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PRACTICAL STABILITY IN SWARMS SYSTEM

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ABSTRACT. Practical stability is a significant practical importance in scientific and engineering problems but less investigated. In this paper, we studied practical stability in swarms system and present new results.

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1. Introduction

Swarming has been studied extensively in biology[1], and there is significant relevant literature in physics where collective behavior of "self-propelled particles" is studied. Swarms have also been studied in the context of engineer applications. For example, the work in on "social potential functions[2]", "intelligent vehicle highway systems[3]", "coordination of groups of mobile autonomous agents[7]".

The notion of practical stability in dynamic stability was discussed by Lasalle and Lefchetz in the 1960s and then was treated by Liao. Practical stability of dynamic systems was studied by Yang and He[5][9]. Generalized practical stability results by perturbing Lyapunov functions were given by Stutson and Vatsala[10].

In this paper, we studied practical stability in swarms system, which is no viewed before. It's significant in the swarms to deal with practical stability.

For readers' convenience, we first introduce the practical stability of equilibrium points of swarms systems.

We consider a swarm of M individuals in a n-dimensional Euclidean space. We model the individuals as points and ignore their dimensions. The position of individual i is described by $x_i \in R^n$. We assume synchronous motion and no time delays.

We consider the equation of motion of each individual i described by [8]:

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$$\dot{x}^i = \sum_{j=1, j\neq i}^{M} g(x^i - x^j),\ i = 1, 2, \cdots, M \tag{1}$$

$$g(x^i - x^j) = -(x^i - x^j)\left(a - bexp\left(\frac{\|x^i - x^j\|^2}{c}\right)\right) \tag{2}$$

where $g(\cdot)$ represents the function of mutual attraction and repulsion between the individuals, a, b, c is positive constant, $a < b$ and $\|\cdot\|$ is the Euclidean norm.

Let the system (1) be under influence of a permanently acting perturbation $p(x^i)$ with $\|p(x^i)\| \leq \delta$. So that the swarms system is

$$\dot{x}^i = \sum_{j=1, j\neq i}^{M} g(x^i - x^j) + p(x^i),\ i = 1, 2, \cdots, M \tag{3}$$

To study practical stability of equilibrium points of swarms system, we define the center of the swarm as $\bar{x} = \frac{1}{M}\sum_{i=1}^{M} x^i$. Then, the motion of the center in model (1) is given by

$$\dot{\bar{x}} = \frac{1}{M}\sum_{i=1}^{M}\sum_{j=1, j\neq i}^{M} g(x^i - x^j) = 0 \tag{4}$$

Which follows from the fact that $g(x^i - x^j)$ is odd function of the form of (2) and $g(x^i - x^j) = -g(x^j - x^i)$ for all pairs (i, j). The equation (4) implies that the center of swarm (1) is stastic. However, this does not imply anything about the motion of the individuals.

From (4) we know that $\bar{x}$ is equilibrium points of swarms system (1).

Definition 1. Swarms system is called *practically stable*, for given estimation (λ, A) and some $t_0 \in A$ if $\|x^i(0) - \bar{x}\| < \lambda$, then $\|x^i(t) - \bar{x}\| < A$ is achieved for all $t \geq t_0$.

Definition 2. We call $Q_0 = \{x^i | \|x^j(0) - \bar{x}\| < \lambda\}$ the initial state set and $Q = \{x^i | \|x^i(t) - \bar{x}\| < A\}$ the permissible state set.

In order to study practical stability in swarms system we should first know:
(i) the scope of the permissible state set;
(ii) the amplitude of $p(x^i)$ (i.e., what the number δ is);
(iii) how large the initial state set is.

2. Practical stability of swarms system

2.1. Practical stability without perturbation

Stability analysis had been studied in Lyapunov sense[8] to Eq.(1). There is no work on practical stability of equilibrium points. In addition, the practical stability should be an intrinsic property of swarms system. Therefore it is reasonable to establish the stability criteria in terms of the original unperturbed system.

