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Elastically-influenced instabilities in Taylor-Couette and other flows with curved streamlines: a review

Susan J. Muller*

Department of Chemical Engineering, University of California, Berkeley, Berkeley, CA 94720-1462, U.S.A.

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Abstract

Viscoelastic instabilities are of fundamental importance to understanding the physics of complex fluids and of practical importance to materials processing and fluid characterization. Significant progress has been made over the past 15 years in understanding instabilities in viscoelastic flows with curved streamlines and is reviewed here. Taylor-Couette flow, torsional flow between a cone and plate, and torsional flow between parallel plates have received special attention due to both the basic significance of these flows and their critical role in rheometry. First, we review the criteria for determining when these flows become unstable due to elasticity in the absence of inertia, and discuss the generalization of these criteria to more complex flows with curved streamlines. Then, focusing on experiments and simulations in the Taylor-Couette problem, we review how thermal sensitivity (*i.e.*, the dependence of fluid viscosity and elasticity on temperature) and inertia affect the stability of viscoelastic flows. Finally, we conclude with some general thoughts on unresolved issues and remaining challenges related to viscoelastic instabilities.

Keywords : viscoelastic instabilities, Taylor-Couette, curvilinear streamlines, thermoelastic instabilities

1. Introduction

Instabilities in flows of viscoelastic liquids are of interest for a number of reasons. The rate at which many polymer processing operations can function is limited by the onset of flow instabilities, instabilities in simple rheometric flows compromise the quality and extent to which viscoelastic material parameters may be measured, and the prediction of instabilities provides stringent tests of numerical methods for flows of complex liquids. The understanding and development of constitutive equations remains an important issue for viscoelastic liquids, and the prediction of elastically-influenced instabilities also provides a powerful means for assessing whether the appropriate physics has been included in these models. In addition, understanding the mechanism by which polymers suppress turbulence in drag reducing applications ultimately involves determining the effect of small amounts of elasticity on hydrodynamic stability. A number of important reviews of earlier work on viscoelastic instabilities are available. Petrie and Denn, in 1976, focused on fiber spinning, draw resonance, melt fracture, and other processing instabilities as well as on flow between concentric, rotating cylinders (*i.e.*, Taylor-Couette flow). Somewhat more recently, Larson (1992) presented a very comprehensive review of instabilities in a range of viscometric flows (Taylor-Couette, cone-and-plate, plate-and-plate, and parallel shear flows) as well as discussions of extrudate distortion, interfacial instabilities, and extensional and multi-dimensional flows. In 1996, Shaqfeh detailed the recent advances made in understanding purely elastic instabilities (*i.e.*, instabilities at vanishing Reynolds number) in viscometric flows. Here, we present a brief and selective update of work in the area of viscoelastic instabilities; due to the author's own research interests, this informal review is somewhat biased towards coverage of work on the Taylor-Couette problem.

*Corresponding author: muller2@berkeley.edu

2. Early work on viscoelastic instabilities

Work on instabilities in viscoelastic flows dates back at least to the 1960's. Many of the early studies focused naturally on the Taylor-Couette problem (Fig. 1), both due to its central importance in studies of hydrodynamic stability and the widespread use of this flow for measurements of fluid viscosity. For Newtonian fluids, the study of this flow between concentric, rotating cylinders dates back to at least 1687 and Newton's *Principia*, and its rich history, including contributions from Stokes, Mallock, Couette, Taylor, and Chandrasekhar has been described by Donnelly (1991). In a now classic paper, Andereck, Liu, and Swinney (1986) used flow visualization and spectral studies to reveal an astonishingly large number of different flow states in the Newtonian Taylor-Couette system, including

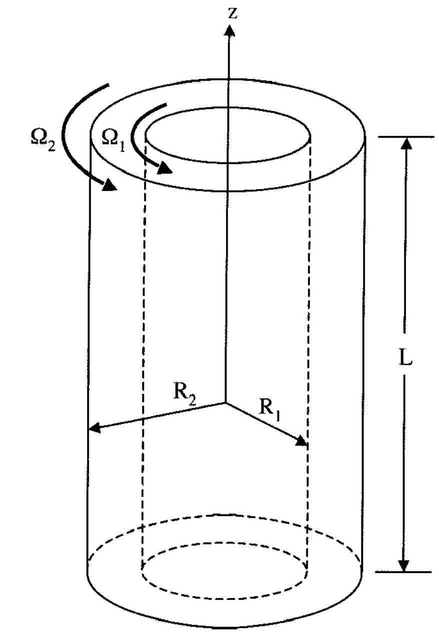

Fig. 1. The Taylor-Couette geometry.

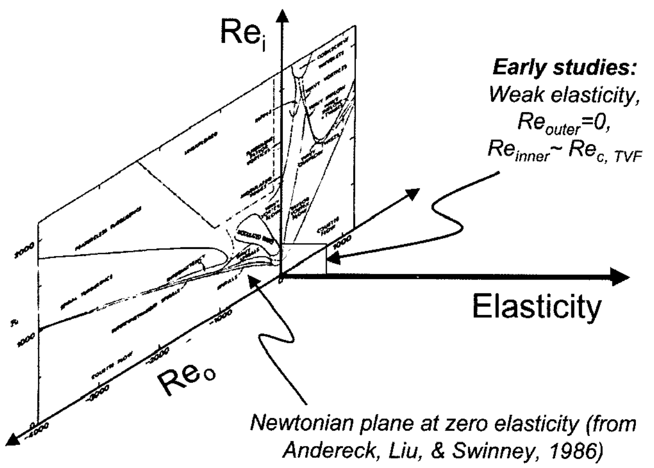

Fig. 2. The stability diagram for Taylor-Couette flow, shown on Re_o-Re_i-Elasticity coordinates. The plane at zero elasticity corresponds to the Newtonian problem; the results of Andereck, Liu, and Swinney (1986) are reproduced on this plane. Early viscoelastic flow studies focused on weak elasticity, a stationary outer cylinder, and the transition from Couette flow to Taylor Vortex flow.

