'[W^T(n)B(n)]B(n)[x(n) - H[W^T(n)B(n)]]] = 0. \quad (6)$$

From Eq.(6), if the input statistics and the information of the nonlinear function H are perfectly known, the nonlinear equation seems to be solved, but in general, the solution are not given. Therefore, it is efficient to use the stochastic gradient algorithm using the input data sequence as follows [9]

$$W(n+1) = W(n) - \mu\nabla_n \quad (7)$$

where μ is a positive constant that determines the convergence and stability. ∇_n is the estimated value of the gradient vector and defined as

$$\nabla_n = \frac{\partial e^2(n)}{\partial W(n)} \quad (8a)$$

$$= -H'[W^T(n)B(n)]e(n)B(n). \quad (8b)$$

From Eq.(7) and Eq.(8), the compensation mechanism of the coefficients can be produced

$$W(n+1) = W(n) + \mu H'[W^T(n)B(n)]B(n)e(n) \quad (9)$$

where H' is the differentiation value of the nonlinear function H and the estimation process of the optimum coefficients is complex due to an estimation of the value.

Therefore, consider the following case. If the gradient function of the nonlinear function, H', is always positive under the boundary condition of the input's amplitude (a,b), or a monotonous function.

$$H'[y] > 0, \qquad a < y < b, \quad (10)$$

then, from Eq.(8), the gradient direction is determined by the $B(n)e(n)$, and a constant μ' is substituted for $\mu H'$, so the following equation can be produced

$$W'(n+1) = W'(n) + \mu' B(n)e'(n) \quad (11)$$

where $e'(n)$ is the error due to $W'(n)$. In the above equation, the function H' is not used and it does not need to estimate the nonlinear function H, so the compensation mechanism is equal to that of the general LMS algorithm. In reality, the condition of Eq.(10) is satisfied in a number of nonlinear systems as nonlinear components having the saturation characteristics, so the simplified algorithm in the above Eq.(11) can be applied to linearize the nonlinear systems.

2.1.1. The bias of the estimated coefficient values

Under the condition of Eq.(10), when $n \to \infty$, then $E[W(n)] \to W_{opt}$ and the following relationship may be produced

$$E[H'[W_{opt}^T(n)B(n)]B(n)[x(n) - H[W_{opt}^T(n)B(n)]]] = 0. \quad (12)$$

$p(x)$ is defined as the probability density function (PDF) of $x(n)$ and Eq.(12) can be reexpressed as follows

$$\int_a^b H'[W_{opt}^T B(n)]B(x(n) - H[W_{opt}^T B(n)])p(x)dx = 0. \quad (13)$$

This is equal to the required condition of Eq.(6), the stability of the adaptive algorithm from Eq.(9) can be guaranteed.

In Eq.(9), when $n \to \infty$, then $E[W'(n)] \to W_{opt}'$, and the following equations can be produced

$$E[B(n)(x(n) - H[W_{opt}^T B(n)])] = 0. \quad (14)$$

$$\int_a^b B(x(n) - H[W_{opt}'^T B])p(x)dx = 0. \quad (15)$$

Eq.(15) can be reexpressed as

$$\int_a^b H'[W_{opt}'^T B]B(x(n) - H[W_{opt}'^T B])\frac{p(x)}{H'[W_{opt}'^T B]}dx$$
$$= 0. \quad (16)$$

This is compared with Eq.(13), and W_{opt}' is considered to be the optimum coefficient when the input data has PDF as follows

$$p'(x) = \frac{\alpha p(x)}{H'[W_{opt}'^T B]} \quad (17)$$

where α is the constant. Therefore, the linearization process is applied to the data weighted by $1/H'$, and if H is not a linear function, then the simplified algorithm in Eq.(11) may have the biased results.

