Bull. Korean Math. Soc. **45** (2008), No. 2, pp. 207–220

SUPRA FUZZY CONVERGENCE OF FUZZY FILTERS

A. A. Ramadan and A. A. Abd El-latif

Abstract. We introduce and study the notions of supra fuzzy convergence of fuzzy filters, $s\gamma$-fuzzy open (closed) sets, $s\gamma(s\gamma^*)$-fuzzy continuous and $s\gamma$-fuzzy open mapping. Also, we investigate some of fundamental properties of these notions.

1. Introduction and preliminaries

Šostak [14], introduce the fundamental concept of a fuzzy topological structure as an extension of both crisp topology and Chang's fuzzy topology [1], in the sense that not only the object were fuzzified, but also the axiomatics. In [15, 16] Šostak gave some rules and showed how such an extension can be realized. Chattopdhyay et al. [2, 3] have redefined the similar concept. In [11] Ramadan gave a similar definition namely "Smooth fuzzy topology" for lattice $L = [0, 1]$, it has been developed in many direction [4, 6, 7, 9]. Ramadan [12], introduce the concept of smooth filter structures in the framework of smooth topology and he establish some of their properties. Also, Ramadan et al. [13] introduce the concept of convergence of smooth fuzzy filter in smooth fuzzy topological spaces. In this paper we introduce and study the notions of supra fuzzy convergence of fuzzy filters, $s\gamma$-fuzzy open (closed) sets, $s\gamma(s\gamma^*)$-fuzzy continuous and $s\gamma$-fuzzy open mapping. Also, we investigate some of fundamental properties of these notions.

Throughout this paper, let X be a nonempty set, $I = [0, 1]$, $I_0 = (0, 1]$ and I^X denote the set of all fuzzy subsets of X. A fuzzy point x_t for $t \in I_0$ is an element of I^X such that, for $y \in X$,

$$x_t(y) = \begin{cases} t & \text{if } y = x, \\ 0 & \text{if } y \neq x. \end{cases}$$

The set of all fuzzy points in X is denoted by $Pt(X)$. A fuzzy point $x_t \in \lambda$ if and only if $t \leq \lambda(x)$ [10].

Received May 6, 2005.

2000 *Mathematics Subject Classification.* 54A40.

Key words and phrases. fuzzy topological space, supra fuzzy topology, fuzzy filter, supra fuzzy convergence, $s\gamma$-fuzzy continuous.

Definition 1.1 ([11]). A mapping $\tau : I^X \longrightarrow I$ is called fuzzy topology on X if it satisfies the following conditions:

(O1) $\tau(\underline{0}) = \tau(\underline{1}) = 1$.

(O2) $\tau(\mu_1 \wedge \mu_2) \geq \tau(\mu_1) \wedge \tau(\mu_2)$ for any $\mu_1, \mu_2 \in I^X$.

(O3) $\tau(\bigvee_{i\in J} \mu_i) \geq \bigwedge_{i \in J} \tau(\mu_i)$ for any $\{\mu_i : i \in J\} \subseteq I^X$.

The pair (X, τ) is called fuzzy topological space (briefly, fts).

Theorem 1.1 ([3]). *Let (X, τ) be a fts. Then, for each $\lambda \in I^X$ and $r \in I_0$ we define an operator $C_\tau : I^X \times I_0 \longrightarrow I^X$ as follows:*

$$C_\tau(\lambda, r) = \bigwedge\{\mu : \lambda \leq \mu, \tau(\underline{1} - \mu) \geq r\}.$$

For each $\lambda, \mu \in I^X$ and $r, s \in I_0$ the operator C_τ satisfies the following conditions:

(i) $C_\tau(\underline{0}, r) = \underline{0}$.

(ii) $\lambda \leq C_\tau(\lambda, r)$.

(iii) $C_\tau(\lambda, r) \vee C_\tau(\mu, r) = C_\tau(\lambda \vee \mu, r)$.

(iv) $C_\tau(\lambda, r) \leq C_\tau(\lambda, s)$ *if* $r \leq s$.

(v) $C_\tau(C_\tau(\lambda, r), r) = C_\tau(\lambda, r)$.

Theorem 1.2 ([8]). *Let (X, τ) be a fts. Then, for each $\lambda \in I^X$ and $r \in I_0$ we define an operator $I_\tau : I^X \times I_0 \longrightarrow I^X$ as follows:*

$$I_\tau(\lambda, r) = \bigvee\{\mu : \mu \leq \lambda, \tau(\mu) \geq r\}.$$

For each $\lambda, \mu \in I^X$ and $r, s \in I_0$ the operator I_τ satisfies the following conditions:

(i) $I_\tau(\underline{1} - \lambda, r) = \underline{1} - C_\tau(\lambda, r)$ *and* $C_\tau(\underline{1} - \lambda, r) = \underline{1} - I_\tau(\lambda, r)$.

(ii) $I_\tau(\underline{1}, r) = \underline{1}$.

(iii) $I_\tau(\lambda, r) \leq \lambda$.

(iv) $I_\tau(\lambda, r) \wedge I_\tau(\mu, r) = I_\tau(\lambda \wedge \mu, r)$.

(v) $I_\tau(\lambda, r) \geq I_\tau(\lambda, s)$ *if* $r \leq s$.

(vi) $I_\tau(I_\tau(\lambda, r), r) = I_\tau(\lambda, r)$.

Definition 1.2 ([5]). A mapping $\tau : I^X \longrightarrow I$ is called supra fuzzy topology on X if it satisfies the following conditions:

(S1) $\tau(\underline{0}) = \tau(\underline{1}) = 1$.

(S2) $\tau(\bigvee_{i\in J} \mu_i) \geq \bigwedge_{i\in J} \tau(\mu_i)$ for any $\{\mu_i : i \in J\} \subseteq I^X$.

The pair (X, τ) is called supra fuzzy topological space (briefly, sfts).

Let τ^* be supra fuzzy topology. Then τ^* ia called the supra fuzzy topology associated with a fuzzy topology τ if $\tau \leq \tau^*$.

Definition 1.3 ([11]). Let (X, τ_1) and (Y, τ_2) be fts's and let τ_1^* and τ_2^* be associated supra fuzzy topologies with τ_1 and τ_2 respectively. Then the mapping $f : X \to Y$ is called fuzzy continuous (resp. supra fuzzy continuous) if $\tau_1(f^{-1}(\mu)) \geq \tau_2(\mu)$(resp. $\tau_1^*(f^{-1}(\mu)) \geq \tau_2^*(\mu)$) for each $\mu \in I^Y$.

