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Characteristic wave detection in ECG using complex-valued Continuous Wavelet Transforms

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Abstract

In this study the complex-valued continuous wavelet transform (CWT) has been applied in detection of Electrocardiograms (ECG) as response to various signal classification methods such as Fourier transforms and other tools of time frequency analysis. Experiments have shown that CWT may serve as a detector of non-stationary signal changes as ECG. The tested signal is corrupted by short time events. We applied CWT to detect short-time event and the result image representation of the signal has showed us that one can easily find the discontinuity at the time scale representation. Analysis of ECG signal using complex-valued continuous wavelet transform is the first step to detect possible changes and alternans. In the second step, modulus and phase must be thoroughly examined. Thus, short time events in the ECG signal, and other important characteristic points such as frequency overlapping, wave onsets/offsets extrema and discontinuities even inflection points are found to be detectable. We have proved that the complex-valued CWT can be used as a powerful detector in ECG signal analysis.

Key words : ECG analysis, Beat Detection, P-QRS-T waves, Wavelet Transforms.

Ⅰ. INTRODUCTION

The electrocardiogram (ECG) is an indirect measure of the electrical activity of the heart. The activity can be measured by placing leads on the surface of the skin. Cardiologists can use features of these signals to obtain important data about the clinical condition of their patients. These features are reflected by the morphology and duration of the individual waves of the ECG (P, QRS complex, and T waves). Thus detection of every component of the ECG signals can be an extra clinical sign and can be very informative with respect to the first manifestation of the signal. In fact, waveform detection is necessary to determine the heart rate, and several related arrhythmias such as Tachycardia, Bradycardia and Heart Rate Variation: it is also necessary for further processing of the signal in order to detect abnormal beats [1]. We consider the clinical criteria for determining the starting points and endpoints of the P[1] wave, QRS complex and T wave as very important. We also consider that in the body surface ECG lays more information.

Producing an algorithm for the detection of ECG signal characteristic points is a difficult problem due to the time-varying morphology of the signal subject to physiological conditions and the presence of noise. Recently, a number of wavelet-based techniques have been proposed to detect these features. Senhadji et al compared the ability of wavelet transform based on three different wavelets (Daubechies, Spline, and Morlet) to recognize and describe isolated cardiac beats [2]. Sahambi et al used a first-order derivative of the Gaussian function as a wavelet for the characterization of the ECG wave forms. They used modulus maxima-based wavelet analysis to detect and measure various parts of signal especially the location of the onset and offset of the QRS complex and P and T waves [3].Since the application of wavelet transformation in electrocardiology is relatively new fields of research and many methodological aspects of the wavelet technique will require further investigations in order to improve the clinical usefulness of this novel signal processing technique.

In this research we applied over complete complex-valued CWT which best suits for the analysis of ECG signals by complex nature of wavelet transforms. We propose the wavelet-based technique for the detection of signals using real-valued and complex valued wavelets. This technique exploits localization property of CWT where narrow signal elements are present across wide range of scales in time scale

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represented signal. So called maximum curves are generated using local maxima of the CWT modulus and tracks CWT modulus ridges. The method shows how modulus and phase of complex-valued CWT can be used in analysis of ECG signal characteristic waves. We demonstrated that complex-valued CWT can be used as a good technique to extract ECG signal characteristic points.

II. MATERIALS AND METHODS

A. Continuous Wavelet Transforms

The wavelet transform enables time-frequency representations of the signal, all with different resolutions: high resolution in time and low resolution in frequency for high frequencies and low resolution in time and high in frequency for low frequencies. The CWT does this by having a variable window width, which is related to the scale of observation. Any signal $f(t)$can be decomposed into a set of base functions $\psi_{s\tau}(t)$ which are called the wavelets. The Continuous Wavelet Transform is obtained by formula [5]:

$$\gamma(s,\tau)=\int_{-\infty}^{\infty} f(t)\psi^{*}_{s,\tau}(t)dt\,, \tag{1}$$

where * denotes complex conjugation. The variables s and τ denotes scale and translation. The wavelets are generated from a single base wavelet ψ, the so-called mother wavelet, by scaling and translation:

$$\psi_{s\tau}(t)=\frac{1}{\sqrt{s}}\psi\left(\frac{1-\tau}{s}\right) \tag{2}$$

In equation (2) s is the scale factor, τ is translation factor and the factor $\sqrt{s}$ is for energy normalization across different scales. Generally speaking $\gamma_{s\tau}(t)$ is obtained by the following process: the basic wavelet (with scale s = 1) is shifted along the signal $f(t)$ and for each value of time shifting τ the integral (1) is computed, then the wavelet window is stretched by factor s (the width of the wavelet window is increased s times) and again shifted along the signal. This process can be repeated over and over again. The larger scale the lower frequency components that are treated.