2.1.2. The average stability of the compensator coefficient vector

2.1.2-1. Lyapunov's indirect method for the stability of the nonlinear system

The nonlinear state space equation for the state vector z can be expressed as follows

$$z(n+1) = F[n, z(n)]. \quad (18)$$

z_{eq} is the equilibrium state and consider the linear system as follows

$$y(n+1) = Q_n y(n) \quad (19)$$

where Q_n is the Jacobian matrix defined as

$$Q_n = \frac{\partial F}{\partial z}\Big|_{z=z_{eq}} . \quad (20)$$

If the system, Q_n is exponentially asymptotically stable at z_{eq}, then the nonlinear state space equation in Eq.(18) is also exponentially asymptotically stable. Therefore, the absolute value of all eigenvalues for Q_n may be given between 0 and 1, and if the initial condition is given near z_{eq}, then the nonlinear state space equation converges to z_{eq} and stable at the equilibrium state.

2.1.2-2. The average stability of the simplified adaptive algorithm

From Eq.(11), the coefficient compensation equation can be reexpressed as

$$W'(n+1) =$$
$$W'(n) + \mu' B(n)[x(n) - H'[W'^T B(n)]] \quad (21)$$

In order to apply the Lyapunov's indirect method, the above equation can be linearized for W_{opt}' as follows

$$W'(n+1) =$$
$$W_{opt}'(n) + \mu' B(n)[x(n) - H'[W_{opt}'^T B(n)]] +$$
$$[I - \mu' H'[W_{opt}'^T B(n)]B(n)B^T(n)][W'(n) - W_{opt}'] \quad (22)$$

where I is the identity matrix. Averaging both sides and assuming that $W'(n)$ and $B(n)$ are independent, the linearized equation can be produced as follows

$$E[V'(n+1)] =$$
$$[I - \mu' E[H'[W_{opt}'^T B(n)]B(n)B^T(n)]]E[V'(n)]$$
$$+ \mu' E[B(n)[x(n) - H[W_{opt}'^T B(n)]]] \quad (23)$$

where $V(n)$ is the difference between the coefficient vector and the convergence state, defined as

$$V(n) = W'(n) - W_{opt}.$$

In Eq.(4-23), all eigenvalues of $\mu' E[H'[W_{opt}'^T B(n)] B(n)B^T(n)]$ must have values between 0 and 2 in order to be stable. In the case that the gradient of H satisfy the following condition,

$$H'[y] > 0, \qquad a < y < b \quad (24)$$

if the boundary of the convergence parameter μ' is as follows, then the average coefficient in Eq.(11) and Eq.(23), can be stable at W_{opt}'

$$0 < \mu' < \frac{2}{H'_{max}\lambda_{max}} \tag{25}$$

where H'_{max} is the largest gradient of H and λ_{max} is the largest eigenvalue of $E[B(n)B^T(n)]$. Therefore, if the gradient function of the nonlinear function H is a monotonous function under the boundary of input, then it is possible for the coefficients to converge using the simplified LMS algorithm in Eq.(11) without estimation of H .

2.2. Proposed Hybrid Algorithm

The block diagram of the adaptive compensator of nonlinearities using the proposed hybrid filter is shown in Fig. 2. The memoryless adaptive compensator, or pre-processor W is located in front of a nonlinear system H . Here, H is assumed to be the linear combination of the nonlinear functions representing overall nonlinear system such as loudspeakers, amplifiers, microphones, converters, and etc. The optimum coefficient value of the compensator is given by minimizing the variance of the distortion [8]

$$\frac{\partial E[e^2(n)]}{\partial B(n)} = 0, \tag{26}$$

where $b_m(n) = f_i[\alpha x(n) - \beta e(n)], \quad i = 1,2,\cdots,P$

Similar to the results in section A, the compensation mechanism of the coefficients can be produced as follows

$$W(n+1) = W(n) + \mu H'[W^T(n)B(n)]B(n)e(n) \tag{27}$$

where H' is the differentiation value of the nonlinear function H and the estimation process of the optimum coefficients is complex due to an estimation of the value.