Definition 1.4 ([12]). A mapping $\mathcal{F}: I^X \longrightarrow I$ is called fuzzy filter on X if it satisfies the following conditions:

(F1) $\mathcal{F}(\underline{0}) = 0$.

(F2) $\mathcal{F}(\lambda \wedge \mu) \geq \mathcal{F}(\lambda) \wedge \mathcal{F}(\mu)$ for each $\lambda, \mu \in I^X$.

(F3) If $\lambda \leq \mu$, $\mathcal{F}(\lambda) \leq \mathcal{F}(\mu)$.

A fuzzy filter is said to be proper if $\mathcal{F}(\tilde{1}) = 1$.

If $\mathcal{F}_1$ and $\mathcal{F}_2$ are fuzzy filters on X, we say $\mathcal{F}_1$ is finer than $\mathcal{F}_2$ (or $\mathcal{F}_2$ is coarser than $\mathcal{F}_1$), denoted by $\mathcal{F}_2 \leq \mathcal{F}_1$, if and only if $\mathcal{F}_2(\lambda) \leq \mathcal{F}_1(\lambda)$ for all $\lambda \in I^X$.

Theorem 1.3 ([13]). *Let $\mathcal{F}$ and $\mathcal{G}$ be proper fuzzy filters on X satisfying the following condition:*

(C) *If $\lambda_1, \lambda_2 \in I^X$ with $\mathcal{F}(\lambda_1) > 0$ and $\mathcal{G}(\lambda_1) > 0$, we have $\lambda_1 \wedge \lambda_2 \neq \underline{0}$.*

Define a mapping $\mathcal{F} \vee \mathcal{G} : I^X \to I$ as

$$\mathcal{F} \vee \mathcal{G}(\lambda) = \bigvee\{\mathcal{F}(\lambda_1) \wedge \mathcal{G}(\lambda_2) : \lambda = \lambda_1 \wedge \lambda_2\}.$$

Then $\mathcal{F} \vee \mathcal{G}$ is the coarsest proper fuzzy filter which is finer than $\mathcal{F}$ and $\mathcal{G}$.

Theorem 1.4 ([13]). *Let a mapping $f : X \to Y$ and $\mathcal{F}$ a fuzzy filter on X. We define a mapping $f(\mathcal{F}) : I^Y \to I$ as:*

$$f(\mathcal{F})(\mu) = \mathcal{F}(f^{-1}(\mu)).$$

Then $f(\mathcal{F})$ is a fuzzy filter on Y.

2. Supra fuzzy filter convergence

Theorem 2.1. *Let (X, τ) be sfts and $x_t \in Pt(X)$. Define $S_{x_t} : I^X \longrightarrow I$ by*

$$S_{x_t}(\lambda) = \begin{cases} \bigvee\{\tau(\nu_i) : \wedge_{i=1}^n \nu_i \leq \lambda\}, & \text{if } \ x_t \in \nu_i \\ 0, & \text{otherwise.} \end{cases}$$

Then S_{x_t} is a fuzzy filter on X, we call it the supra neighborhood fuzzy filter at x_t.

Proof. (F1) is easy.

(F2) Suppose that there exist $\lambda_1, \lambda_2 \in I^X$ and $r \in I_0$ such that

$$S_{x_t}(\lambda_1 \wedge \lambda_2) < r \leq S_{x_t}(\lambda_1) \wedge S_{x_t}(\lambda_2).$$

Since $S_{x_t}(\lambda_1) \geq r$ and $S_{x_t}(\lambda_2) \geq r$ and by definition of S_{x_t} there exist $\nu_i, \mu_j \in I^X$, $i = 1, 2, \ldots, n$, $j = 1, 2, \ldots, m$ such that

$$\wedge_{i=1}^n \nu_i \leq \lambda_1, \quad x_t \in \nu_i, \quad \tau(\nu_i) \geq r, \quad i = 1, 2, \ldots, n$$

and

$$\wedge_{j=1}^m \mu_j \leq \lambda_2, \quad x_t \in \mu_j, \quad \tau(\mu_j) \geq r, \quad j = 1, 2, \ldots, m.$$

Then, $\wedge_{i=1}^n \nu_i \wedge \wedge_{i=1}^m \mu_j \leq \lambda_1 \wedge \lambda_2$ and since for each $i = 1, 2, \ldots, n$, $j = 1, 2, \ldots, m$, $x_t \in \nu_i$, $x_t \in \mu_j$, $\tau(\nu_i) \geq r$ and $\tau(\mu_j) \geq r$ we have $S_{x_t}(\lambda_1 \wedge \lambda_2) \geq r$.

It is a contradiction. Hence, $S_{x_t}(\lambda_1 \wedge \lambda_2) \geq S_{x_t}(\lambda_1) \wedge S_{x_t}(\lambda_2)$ for each $\lambda_1, \lambda_2 \in I^X$.

(F3) Let $\lambda_1, \lambda_2 \in I^X$ such that $\lambda_1 \leq \lambda_2$. Suppose that there exists $r \in I_0$ such that

$$S_{x_t}(\lambda_1) \geq r > S_{x_t}(\lambda_2).$$

Since $S_{x_t}(\lambda_1) \geq r$, and by definition of S_{x_t}, there exist $\nu_i \in I^X$, $i = 1, 2, \ldots, n$ such that

$$\wedge_{i=1}^{n} \nu_i \leq \lambda_1 \leq \lambda_2, \quad x_t \in \nu_i, \quad \tau(\nu_i) \geq r, \quad i = 1, 2, \ldots, n.$$

Then $S_{x_t}(\lambda_2) \geq r$. It is a contradiction. Hence, $S_{x_t}(\lambda_1) \leq S_{x_t}(\lambda_2)$. □

Definition 2.1. Let (X, τ) be a sfts and let $\mathcal{F}$ be a fuzzy filter on X. We say that $\mathcal{F}$ is supra fuzzy converges to $x_t \in Pt(X)$ if $\mathcal{F}$ is finer than the supra neighborhood fuzzy filter S_{x_t}.

Definition 2.2. Let (X, τ) be a sfts and let $\mathcal{F}$ be a fuzzy filter on X. For, $r \in I_0$ we say that $x_t \in Pt(X)$ is r-supra fuzzy cluster point of $\mathcal{F}$ if for every $\lambda, \mu \in I^X$ with $x_t \in \lambda$, $\tau(\lambda) \geq r$ and $\mathcal{F}(\mu) \geq r$, we have $\lambda \wedge \mu \neq \underline{0}$.