Couette flow, Taylor vortex flow, wavy vortex flow, modulated waves, turbulent Taylor vortices, spirals, wavy spirals, interpenetrating spirals, spiral turbulence, corkscrew wavelets, featureless turbulence, and a rather large region of unexplored parameter space. Andereck *et al.* plotted transition boundaries as a function of inner and outer-cylinder Reynolds numbers, Re_i and Re_o. Considering this Re_o-Re_i plot as the plane at zero elasticity in Re_o-Re_i-Elasticity space provides a convenient starting point for discussing viscoelastic Taylor-Couette flow (Fig. 2).

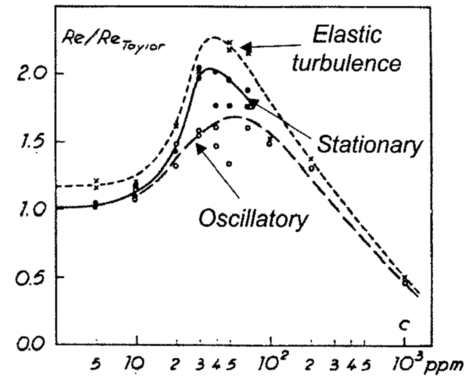

Fig. 3. Early Taylor-Couette experiments for a solution of polyacrylamide (Praestol 2935) in a 1:1 mixture of dimethylformamide and water. (reproduced from Giesekus, 1972).

Initial studies of viscoelastic Taylor-Couette flow focused on a small section of the Re_i-Elasticity plane corresponding to $Re_o=0$ (stationary outer cylinder), weak elasticity, and the first transition from Couette flow (Fig. 2). Experiments by Rubin and Elata (1966), Denn and Roisman (1969), Giesekus (1966; 1972) and others indicated that the addition of small concentrations of polymer, or weak viscoelasticity, stabilizes Taylor-Couette flow against the formation of inertial Taylor vortices. That is, the critical Taylor number or Re increases relative to a Newtonian fluid. The form of the disturbance flow was unaffected - vortices were stationary and the wavenumber was unchanged from the Newtonian case. However, with increasing polymer concentration, Giesekus (1966; 1972) noted the first transition was, for certain polymers, to oscillatory vortices, and that at Re above that required to form stationary vortices, elastic turbulence was observed (Fig. 3). At still higher polymer concentrations, the critical condition for the primary transition decreased with concentration, so that at concentrations of order a few thousand ppm destabilization of the base flow was observed. Unfortunately in most of these early experiments, fluid rheological characterization was either entirely absent or incomplete (*e.g.*, in some cases, viscosity was measured as a function of shear rate, but no normal stress coefficient data were reported).

Early theoretical work on viscoelastic Taylor-Couette flow, like most early experiments, focused on the modifications of the primary inertial instability due to weak viscoelasticity. Early analyses by Giesekus (1966) and Ginn and Denn (1969) of the stability of a second order fluid pointed to the important role of the second normal stress coefficient in determining whether Taylor vortex flow was stabilized or destabilized. Negative values of the second normal stress coefficient Ψ_2, particularly in the small gap limit, were found to stabilize the base flow. However, meaningful comparison between these early analyses and experiments was seldom possible because the experimental fluids were incompletely characterized or displayed a shear-thinning viscosity and hence were poorly described by the second order fluid model.

Beard, Davies, and Walters (1966) were among the first to give theoretical consideration of stronger elastic effects in the Taylor-Couette problem. These authors computed the linear stability of an upper-convected Maxwell fluid as a function of an elasticity parameter. The upper-convected Maxwell fluid can be considered as a special case of the three-parameter Oldroyd-B constitutive equation. The latter equation is derivable from a simple Brownian bead-Hookean spring dumbbell model of the polymer and predicts constant (*i.e.* non-shear-thinning) shear viscosity η, constant first normal stress coefficient Ψ_1, and $\Psi_2=0$. The Oldroyd-B fluid is characterized by a single polymeric relaxation time λ, a solvent viscosity η_s, and a polymer contribution to the viscosity η_p $(=\eta-\eta_s)$. The two additional dimensionless parameters introduced by the Oldroyd-B model (relative to a Newtonian fluid) are typically defined as a Deborah number De (typically given as the product of λ and a characteristic shear rate) that characterizes the ratio of elastic to viscous forces, and the ratio of η_s to total viscosity η. For $\eta_s=0$, the Oldroyd-B model reduces to the upper-convected Maxwell model. Beard *et al.* found that for low values of elasticity as given by E $=$De/Re, increasing elasticity resulted in a systematic decrease in the critical Reynolds number for the stationary Taylor mode. However, at higher values of elasticity, a new overstable or oscillatory mode becomes more unstable than the stationary, inertial mode. The critical Reynolds number associated with this oscillatory or inertio-elastic mode decreases monotonically with increasing elasticity over the range examined ($0<E<1$).

Stability analyses for more general constitutive models, which could describe shear-thinning in the viscosity and the normal stress coefficients, realistic extensional rheological behavior, a spectrum of relaxation times, *etc.*, were considered by a number of early researchers. The Taylor-Couette stability with respect to infinitesimal disturbances of a "simple fluid", in which the stress in a fluid element depends only on the kinematic history of that element alone, was studied by Lockett and Rivlin (1968), Smith and Rivlin (1972), and Miller and Goddard (1979). However, comparison of these analyses to experiments was again essentially precluded by the large number (~10) of material parameters contained in this very general constitutive model and the lack of adequate fluid characterization in at least the early experiments.