Theorem 1. *Swarms (1) is practically stable with respect t_0, Q and Q_0 , where*

$$Q = \{x^i \| \|x^i - \bar{x}\| < A\}, \ A = \frac{b}{a}\sqrt{\frac{c}{2}} exp\left(-\frac{1}{2}\right),$$

$$t_0 = \max_i \left\{ -\frac{1}{2a} \ln \frac{A^2}{2V_i(0)} \right\}$$

Q_0 is unrestrained.

Proof. Defining a Lyapunov function as $V_i = \frac{1}{2}\|e^i\|^2, e^i = x^i - \bar{x}$, we have

$$\begin{aligned}
\dot{V}_i =& (x^i - \bar{x})(\dot{x}^i - \dot{\bar{x}})^T \\
=& -(x^i - \bar{x}) \left(\sum_{j=1, j\neq i}^{n} (x^i - x^j)^T \left(a - b\exp\left(\frac{\|x^i - x^j\|^2}{c} \right) \right) \right) \\
=& -(x^i - \bar{x}) \left[a \sum_{j=1, j\neq i}^{M} (x^i - x^j)^T - b \sum_{j=1, j\neq i}^{M} exp(-\frac{\|x^i - x^j\|^2}{c})(x^i - x^j)^T \right] \\
\leq& - aM\|x^i - \bar{x}\|^2 + b\|x^i - \bar{x}\| \sum_{j=1, j\neq i}^{M} \|x^i - x^j\| exp\left(-\frac{\|x^i - x^j\|^2}{c} \right)
\end{aligned}$$

Let $Z(x^i - x^j) = \|x^i - x^j\| exp\left(-\frac{\|x^i - x^j\|^2}{c} \right)$. Since $Z_{\max} = \sqrt{\frac{c}{2}} exp\left(-\frac{1}{2}\right)$, we obtain

$$\dot{V}_i \leq - aM\|x^i - \bar{x}\|^2 + b(M-1)\sqrt{\frac{c}{2}} exp\left(-\frac{1}{2}\right) \|x^i - \bar{x}\|$$

which implies that as long as $\|x^i - \bar{x}\| > A$, and we have $\dot{V}_i < 0$. So we obtain $\|x^i - \bar{x}\| \leq A$ with time passing.

In order to achieve t_0, we consider function by

$$h(\|x^i - \bar{x}\|) = -(M-1)\left(a\|x^i - \bar{x}\|^2 + b\sqrt{\frac{c}{2}} exp\left(-\frac{1}{2}\right) \|x^i - \bar{x}\|\right).$$

It's clear that if $\|x^i - \bar{x}\| > A$, then $h(\|x^i - \bar{x}\|) < 0$ holds, and we get $\dot{V}_i \leq - a\|x^i - \bar{x}\|^2 = -2aV_i$, which implies that $V_i(t) \leq V_i(0)\exp(-2at)$.
Solving inequality above, we known

$$t \leq -\frac{1}{2a} \ln \frac{A^2}{2V_i(0)}$$

Let $t_0 = \max_i \left\{ -\frac{1}{2a} \ln \frac{A^2}{2V_i(0)} \right\}$. Then the proof is thus completed. □

Theorem 1 tell us as long as $t \geq t_0$, we achieve $\|x^i - \bar{x}\| \leq A$, from which we know E.q.(1) is practically stable.

2.2. Practical stability with perturbation

It is natural to consider perturbation in swarms. Because of effects of all kinds of unpredictable factors, perturbation is seen in swarms. Below we consider practical stability to E.q.(3).