Definition 2.3. Let (X, τ) be a sfts and let $\mathcal{F}$ be a fuzzy filter on X. For, $r \in I_0$ we say that $x_t \in Pt(X)$ is r-supra fuzzy strong cluster point of $\mathcal{F}$ if for every $\lambda, \mu \in I^X$ with $S_{x_t}(\lambda) \geq r$ and $\mathcal{F}(\mu) \geq r$, we have $\lambda \wedge \mu \neq \underline{0}$.

Remark 2.1. Every r-supra fuzzy strong cluster point of a fuzzy filter is also r-supra fuzzy cluster point but the converse is not true in general as the following example shows.

Example 2.1. Let $X = \{x, y, z\}$ be a set. Define $\lambda_1, \lambda_2 \in I^X$ as follows:

$$\lambda_1(x) = 0.8 \qquad \lambda_1(y) = 0.5 \qquad \lambda_1(z) = 0.0$$

$$\lambda_2(x) = 0.8 \qquad \lambda_2(y) = 0.0 \qquad \lambda_2(z) = 0.5.$$

We define a supra fuzzy topology $\tau : I^X \to I$ as follows:

$$\tau(\lambda) = \begin{cases} 1, & \text{if } \lambda = \underline{0}, \underline{1} \\ 0.5, & \text{if } \lambda = \lambda_1 \vee \lambda_2 \\ 0.3, & \text{if } \lambda \in \{\lambda_1, \lambda_2\} \\ 0, & \text{otherwise.} \end{cases}$$

Let $t \leq 0.8$ and $0 < r < 0.3$. Then

$$S_{x_t}(\lambda) = \begin{cases} 1, & \text{if } \lambda = \underline{1} \\ 0.5, & \text{if } \lambda_1 \vee \lambda_2 \leq \lambda < \underline{1} \\ 0.3, & \text{if } \lambda_1 \leq \lambda < \lambda_1 \vee \lambda_2 \quad \text{or} \quad \lambda_2 \leq \lambda < \lambda_1 \vee \lambda_2 \\ 0.3, & \text{if } \lambda_1 \wedge \lambda_2 \leq \lambda < \lambda_1 \quad \text{or} \quad \lambda_1 \wedge \lambda_2 \leq \lambda < \lambda_2 \\ 0, & \text{otherwise.} \end{cases}$$

Define a fuzzy filter $\mathcal{F} : I^X \to I$ as follows:

$$\mathcal{F}(\lambda) = \begin{cases} 1, & \text{if } \lambda = \underline{1} \\ 0.6, & \text{if } \chi_{\{y,z\}} \le \lambda < \underline{1} \\ 0, & \text{otherwise.} \end{cases}$$

Then x_t is r-supra fuzzy cluster point of a fuzzy filter $\mathcal{F}$ but it is not r-supra fuzzy strong cluster point of $\mathcal{F}$. Since, $S_{x_t}(\lambda_1 \wedge \lambda_2) = 0.3 > r$ and $\mathcal{F}(\chi_{\{y,z\}}) = 0.6 > r$ but $(\lambda_1 \wedge \lambda_2) \wedge \chi_{\{y,z\}} = \underline{0}$.

Theorem 2.2. *Let (X, τ) be a sfts and let $\mathcal{F}$ be a fuzzy filter on X. Then $\mathcal{F}$ has $x_t \in Pt(X)$ as r-supra fuzzy strong cluster point if and only if there is a finer fuzzy filter $\mathcal{G}$ than $\mathcal{F}$ such that $\mathcal{G}$ supra fuzzy converges to x_t.*

Proof. If x_t is r-supra fuzzy strong cluster point of $\mathcal{F}$, then for each $\lambda, \mu \in I^X$ with $S_{x_t}(\lambda) \ge r$ and $\mathcal{F}(\mu) \ge r$, we have $\lambda \wedge \mu \ne \underline{0}$. From Theorem 1.3, we can define

$$\mathcal{G} = S_{x_t} \vee \mathcal{F}$$

such that $S_{x_t} \le \mathcal{G}$ and $\mathcal{F} \le \mathcal{G}$. Thus $\mathcal{G}$ is supra fuzzy converges to x_t.

Conversely, if $S_{x_t} \le \mathcal{G}$ and $\mathcal{F} \le \mathcal{G}$, then for each $\lambda, \mu \in I^X$ and $r \in I_0$ with $S_{x_t}(\lambda) \ge r$ and $\mathcal{F}(\mu) \ge r$, we have $\mathcal{G}(\lambda) \ge r$ and $\mathcal{G}(\mu) \ge r$. Since $\mathcal{G}$ is a fuzzy filter, $\mathcal{G}(\lambda \wedge \mu) \ge \mathcal{G}(\lambda) \wedge \mathcal{G}(\mu) \ge r$, then $\lambda \wedge \mu \ne \underline{0}$. Thus x_t is r-supra fuzzy strong cluster point of $\mathcal{F}$. □

Theorem 2.3. *Let (X, τ_1) and (Y, τ_2) be sfts's and let $f : (X, \tau_1) \to (Y, \tau_2)$ be a supra fuzzy continuous mapping. Then we have the following statements*:

(i) $S_{f(x)_t}(\mu) \le S_{x_t}(f^{-1}(\mu))$ *for each* $\mu \in I^Y$.

(ii) *For every fuzzy filter $\mathcal{F}$ on X and $x_t \in Pt(X)$, if $\mathcal{F}$ supra fuzzy converges to x_t, then $f(\mathcal{F})$ supra fuzzy converges to $f(x)_t$ in Y.*

Proof. (i) Suppose that there exist $\mu \in I^Y$ and $r \in I_0$ such that

$$S_{f(x)_t}(\mu) \ge r > S_{x_t}(f^{-1}(\mu)).$$

Since $S_{f(x)_t}(\mu) \ge r$, there exist $\nu_i \in I^Y$ with $f(x)_t \in \nu_i$, $\tau_2(\nu_i) \ge r$, $i = 1, 2, \ldots, n$ such that $\wedge_{i=1}^n \nu_i \le \mu$. Then,

$$f^{-1}(\wedge_{i=1}^n \nu_i) = \wedge_{i=1}^n f^{-1}(\nu_i) \le f^{-1}(\mu) \quad \text{and} \quad x_t \in f^{-1}(\nu_i), i = 1, 2, \ldots, n.$$

Also, $\tau_1(f^{-1}(\nu_i)) \ge \tau_2(\nu_i) \ge r$, hence $S_{x_t}(f^{-1}(\mu)) \ge r$. It is a contradiction. Thus $S_{f(x)_t}(\mu) \le S_{x_t}(f^{-1}(\mu))$ for each $\mu \in I^Y$.