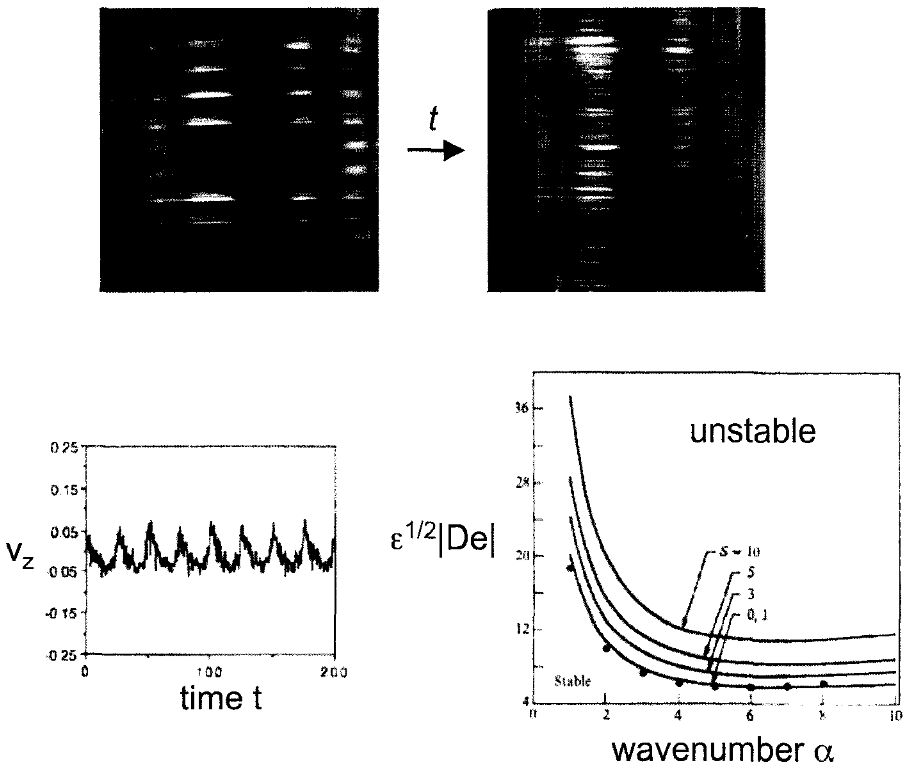

Fig. 4. Flow visualization of the onset and development of purely elastic instabilities in a Boger fluid in Taylor-Couette flow (top, reproduced from Larson *et al.* 1990) and laser Doppler velocimetry measurements of the oscillatory flow just above the critical De (bottom left, reproduced from Muller *et al.* 1993). Bottom right: Neutral stability curves for an Oldroyd-B fluid; ε is the gap, S is the ratio of solvent to polymeric contributions to the viscosity (reproduced from Larson *et al.* 1990).

3. Boger fluids and purely elastic instabilities

Advances in fluid characterization methods, and especially advances in understanding how to create non-shear-thinning, highly elastic dilute polymer solutions (so called "Boger fluids", see Boger 1977/78; Prilutski *et al.*, 1983) that are well-described by the Oldroyd-B model led eventually to meaningful comparisons between stability theory and experiments. That is, Boger fluids - typically created by dissolving a trace amount of a very high molecular weight polymer in a viscous, Newtonian solvent - allow at least semi-quantitative comparison between experiments and predictions using the relatively simple Oldroyd-B constitutive equation. Using this approach, Larson, Shaqfeh, and Muller (Muller *et al.*, 1989; Larson *et al.*, 1990) discovered a purely elastic (*i.e.*, at vanishing Re) instability in Taylor-Couette flow using linear stability analysis, flow visualization, and rheometric measurements. (In terms of Fig. 2, these instabilities occur on the "Elasticity" axis). Above a critical Deborah number, a coupling of radial velocity fluctuations to nonzero θθ-normal (or hoop) stresses leads to amplification of the hoop stresses and hence the radial perturbations, leading to an oscillatory disturbance flow. Since the instability is unrelated to centrifugal destabilization, it occurs even with rotation of only the outer cylinder. Rheometric, flow visualization, and laser Doppler velocimetry experiments revealed a time-dependent disturbance flow with a wavenumber comparable to predictions (*cf.* Fig. 4; Larson *et al.*, 1990; Muller *et al.*, 1993). Critical conditions also varied with the gap size as predicted (Shaqfeh *et al.*, 1992). Avgousti and Beris (1993) and Joo and Shaqfeh (1994) subsequently determined that a non-axisymmetric, oscillatory mode was more unstable than the axisymmetric mode initially identified by Larson *et al.* While the predicted critical Deborah number varies somewhat with details of the fluid rheology (*e.g.*, altering the constitutive equation to allow for a spectrum of relaxation times, shear-thinning, *etc.*), the form of the disturbance flow - nonaxisymmetric, oscillatory vortices - is insensitive to these variations (Larson *et al.*, 1994; Al-Mubaiyedh *et al.*, 2000a).

The mechanism for the purely elastic instability described by Larson *et al.* (1990) suggested that related instabilities should occur, by a similar mechanism, in any rotational shearing flow. Shortly thereafter, Joo and Shaqfeh (1991) predicted a related instability in pressure driven flow in a curved channel (Dean flow). These authors subsequently published an energy analysis for the stability of viscoelastic Taylor-Dean flows - *i.e.*, the spectrum of flows from pure pressure driven flow in a curved channel (*i.e.*, Dean flow) to Taylor-Couette flow (Joo and Shaqfeh, 1994). In the latter work the authors show that energy transfer to and from the perturbation is controlled by different terms in the two limiting cases. In Taylor-Couette flow energy transfer is dominated by the coupling of the perturbation stresses with the base state velocity gradients, while in Dean flow the coupling of the perturbation velocity and the base state stress gradients controls the onset of instability. Moreover, their analysis for an Oldroyd-B fluid reveals that when both pressure and Couette flow driving forces are present, two separate modes compete leading to either stationary (in the Dean flow limit) or oscillatory (in