The advantage of CWT over other time-frequency transformations is that the CWT is not limited to using sinusoidal analysing functions. Rather, a large selection of localized wave forms can be employed as long as they satisfy predefined mathematical criteria. Coefficients of the CWT are denoted as $\gamma(s,\tau)$ for particular scale (s) and translation(τ). Scale can be treated as frequency, and translation as time, but considering that larger scales represent the lower frequencies.

Wavelets are basis functions used for expansion. They are characterized by a number of properties that determine their use in the frame of time-frequency localization. Formally, a real valued function $\psi(t)$ is called a wavelet if it satisfies two constraints defined by

$$\int_{-\infty}^{\infty}\psi(t)dt=0 \text{ and } \int_{-\infty}^{\infty}\psi^{2}(t)dt=1 \tag{3}$$

The first part of Eq.3states that the wavelet oscillates, the second part says the wavelet must be nonzero somewhere. The properties of wavelets may serve as a key for selection of function for a specific application. Briefly, while analysis needs even non-orthogonal wavelets, compression requires orthogonal and smooth wavelets. Filtering may require symmetrical functions and rational coefficients of filters corresponding to wavelets. The following properties are most discussed in literature: orthogonality, compact (finite) support, rational coefficients of corresponding filters, symmetry, smoothness , and analytic expression[8].

a) Real valued wavelets

The most used and/or discussed real-valued wavelets are: Haar wavelet, family of Daubechies wavelets, Morlet wavelet, Meyer wavelet, Mexican hat wavelet, family of Coiflet wavelets, family of Symlet wavelets, and biorthogonal wavelets. Time

Table 1. Time resolution, frequency resolution and time-frequency resolution of selected real-valued wavelets.

wavelet	Δ_t^2	Δ_w^2	$\Delta_t^2\,\Delta_w^2$
Morlet	0.7071	0.07081	0.5007
Gaussian No.2	0.7637	0.6889	0.5261
Meyer	0.8418	0.9824	0.8271
Daubechies No.2	1.540	9.424	14.51
Haar	0.5775	130.6	75.44

Theoretical minimum of $\Delta_t^2\,\Delta_w^2$ is 0.5.

Table 2. Time resolution, frequency resolution and time-frequency resolution of selected complex -valued wavelets. Theoretical minimum of $\triangle_t^2 \triangle_w^2$ is 0.5.

wavelet	$\triangle_t^2$	$\triangle_w^2$	$\triangle_t^2 \triangle_w^2$
cpx Morlet No.1-0.5 (real or imaginary part)	0.3533	1.416	0.5006
cpx Kingsbury (real part)	1.552	3.418	5.306
cpx Kingsbury (imaginary part)	1.565	3.681	5.765
cpx Daubechies No.6. (real part)	2.526	3.066	7.749
cpx Daubechies No.6 (imaginary part)	2.647	3.363	8.904

and frequency resolution of various wavelets differ. The ideal resolution value is represented by an equality curve $\triangle_t^2 \triangle_w^2 = 0.5$. Results for all wavelets lay right and above the equality curve. The closer to the equality curve, the better time resolution, frequency resolution, or both resolutions are. The results for selected wavelets are summarized in Table1 [5].

b) Complex valued wavelets

The most used and/or discussed complex-valued wavelets are: Complex Gaussian wavelets, Complex Daubechies wavelets, Complex Kingsbury wavelet, Complex Morlet wavelets, Complex Frequency B-spline wavelets, Complex Shannon wavelets. Time and frequency resolution of various complex-valued wavelets differ too. The results for selected wavelets are summarized in Table 2.

Complex-valued wavelet transform plays a special role in signal analysis. Complex nature of wavelets provides further improvement in signal detection compared to real-valued wavelet analysis. This is possible by using so called dual-tree processing through cross-correlation with real and imaginary parts of wavelets. The resulted complex-valued time-frequency image (CWT) can be further analyzed by detection of significant attributes in its modulus and phase. In this way, not only the waves can be detected but also various shapes of the waves can be distinguished.