(ii) Let $\mathcal{F}$ be a fuzzy filter on X and $x_t \in Pt(X)$ such that $\mathcal{F}$ is supra fuzzy converges to x_t. Then for each $\mu \in I^Y$ we have

$$S_{x_t}(f^{-1}(\mu)) \le \mathcal{F}(f^{-1}(\mu)).$$

Since f is supra fuzzy continuous and by using (i) we have

$$S_{f(x)_t}(\mu) \le S_{x_t}(f^{-1}(\mu)) \le \mathcal{F}(f^{-1}(\mu)) = f(\mathcal{F})(\mu).$$

Then, $S_{f(x)_t} \le f(\mathcal{F})$. Hence $f(\mathcal{F})$ is supra fuzzy converges to $f(x)_t$. □

3. r-$s\gamma$-fuzzy open sets and r-$s\gamma$-fuzzy open sets

Definition 3.1. Let (X,τ) be sfts, $\nu \in I^X$ and $r \in I_0$. Then ν is called:

(i) r-$s\gamma$-fuzzy open (briefly r-$s\gamma$fo) set if either $\nu = \underline{0}$ or $S_{x_t}(\nu) \geq r$ for all $x_t \in \nu$.

(ii) r-$s\gamma$-fuzzy closed (briefly r-$s\gamma$fc) set if $\underline{1} - \nu$ is r-$s\gamma$fo set.

Remark 3.1. Let (X,τ) be sfts and $r \in I_0$. Then for every $\lambda \in I^X$ with $\tau(\lambda) \geq r$, λ is r-$s\gamma$fo but the converse is not true in general as the following example shows.

Example 3.1. Let $X = \{x,y,z\}$ be a set. Define $\lambda_1, \lambda_2, \mu \in I^X$ as follows:

$$\lambda_1(x) = 1.0 \qquad \lambda_1(y) = 0.6 \qquad \lambda_1(z) = 1.0$$

$$\lambda_2(x) = 0.6 \qquad \lambda_2(y) = 1.0 \qquad \lambda_2(z) = 0.0$$

$$\mu(x) = 1.0 \qquad \mu(y) = 0.9 \qquad \mu(z) = 1.0.$$

We define a supra fuzzy topology $\tau : I^X \to I$ as follows:

$$\tau(\lambda) = \begin{cases} 1, & \text{if } \lambda = \underline{0}, \underline{1} \\ 0.6, & \text{if } \lambda = \lambda_1 \\ 0.4, & \text{if } \lambda = \lambda_2 \\ 0, & \text{otherwise.} \end{cases}$$

Let $0 < r \leq 0.4$. If $t > 0.6$ we have

$$S_{x_t}(\lambda) = \begin{cases} 1, & \text{if } \lambda = \underline{1} \\ 0.6, & \text{if } \lambda_1 \leq \lambda < \underline{1} \\ 0, & \text{otherwise.} \end{cases}$$

$$S_{y_t}(\lambda) = \begin{cases} 1, & \text{if } \lambda = \underline{1} \\ 0.4, & \text{if } \lambda_2 \leq \lambda < \underline{1} \\ 0, & \text{otherwise.} \end{cases}$$

$$S_{z_t}(\lambda) = \begin{cases} 1, & \text{if } \lambda = \underline{1} \\ 0.6, & \text{if } \lambda_1 \leq \lambda < \underline{1} \\ 0, & \text{otherwise.} \end{cases}$$

If $t \leq 0.6$ we have

$$S_{x_t}(\lambda) = \begin{cases} 1, & \text{if } \lambda = \underline{1} \\ 0.6, & \text{if } \lambda_1 \leq \lambda < \underline{1} \\ 0.4, & \text{if } \lambda_2 \leq \lambda < \underline{1} \\ 0.6, & \text{if } \lambda_1 \wedge \lambda_2 \leq \lambda < \lambda_1 \quad \text{or} \quad \lambda_1 \wedge \lambda_2 \leq \lambda < \lambda_2 \\ 0, & \text{otherwise.} \end{cases}$$

$$S_{y_t}(\lambda) = \begin{cases} 1, & \text{if} \quad \lambda = \underline{1} \\ 0.6, & \text{if} \quad \lambda_1 \leq \lambda < \underline{1} \\ 0.4, & \text{if} \quad \lambda_2 \leq \lambda < \underline{1} \\ 0.6, & \text{if} \quad \lambda_1 \wedge \lambda_2 \leq \lambda < \lambda_1 \quad \text{or} \quad \lambda_1 \wedge \lambda_2 \leq \lambda < \lambda_2 \\ 0, & \text{otherwise.} \end{cases}$$

$$S_{z_t}(\lambda) = \begin{cases} 1, & \text{if} \quad \lambda = \underline{1} \\ 0.6, & \text{if} \quad \lambda_1 \leq \lambda < \underline{1} \\ 0, & \text{otherwise.} \end{cases}$$

Then, μ is r-$s\gamma$fo set of X but $\tau(\mu) = 0.0 \not\geq r$.

Theorem 3.1. *Let (X, τ) be a sfts and $r \in I_0$. Then,*

(i) *Any union of r-$s\gamma fo$ sets is r-$s\gamma fo$ set.*

(ii) *Any intersection of r-$s\gamma fc$ sets is r-$s\gamma fc$ set.*

Proof. (i) Let $\{\lambda_i : i \in J\}$ be a family of r-$s\gamma$fo sets. Then for each $i \in J$ we have $S_{x_t}(\lambda_i) \geq r$ for each $x_t \in \lambda_i$. So, there exist $\nu_{ik} \in I^X$ with $x_t \in \nu_{ik}$ and $\tau(\nu_{ik}) \geq r$, $k = 1, 2, \ldots, n_i$ such that $\bigwedge_{k=1}^{n_i} \nu_{ik} \leq \lambda_i$ then

$$\bigvee_{i \in J} (\bigwedge_{k=1}^{n_i} \nu_{ik}) \leq \bigvee_{i \in J} \lambda_i.$$

Thus

$$\bigwedge_{k=1}^{n_i} (\bigvee_{i \in J} \nu_{ik}) \leq \bigvee_{i \in J} \lambda_i.$$

Let $\nu_{i_0 k} = \bigvee_{i \in J} \nu_{ik}$. Then

$$\tau(\nu_{i_0 k}) = \tau(\bigvee_{i \in J} \nu_{ik}) \geq \bigwedge_{i \in J} \tau(\nu_{ik}) \geq r.$$

Since $x_t \in \nu_{i_0 k}$ for each $k = 1, 2, \ldots, n_i$ and $\bigwedge_{k=1}^{n_i} \nu_{i_0 k} \leq \bigvee_{i \in J} \lambda_i$,

$$S_{x_t}(\bigvee_{i \in J} \lambda_i) \geq r \quad \text{for each} \quad x_t \in \bigvee_{i \in J} \lambda_i.$$

Thus $\bigvee_{i \in J} \lambda_i$ is r-$s\gamma$fo set on X.