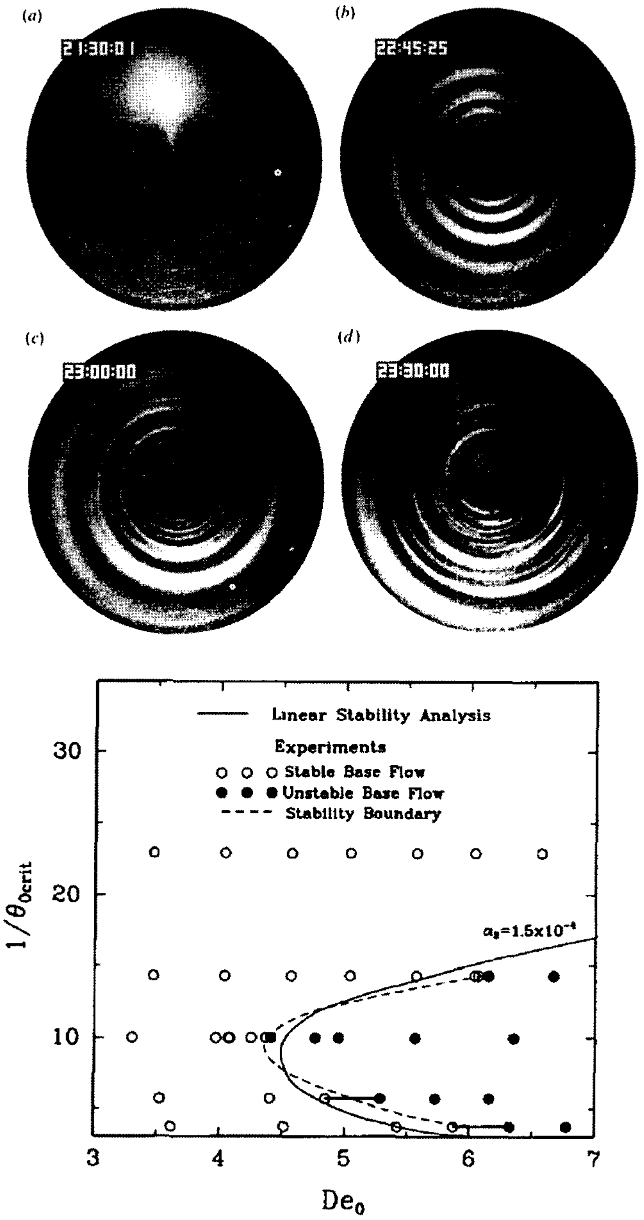

Fig. 5. In cone-and-plate flow, the instability forms as a Bernoulli spiral, as seen in flow visualization with a polyisobutylene-based Boger fluid (top, reproduced from McKinley *et al.*, 1995). The stability curve, plotted as the reciprocal of the critical cone angle versus De, is quantitatively predicted by a linear stability analysis for a four-mode Giesekus model. (bottom, reproduced from Oztekin *et al.*, 1994).

the Taylor-Couette limit) disturbance flows.

McKinley and Brown and co-workers (McKinley *et al.*, 1991; Oztekin and Brown, 1993; Byars *et al.*, 1994; McKinley *et al.*, 1995) used linear stability analyses and experiments to examine purely elastic instabilities in torsional flows between parallel plates and between a cone and plate. These latter two flows are pervasive in rheological characterization of viscoelastic fluids, so understanding the critical conditions for flow transitions is crucial to interpreting measurements of fluid viscosity and normal stresses. Through the use of Oldroyd-B as well as more accurate models (Chilcott-Rallison, multi-mode Giesekus models) that incorporate shear-thinning and a spectrum of relaxation times, these authors were able to quantitatively predict the critical Deborah number as a function of gap or cone angle. As seen in Fig. 5, in cone and plate flow, the instability forms as an outward propagating Bernoulli spiral; quantitative prediction of the onset of these spirals was achieved for a four-mode Giesekus model fit to the shear rheological data for the 0.31 wt% polyisobutlyene-based Boger fluid used in the experiments. The mechanism of both the parallel plate and cone-and-plate instabilities is again related to a coupling of the base state velocity gradients to the perturbation stresses, and so is essentially the same mechanism presented by Larson *et al.* (1990).

Indeed, Pakdel and co-workers (1996; McKinley *et al.*, 1996) developed a universal criterion, based on dimensional arguments, theory, and experiments, for purely elastic instabilities that describes a broad range of two-dimensional, isothermal, single phase flows of viscoelastic fluids. They predict the onset of elastic instabilities in the limit of vanishing Re when:

$$\left[\frac{\lambda V}{\Re}\frac{\tau_{11}}{\eta\gamma}\right]^{1/2} \geq M_{crit}$$

where λ is the fluid relaxation time, V is the characteristic velocity, $\Re$ is a characteristic radius of curvature of a streamline, τ_{11} is the normal stress in the "1" or flow direction, η is the viscosity, and γ is a characteristic local shear rate. The first ratio on the left is a ratio of the length scale over which perturbations to the base viscoelastic stress and velocity field relax (λV) to the local curvature of the flow $\Re$; the second characterizes the coupling of these perturbations to the elastic stresses in the base flow. Thus the criterion captures both the curvature of the streamlines and the influence of normal stresses in the fluid. Although the numerical value of M_{crit} cannot be captured by this dimensional analysis, the scaling works for flows including Taylor-Couette flows, cone and plate flows, and flow between eccentric rotating cylinders. Despite the fact that the local radius of curvature varies throughout a complex, two-dimensional flow, the radius of curvature should still scale with the rheological and geometric parameters in the problem. McKinley *et al.* (1996) demonstrate a scheme for quantifying instabilities based on the above criterion for two-dimensional viscoelastic flow past a cylinder as well as for lid-driven cavity flows for a broad range of cavity aspect ratios and for different fluids. While the analysis cannot capture the precise value of M_{crit} for a particular flow, predict the wavenumber or spatio-temporal symmetry of the disturbance flow, nor distinguish between purely elastic instabilities observed in pure shearing flows and those in mixed flows in which extension plays an important role, it provides a simple, powerful means for characterizing purely elastic instabilities in a wide range of complex geometries.

4. Boger fluids and non-isothermal effects on stability

A necessary feature of experiments on purely elastic instabilities is high fluid viscosity η; high viscosity insures both vanishing inertial forces (Re<<1) and high fluid elasticity, since the fluid relaxation time λ scales with η for dilute polymer solutions. High fluid viscosity also leads to the increased importance of viscous dissipation and associated non-isothermal effects. Viscous heating can produce temperature gradients that in turn result in gradients in fluid viscosity and fluid elasticity. These gradients can also