Theorem 2. *Swarms (3) is practically stable with respect* t_0, Q *and* Q_0 *, where*

$$Q = \{x^i | \|x^i - \bar{x}\| \leq A\}, \ A = \frac{b}{a}\sqrt{\frac{c}{2}} exp(-\frac{1}{2}) + \frac{2\delta}{aM}$$

$$t_0 = \max_i \left\{ -\frac{1}{2a} \ln \frac{A^2}{2V_i(0)} \right\}$$

Q_0 *is unrestrained.*

Proof. Using $V_i = \frac{1}{2}\|x^i - \bar{x}\|^2$, we have

$$\begin{aligned} \dot{V}_i &= (x^i - \bar{x})(\dot{x}^i - \dot{\bar{x}})^T \\ &\leq -aM\|x^i - \bar{x}\|^2 + b(M-1)\sqrt{\frac{c}{2}} exp(-\frac{1}{2})\|x^i - \bar{x}\| \\ &\quad + \|p(x^i) - \frac{1}{M}\sum_{i=1}^{M} p(x^i)\| \|x^i - \bar{x}\| \\ &\leq -aM\|x^i - \bar{x}\| \left(\|x^i - \bar{x}\| - \frac{b(M-1)\sqrt{\frac{c}{2}} exp(-\frac{1}{2})}{aM} \right. \\ &\quad \left. - \frac{2(M-1)\delta}{aM^2} \right) \end{aligned}$$

which implies that as long as $\|x^i - \bar{x}\| > A$, and we have $\dot{V}_i < 0$.Using the same method as theorem1, we achieve

$$t_0 = \max_i \left\{ -\frac{1}{2a} \ln \frac{A^2}{2V_i(0)} \right\}$$

□

We proved swarms practical stability with perturbation in theorem 2. There are no restricts for the initial state set in both theorems, but permission state set $Q = \{x^i | \|x^i - \bar{x}\| < A\}$,with $A = \frac{b}{a}\sqrt{\frac{c}{2}} exp\left(-\frac{1}{2}\right)$ in theorem1 and $A = \frac{b}{a}\sqrt{\frac{c}{2}} exp\left(-\frac{1}{2}\right) + \frac{2\delta}{aM}$ in theorem 2, which explains the motions of the individuals.

Theorem 3. *If* $\bar{x}$ *is practically stable with respect to* Q*,* Q_0 *and* δ*, then*

$$\delta \leq B\|f\|_Q \tag{5}$$

where

$$B = \max_{(i,t)}\{\|x^i(t) - \bar{x}\|\},\ \ f(x^i) = -\sum_{j=1, j\neq i}^{M} g(x^i - x^j),$$

$$\|f\|_Q = \max_{x^i \in Q} \frac{\|f(x^i)\|}{\|x^i - \bar{x}\|}.$$

Proof. We use the method as in [5]. If this is not the case, then $\delta > B\|f\|_Q$ and we can take a constant perturbation p with $\|p\| = \delta$. Now there are two cases to consider here. (a)there exists an equilibrium point x_e^i to E.q. (3). Since $\bar{x}$ to E.q. (1) is stable in Lyapunov sense[8], the equilibrium point x_e^i to E.q. (3). is stable in Lyapunov sense. Nonetheless, from $f(x_e^i) + p = 0$, it follows that

$$\|f\|_Q \|x^i(0) - \bar{x}\| \geq \|f(x^i(0))\| = \|p\| = \delta.$$

Therefore, $\|x^i(0) - \bar{x}\| \geq \dfrac{\delta}{\|f\|_Q} > B$ which implies that $x_e^i \notin Q$. Leading to a contradiction by definition. (b) there exists no point satisfying $f(x^i) + p = 0$. In this case, one constructs a function $V_i = px^i$ and its derivative along the trajectories to E.q. (3). is

$$\dot{V}_i = pf(x^i) + \|p\|^2 \neq 0$$

If there exist a point x_0^i such that

$$pf(x_0^i) + \|p\|^2 = pf(x_0^i) + \delta^2 = 0,$$

then $\delta^2 = -pf(x_0^i) \leq \|p\| \|f(x_0^i)\| \leq \delta \|f\|_Q \|x_0^i - \bar{x}\|$. It follows that

$$\|x_0^i - \bar{x}\| \geq \frac{\delta}{\|f\|_Q} \geq B.$$

Therefore one sees that $V_i > 0$ or $V_i < 0$ for trajectories in Q. It follows that no solution to E.q. (3) is practically stable, contrary to the hypothesis. The proof is thus completed. □

3. Practical stability with different perturbation

In this section, we discuss several different perturbations in swarms. Supposed that theorem 3 is satisfied for all perturbations.