Also, if the gradient function of the nonlinear function, H', is always positive under the boundary

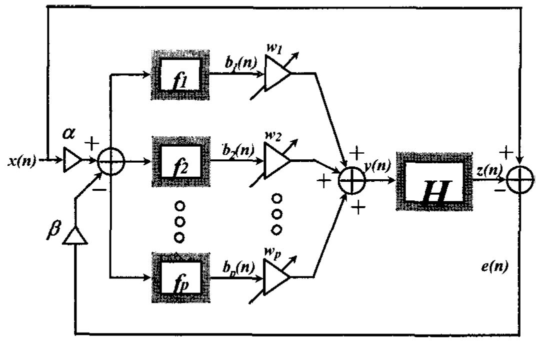

Fig. 2. The linearization scheme for a nonlinear function using the proposed hybrid filter.

condition of the input's amplitude (a,b), or a monotonous function, [9]

$$H'[y] > 0, \qquad a < y < b, \tag{28}$$

then, the gradient direction is determined by the $B(n)e(n)$, and a constant μ' is substituted for $\mu H'$, so the following equation can be produced

$$W'(n+1) = W'(n) + \mu' B(n)e'(n) \tag{29}$$

where $e'(n)$ is the error due to $W'(n)$. In the above equation (29), the function H' is not used and it does not need to estimate the nonlinear function H , so the compensation mechanism is equal to that of the general LMS algorithm. In reality, the condition of Eq.(28) is satisfied in a number of nonlinear systems as nonlinear components having the saturation characteristics, so the simplified proposed hybrid filter in the above Eq.(29) can be applied to linearize the nonlinear systems.

2.2.1. The bias of the estimated coefficient values

Similar to the results in section A-1, the linearization process is applied to the data weighted by $1/H'$, and if H is not a linear function, then the simplified proposed hybrid filter in Eq.(29) may have the biased results.

2.2.2. The average stability of the compensator coefficient vector

2.2.2-1. Lyapunov's indirect method for the stability of the nonlinear system

Similar to the results in section A−2−1, the nonlinear system may be stable at the equilibrium state.

2.2.2−2. The average stability of the simplified adaptive algorithm

Similar to the results in section A−2−2, in the case that the gradient of H satisfy the condition in Eq.(28), if the boundary of the convergence parameter μ' is as follows, then the average coefficient can be stable at W_{opt}'

$$0 < \mu' < \frac{2}{H'_{max}\lambda_{max}} \tag{30}$$

where H'_{max} is the largest gradient of H and λ_{max} is the largest eigenvalue of $E[B(n)B^T(n)]$. Therefore, if the gradient function of the nonlinear function H is a monotonous function under the boundary of input, then it is possible for the coefficients to converge using the simplified proposed hybrid filter in Eq.(29) without estimation of H.

III. Computer Simulation and Results

3.1. The Computer Simulations 1

For the computer simulations, a traveling wave tube (TWT) is selected as the nonlinear object model, which may be used in a satellite communication as a power amplifier [10]. In the TWT model, the relation−ship between an input and an output is as follows and it represents the saturation characteristics, illustrated as Fig. 3,

$$H(y) = \frac{2y}{1+y^2}. \tag{31}$$

$$y(n) = w_1(n)x(n) + w_2(n)x^3(n) \tag{32}$$

Because the TWT characteristic function is a odd function, the Taylor's series, composed of odd functions,

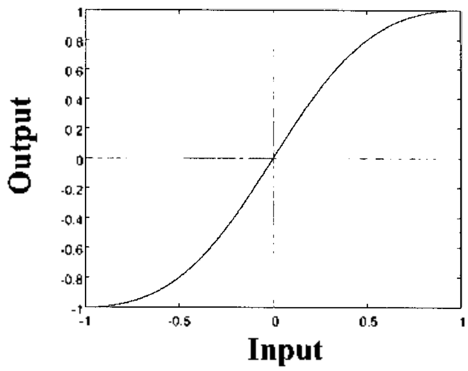

Fig. 3. The input-output characteristics of TWT.

can be used as a function of the compensator for estimation of the inverse function.

Random signals are used as input signals, which are distributed uniformly between −1 and +1.

$$p(x) = \frac{1}{2}, \qquad -1 < x < 1. \tag{33}$$

For the computer simulations, the conventional Volterra filter, from Eq.(11) and the proposed hybrid filter, from Eq.(29), are used to be compared with each other, and the results are expressed as average of the independent simulations.