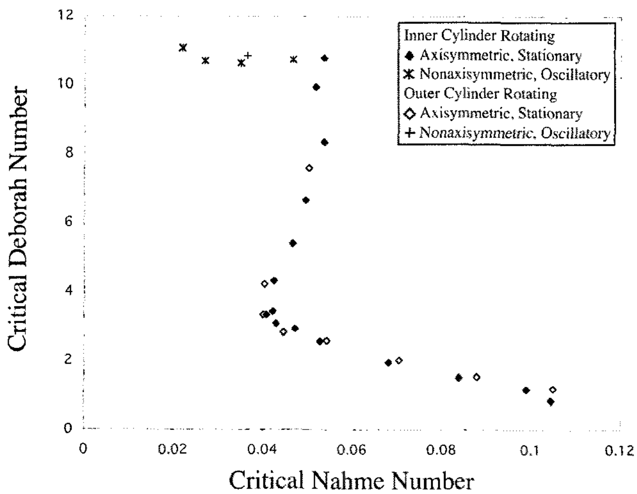

Fig. 6. Experimentally determined stability boundaries for PIB-based Boger fluids in Taylor-Couette flow. In the limit of Nahme number approaching zero, the isothermal non-axisymmetric, oscillatory mode is observed at a critical De near 11 for either rotation of the inner or outer cylinder. As the Nahme number increases, non-isothermal effects become increasingly important, the critical De decreases, and an axisymmetric, stationary mode is observed. At the highest Nahme number plotted (which corresponds to the lowest value of elasticity number E), a small inertial influence becomes apparent in the separation of the critical De for inner versus outer cylinder rotation. (Reproduced from White and Muller, 2003).

lead to flow instabilities, and are best parameterized by the Nahme number, Na, the ratio of the temperature change from viscous heating to the temperature change required to alter the fluid viscosity. In the viscoelastic Taylor-Couette problem at vanishing Re, viscous heating leads to significant destabilization through a coupling between base state temperature gradients and the radial perturbation velocity. This new thermo-elastic mode results in a sharp decrease in the critical De and a change in the type of disturbance flow, from non-axisymmetric and time dependent at vanishing Nahme number to stationary and axisymmetric with increasing Na (Al-Mubaiyedh *et al.*, 1999; 2000a; White and Muller, 2000; 2003). Experimental results showing this dramatic decrease in critical De and the change in the form of the disturbance flow are presented in Fig. 6 (White and Muller, 2003). The stability boundary in the De-Na plane shown is multi-valued because this parameter space was traversed experimentally by changing the fluid temperature, hence the Peclet number and the elasticity number vary along the curve. In other words, if one thinks in terms of Re_i - Na - De space, the experiments in Fig. 6 represent a path along a curved surface beginning on the De axis. Experiments at all but the highest Na probed (which corresponds to the lowest elasticity number, $E \leq 0.02$) reveal that inertial influences are negligible for both the isothermal and non-isothermal modes: the critical conditions are independent of which cylinder is rotated except at E 0.02. The experiments are in semi-quantitative agreement with non-isothermal predictions for an Oldroyd-B model (Al-Mubaiyedh *et al.*, 2002a; Thomas *et al.*, 2004).

We note that because both the viscous heating time scale ($\sim(R_2 - R_1)^2/\alpha$, where α is the thermal diffusivity) and the relaxation time can be long, extremely slow ramp rates must be used to determine the critical conditions when viscous heating and/or elastic effects are important in viscoelastic flows. This can be used to advantage, as ramps that are rapid with respect to the development of the thermal field (but slow with respect to the polymer time scale) will access the isothermal critical conditions, while slow ramps will allow the thermal field to become fully-developed and will thus access the non-isothermal critical conditions. Finally, we note that viscous-heating effects in *Newtonian* Taylor-Couette flow are also profoundly destabilizing and lead to a change in the primary instability mode from stationary to oscillatory (Al-Mubaiyedh *et al.*, 2002b; White and Muller, 2002a; 2002b).

Viscous heating effects on the stability of other flows have also been investigated. Al-Mubaiyedh *et al.* (2000b) considered the effects of viscous heating on the stability of purely elastic Dean flow of an Oldroyd-B fluid. These authors found that for Peclet numbers (Pe) less than 10^5, viscous heating is stabilizing but the temperature boundary conditions have a significant effect on the spatio-temporal characteristics of the disturbance flow. For $Pe > 10^5$, a new axisymmetric and time-dependent mode is found to be the most unstable mode when viscous heating is present. Experiments by Rothstein and McKinley (2001) probed the effects of viscous heating on purely elastic instabilities in cone-and-plate and plate-and-plate flows. In contrast to the Taylor-Couette and Dean cases, they found no new thermo-elastic mode in these flows and, due to the decrease in relaxation time with increased temperature, observed that viscous heating effects were monotonically stabilizing. Olagunju and co-workers (2002) subsequently considered the effects of viscous heating on viscoelastic cone-and-plate flows through a linear stability analysis for an Oldroyd-B fluid. Their results were in qualitative agreement with the experiments of Rothstein and McKinley: in the limits $Re \ll Na \ll 1$ and small cone angle, no new disturbance modes appeared and viscous heating monotonically stabilized the base flow to both small and long wavelength disturbances. Olagunju (2005) subsequently considered non-isothermal, creeping viscoelastic flow between parallel plates via a singular perturbation expansion in the gap between the plates, and showed that thermally-induced normal stress stratification leads to secondary flows characterized by roll cells normal to the direction of rotation. Due to the widespread use of Taylor-Couette, cone and plate, and parallel plate geometries for rheological characterization of polymer solutions and melts, understanding these effects of viscous heating on the form and stability of the base flows is critical to the interpretation of rheological measurements.

5. Inertio-elastic effects in isothermal flows

For isothermal flows of viscoelastic liquids, when both elasticity and inertial effects are important, linear stability predictions for an Oldroyd-B fluid have been generated for a small number of cases of fixed outer to inner cylinder rotation. When $De/Re \ll 1$, modest destabilization of the stationary, axisymmetric, inertial mode with De at low De is predicted, as first noted by Beard *et al.* (1966). For $De/Re \sim 0.1$ or greater, an oscillatory, nonaxisymmetric inertio-elastic mode becomes the most unstable mode, and the critical Re drops to a fraction of the Newtonian value (Beard *et al.*, 1966; Khayat, 1999; Avgousti and Beris, 1993; Joo and Shaqfeh, 1992). As $De/Re \rightarrow \infty$, the purely elastic mode described by Larson and co-workers is the most unstable one. While some theoretical attention has been given to nonlinear analysis of viscoelastic Taylor-Couette flow (Sureshkumar *et al.*, 1994; Khayat, 1999; Kumar and Graham, 2000; 2001), the introduction of six additional stress variables through the constitutive equation, the strong nonlinear interactions between eigenmodes and the need for very fine meshes to capture fine spatial structures, and the mixed elliptic-hyperbolic nature of the equations have hampered progress. Thomas and co-workers (2006) have recently presented the first time-dependent simulations of three-dimensional Taylor-Couette flows of an Oldroyd-B fluid. Their results, for two radius ratios, a modest range of elasticity ($0<De/Re<0.2$), and rotation of the inner cylinder only, reveal that ribbons and spirals emerge near the bifurcation point. The nature of the bifurcation depends on De/Re as does the pattern selection, with ribbons generally being the stable pattern above the linear stability threshold.