3.1. constant perturbation (i.e., $\mathbf{p(x^i) = p}$ is constant vector)

E.q. (3) is described by

$$\dot{x}^i = \sum_{j=1, j\neq i}^{M} g(x^i - x^j) + p. \tag{6}$$

From the $\dot{V}_i$ equation we obtain

$$\begin{aligned}\dot{V}_i &= (x^i-\bar{x})(\dot{x}^i-\dot{\bar{x}})^T = (x^i-\bar{x})\left(\sum_{j=1,j\neq i}^{M} g(x^i-x^j)+p-p\right)\\ &= (x^i-\bar{x})\left(\sum_{j=1,j\neq i}^{M} g(x^i-x^j)\right)\\ &\leq -aM\|x^i-\bar{x}\|^2+b(M-1)\sqrt{\frac{c}{2}}\exp(-\frac{1}{2})\|x^i-\bar{x}\|.\end{aligned}$$

We can achieve the same results as theorem 1, namely: as long as

$$t>\max_i\left\{-\frac{1}{2a}\ln\frac{A^2}{2V_i(0)}\right\},$$

one get

$$\|x^i-\bar{x}\|\leq A,\ \ A=\frac{b}{a}\sqrt{\frac{c}{2}}exp(-\frac{1}{2}).$$

This result is natural, because each individual has the same constant perturbation. Only difference from system (1) is the center of swarm, which drifts here. We can see this fact below: From(6) and definition of swarms center, we know

$$\dot{\bar{x}}=\frac{1}{M}\sum_{i=1}^{M}\dot{x}^i=\frac{1}{M}\sum_{i=1}^{M}\sum_{j=1,j\neq i}^{M} g(x^i-x^j)+\frac{1}{M}\sum_{i=1}^{M}p=p \tag{7}$$

E.q.(7) implies that the center of swarms with the constant perturbation drifts with the constant velocity vector p.

3.2. Practical stability with line perturbation

E.q. (3) is described by

$$\dot{x}^i=\sum_{j=1,j\neq i}^{n} g(x^i-x^j)+a_l(x^i+b_l) \tag{8}$$

where $a_l\in R, b_l\in R, \|a_l\|\leq aM$. Defining the same Lyapunov function as $V_i=\frac{1}{2}\|x_i-\bar{x}\|^2$, we have

$$\begin{aligned}\dot{V}_i &= (x^i-\bar{x})(\dot{x}^i-\dot{\bar{x}})^T\\ &= (x^i-\bar{x})\Big(\sum_{j=1,j\neq i}^{M} g(x^i-x^j)+a_l(x^i+b_l)-a_l(\bar{x}+b_l)\Big)\\ &= (x^i-\bar{x})\Big(\sum_{j=1,j\neq i}^{M} g(x^i-x^j)+a_l(x^i-\bar{x})\Big)^T\end{aligned}$$

$$= (x^i - \bar{x})\Big(\sum_{j=1,j\neq i}^{M} g(x^i - x^j)\Big)^T + a_l\|(x^i - \bar{x})\|^2$$

$$\le -aM\|x^i - \bar{x}\|^2 + b(M-1)\sqrt{\frac{c}{2}}exp(-\frac{1}{2})\|x^i - \bar{x}\| + \|a_l\|\|x^i - \bar{x}\|^2$$

$$= (\|a_l\| - aM)\|x^i - \bar{x}\|^2 + b(M-1)\sqrt{\frac{c}{2}}exp(-\frac{1}{2})\|x^i - \bar{x}\|.$$

It implies that as long as $\|x^i - \bar{x}\| > -\frac{\tilde{b}}{\tilde{a}}$, we achieve $\dot{V}_i < 0$ where $\tilde{a} = \|a_l\| - aM$, $\tilde{b} = b(M-1)\sqrt{\frac{c}{2}}exp(-\frac{1}{2})$.