B. ECG characteristic wave detection algorithm

The algorithm presented in this section is applied directly at one run over the whole digitized ECG signal which we acquired using from subject. There are actually four separate algorithms, each of which is designed to extract certain features of the ECG signal. The description of the ECG wave detection algorithm is shown in Fig.1.

The first, the peak of the *QRS complex* with its high dominated amplitude in the signal is detected. Then *Q* and *S* waves are detected.

The Zero voltage level of the signal is found. P and T waves along with their onsets and offsets are the last things to be detected.

Ⅲ. RESULTS AND DISCUSSION

In this paper, we also mention best basis wavelet functions that are well suited for detecting and localization of important ECG events. This is an important point to be discussed how to choose the mother functions to be compared with the signal. In principle, the wavelet function should have a certain shape that we would like to localize in the original signal. However, due to mathematical restrictions, not every function can be used as a wavelet. Then, one criterion for choosing the wavelet function is that *"it looks similar"* to the patterns of the original signal. In respect, our choice is motivated by the shape of the waveforms to be detected in the ECG signal. These wavelet functions are Morlet and complex Morlet No.1-05 and complex Gaussian No.1-05. We will see the performance of these wavelets on ECG wave detection in this section.

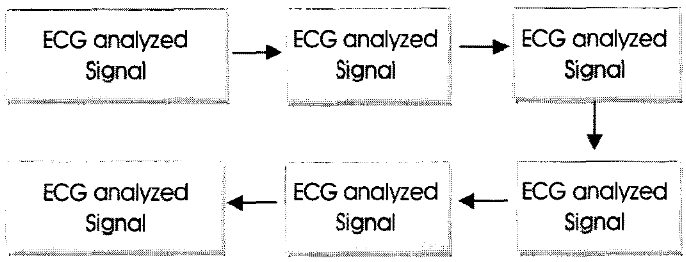

Fig. 1. The flowchart of the proposed detection algorithm

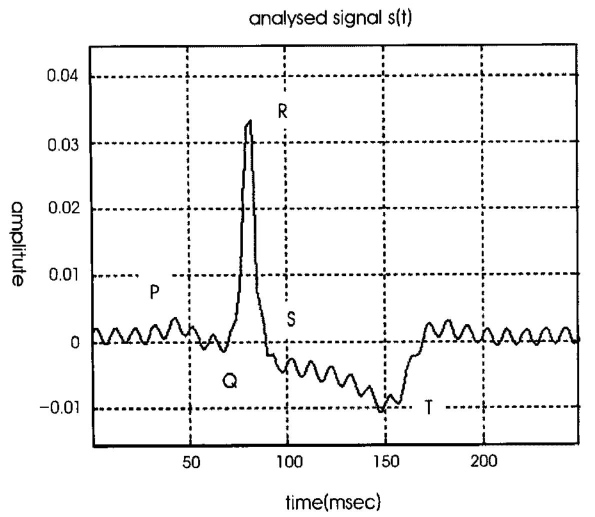

Fig. 2. The analyzed ECG signal and its segments with three main waves denoted by R, QRS-complex and T.

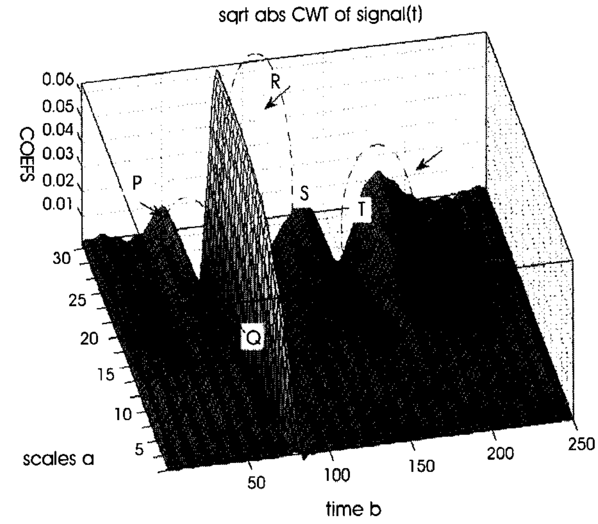

Fig. 3. CWT 3D time scale representation of the analyzed signal using Morlet wavelet.