Experiments have uncovered a much broader range of flow states in Taylor-Couette flow influenced by both inertia and elasticity; however, as the elasticity number decreases, rheological characterization of the fluids tends to become more difficult and shear-thinning frequently becomes a complicating factor. In most studies, elasticity has been varied by varying the concentration of polymer, and the outer cylinder was held stationary. Giesekus (1972) was perhaps the first to note that the primary transition from the base flow was, under certain conditions, to an oscillatory flow state, and that further increasing Re resulted in subsequent transitions to a stationary state and then to elastic turbulence (Fig. 3). Friebe (1976) reported a similar sequence of transitions, preceded in some cases by a spiral instability as the first transition from Couette flow. The importance of shear-thinning and extensional viscosity were suggested by Haas and Bühler (1989). With increasing Re, these authors reported the same sequence of transitions as reported by Friebe for some solutions, but a transition from Couette flow to stationary Taylor vortices to wavy vortices to stationary vortices to turbulent flow for other fluids. Yi and Kim (1997) examined a series of ultradilute polymer solutions, including varying concentrations of polyacrylaminde, xanthan gum, and polyacrylic acid. For all solutions, Yi and Kim observed the transition sequence from Couette flow to turbulent flow was qualitatively similar to that for a Newtonian fluid. In addition, they found stabilization of Couette flow by the addition of polymer for all solutions. By expanding the critical Reynolds number for Taylor vortex flow in terms of the shear-thinning index and the normal stress coefficients, Yi and Kim were able to isolate the effects of n, Ψ_1, and Ψ_2 on the critical conditions, and identify the importance of even a weak shear-rate dependence in determining the flow stability.

Baumert and Muller (1995; 1997; 1999) examined the effects of varying fluid elasticity by using a single concentration of polymer and varying the solvent viscosity. By following the evolution of the flow for extended times following steps in De, Baumert and Muller revealed a series of additional transitions in non-shear-thinning elastic fluids, including axially traveling vortices and a disordered, oscillating "flame pattern" in flows at two different De/Re and considering $\Omega_2/\Omega_1=0$, 0.5, and ∞. Groisman and Steinberg (1996; 1997; 1998), by varying temperature and solvent composition, systematically controlled De/Re over three decades in experiments with a fixed outer cylinder in which the stability was mapped via abrupt steps in inner cylinder speed. At low De/Re, minor destabilization of Taylor vortex flow was followed by rotating standing waves and disordered oscillations as Re was increased; at moderate De/Re they identified a point at which the flow became simultaneously unstable to Taylor vortices and disordered oscillations. However, transitions and flow states were sensitive to η_p/η. For $De/Re>2.5$, all transitions appeared independent of Re and were hysteretic; subsequent work by Groisman and Steinberg (2004) in this limit revealed that the flows above the critical condition exhibited the main features of turbulence, including motion characterized by a broad range of spatial and temporal scales.

Crumeyrolle, Mutabazi, and Grisel (2002) used varying concentrations of polyethylene oxide to span three decades in elasticity in Taylor-Couette flows with a stationary outer cylinder. These authors identified stabilization of Taylor vortices at low De/Re when shear-thinning was negligible; in this low concentration, low elasticity limit, Taylor vortex flow was followed by wavy vortex flow at higher Re; this contrasts with the transition to rotating standing waves reported by Groisman and Steinberg under similar conditions. At higher concentrations, Crumeyrolle *et al.* demonstrated that shear-thinning becomes important and the critical mode for the primary transition is standing waves; under these conditions the critical Reynolds number decreases with polymer concentration. More recently, Crumeyrolle and co-workers (2005) used 2D Fourier analysis and complex demodulation of space-time plots to extract the amplitude and phase of left and right propagating spiral modes and harmonics in weakly elastic Taylor-Couette flows, and linked the appearance of harmonics to shear-thinning. This type of analysis allows for quantitative determination of stability boundaries and results can be interpreted, as noted by the authors, within the framework of coupled complex Ginzburg-Landau equations and may serve as further tests of constitutive equations through comparisons with direct numerical simulations.

6. Concluding thoughts

Observations of viscoelastic Taylor-Couette flow reveal rich stability diagrams that vary from polymer to polymer and with rheological material properties. Many rheological parameters differ across existing viscoelastic experiments and subtle differences in rheological behavior (shear-thinning, η_p/η, chain stiffness, elasticity number) lead to large stability changes. Careful and complete rheological characterization of the solutions used is often daunting technically but is crucial to interpretation of the results in terms of stability calculations.

There are a number of potentially long time scales in the problem that one must be aware of: the viscous time scale $(R_2-R_1)^2/\nu$, a phase diffusion time (associated with diffusion of vorticity from the ends of the system) $(L/2)^2/\nu$, the polymer relaxation time λ, and the thermal time scale $(R_2-R_1)^2/\alpha$. These sometimes long time scales are important in determining appropriate quasi-static acceleration rates for experimentally mapping critical conditions. That is, experimental identification of stability boundaries must be accomplished through appropriately slow adiabatic ramps in rotation speed or Re.

Finally, we note that almost all experimental studies to date have focused on the case of no outer cylinder rotation. Despite significant progress in understanding viscoelastic Taylor-Couette flow, the sort of comprehensive, detailed mapping of the Re_i vs Re_o plane via adiabatic ramps that Andereck, Swinney, and Liu performed for Newtonian fluids in 1986 has yet to be published for even one viscoelastic fluid (*cf.* Fig. 2). Thus, the problem presents a huge parameter space, much of which remains unexplored. This is also true for other viscoelastic flows with curved streamlines; despite significant progress in our understanding and prediction of elastically-influenced instabilities, much work remains.

