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Analysis of a Geometrically Asymmetric Trapezoidal Fin with Variable Fin Base Thickness and Height

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Abstract

A geometrically asymmetric trapezoidal fin is analyzed using the one-dimensional analytic method. Heat loss and thermal resistance are represented as a function of the fin base thickness, base height, fin shape factor, inside fluid convection characteristic number, convection characteristic numbers ratio, fin length and ambient convection characteristic number. The relationship between the fin base height and the shape factor for equal amounts of heat loss is presented. One of the results shows that the variations of the fin base thickness and the inside fluid convection characteristic number give no effect on the thermal resistance.

Key words: Heat loss, Thermal resistance, Fin base temperature, Fin shape factor

Nomenclature

A_c : cross sectional area of the fin [m^2]
A_s : surface area of the fin [m^2]
h : ambient heat transfer coefficient [W/m^2℃]
h_e : fin tip heat transfer coefficient [W/m^2℃]
h_f : inside fluid heat transfer coefficient [W/m^2℃]
I_0 : modified Bessel function of the first kind order of 0
I_1 : modified Bessel function of the first kind order of 1
k : thermal conductivity [W/m℃]
K_0 : modified Bessel function of the second kind order of 0
K_1 : modified Bessel function of the second kind order of 1
l_b : fin base thickness [m]
L_b : dimensionless fin base thickness, l_b/l_c
l_c : characteristic length [m]
l_e : fin tip length [m]
L_e : dimensionless fin tip length, l_e/l_c
l_h : fin base height [m]
L_h : dimensionless fin base height, l_h/l_c
l_w : fin width [m]
M : ambient convection characteristic number, $(hl_c)/k$
M_e : fin tip convection characteristic number, $(h_e l_c)/k$
M_f : inside fluid convection characteristic number, $(h_f l_c)/k$
q : heat loss from the fin [W]
Q : dimensionless heat loss from the fin, $q/(kl_w\phi_f)$
r_t : thermal resistance [℃/W], $(T|_{x=l_b}-T_\infty)/q$
R_t : dimensionless thermal resistance, $(\theta|_{X=L_b})/Q$
s : upper fin lateral surface slope, $(1-\xi)l_h/(l_e-l_b)$
T : fin temperature [℃]
T_f : inside fluid temperature [℃]
T_∞ : ambient temperature [℃]
x : length directional variable [m]
X : dimensionless length directional variable, x/l_c
y : height directional variable [m]

Greek characters

β : ratio of convection characteristic numbers, M_e/M
θ : dimensionless temperature, $(T-T_\infty)/(T_f-T_\infty)$
ξ : fin shape factor, $1-s(l_e-l_b)/l_h$, $(0<\xi\le 1)$
ϕ_f : adjusted inside fluid temperature [℃], (T_f-T_∞)

Subscripts

b : fin base
c : characteristic

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e : fin tip
h : fin height at the base
f : inside fluid
t : thermal
w : fin width
∞ : ambient

1. Introduction

Extended surfaces or fins play very important roles in thermal systems. Therefore, many investigators have analyzed the fin problems and a number of studies for the various shapes of fins have been presented. The most commonly studied fins are longitudinal rectangular, triangular, trapezoidal fins and the annular or circular fins. For example, Casarosa and Franco [1] presented the optimum design of single longitudinal fins with constant thickness, considering different uniform heat transfer coefficients by means of an accurate mathematical method while heat conduction in an array of triangular fins with an attached wall was analyzed using the finite element method [2]. Razelos and Satyaprakash [3] presented an analysis of trapezoidal profile longitudinal fins that delineates their thermal performance and an improved solution of the optimal problem. The optimization of a convective and radiating annular fin under thermally asymmetric condition was reported [4]. An analysis of the heat transfer characteristics of a circular fin dissipating heat from its surface by convection and radiation was made [5].

Also, performance or optimization of the rather unique shape of the fin has been presented. Hashizume et al. [6] analyzed fin efficiency of serrated fins and derived an analytical solution of the theoretical fin efficiency in the form of modified Bessel functions. The effect of a wing on the heat loss from a modified rectangular fin is investigated [7]. The geometric optimization of T-shaped fin assemblies subject to total volume and fin-material constraints was reported [8]. Recently, elliptical disk fins were analyzed and optimized using a semi-analytical technique [9]. In their studies, fin base temperature is given as a constant.