In order to get t_0, we consider inequality

$$\dot{V}_i(t) \le 2\tilde{a}V_i(t) + \tilde{b}\sqrt{2V_i(t)}. \tag{9}$$

From inequality (9), we can get

$$t \le \frac{1}{2\tilde{a}}\ln\frac{-2\sqrt{2}\tilde{b} + \sqrt{2\tilde{b}}}{-2\sqrt{V_i(0)}\tilde{a} + \sqrt{2\tilde{b}}}.$$

We get practical stability to system (8) with respect to Q, Q_0 and t_0 ,where

$$Q = \left\{x^i \Big| \|x^i - \bar{x}\| < -\frac{\tilde{b}}{\tilde{a}}\right\}, \quad t_0 = \max_i\left\{\frac{1}{2\tilde{a}}\ln\frac{-2\sqrt{2}\tilde{b} + \sqrt{2\tilde{b}}}{-2\sqrt{V_i(0)}\tilde{a} + \sqrt{2\tilde{b}}}\right\},$$

Q_0 is unrestrained.

3.3. Practical stability with exponent perturbation

We consider exponent perturbation described by:

$$\dot{x}^i = \sum_{j=1,j\neq i}^{M} g(x^i - x^j) + a_e exp\left(-\frac{\|x^i - \bar{x}\|^2}{b_e}\right) \tag{10}$$

where $b_e > 0$.

From the $\dot{V}_i$ equation we obtain

$$\begin{aligned}
\dot{V}_i &= (x^i - \bar{x})(\dot{x}^i - \dot{\bar{x}})^T \\
&= (x^i - \bar{x})\Big\{\sum_{j=1,j\neq i}^{M} g(x^i - x^j) + a_e exp\Big(-\frac{\|x^i - \bar{x}\|^2}{b_e}\Big) \\
&\quad -\frac{1}{M}a_e\sum_{i=1}^{M} exp\left(-\frac{\|x^i - \bar{x}\|^2}{b_e}\right)\Big\}^T \\
&\le -aM\|x^i - \bar{x}\|^2 + b(M-1)\sqrt{\frac{c}{2}}exp(-\frac{1}{2})\|x^i - \bar{x}\| + \frac{2(M-1)\|a_e\|}{M}
\end{aligned}$$

which implies that as long as

$$\|x^i - \bar{x}\| > \frac{\hat{b} + \sqrt{\hat{b}^2 + 4\hat{a}\hat{b}}}{2\hat{a}},$$

we have $\dot{V}_i < 0$ where

$$\hat{a} = aM,\ \hat{b} = b(M-1)\sqrt{\frac{c}{2}}exp(-\frac{1}{2}), \quad \hat{c} = \frac{2(M-1)\|a_e\|}{M}.$$

Denote $A = \frac{\hat{b}}{\hat{a}} + \sqrt{\frac{\hat{c}}{\hat{a}}}$ and note that $A > \frac{\hat{b} + \sqrt{\hat{b}^2 + 4\hat{a}\hat{b}}}{2\hat{a}}$. This implies that as $\|x^i - \bar{x}\| > A$, $\dot{V}_i < 0$ is satisfied.