References

Al-Mubaiyedh, U. A., R. Sureshkumar and B. Khomami, 1999, Influence of energetics on the stability of viscoelastic Taylor-Couette flow, *Phys. Fluids* **11**, 3217-3226.

Al-Mubaiyedh, U. A., R. Sureshkumar and B. Khomami, 2000a, Linear stability of viscoelastic Taylor-Couette flow: influence of fluid rheology and energetics, *J. Rheol.* **44**, 1121-1138.

Al-Mubaiyedh, U. A., R. Sureshkumar and B. Khomami, 2000b, Energetic effects on the stability of viscoelastic Dean flow, *J. Non-Newtonian Fluid Mech.* **95**, 277-293.

Al-Mubaiyedh, U. A., R. Sureshkumar and B. Khomami, 2002a, Nonlinear stability analysis of viscoelastic Taylor-Couette flow in the presence of viscous heating, *Phys. Fluids* **14**, 1056-1064.

Al-Mubaiyedh, U. A., R. Sureshkumar and B. Khomami, 2002b, Effect of viscous heating on the stability of Taylor-Couette flow, *J. Fluid Mech.* **462**, 100-132.

Andereck, C. D., S. S. Liu and H. L. Swinney, 1986, Flow regimes in a circular Couette system with independently rotating cylinders, *J. Fluid Mech.* **164**, 155-83.

Avgousti, M. and A. N. Beris, 1993, Non-axisymmetric modes in viscoelastic Taylor-Couette flow, *J. Non-Newtonian Fluid Mech.* **50**, 225-251.

Baumert, B. M. and S. J. Muller, 1995, Flow visualization of the elastic Taylor-Couette instability in Boger fluids, *Rheol. Acta* **34**, 147-159.

Baumert, B. M. and S. J. Muller, 1997, Flow regimes in model viscoelastic fluids in a circular Couette system with independently rotating cylinders, *Phys. Fluids* **9**, 566-586.

Baumert, B. M. and S. J. Muller, 1999, Axisymmetric and non-axisymmetric elastic and inertio-elastic instabilities in Taylor-Couette flow, *J. Non-Newtonian Fluid Mech.* **83**, 33-69.

Beard, D. E., M. H. Davies and K. Walters, 1966, The stability of elastico-viscous flow between rotating cylinders. Part 3. overstability in viscous and Maxwell fluids, *J. Fluid Mech.* **24**, 321-334.

Boger, D. V., 1977/1978, A highly elastic constant viscosity fluid, *J. Non-Newtonian Fluid Mech.* **3**, 87-91.

Byars, J. A., A. Oztekin, R. A. Brown and G. H. McKinley, 1994, Spiral instabilities in the flow of highly elastic fluids between rotating parallel disks, *J. Fluid Mech.* **271**, 173-218.

Crumeyrolle, O., I. Mutabazi and M. Grisel, 2002, Experimental study of inertioelastic Couette-Taylor instability modes in dilute and semidilute polymer solutions, *Phys. Fluids* **14**, 1681-1688.

Crumeyrolle, O., N. Latrache, I. Mutabazi and A. B. Ezersky, 2005, Instabilities with shear-thinning polymer solutions in the Couette-Taylor system, *J. Phys.: Conf. Ser.* **14**, 78-93.

Denn, M. M. and J. J. Roisman, 1969, Rotational stability and measurement of normal stress functions in dilute polymer solutions, *AIChE J.* **15**, 454-459.

Donnelly, R. J., 1991, Taylor-Couette flow: the early days, *Physics Today* **44**, 32-39.

Friebe, H. W., 1976, Das stabilitätsverhalten verdünnter lösungen sehr langkettiger hochpolymerer in der Couette-strömung, *Rheol. Acta* **15**, 329-355.

Giesekus, H., 1966, Zur stabilität von Strömungen viskoelastischer Flüssigkeiten: 1. Ebene und kreisförmige Couette-Strömung, *Rheol. Acta* **5**, 239-252.

Giesekus, H., 1972, On instabilities in Poiseuille and Couette flows of viscoelastic fluids, in *Progress in Heat and Mass Transfer* 5, ed. W. R. Schowalter, A. V. Luikov, W. J. Minkowycz and N.H. Afgan, Permagon, NY, pp. 187-193.

Ginn, R. F. and M. M. Denn, 1969, Rotational stability in viscoelastic liquids: theory, *AIChE J.* **15**, 450-454.

Groisman, A. and V. Steinberg, 1996, Couette-Taylor flow in a dilute polymer solution, *Phys. Rev. Lett.* **77**, 1480-1483.

Groisman, A. and V. Steinberg, 1997, Solitary vortex pairs in viscoelastic Couette flow, *Phys. Rev. Lett.* 78, 1460-1463.

Groisman, A. and V. Steinberg, 1998, Mechanism of elastic instability in Couette flow of polymer solutions: experiment, *Phys. Fluids* **10**, 2451-2463.

Groisman, A. and V. Steinberg, 2004, Elastic turbulence in curvilinear flows of polymer solutions, *New J. Phys.* **6**, 1-48.

Haas, R. and K. Bühler, 1989, Einfluß nichtnewtonsher stoffeigenschaften auf die Taylor-wirbelströmung, *Rheol. Acta* **28**, 402-413.

Joo, Y. L. and E. S. G. Shaqfeh, 1991, Viscoelastic Poiseuille flow through a curved channel: a new elastic instability, *Phys. Fluids A* **3**, 2043-2046.

Joo, Y. L. and E. S. G. Shaqfeh, 1992, The effects of inertia on the viscoelastic Dean and Taylor-Couette flow instabilities with application to coating flows, *Phys. Fluids A* **4**, 2415-2431.

Joo, Y. L. and E. S. G. Shaqfeh, 1994, Observations of purely elastic instabilities in the Taylor-Dean flow of a Boger fluid, *J.*

Fluid Mech. **262**, 27-73.

Khayat, R. E., 1999, Finite-amplitude Taylor-vortex flow of viscoelastic fluids, *J. Fluid Mech.* **400**, 33-58.

Kumar, K. A. and M. D. Graham, 2000, Solitary coherent structures in viscoelastic shear flow: computation and mechanism, *Phys. Rev. Lett.* **85**, 4056-4059.