In this study, the analysis of a geometrically asymmetric trapezoidal fin with various upper fin lateral surface slopes is analyzed using a one dimensional analytic method. For this analysis, the convection from inside fluid to the inside wall, the conduction from the inside wall to the fin base, and the conduction through the fin base are considered simultaneously and heat transfer through the fin tip is not ignored. Under these conditions, heat loss and thermal resistance are presented as a function of the fin base thickness, base height, fin length, inside fluid convection characteristic number, ambient convection characteristic number, ratio of convection characteristic numbers and fin shape factor.

2. One-dimensional analysis

A geometrically asymmetric trapezoidal fin is shown in Fig. 1. In this diagram, upper lateral surface slope of the fin s is denoted by $l_h(1-\xi)/(l_e-l_b)$ and the shape of the fin is rectangular for $\xi=1$ and that becomes a geometrically asymmetric triangular fin as the value of ξ approaches 0. For this geometrically asymmetric fin, dimensionless energy balance equation under steady state is expressed by Eq. (1).

$$\frac{d^2\theta}{dX^2}-\frac{s}{L_h-s(X-L_b)}\frac{d\theta}{dX}-M\frac{\sqrt{1+s^2}+1}{L_b-s(X-L_b)}\theta=0 \tag{1}$$

Eq. (1) is derived from a general form of the energy equation for one-dimensional conditions in an extended surface, Eq. (1-1) through Eq. (1-2).

$$\frac{d^2T}{dx^2}+\left(\frac{1}{A_c}\frac{dA_c}{dx}\right)\frac{dT}{dx}-\left(\frac{1}{A_c}\frac{h}{k}\frac{dA_s}{dx}\right)(T-T_\infty)=0 \tag{1-1}$$

$$\frac{d^2T}{dx^2}+\frac{1}{[-s(x-l_b)+l_h]\,l_w}(-sl_w)\frac{dT}{dx}-\frac{1}{[-s(x-l_b)+l_h]\,l_w}\frac{h}{k}\left[\sqrt{1+\left(\frac{dy}{dx}\right)^2}+1\right]l_w(T-T_\infty)=0 \tag{1-2}$$

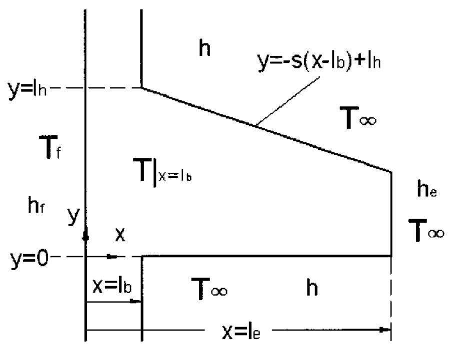

Fig. 1. Schematic diagram of a geometrically asymmetric trapezoidal fin.

Two boundary conditions are required to solve the energy balance equation and these conditions are given as Eqs. (2) and (3). Physically Eq. (2) means that the convection from the inside fluid to the inside wall, conduction from the inside wall to the fin base and the conduction through the fin base are all the same and Eq. (3) explains that the conduction through the fin tip is the same as the convection from the fin tip to the ambient air.

$$-\left.\frac{d\theta}{dX}\right|_{X=L_b} = \frac{1-\left.\theta\right|_{X=L_b}}{1/M_f + L_b} \quad (2)$$

$$\left.\frac{d\theta}{dX}\right|_{X=L_e} + M_e \left.\theta\right|_{X=L_e} = 0 \quad (3)$$

When Eq. (1) with two boundary conditions (2) and (3) are solved, the temperature distribution within the geometrically asymmetric trapezoidal fin can be obtained. The dimensionless form $\theta(X)$ is given by Eq. (4).

$$\theta(X) = \frac{C_2 I_0\{f(X)\} + C_3 K_0\{f(X)\}}{C_1} \quad (4)$$

where,

$$C_1 = C_2\{I_0(D) + C_4 I_1(D)\} + C_3\{K_0(D) - C_4 K_1(D)\} \quad (5)$$

$$C_2 = C_5 K_1(E) + M_e K_0(E) \quad (6)$$

$$C_3 = C_5 I_1(E) - M_e I_0(E) \quad (7)$$

$$C_4 = \frac{sF}{\sqrt{L_h}}\left(\frac{1}{M_f} + L_b\right) \quad (8)$$

$$C_5 = \frac{sF}{\sqrt{L_h - s(L_e - L_b)}} \quad (9)$$

$$D = 2F\sqrt{L_h} \quad (10)$$

$$E = 2F\sqrt{L_h - s(L_e - L_b)} \quad (11)$$

$$F = \frac{\sqrt{M}}{s}\{(1+s^2)^{1/2} + 1\}^{1/2} \quad (12)$$

$$f(X) = 2F\sqrt{L_h - s(X - L_b)} \quad (13)$$

The heat loss conducted into the fin through the fin base is calculated by Eq. (14).