In order to get t_0, note that for $\|x^i - \bar{x}\| > A$, we have

$$\dot{V}_i \leq -\frac{2\hat{a}\hat{b}\sqrt{\hat{a}\hat{c}}}{(\hat{b} + \sqrt{\hat{a}\hat{c}})^2} V_i \tag{11}$$

From inequality (11), we achieve

$$t \leq -\frac{(\hat{b} + \sqrt{\hat{a}\hat{c}})^2}{2\hat{a}\hat{b}\sqrt{\hat{a}\hat{c}}} \ln \frac{A^2}{2V_i(0)}.$$

We get practical stability to system (10) with respect to Q, Q_0 and t_0, where

$$Q = \{x^i | \|x^i - \bar{x}\| < A\}, \quad t_0 = \max_i \left\{ -\frac{(\hat{b} + \sqrt{\hat{a}\hat{c}})^2}{2\hat{a}\hat{b}\sqrt{\hat{a}\hat{c}}} \ln \frac{A^2}{2V_i(0)} \right\},$$

Q_0 is unrestrained.

4. Examples

In this section, we will provide some simulation examples to illustrate the theory developed in preceding sections. We chose an n=3 dimensional space for ease of visualization of results and used the region $[0,10]\times[0,10]\times[0,10]$ in the space. In all the simulations performed below we used $M = 10$ individuals. As parameters of the attraction/repulsion function $g(\cdot)$ in (2) we used $a = 0.1, b = 0.4, c = 50$ for most of the simulations. We performed simulations in this section.

Results of practical stability were obtained for all kinds of perturbations as shown in Table 1,where we chose parameters as p=[0.1 0.1 0.1],$\|a_l\| = 0.001$,$b_l = [0.5\ 0.2\ 0.1]$,$\|a_e\| = 0.5$,$b_e = 20$. One easily can see that the case, as expected, the swarm will move the region given wherever the initial position of individuals is. This fact is fit to definition of practical stability. Moreover, $\|x^i(t_0) - \bar{x}\|$ is much more smaller than A, which implied that A is conservative, because in the aforementioned proof, we enlarged $\dot{V}_i$. Therefore, A is, in general, much smaller than that above.

5. Conclusion

In this article, we developed a model of the swarms and analyzed its practical stability properties for different perturbations. The study to practical stability

Table 1. The simulation results to practical stability with 10 individuals in 3 dimensional space

10 individuals initial position	$\|x^i(0)-\bar{x}\|$	No P.B. $A=0.2426$ $t_0=32.170$ $\|x^i(0)-\bar{x}\|$	const P.B. $A=0.2426$ $t_0=32.170$ $\|x^i(0)-\bar{x}\|$	Line P.B. $A=0.2186$ $t_0=52.361$ $\|x^i(0)-\bar{x}\|$	Exp P.B. $A=1.3295$ $t_0=68.013$ $\|x^i(0)-\bar{x}\|$
[0.7962 8.1372 9.4524]	6.2732	0.0579	0.2632	0.1987	0.9585
[7.2092 4.6623 6.1327]	2.8026	0.0414	0.2178	0.2010	0.2345
[7.6491 7.2229 7.8293]	3.8930	0.0579	0.2178	0.1703	1.1315
[6.5794 9.9487 0.0351]	6.7000	0.0392	0.0825	0.0459	0.2345
[8.1041 3.6250 7.9696]	4.8267	0.1092	0.2178	0.0587	0.2345
[3.7424 7.3080 6.4182]	2.0792	0.0579	0.0825	0.2010	0.9890
[3.0623 6.4967 1.7848]	3.9782	0.0414	0.0825	0.1703	0.5952
[3.7070 6.8134 5.2940]	1.4484	0.0414	0.0825	0.1703	1.1315
[7.0675 0.0761 2.1874]	7.0687	0.0414	0.2178	0.0459	0.2345
[1.6837 6.5415 5.4805]	3.3131	0.0414	0.1325	0.0459	0.5952

in the swarms is a significant practical importance topic. The model here is a simple and possible future extensions of the work here could be done by more really model.

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