Kumar, K. A. and M. D. Graham, 2001, Finite-amplitude solitary states in viscoelastic shear flow: computation and mechanism, *J. Fluid Mech.* **443**, 301-328.

Larson, R. G., 1992, Instabilities in viscoelastic flows, *Rheol. Acta* **31**, 213-263.

Larson, R. G., E. S. G. Shaqfeh and S.J. Muller, 1990, A purely elastic instability in Taylor-Couette flow, *J. Fluid Mech.* **218**, 573-600.

Larson, R. G., E. S. G. Shaqfeh and S.J. Muller, 1994, The effect of fluid rheology on the elastic Taylor-Couette instability, *J. Non-Newtonian Fluid Mech.* **51**, 195-225.

Lockett, F. J. and R. S. Rivlin, 1968, Stability in Couette flow of a viscoelastic fluid. Part I, *Journal de Mecanique* **7**, 475-498.

McKinley,G. H., J. A. Byars, R. A. Brown and R. C. Armstrong, 1991, Observations on the elastic instability in cone-and-plate and parallel-plate flows of a polyisobutylene Boger fluid, *J. Non-Newtonian Fluid Mech.* **40**, 201-229.

McKinley, G. H., A. Oztekin, J. A. Byars and R. A. Brown, 1995, Self-similar spiral instabilities in elastic flows between a cone and a plate, *J. Fluid Mech.* **285**, 123-164.

McKinley, G. H., P. Pakdel and A. Oztekin, 1996, Rheological and geometric scaling of purely elastic flow instabilities, *J. Non-Newtonian Fluid Mech.* **67**, 19-47.

Miller, C. and J. D. Goddard, 1979, Polymer fluid mechanics, *Advances Appl. Mech.* **19**, 143-219.

Muller, S. J., R. G. Larson and E. S. G. Shaqfeh, 1989, A purely elastic transition in Taylor-Couette flow, *Rheol. Acta* **28**, 499-503.

Muller, S. J., E. S. G. Shaqfeh and R. G. Larson, 1993, Experimental studies of the onset of oscillatory instability in viscoelastic Taylor-Couette flow, *J. Non-Newtonian Fluid Mech.* **46**, 315-330.

Olagunju, D. O., L. P. Cook and G. H. McKinley, 2002, Effect of viscous heating on linear stability of viscoelastic cone-and-plate flow: axisymmetric case, *J. Non-Newtonian Fluid Mech.* **102**, 321-342.

Olagunju, D. O., 2005, Secondary flow in non-isothermal viscoelastic parallel-plate flow, *J. Eng. Mathematics* **51**, 325-338.

Oztekin, A. and R. A. Brown, 1993, Instability of a viscoelastic fluid between rotating parallel disks: analysis for the Oldroyd-B fluid, *J. Fluid Mech.* **255**, 473-502.

Oztekin , A., R. A. Brown and G. H. McKinley, 1994, Quantitative prediction of the viscoelastic instability in cone-and-plate flow of a Boger fluid using a multi-mode Giesekus model, *J. Non-Newtonian Fluid Mech.* **54**, 351-377.

Pakdel, P. and G. H. McKinley, 1996, Elastic instability and curved streamlines, *Phys. Rev. Lett.* **77**, 2459-2462.

Petrie, C. J. S. and M. M. Denn, 1976, Instabilities in polymer processing, *AIChE J.* **22**, 209-236.

Prilutski, G., R. K. Gupta, T. Sridhar and M. E. Ryan, 1983, Model viscoelastic liquids, *J. Non-Newtonian Fluid Mech.*, **12**, 233-241.

Rothstein, J. P. and G. H. McKinley, 2001, Non-isothermal modification of purely elastic flow instabilities in torsional flows of polymeric fluids, *Phys. Fluids* **13**, 382-396.

Rubin, H. and C. Elata, 1966, Stability of Couette flow of dilute polymer solutions, *Phys. Fluids* **9**, 1929-1933.

Shaqfeh, E. S. G., 1996, Purely elastic instabilities in viscometric flows, *Annual Reviews of Fluid Mechanics* **28**, 129-185.

Shaqfeh, E. S. G., S. J. Muller and R. G. Larson, 1992, The effects of gap width and dilute solution properties on the viscoelastic Taylor-Couette instability, *J. Fluid Mech.* **235**, 285-317.

Smith, M. M. and R. S. Rivlin, 1972, Stability in Couette flow of a viscoelastic fluid Part II, *Journal de Mecanique* **11**, 69-94.

Sureshkumar, R., A. N. Beris and M. Avgousti, 1994, Non-axisymmetric subcritical bifurcations in viscoelastic Taylor-Couette flow, *Proc. R. Soc. Lond. A* **447**, 135-153.

Thomas, D. G., U. A. Al-Mubaiyedh, R. Sureshkumar and B. Khomami, 2004, Effect of inertia on thermoelastic flow instability, *J. Non-Newtonian Fluid Mech.* **120**, 93-100.

Thomas, D. G., U. A. Al-Mubaiyedh, R. Sureshkumar and B. Khomami, 2006, Time-dependent simulations of non-axisymmetric patterns in Taylor-Couette flow of dilute polymer solutions, *J. Non-Newtonian Fluid Mech.* **138**, 111-133.

White, J. M. and S. J. Muller, 2000, Viscous heating and the stability of Newtonian and viscoelastic Taylor-Couette flows, *Phys. Rev. Lett.* **84**, 5130-5133.

White, J. M. and S. J. Muller, 2002a, Experimental studies on the stability of Newtonian Taylor-Couette flow in the presence of viscous heating, *J. Fluid Mech.* **462**, 133-159.

White, J. M. and S. J. Muller, 2002b, The role of thermal sensitivity of fluid properties, centrifugal destabilization, and nonlinear disturbances on the viscous heating instability in Newtonian Taylor-Couette flow, *Phys. Fluids* **14**, 3880-3890.

White, J. M. and S. J. Muller, 2003, Experimental studies on the effect of viscous heating on the hydrodynamic stability of viscoelastic Taylor-Couette flow, *J. Rheol.* **47**, 1467-1492.

Yi, M. K. and C. Kim, 1997, Experimental studies on the Taylor instability of dilute polymer solutions, *J. Non-Newtonian Fluid Mech.* **72**, 113-139.