(ii) It is easy from(i) and the fact, $\bigvee_{i \in J} (\underline{1} - \lambda_i) = \underline{1} - \bigwedge_{i \in J} \lambda_i$. □

Definition 3.2. Let (X, τ) be a sfts, $\lambda \in I^X$ and $r \in I_0$. Then,

(i) The r-$s\gamma$-interior of λ denoted by $sI_\gamma(\lambda, r)$ is defined by

$$sI_\gamma(\lambda, r) = \bigvee \{\mu \in I^X : \mu \leq \lambda, \mu \quad \text{is} \quad r\text{-}s\gamma\text{fo}\}.$$

(ii) The r-$s\gamma$-closure of λ denoted by $sC_\gamma(\lambda, r)$ is defined by

$$sC_\gamma(\lambda, r) = \bigwedge \{\mu \in I^X : \mu \geq \lambda, \mu \quad \text{is} \quad r\text{-}s\gamma\text{fc}\}.$$

Theorem 3.2. *Let (X,τ) be a sfts, $\lambda \in I^X$ and $r \in I_0$. Then,*

$$sI_\gamma(\lambda,r) = \bigvee\{x_t \in Pt(X) : S_{x_t}(\lambda) \geq r\}.$$

Proof. For each $x_t \in sI_\gamma(\lambda,r)$, there exists r-$s\gamma$fo set $\mu \in I^X$ such that $x_t \in \mu$ and $\mu \leq \lambda$. Then $S_{x_t}(\mu) \geq r$. Since S_{x_t} is fuzzy filter, $S_{x_t}(\lambda) \geq S_{x_t}(\mu) \geq r$. Thus

$$sI_\gamma(\lambda,r) \leq \bigvee\{x_t \in Pt(X) : S_{x_t}(\lambda) \geq r\}. \tag{3.1}$$

Conversely, for each $x_t \in Pt(X)$ and $S_{x_t}(\lambda) \geq r$, there exist $\nu_i \in I^X$ with $x_t \in \nu_i$, $\tau(\nu_i) \geq r$, $i = 1,2,\ldots,n$ such that $\nu = \bigwedge_{i=1}^{n} \nu_i \leq \lambda$. Thus $S_{x_t}(\nu) \geq r$, since $x_t \in \nu \leq \lambda$, $x_t \in sI_\gamma(\lambda,r)$. Then

$$\bigvee\{x_t \in Pt(X) : S_{x_t}(\lambda) \geq r\} \leq sI_\gamma(\lambda,r). \tag{3.2}$$

From (3.1) and (3.2) we have

$$sI_\gamma(\lambda,r) = \bigvee\{x_t \in Pt(X) : S_{x_t}(\lambda) \geq r\}.$$

□

Theorem 3.3. *Let (X,τ) be a sfts, $\lambda \in I^X$ and $r \in I_0$. Then we have*
(i) $sI_\gamma(\underline{1} - \lambda, r) = \underline{1} - sC_\gamma(\lambda,r)$.
(ii) $sC_\gamma(\underline{1} - \lambda, r) = \underline{1} - sI_\gamma(\lambda,r)$.

Proof. For $\lambda \in I^X$ and $r \in I_0$ we have the following:

$$\begin{aligned}\underline{1} - sC_\gamma(\lambda,r) &= \underline{1} - \bigwedge\{\mu \in I^X : \mu \geq \lambda, \mu \quad \text{is} \quad r\text{-}s\gamma\text{fc}\}\\ &= \bigvee\{\underline{1} - \mu : \underline{1} - \mu \leq \underline{1} - \lambda, \underline{1} - \mu \quad \text{is} \quad r\text{-}s\gamma\text{fo}\}\\ &= sI_\gamma(\underline{1} - \lambda, r).\end{aligned}$$

(ii) Similar to (i). □

Theorem 3.4. *Let (X,τ) be a sfts, $\lambda \in I^X$ and $r \in I_0$. Then*
(i) *λ is r-$s\gamma$fo set if and only if $\lambda = sI_\gamma(\lambda,r)$.*
(ii) *λ is r-$s\gamma$fc set if and only if $\lambda = sC_\gamma(\lambda,r)$.*

Proof. Obvious. □

Theorem 3.5. *Let (X,τ) be a sfts. For $\lambda,\mu \in I^X$ and $r \in I_0$ the following statements are valid:*
(i) $sI_\gamma(\lambda,r) \leq \lambda \leq sC_\gamma(\lambda,r)$.
(ii) $sI_\gamma(\lambda,r) \leq sI_\gamma(\nu,r)$, *if* $\lambda \leq \mu$.
(iii) $sC_\gamma(\lambda,r) \leq sC_\gamma(\nu,r)$, *if* $\lambda \leq \mu$.
(iv) $sI_\gamma(sI_\gamma(\lambda,r),r) = sI_\gamma(\lambda,r)$.
(v) $sC_\gamma(sC_\gamma(\lambda,r),r) = sC_\gamma(\lambda,r)$.

Proof. Straightforward. □

Theorem 3.6. *Let (X,τ) be a sfts. Then the mapping $T: I^X \to I$ which defined by*

$$T(\lambda)=\begin{cases}\bigvee\{r: r\in I_0\}, & \text{if } \lambda \text{ is } r-s\gamma fc\\ 0, & \text{otherwise,}\end{cases}$$

is a fuzzy topology on X.

Proof. (T1) Since $\underline{0}$ and $\underline{1}$ are r-$s\gamma$fo set on X for each $r\in I_0$, $T(\underline{0})=T(\underline{1})=1$.