Fig. 4 represents results of the nonlinear distortion attenuation in the system H, in which the conventional Volterra filter and the proposed hybrid filter are used to be compared with each other. In the figure, (a) and (b) represent the results of both the conventional Volterra filter and the proposed hybrid filter, respectively and it is verified that the latter converges faster than the former.

Fig. 5 represents the trajectories of filter coefficients for both filters, in which the conventional Volterra filter may adapt more slowly than the proposed hybrid filter. Both algorithms converge to the coefficients value of about 0.5075 and 0.1018, and 0.5040 and 0.0267, for the first−order and the third−order coefficients, respectively.

Fig. 8 represents the results of the linearization, in which the filter coefficients of the steady state are used when both algorithms converge to the optimum solution sufficiently. In the figure, (a) represents the

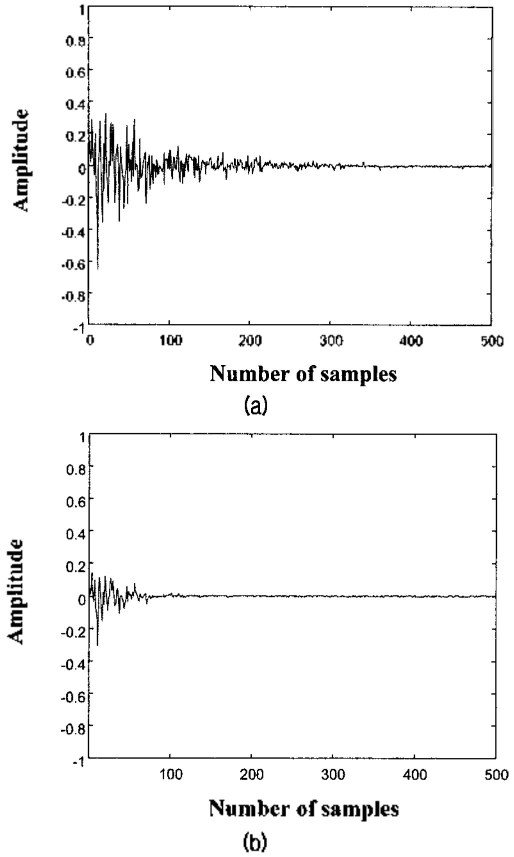

Fig. 4. The mean sample error signal of nonlinear distortion attenuation using (a) the conventional Volterra filter (b) the proposed hybrid filter.

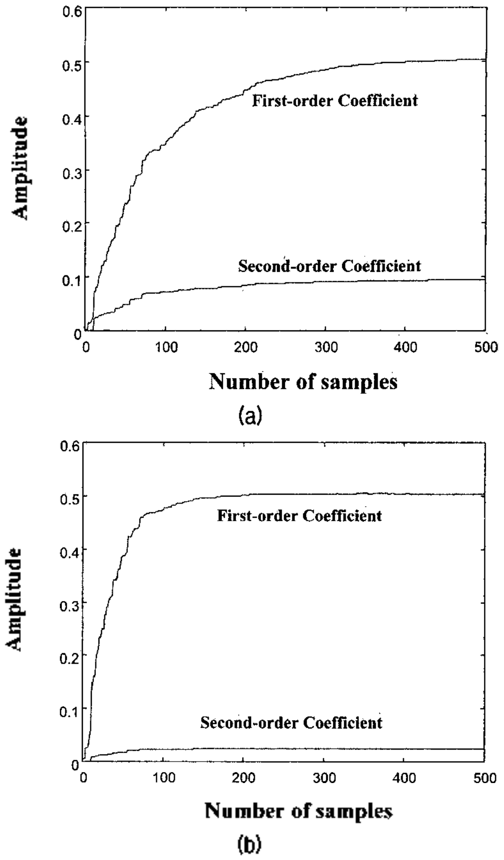

Fig. 5. The mean trajectories of the filter coefficients using (a) the conventional Volterra filter (b) the proposed hybrid filter.

characteristics of H before compensation and (b) and (c) represents the characteristics after compensation by the conventional Volterra filter and the proposed hybrid filter, respectively, and it is verified that both filters can compensate the nonlinear distortion to linearize the nonlinear system with a little difference. Therefore, the simulations results demonstrated that the proposed hybrid filter may have faster convergence speed and better capability of compensating the non-linear distortion than the conventional Volterra filter.