We use a typical example of a discrete-time biological (ECG) signal as shown in Fig.2 in our experiment. The signal is sampled at rate of $f = 500\text{Hz}$. One can see the signal has relatively flat segments that alternate with low-amplitude and high amplitude waves of different frequency contents. Further the signal contains small colour (13) noise over the whole signal curve (signal-to- noise ratio is relatively high). From a physiological point of view, the test signal represents a recording of heart electrical activity during a single heart cycle. The cycle is composed of *P-wave, QRS* - complex, *ST-segment, T-wave*, other minor waves and segments. The described ECG signal has been chosen for demonstration of wavelet real-valued and complex-valued wavelet analysis for easier discussion on the application of complex-valued CWT.

The three main waves/peaks can be located in time. The first wave is located at around $t = 400msec$(small +/- bipolar wave marked by *P* in Fig.2, the second one is at around $t = 85msec$(high +/- bipolar wave marked by *QRS*), and the last one at around $t - 145msec$ (high negative wave marked by *T*). These characteristic waves will be analyzed in the following part.

In the first step we analyzed the signal by applying complex-valued continuous wavelet transform to detect possible changes and alternans of ECG signal. As the second step, modulus and phase has been thoroughly examined. Thus, the main signal wave can be detected and the maxima, minima, or even inflection points found as detectable with wave onsets and offsets.

We start analyzing the signal by using CWT of Morlet wavelet. The CWT time frequency plane shown as 3D plot of absolute value of the wavelet coefficients is depicted in Fig.3. One can easily find the three wave components mentioned above *P, QRS* and *T* waves. The resulted time frequency plane shows that significant waves are represented by peaks that have ripple nature. This is caused by localization property of the wavelet transform and follows character of the analysis wavelet. In addition we can see that the time events (waves) can be easily located in the time-frequency domain and the events can further be well bounded. The other characteristic points of the signal can be detected such as inflection and maxima and minima points represented in wavelet coefficients. The output result demonstrates the powerful capability of continuous wavelet transform to extract several characteristics at once.

Next we present the abilities of complex-valued CWT to analyse the signal. The analyzed signal has not been filtered to remove unwanted noise due to the possible damage of useful signal components. For the demonstrations of the wavelet transform the analyzed signal has been artificially corrupted by a short-time event - a discontinuity in first derivative at $t = 132msec$. Such discontinuity can hardly be seen in the time domain without further processing. The signal has been transformed using the complex Morlet wavelet No.1-0.5.

As the CWT promises to detect short-time events regardless their frequency contents, we should obtain significant differences between resulting time-frequency images. The images are complex-valued. Thus possibilities to visualize the results are boarder than in the real-valued CWT case. First, the modulus has been computed and visualized as a 2D shaded contour plot with increased dynamics by applying square root

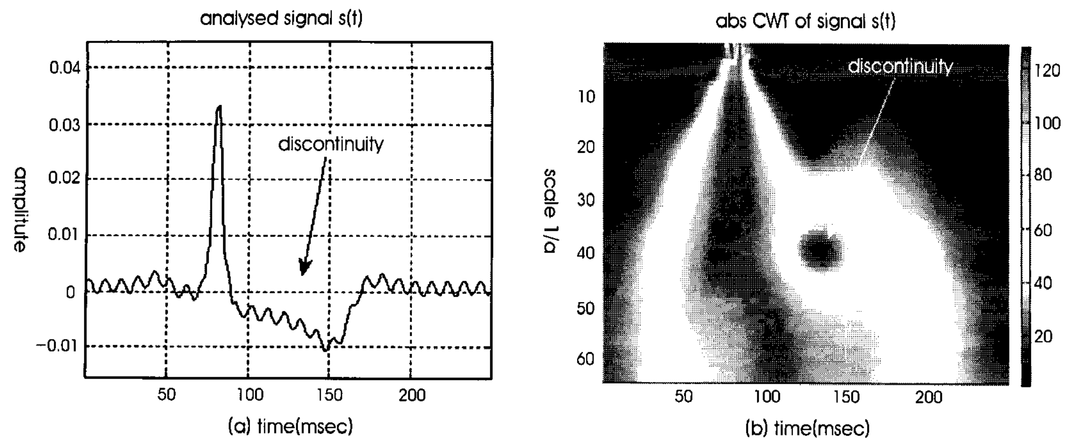

Fig. 4. (a) Signal $s(t)$ with artificial discontinuity in first derivative and (b) its modulus of CWT using complex Morlet wavelet No.1-0.5

(see Fig.4 (b)). Studying the modulus of CWT output depicted at time around $t = 132msec$ one can easily find a circular object located at scales $a = 35 - 45$ (mid-frequencies). Although the object is low in value, it is detectable with relatively good time resolution. This is also very important information to extract when studying the ECG signals.