(T2) Suppose that there exist $\lambda_1,\lambda_2\in I^X$ and $r_0\in I_0$ such that

$$T(\lambda_1\wedge\lambda_2)<r_0\le T(\lambda_1)\wedge T(\lambda_2).$$

Then $T(\lambda_1)\ge r_0$ and $T(\lambda_2)\ge r_0$ which implies that

$$S_{x_t}(\lambda_1)\ge r_0 \text{ for each } x_t\in\lambda_1 \text{ and } S_{x_t}(\lambda_2)\ge r_0 \text{ for each } x_t\in\lambda_2.$$

Since S_{x_t} is a fuzzy filter we have

$$S_{x_t}(\lambda_1\wedge\lambda_2)\ge S_{x_t}(\lambda_1)\wedge S_{x_t}(\lambda_2)\ge r_0 \text{ for each } x_t\in\lambda_1\wedge\lambda_2.$$

Then $\lambda_1\wedge\lambda_2$ is r_0-$s\gamma$fo set on X. Thus $T(\lambda_1\wedge\lambda_2)\ge r_0$. It is a contradiction. Hence,

$$T(\lambda_1\wedge\lambda_2)\ge T(\lambda_1)\wedge T(\lambda_2) \text{ for each } \lambda_1,\lambda_2\in I^X.$$

(T3) Suppose that there exist $\lambda=\bigvee_{i\in J}\lambda_i\in I^X$ and $r\in I_0$ such that

$$T(\lambda)<r_0\le\bigwedge_{i\in J}T(\lambda_i).$$

Then $T(\lambda_i)\ge r_0$ for each $i\in J$, this implies that $S_{x_t}(\lambda_i)\ge r_0$ for each $x_t\in\lambda_i$, $i\in J$. By using Theorem 3.1, we have $S_{x_t}(\bigvee_{i\in J}\lambda_i)\ge r_0$ for each $x_t\in\bigvee_{i\in J}\lambda_i$. Thus $S_{x_t}(\lambda)\ge r_0$, a contradiction. Thus

$$T(\bigvee_{i\in J}\lambda_i)\ge\bigwedge_{i\in J}T(\lambda_i) \quad \text{for each } \{\lambda_i:\lambda_i\in I^X\}.$$

□

4. $s\gamma$-fuzzy continuous and $s\gamma^*$-fuzzy continuous mappings

Definition 4.1. Let (X,τ_1) and (Y,τ_2) be fts's and let τ_1^* be an associated supra fuzzy topology with τ_1. Then the mapping $f: X\to Y$ is called $s\gamma$-fuzzy continuous if $f^{-1}(\lambda)$ is r-$s\gamma$-fo set on X for each $\lambda\in I^Y$ with $\tau_2(\lambda)\ge r$.

Theorem 4.1. *Let $f:(X,\tau_1)\to(Y,\tau_2)$ be a mapping from a fts (X,τ_1) to another fts (Y,τ_2) and let τ_1^* be an associated supra fuzzy topology with τ_1. Then the following statements are equivalent:*

(i) *f is $s\gamma$-fuzzy continuous;*

(ii) *$f^{-1}(\lambda)$ is r-$s\gamma$-fc set on X for each $\lambda\in I^X$ and $r\in I_0$ with $\tau_2(\underline{1}-\lambda)\ge r$;*

(iii) *$f(sC_\gamma(\nu,r))\le C_{\tau_2}(f(\nu),r)$ for each $\nu\in I^X$, $r\in I_0$;*

(iv) *$sC_\gamma(f^{-1}(\lambda),r))\le f^{-1}(C_{\tau_2}(\lambda,r))$ for each $\lambda\in I^Y$, $r\in I_0$;*

(v) *$f^{-1}(I_{\tau_2}(\lambda,r))\le sI_\gamma(f^{-1}(\lambda),r))$ for each $\lambda\in I^Y$, $r\in I_0$.*

Proof. (i)⇔(ii) It is easily proved from Definition 3.1, and $f^{-1}(\underline{1}-\lambda)=\underline{1}-f^{-1}(\lambda)$.

(ii)⇒(iii) Suppose that there exist $\nu \in I^X$ and $r \in I_0$ such that

$$f(sC_\gamma(\nu,r)) \not\leq C_{\tau_2}(f(\nu),r).$$

Then there exist $y \in Y$ and $t \in I_0$ such that

$$f(sC_\gamma(\nu,r))(y) > t > C_{\tau_2}(f(\nu),r)(y).$$

If $f^{-1}(\{y\})=\phi$, then $f(sC_\gamma(\nu,r))(y)=0$, it is a contradiction. If $f^{-1}(\{y\})\neq\phi$, then

$$f(sC_\gamma(\nu,r))(y)=\sup_{x\in f^{-1}(\{y\})} sC_\gamma(\nu,r)(x) > t > C_{\tau_2}(f(\nu),r)(f(x)).$$

Then there exist $x_0 \in f^{-1}(\{y\})$ such that

$$f(sC_\gamma(\nu,r))(y) \geq sC_\gamma(\nu,r)(x_0) > t > C_{\tau_2}(f(\nu),r)(f(x_0)). \tag{4.1}$$

Since $C_{\tau_2}(f(\nu),r)(f(x_0)) < t$, there exists $\mu \in I^Y$ with $\tau_2(\underline{1}-\mu)\geq r$ and $f(\nu)\leq\mu$ such that

$$C_{\tau_2}(f(\nu),r)(f(x_0)) \leq \mu(f(x_0)) < t.$$

Moreover, $f(\nu)\leq\mu$ implies $\nu\leq f^{-1}(\mu)$. By (ii) $f^{-1}(\mu)$ is r-$s\gamma$-fc set on X. Thus

$$sC_\gamma(\nu,r))(x_0) \leq sC_\gamma(f^{-1}(\mu),r))(x_0) = f^{-1}(\mu)(x_0) = \mu(f(x_0)) < t.$$

It is a contradiction with (4.1).

(iii)⇒(iv) Let $\lambda \in I^Y$ be arbitrary. Put $\nu = f^{-1}(\lambda)$, by (iii) we have

$$f(sC_\gamma(f^{-1}(\lambda),r))) \leq C_{\tau_2}(f(f^{-1}(\lambda,r))) \leq C_{\tau_2}(\lambda,r).$$

This implies that

$$sC_\gamma(f^{-1}(\lambda),r) \leq f^{-1}(f(sC_\gamma(f^{-1}(\lambda),r)))) \leq f^{-1}(C_{\tau_2}(\lambda,r)).$$

(iv)⇒(v) It easily proved from Theorem 1.2 (i) and Theorem 3.3.