$$q = -k\, l_h\, l_w \left.\frac{dT}{dx}\right|_{x=l_b} \quad (14)$$

Then, the dimensionless heat loss from the geometrically asymmetric trapezoidal fin is obtained by Eq. (15).

$$Q = \frac{sF\sqrt{L_h}\{C_2 I_1(D) - C_3 K_1(D)\}}{C_1} \quad (15)$$

The thermal resistance for a fin is defined by Eq. (16).

$$r_t = \frac{\left.T\right|_{x=l_b} - T_\infty}{q} \quad (16)$$

The dimensionless thermal resistance for a fin can be expressed by Eq. (17).

$$R_t = kl_w r_t = \frac{\left.\theta\right|_{X=L_b}}{Q} \quad (17)$$

3. Results and discussions

Figure 2 represents the temperature profile along the normalized X position of the fin for different values of the fin base thickness and ambient convection characteristic number. The normalized X position NPX is defined as $(X-L_b)/(L_e-L_b)$ so that NPX=0 means the fin base and NPX=1 represents the fin tip position. It shows that the fin base temperature decreases as both the fin base thickness and ambient convection characteristic number increases. It can be noted that the effect of fin base thickness on the temperature decreases as NPX increases for the same value of ambient convection characteristic number.

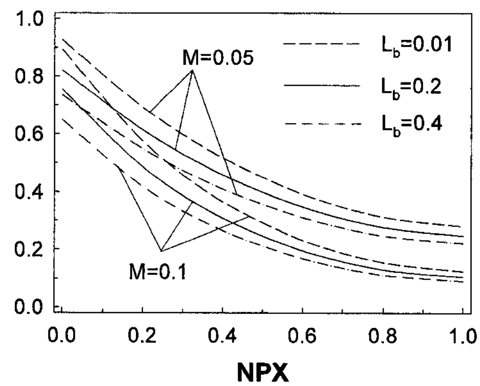

Fig. 2. Temperature profile along the normalized X position (ξ=0.5, M_f=10, L_h=0.15, β=1, L_e=2+L_b).

Table 1 lists the effect of M_f and L_e on the fin base temperature. The fin base temperature increases as the inside fluid convection characteristic number increases since heat transfer increases from the inside fluid through the inside wall to the fin base with increase of the inside fluid convection characteristic number. The fin base temperature decreases as the fin length increases since heat transfer increases from the fin base to the ambient air with increase of the fin length.

Heat loss and thermal resistance versus the fin base thickness are presented in Fig. 3. Heat loss decreases linearly as the fin base thickness increases and the decreasing rate is remarkable in the case of M=0.05. The thermal resistance is independent on the variation of the fin base thickness. Physically it means that the decreasing rate of the adjusted fin base temperature is the same as that of heat loss with increase of the fin base thickness.

Figure 4 represents the heat loss and thermal resistance as a function of the fin base height. It shows that heat loss increases somewhat rapidly first and then levels off as the fin base height increases. The thermal resistance decreases very rapidly as the fin base height increases from 0.01 to about 0.05 and the effect of the fin base height on the thermal resistance seems to be not so much for $L_h > 0.1$.

Table 1. The effect of M_f and L_e on the fin base temperature (ξ=0.5, M=0.05, L_b=0.1, L_h=0.15, β=1)

M_f	L_e	$\theta\|_{X=L_b}$
1	1	0.6386
	3	0.5423
10	1	0.9067
	3	0.8670
100	1	0.9464
	3	0.9222

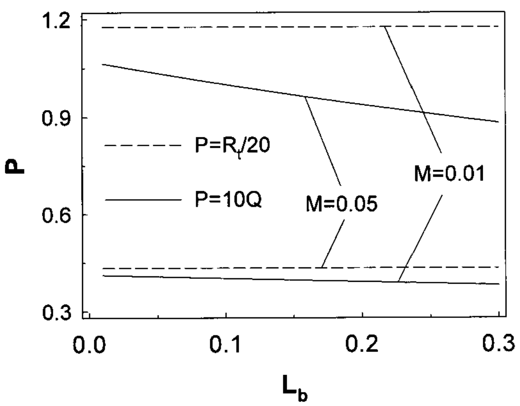

Fig. 3. Heat loss and thermal resistance versus the fin base thickness (ξ=0.5, M_f=10, L_h=0.15, β=1, L_e=3+L_b).