The output of the complex-valued wavelet analysis can be presented in another way. Modulus and phase of the CWT may be replaced by a real part and imaginary part of the CWT. Thus the complex-valued CWT is computed via the dual-tree algorithm. This corresponds to pure CWT analysis by two different wavelets (a real part and an imaginary part of the same complex-valued wavelet).

The results, absolute value of the real part and the imaginary part of the complex-valued CWT is shown in Fig.5 using Gaussian No.3 wavelet. One can see differences between the real and imaginary part of the CWT. This is caused by different shape of the real and the imaginary part of the used wavelet. The used Gaussian No.3 wavelet is displayed in Fig.6.We can see the difference of the shape of the wavelet

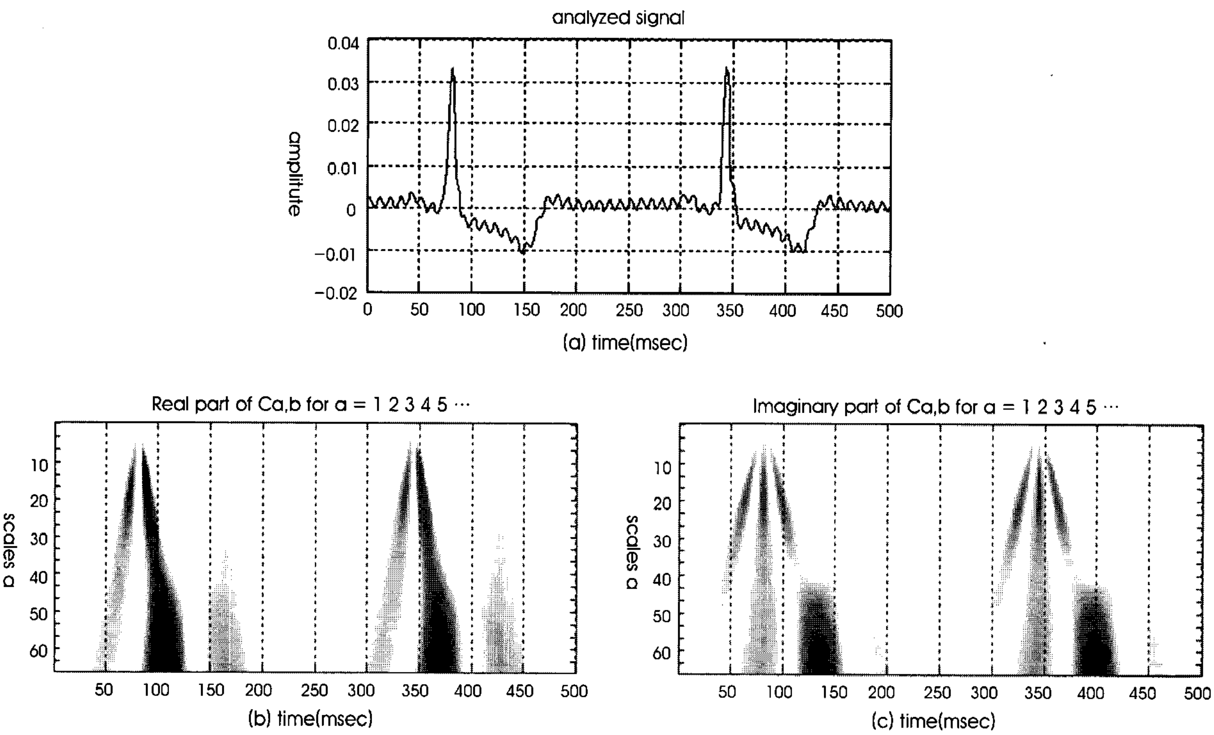

Fig. 5. Absolute value of real (b) and imaginary part (c) ofCWT from two cycle ECG signal (a) using complex Gaussian wavelet No.3.