(v)⇒(i) Let $\mu \in I^Y$ be arbitrary and $\tau_2(\mu)\geq r$. By (v) we have

$$f^{-1}(\mu) = f^{-1}(I_{\tau_2}(\mu,r)) \leq sI_\gamma(f^{-1}(\mu),r).$$

On the other hand, by Theorem 3.5, $f^{-1}(\mu)\geq sI_\gamma(f^{-1}(\mu),r)$.
Thus $f^{-1}(\mu)=sI_\gamma(f^{-1}(\mu),r)$. By Theorem 3.4(i), $f^{-1}(\mu)$ is r-$s\gamma$-fo set on X. □

Definition 4.2. Let $f: X \to Y$ be a mapping from a sfts (X,τ_1) to another sfts (Y,τ_2). Then f is said to be $s\gamma^*$-fuzzy continuous if $f^{-1}(\lambda)$ is r-$s\gamma$-fo set on X for each r-$s\gamma$-fo set λ on Y.

Remark 4.1. Every $s\gamma^*$-fuzzy continuous mapping and every fuzzy continuous mapping is $s\gamma$-fuzzy continuous mapping but the converse may not be true as we shows in the following example.

Example 4.1. Let $X = \{x, y\}$ and $Y = \{a, b\}$ be sets. Define $\lambda_1, \lambda_2 \in I^X$ and $\mu_1, \mu_2, \nu \in I^Y$ as follows:

$$\lambda_1(x) = 1.0 \qquad \lambda_1(y) = 0.6$$
$$\lambda_2(x) = 0.6 \qquad \lambda_2(y) = 1.0$$
$$\mu_1(a) = 1.0 \qquad \mu_1(b) = 0.7$$
$$\mu_2(a) = 0.0 \qquad \mu_2(b) = 1.0$$
$$\nu(a) = 0.8 \qquad \nu(b) = 0.8.$$

Define the fuzzy topologies $\tau_1 : I^X \to I$ and $\tau_2 : I^Y \to I$ as follows:

$$\tau_1(\lambda) = \begin{cases} 1, & \text{if} \quad \lambda = \underline{0}, \underline{1} \\ 0.5, & \text{if} \quad \lambda = \lambda_2 \\ 0, & \text{otherwise.} \end{cases}$$

$$\tau_2(\mu) = \begin{cases} 1, & \text{if} \quad \mu = \underline{0}, \underline{1} \\ 0.4, & \text{if} \quad \mu = \mu_1 \\ 0, & \text{otherwise.} \end{cases}$$

Define their associated supra fuzzy topologies $\tau_1^* : I^X \to I$ and $\tau_2^* : I^Y \to I$ as follows:

$$\tau_1^*(\lambda) = \begin{cases} 1, & \text{if} \quad \lambda = \underline{0}, \underline{1} \\ 0.7, & \text{if} \quad \lambda = \lambda_1 \\ 0.6, & \text{if} \quad \lambda = \lambda_2 \\ 0, & \text{otherwise.} \end{cases}$$

$$\tau_2^*(\mu) = \begin{cases} 1, & \text{if} \quad \mu = \underline{0}, \underline{1} \\ 0.6, & \text{if} \quad \mu = \mu_1, \mu_2 \\ 0, & \text{otherwise.} \end{cases}$$

Let $0 < r \leq 0.4$ and $0 < t \leq 0.6$. Then we have

$$S_{x_t}(\lambda) = \begin{cases} 1, & \text{if} \quad \lambda = \underline{1} \\ 0.7, & \text{if} \quad \lambda_1 \leq \lambda < \underline{1} \\ 0.6, & \text{if} \quad \lambda_2 \leq \lambda < \underline{1} \\ 0.7, & \text{if} \quad \lambda_1 \wedge \lambda_2 \leq \lambda < \lambda_1 \\ 0, & \text{otherwise.} \end{cases}$$

$$S_{y_t}(\lambda) = \begin{cases} 1, & \text{if} \quad \lambda = \underline{1} \\ 0.7, & \text{if} \quad \lambda_1 \leq \lambda < \underline{1} \\ 0.6, & \text{if} \quad \lambda_2 \leq \lambda < \underline{1} \\ 0.7, & \text{if} \quad \lambda_1 \wedge \lambda_2 \leq \lambda < \lambda_1 \\ 0, & \text{otherwise.} \end{cases}$$

$$S_{a_t}(\mu) = \begin{cases} 1, & \text{if } \mu = \underline{1} \\ 0.6, & \text{if } \mu_1 \leq \mu < \underline{1} \\ 0, & \text{otherwise.} \end{cases}$$

$$S_{b_t}(\mu) = \begin{cases} 1, & \text{if } \mu = \underline{1} \\ 0.6, & \text{if } \mu_1 \wedge \mu_2 \leq \mu < \underline{1} \\ 0, & \text{otherwise.} \end{cases}$$

Define the mapping $f : X \to Y$ as follows:

$$f(x) = a, \qquad f(y) = b.$$

Then f is $s\gamma$-fuzzy continuous but it is neither $s\gamma^*$-fuzzy continuous nor fuzzy continuous.

Theorem 4.2. *Let $f : (X, \tau_1) \to (Y, \tau_2)$ be a mapping from a sfts (X, τ_1) to another sfts (Y, τ_2). Then the following statements are equivalent:*

(i) *f is $s\gamma^*$-fuzzy continuous;*
(ii) *$f^{-1}(\lambda)$ is r-$s\gamma$-fc set on X for each r-$s\gamma$-fc set on Y , $r \in I_0$;*
(iii) *$f(sC_\gamma(\nu, r)) \leq sC_\gamma(f(\nu), r)$ for each $\nu \in I^X$, $r \in I_0$;*
(iv) *$sC_\gamma(f^{-1}(\lambda), r)) \leq f^{-1}(sC_\gamma(\lambda, r))$ for each $\lambda \in I^Y$, $r \in I_0$;*
(v) *$f^{-1}(sI_\gamma(\lambda, r)) \leq sI_\gamma(f^{-1}(\lambda), r))$ for each $\lambda \in I^Y$, $r \in I_0$.*

Proof. Similar to the proof of Theorem 4.1. □

Definition 4.3. Let (X, τ_1) and (Y, τ_2) be fts's and let τ_2^* be an associated supra fuzzy topology with τ_2. Then the mapping $f : X \to Y$ is called $s\gamma$-fuzzy open if $f(\nu)$ is r-$s\gamma$-fo set on Y for each $\nu \in I^X$ with $\tau_1(\nu) \geq r$.