3.2. The Computer Simulations 2

For the computer simulations, the sigmoid function is selected as the nonlinear object model, which may be used in modeling of the nonlinear system functions in many nonlinear applications. In the sigmoid function model, the relationship between an input and an output is as follows and it represents the saturation charac-teristics, illustrated as Fig. 7,

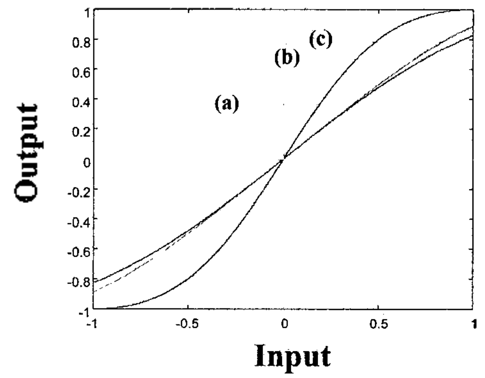

Fig. 6. The linearized TWT function after compensation. (a) no compensation (b) the conventional Volterra filter (c) the proposed hybrid filter

$$H(y) = A\frac{1-e^{-x/B}}{1+e^{-x/B}}, \qquad (34)$$

where A and B are the scaling parameters, and 1 and 0.3 are used for A and B in the simulations, respectively. Because the sigmoid characteristic function is a odd function, the Taylor's series, composed of

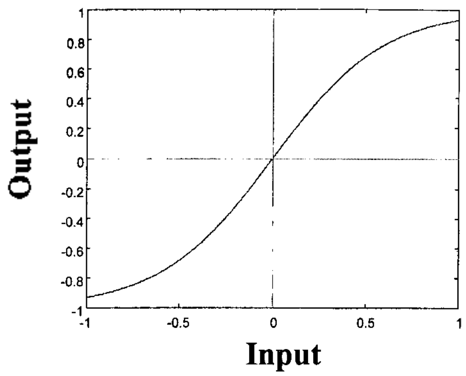

Fig. 7. The input-output characteristics of the sigmoid function.

odd functions, can be used as a function of the compensator for estimation of the inverse function.

$$y(n) = w_1(n)x(n) + w_2(n)x^3(n) \qquad (35)$$

Random signals are used as input signals, which are distributed uniformly between -1 and $+1$.

$$p(x) = \frac{1}{2}, \qquad -1 < x < 1. \qquad (36)$$

For the computer simulations, the simplified algorithm, from Eq.(11) and the proposed hybrid filter, from Eq.(29), are used to be compared with each other, and the results are expressed as average of the independent simulations.

Fig. 8 represents results of the nonlinear distortion attenuation in the system H , in which the conventional Volterra filter and the proposed hybrid filter are used to be compared with each other. In the figure, (a) and (b) represent the results of both the conventional Volterra filter and the proposed hybrid filter, respectively and it is verified that the latter converges faster than the former. Fig. 9 represents the trajectories of both algorithms, in which the proposed hybrid filter may adapt faster than the conventional Volterra filter. Both algorithmsconverge to the coefficients values of about 0.6192 and 0.1257, and 0.6070 and 0.0322, for the