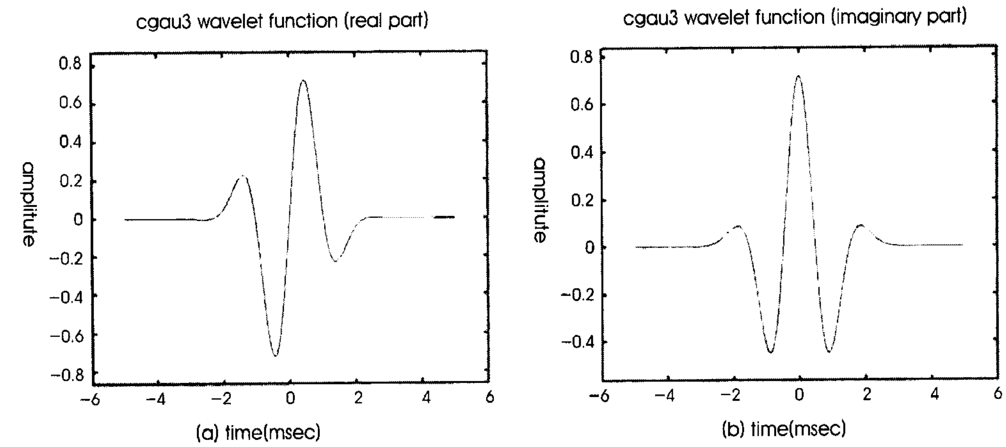

Fig. 6. (a) Real part and (b) imaginary part of complex Gaussian wavelet No.3

basis functions which yields different results in the CWT output representation.

The difference between modulus, the real part and the imaginary part of the CWT are below discussed on detail of the analyzed signal (Fig.7 (a)).

Let's take the same segment as in real-valued analysis case: 85 msec to 150msec in Fig.7. (a). The important wave is the T-wave (marked by circle above) at time around$t = 150msec$. Modulus of time-frequency image (see Fig.7.(b)) displays this wave as a single peak centered at $t = 155msec$ and $a = 21$. The peak monotonically decays to all directions. The slowest decay is along scale axis towards low scales (high frequencies).

The real part of the CWT (Fig.8 (a)) displays the wave T similarly as a peak of almost same time duration and same frequency contents. The only difference is that the peak is composed of three "bumps" spread along the scale axis. The middle "bump"is located at the inflection point of the wave at time $t = 155msec$. The location is equal to location of the center of the peak in modulus of the CWT. The reason is that both the wave and the real part of the wavelet have anti-symmetrical shape. CWT provides correlation between the signal and the wavelet that naturally results in a peak located at the center of the wave being detected.

The imaginary part of the CWT (Fig.8 (b)) displays the wave T similarly as the real part of the CWT. The only difference is that the center of the wave being detected is in between two "bumps". The reason is that the imaginary part of the wavelet is symmetrical contrary to the wave shape.

Phase of the CWT is sensitive to different events than modulus of the CWT. Therefore, different time stamps have been chosen. Panel (a) of Fig.9 contains four time stamps at $t = 45msec$, $t = 85msec$, $150msec$, and $175msec$, marked by four vertical dotted lines. The stamps stand for the following significant points: the wave P inflection point, the

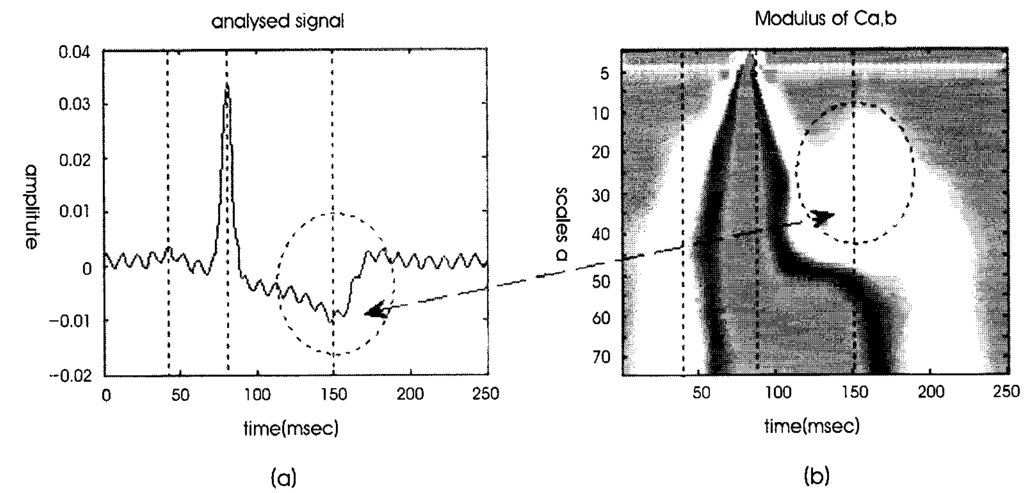

Fig. 7. (a) Detail of the ECG signal, and (b) its modulus of CWT using the complex Gaussian wavelet No.3.