Figure 5 shows the heat loss and thermal resistance as a function of the fin shape factor. It can be known that heat loss increases linearly while the thermal resistance decreases linearly with increase of the fin shape factor. Physically, it means that the decreasing rate of the adjusted fin base temperature is larger than the increasing rate of the heat loss as the shape of the fin changes from a geometrically triangular fin through a geometrically trapezoidal fin to a rectangular fin.

Heat loss and thermal resistance as a function of the inside fluid convection characteristic number are depicted in Fig. 6. Heat loss increases somewhat remarkably in the range of $1 < M_f < 5$ and the effect of the inside fluid convection characteristic number on the heat loss is not so much for $M_f > 10$. It

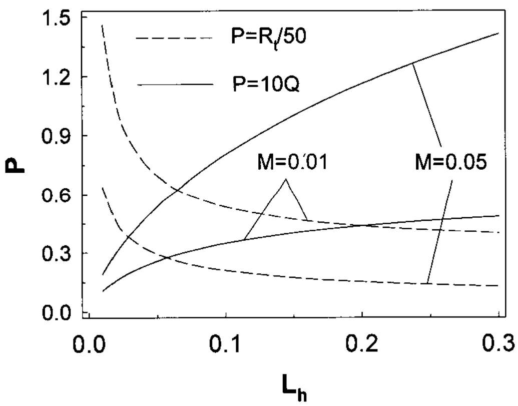

Fig. 4. Heat loss and thermal resistance versus the fin base height (ξ=0.5, M_f=10, L_b=0.1, β=1, L_e=3).

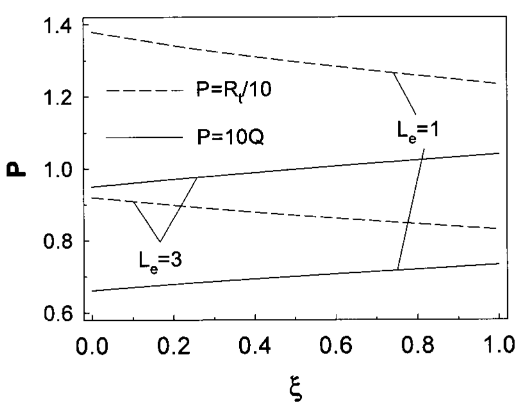

Fig. 5. Heat loss and thermal resistance versus the fin shape factor (M_f=10, L_b=0.1, L_h=0.15, M=0.05, β=1).

shows that the variation of the inside fluid convection characteristic number gives no effect on the thermal resistance.

Figure 7 represents the heat loss and thermal resistance versus the convection characteristic numbers ratio. It shows that heat loss increases linearly while the thermal resistance decreases linearly as the convection characteristic numbers ratio increases and this phenomenon is more remarkable for higher fin base height.

Figure 8 depicts the heat loss and thermal resistance as a function of the fin length. It shows that the heat loss increases while the thermal resistance decreases very rapidly first and then revels off with increase of the fin length. The effect of fin length on the heat loss and thermal resistance seems to be very small for $L_e > 2$ in the case of M=0.05. This phenomenon means that the fin length needs not to be greater than 8 cm under free convection condition {i.e. h=18.9 W/(m^2 · k)} if lc is given as 4 cm and fin material is AISI 302 {i.e. k=15.1 W/(m · k)} for given conditions in Fig. 8.

Heat loss and thermal resistance versus the ambient fluid convection characteristic number is presented in Fig. 9. Both heat loss and the thermal resistance vary parabolically with variation of the ambient convection characteristic number. It can be noted that the difference value of heat loss between ξ =0.05 and ξ =0.75 increases while that of thermal resistance between ξ =0.05 and ξ =0.75 increases first and then decreases as the ambient convection characteristic number increases.