Theorem 4.3. *Let $f : (X, \tau_1) \to (Y, \tau_2)$be a mapping from a fts (X, τ_1) to another fts (Y, τ_2) and let τ_2^* be an associated supra fuzzy topology with τ_2. Then the following statements are equivalent:*

(i) *f is $s\gamma$-fuzzy open;*
(ii) *$f(I_{\tau_1}(\nu, r)) \leq sI_\gamma(f(\nu), r)$ for each $\nu \in I^X$, $r \in I_0$;*
(iii) *$I_{\tau_1}(f^{-1}(\lambda), r)) \leq f^{-1}(sI_\gamma(\lambda, r))$ for each $\lambda \in I^Y$, $r \in I_0$.*

Proof. (i)⇒(ii) For all $\nu \in I^X$, $r \in I_0$, since $\tau_1(I_{\tau_1}(\nu, r)) \geq r$, $f(I_{\tau_1}(\nu, r))$ is r-$s\gamma$-fo set on Y. From Theorem 3.4, we have

$$f(I_{\tau_1}(\nu, r)) = sI_\gamma(f(I_{\tau_1}(\nu, r)), r) \leq sI_\gamma(f(\nu), r).$$

(ii)⇒(i) For all $\nu \in I^X$, $r \in I_0$ with $\tau_1(\nu) \geq r$ we have $I_{\tau_1}(\nu, r) = \nu$. By using (ii) we have

$$f(\nu) = f(I_{\tau_1}(\nu, r)) \leq sI_\gamma(f(\nu), r).$$

Then, $f(\nu) = sI_\gamma(f(\nu), r)$. By Theorem 3.4, $f(\nu)$ is r-$s\gamma$-fo set on Y. Thus f is $s\gamma$-fuzzy open.

(ii)⇒(iii) For all $\lambda \in I^Y$, $r \in I_0$, by (ii) we have

$$f(I_{\tau_1}(f^{-1}(\lambda), r)) \leq sI_\gamma(f(f^{-1}(\lambda)), r) \leq sI_\gamma(\lambda, r).$$

This implies that

$$I_{\tau_1}(f^{-1}(\lambda), r) \leq f^{-1}(f(I_{\tau_1}(f^{-1}(\lambda), r))) \leq f^{-1}(sI_\gamma(\lambda, r)).$$

(iii)$\Rightarrow$(ii) For all $\nu \in I^X$, $r \in I_0$, by (ii) we have

$$I_{\tau_1}(\nu, r) \leq I_{\tau_1}(f^{-1}(f(\lambda)), r) \leq f^{-1}(sI_\gamma(f(\nu), r)).$$

This implies that

$$f(I_{\tau_1}(\nu, r)) \leq f(f^{-1}(sI_\gamma(f(\nu), r))) \leq sI_\gamma(f(\nu), r).$$

□

Theorem 4.4. *Let $f : (X, \tau_1) \to (Y, \tau_2)$ be a mapping from a fts (X, τ_1) to another fts (Y, τ_2) and let τ_2^* be an associated supra fuzzy topology with τ_2. Then the following statements are equivalent:*

(i) *f is $s\gamma$-fuzzy open;*

(ii) *For each $x_t \in Pt(X)$ and for each $\nu \in I^X$, $r \in I_0$ with $\tau_1(\nu) \geq r$ and $x_t \in \nu$, we have $S_{x_t}(f(\nu)) \geq r$.*

Proof. It is easy. □

References

[1] C. L. Chang, *Fuzzy topological spaces*, J. Math. Anal. Appl. **24** (1968), 182–190.

[2] K. C. Chattopadhyay, R. N. Hazra, and S. K. Samanta, *Gradation of openness: fuzzy topology*, Fuzzy Sets and Systems **49** (1992), no. 2, 237–242.

[3] K. C. Chattopadhyay and S. K. Samanta, *Fuzzy topology: fuzzy closure operator, fuzzy compactness and fuzzy connectedness*, Fuzzy Sets and Systems **54** (1993), no. 2, 207–212.

[4] M. Demirci, *Neighborhood structures of smooth topological spaces*, Fuzzy Sets and Systems **92** (1997), no. 1, 123–128.

[5] M. H. Ghanim, O. A. Tantawy, and F. M. Selim, *Gradation of supra-openness*, Fuzzy Sets and Systems **109** (2000), no. 2, 245–250.

[6] M. Güloglu and D. Çoker, *Convergence in I-fuzzy topological spaces*, Fuzzy Sets and Systems **151** (2005), no. 3, 615–623.

[7] U. Höhle and A. P. Šostak, *A general theory of fuzzy topological spaces*, Fuzzy Sets and Systems **73** (1995), no. 1, 131–149.

[8] Y. C. Kim, *r-fuzzy semi-open sets in fuzzy bitopological spaces*, Far East J. Math. Sci. (FJMS) 2000, Special Volume, Part II, 221–236.

[9] ———, *Initial L-fuzzy closure spaces*, Fuzzy Sets and Systems **133** (2003), no. 3, 277–297.

[10] P. M. Pu and Y. M. Liu, *Fuzzy topology. I. Neighborhood structure of a fuzzy point and Moore-Smith convergence*, J. Math. Anal. Appl. **76** (1980), no. 2, 571–599.

[11] A. A. Ramadan, *Smooth topological spaces*, Fuzzy Sets and Systems **48** (1992), no. 3, 371–375.

[12] ———, *Smooth filter structures*, J. Fuzzy Math. **5** (1997), no. 2, 297–308.

[13] A. A. Ramadan, M. A. Abd El-Satter, and Y. C. Kim, *Some properties of smooth filters*, J. Fuzzy Math. **11** (2003), no. 2, 341–354.

[14] A. P. Šostak, *On a fuzzy topological structure*, Rend. Circ. Mat. Palermo (2) Suppl. No. **11** (1985), 89–103.

[15] ———, *On the neighborhood structure of fuzzy topological spaces*, Zb. Rad. No. **4** (1990), 7 14.

[16] ———, *Basic structures of fuzzy topology*, J. Math. Sci. **78** (1996), no. 6, 662–701.

A. A. RAMADAN
DEPARTMENT OF MATHEMATICS
FACULTY OF SCIENCE
AL-QASSEM UNIVERSITY
P.O. BOX 237, BURIEDA 81999, SAUDI ARABIA
E-mail address: aramadan58@yahoo.com

A. A. ABD EL-LATIF
DEPARTMENT OF MATHEMATICS
FACULTY OF SCIENCE
BENI-SUEF UNIVERSITY, EGYPT
E-mail address: ahmeda73@yahoo.com