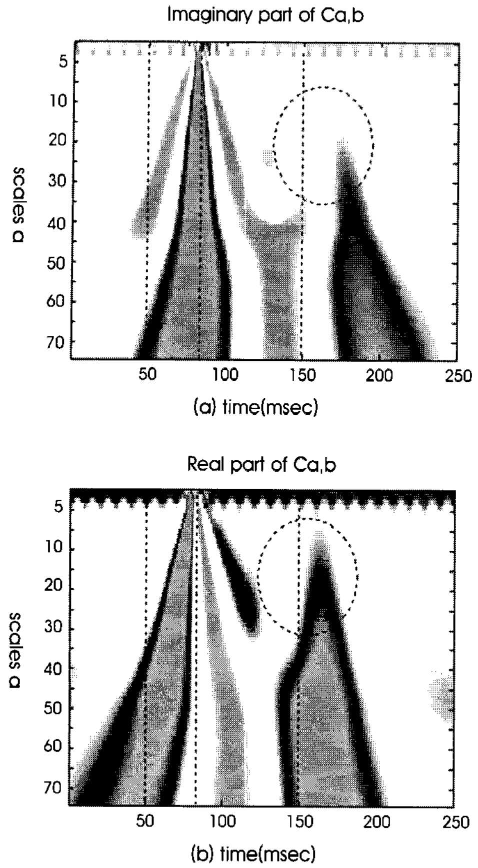

Fig. 8. (a) Real part, and (b) imaginary part of CWT of the ECG signal from the complex Gaussian wavelet No.3

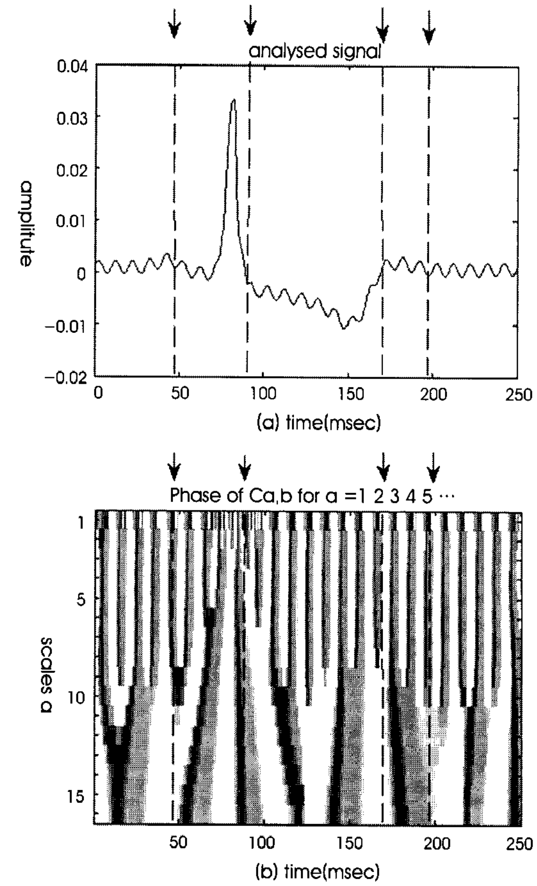

Fig. 9. (a) Detail of the ECG signal and (b) its phase of CWT using the complex Gaussian wavelet No.3.

wave *QRS peak*, the wave *T*, and its inflection point. The same stamps are shown in the panel (b).We can see that number of parameters can be observed in the CWT images/plots: presence and position of the peaks slope of the peaks in various directions, etc.

Detection of waves and short-time events is an important part of ECG signal analysis. Thus, the signal can be examined to find differences from a reference signal, track long-term trends, and multiple time overlapping and/or frequency overlapping changes. Traditional time-domain and frequency-domain detection methods are based on correlation and cross-correlation, coherence, cross-spectra, cepstra, and many other signal processing tools. Time-frequency approach exploits expansion on series to decompose the signal into multiple frequency bands. Further, time and frequency resolution can be individually changed in the bands and thus the analysis algorithm can be adapted to the discontinuity being detected.