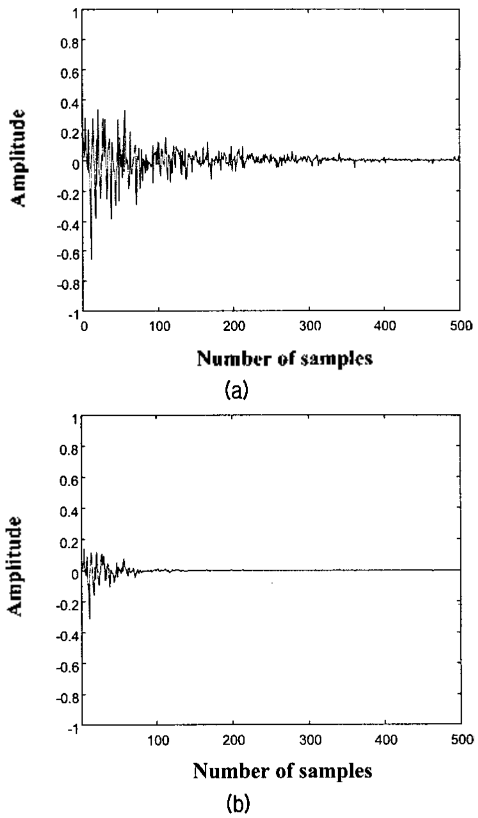

Fig. 8. The mean sample error signal of nonlinear distortion attenuation using.
(a) the conventional Volterra filter (b) the proposed hybrid filter

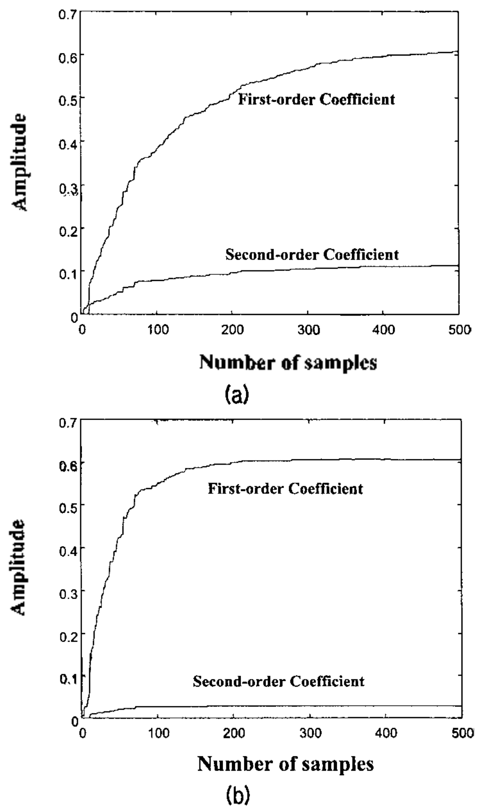

Fig. 9. The mean trajectories of the filter coefficients using.
(a) the conventional Volterra filter (b) the proposed hybrid filter

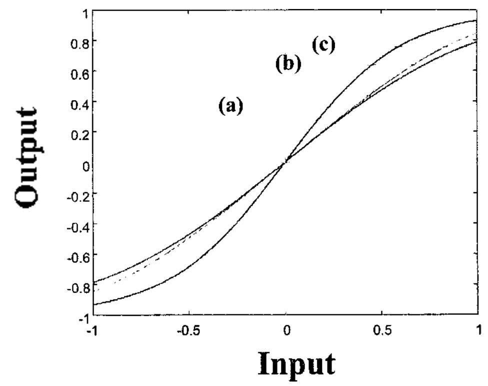

Fig. 10. The linearized function after compensation.
(a) no compensation (b) the conventional Volterra filter (c) the proposed hybrid filter

first-order and the third-order coefficients, respectively.

Fig. 10 represents the results of the linearization, in which the filter coefficients of the steady state are used when both algorithms converge to the optimum solution sufficiently. In the figure, (a) represents the characteristics of H before compensation and (b) and (c) represents the characteristics after compensation by the conventional Volterra filter and the proposed hybrid filter, respectively, and it is verified that both algorithms can compensate the nonlinear distortion to linearize the system. Also, the simulations results demonstrated that the proposed hybrid filter may have faster convergence speed and better capability of compensating the nonlinear distortion than the conventional Volterra filter.

IV. Conclusions

In this paper, the new hybrid adaptive Volterra filter was proposed to be applied for compensating the nonlinear distortion of memoryless nonlinear systems with saturation characteristics. Through computer simulations as well as the analytical analysis, it could be shown that it is possible for both conventional Volterra filter and proposed hybrid Volterra filter, to be applied for linearizing the memoryless nonlinear system with nonlinear distortion. Also, the simulations results demonstrated that the proposed hybrid filter may have faster convergence speed and better capability of compensating the nonlinear distortion than the conventional Volterra filter.

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