Figure 10 presents the relationship between the fin base height and the fin shape factor for equal amounts of heat loss based on the value of L_h=0.1 and ξ =0.5 for three values of the fin length. The value of L_h decreases a little curved with the increase of ξ and

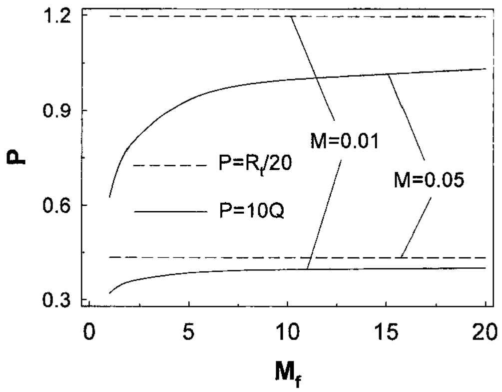

Fig. 6. Heat loss and thermal resistance versus the inside fluid convection characteristic number (ξ=0.5, L_b=0.1, L_h=0.15, β =1, L_e=3).

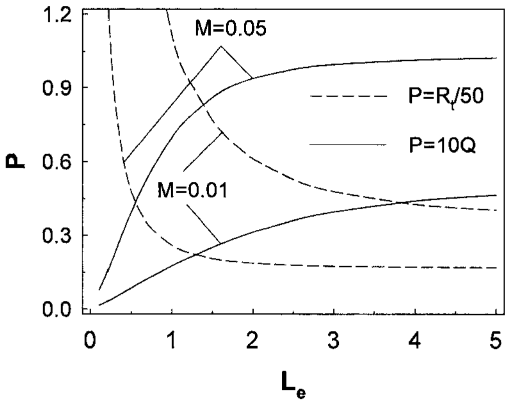

Fig. 8. Heat loss and thermal resistance versus the fin length (ξ=0.5, L_b=0.1, L_h=0.15, M_f=10, β =1).

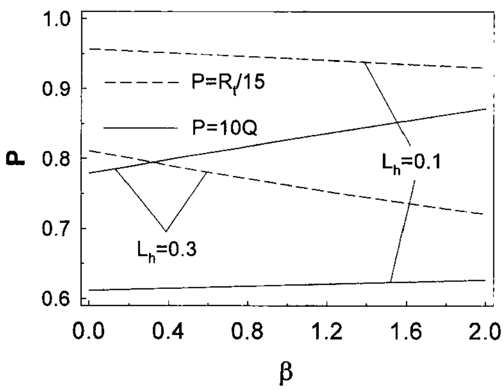

Fig. 7. Heat loss and thermal resistance versus the convection characteristic numbers ratio (ξ=0.5, L_b=0.1, L_e=1, M=0.05, M_f=10).

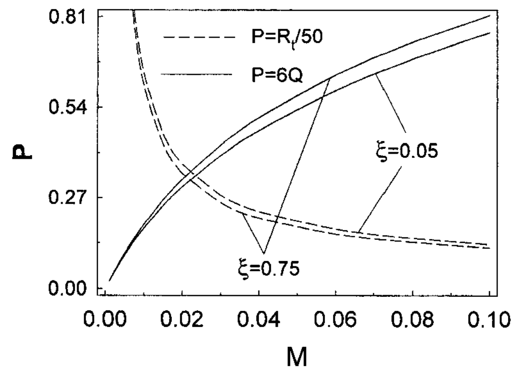

Fig. 9. Heat loss and thermal resistance versus the ambient convection characteristic number (L_b=0.1, L_h=0.15, L_e=2, M_f=10, β =1).

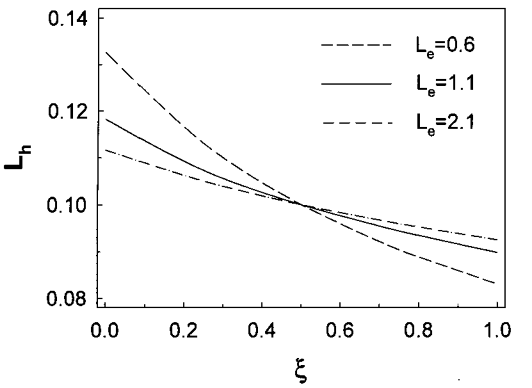

Fig. 10. Relationship between the fin base height and shape factor for equal amounts of heat loss (L_b=0.1, M=0.05, M_f=10, β =1).

the slope of the curve deeper as the fin length decreases.

4. Conclusions

The following conclusions can be made from the results.

(1) The variations of the fin base thickness and the inside fluid convection characteristic number give no effect on the thermal resistance.

(2) Heat loss increases linearly while the thermal resistance decreases linearly with increase of both the fin shape factor and the convection characteristic numbers ratio.

(3) The value of fin base height L_h slightly decreases with increase of the fin shape factor ξ for equal amounts of heat loss based on the value of L_h=0.1 and ξ =0.5

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