In this paper we demonstrated the detection ability of

Table 3. Validation results for the proposed ECG detection algorithm applied to six records from the MIT-BIH.

ECG record number	Beats	False Positive	False Negative	False detections	
				Beats	%
100	2272	0	1	1	0.04
101	1864	0	1	1	0.05
102	2187	0	0	0	0
103	2084	17	4	21	0.9
104	2229	0	10	13	0.7
105	2571	0	1	1	0.04
Total	13207	30	19	49	0.28

complex-valued CWT transforms in analysis of ECG signal by comparing to the real valued case. Together they can be used as a good detector algorithm for diagnosing the normal and abnormal ECG characteristic waves.

As the final step, the overall performance of the algorithm has been tested on six 30min recordings from the MIT-BIH arrhythmia database [10], in where only channel 1 of the two-channel ECG recordings was used. The selected recordings included noise bursts, baseline drifts and movement artifacts. Table 3 shows the detection performance of the algorithm for QRS waveform. In results we achieved 49 false detections where 30 of them were false positives and 19 were false negatives. The table 3 shows us only false detections since it's also important to decrease the number of false detections of the ECG waves. Here false positive defines the detection of noise as QRS wave while false negative means the number of misses of real waveforms. We can evaluate the false detection in percentage where we achieved0.28% which is very low in value. Thus the proposed work achieves the accuracy of true detections almost for 98% for the case of QRS wave. We have tested the algorithm in other ECG waves which resulted in almost same accuracy in detection.

Ⅳ. CONCLUSION

Behavior of the complex-valued continuous wavelet transform (CWT) as response to various signal types has been discussed in this work. Experiments have shown the complex-valued CWT can be used for detection of waves representing the characteristic points of the signal. Studying CWT of the artificially corrupted signals, one can find detectable characteristic points in modulus as well as phase of CWT. The modulus reveals the differences as additional peaks in its image. Although the differences are small in value, they change shape of original peaks in modulus image or they generate separated peaks. The phase responses even more sensitively regardless the noise wave amplitude. Any new signal component is revealed as new phase step along time axis. Analysis of signals using complex-valued continuous wavelet transform is the first step to detect possible changes or alternans. In the second step, modulus and phase must be thoroughly tested. The proposed method can be used as an alternative approach in analyzing ECG signals with good detection ability.

REFERENCES

[1] Lilly LS, *Pathophysiology of Heart Disease.* 3rd Ed. Philadelphia, Lippincott Williams & Wilkins, pp.57-90, 2003.

[2] L. Senhadji, G. Carrault, et al."Comparing Wavelet Transforms for Recognizing Cardiac Patterns", *IEEE Eng Med Biol*, vol.14, no.2, pp. 167-173, May 1995.

[3] Sahambi J.S, Tandon S.M and Bhatt R. K. P, "Using wavelet transforms for ECG characterization: An on-line digital signal processing system", *IEEE Eng. Med. Biol.* vol.16, no.1,pp.7783, Jan,1997.

[4] Addison P.S., Watson J.N. et al. "Evaluating arrhythmias in ECG signals using wavelet transforms", *IEEE Eng. Med. Biol.*, vol.19. pp.104-109. September/October 2000.

[5] S G Mallat, "A Theory for Multiresolution Signal Decomposition: The Wavelet Representation", *IEEE Trans. Pattern Reco. and Machine Int.*, vol.11,no.7, pp.674-693, 1989.

[6] I. W. Selesnick, "The double-density dual-tree DWT," *IEEE Trans. Signal Proc.* 52, pp. 13041314, May. 2004.

[7] K. Dubowik, and J. E. Larsson, "ECG Features Extraction - QRS Detection and Shape Recognition", *Journal of Applied Computer Science,* vol. 9, no. 1, pp 7-21, June 2002..

[8] S. Mallat and W. L. Hwang. "Singularity detection and processing with wavelets", *IEEE Trans. Inf. Theory*, vol.38, no.2, pp. 617643, March 1992.

[9] S. Mallat, "A theory for multiresolution signal decomposition: the wavelet representation", *IEEE Trans. Patt. Anal. Machine Intell.*, vol.11, no.7, pp. 674693, July 1989

[10] G.B. Moody and R.G. Mark, "The MIT-BIH arrhythmia database on CD-ROM and software for use with it", *in Computers in Cardiology, IEEE Computer Society Press*, pp.185-188, 1990.