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Zeno Series, Collective Causation, and Accumulation of Forces*†

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【ABSTRACT】This paper aims to present solutions to three intriguing puzzles on causation that Benardete presents by considering the results of infinite series of telescoping events. The main conceptual tool used in the solutions is the notion of *collective causation*, what many events cause *collectively*. It is straightforward to apply the notion to resolve two of the three puzzles. It does not seem as straightforward to apply it to the other puzzle. After some preliminary clarifications of the situation that Benardete describes to present the puzzle, however, we can apply the notion to resolve it as well.

【Key words】: causation, infinity, Zeno's paradoxes, Benardete's paradoxes collective causation

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José Benardete(1964) presents three intriguing puzzles on causation by considering the results of infinite series of telescoping events. I aim to present solutions to the puzzles in this paper. Benardete takes them to be different versions of the same underlying puzzle, which he calls "the paradox of before-effect" using the "neologism of a *before-effect*" for an effect "temporally prior ... to its cause"(1964, p. 259).[1] But it is useful to divide the puzzles into two groups. One of them, the third puzzle that he calls "the paradox of gods"(*ibid*., p. 260), is somewhat different from the other, first two puzzles. These might be considered paradoxes(or puzzles) of before-effect, because they concern the temporal order between series of telescoping events and their results. The third puzzle, the so-called paradox of gods, however, does not concern temporal order. The apparent difficulty in tackling it lies not in establishing a proper temporal order between an event and its causes, but in figuring out what might possibly cause the given event. Accordingly, it is not straightforward to apply the solutions that I present for the first two puzzles to the third puzzle, which makes it more challenging. So I shall consider the puzzles in two stages. First, I shall present solutions to the first two puzzles (section 1). Then I shall discuss the third puzzle. After a preliminary discussion of the puzzle (section 2), I shall examine and resolve Graham Priest's recent reformulation thereof (section 3)[2] and present my final

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1) He calls the effects in the usual cases, where the effect is temporally posterior to the cause (or causes), "after-effects" (*ibid*., p. 259).

2) Benardete's puzzles, intriguing as they are, fall short of paradoxes. He does not draw straightforward contradictions from the descriptions of the

solution to it (section 4).

The main conceptual tool that I apply to solve the puzzles is the notion of collective causation, what many events cause collectively. Two or more events can cause something that none of them causes individually, just as two or more people (e.g., Watson and Crick) can do something (e.g., discover the structure of DNA) that none of them does individually. We can solve the first two puzzles by directly applying the notion to the situations that Benardete describes to present them. It does not seem as straightforward to apply the notion to the situation that he describes to present the third puzzle. After some preliminary clarifications of the situation, however, we can apply the notion to resolve this puzzle as well.[3]

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situations that he imagines to present them. (He gets close to drawing a contradiction in his further discussion of one of the puzzles [*ibid*., p. 261]. See my discussion in note .) Priest (1999) turns the third puzzle into a paradox: he argues that a situation similar to the one that Benardete considers to present the puzzle is a *contradictory* but *possible* situation, a physically possible situation that satisfies a contradiction.

3) In many natural languages (e.g., English), statements of collective causation or agency (e.g., 'The walls (collectively) arrest a man at a point') rest on use of *plural constructions* (e.g., the plural term 'the walls'), and belong to a larger group of statements that includes what I call *plural predications* (e.g., "Watson and Crick discovered the structure of DNA' or 'Russell and Whitehead wrote *Principia Mathematica*'). On my view, plural constructions are devices for talking about many things (e.g., Watson and Crick; or Russell and Whitehead) as such. For an account of plural constructions based on this view, see, e.g., Yi (1999a), (1999b), (2005), and (2006). (Some languages (e.g., Korean) do not have plural constructions, but have alternative devices for talking about many things as such.)

1. Paradoxes of Before-Effect and Collective Causation

Benardete presents the first of his three puzzles as follows:

> Let the peal of a gong be heard in the last half of a minute, a second peal in the preceding 1/4 minute, a third peal in the 1/8 minute before that, etc. ad infinitum. (Ibid., p.255)

Let us assume that each peal is so very loud that, upon hearing it, anyone is struck deaf – totally and permanently. At the end of the minute we shall be completely deaf ... but we shall not have heard a single peal! For at most we could have heard only *one* of the peals (any single peal striking one deaf *instantly*), and which peal could we have heard? There simply was no first peal. ... we must be in a state of deafness prior to each peal. Here the effect is temporally prior to its cause. (*Ibid*., p.259; original italics)

The second puzzle has a similar structure:

> A man is shot through the heart during the last half of a minute by A. B shoots him through the heart during the preceding 1/4 minute, C during the 1/8 minute before that, &c. *ad infinitum*. Assuming that each shot kills instantly (if the man were alive), the man must be already dead before each shot. Thus he cannot be said to have died of a bullet wound. Here again the infinite sequence logically entails a before-effect. (*Ibid*., p.259)

In both cases, Benardete argues that the effect (the man's being deaf or dead) is temporally prior to a sequence of events that might be considered its cause (the infinite sequence of peals of gong or of shots). If this is right, how can the effect take place *before* its cause?

The third puzzle, the paradox of gods, raises different issues. Benardete presents the puzzle, which he considers "more radical" (*ibid*., p. 259) than the others, as follows:

> [*a*] A man decides to walk one mile from A to B. A god waits in readiness to throw up a wall blocking the man's further advance when the man has travelled ½ mile. A second god (unknown to the first) waits in readiness to throw up a wall of his own blocking the man's further advance when the man has travelled 1/4 mile. A third god ... &c. *ad infinitum*. [*b*] It is clear that this sequence of mere *intentions* (assuming the contrary-to-fact conditional that each god would succeed in executing his intention if given the opportunity) logically entails the consequence that the man will be arrested at point A; he will not be able to pass beyond it, even though not a single wall will in fact be thrown down in his path. [*c*] The before-effect here will be described by the man as a strange field of force blocking his passage forward.(*Ibid*., p.259f; original italics; letters for paragraphs added)

In the situation described in [*a*], the man will be arrested at point A. Benardete takes the arrest to be a before-effect, assimilating this puzzle to the other two. But what is puzzling in this case is not the temporal order between the arrest and its putative cause or causes, the walls that the gods plan to throw up under certain conditions. It is not quite right to say that the man will be arrested *before* the walls are thrown up. This would suggest that the walls will be thrown up after the arrest. But no wall will ever be thrown up because the man cannot reach any point beyond A. As Benardete suggests, however, this seems to make the arrest even more puzzling. If no wall is ever thrown up, why must the man still be arrested? What can block his further advance?

It is straightforward to resolve the first two puzzles by attending to collective causation, what many events cause collectively. To present the second puzzle, recall, Benardete considers the following situation:

> *The Shooting Situation*: "A man is shot through the heart during the last half of a minute by A. B shoots him through the heart during the preceding 1/4 minute, C during the 1/8 minute before that, &c. *ad infinitum*. Assuming that each shot kills instantly (if the man were alive) ...." (*Ibid*., p.259)

Then he argues: "the man must be already dead before *each shot*. Thus he cannot be said to have died of *a bullet wound*" (*ibid*., p.259; my italics). If so, can we conclude, as Benardete does, that the man's death is "a before-effect" of "the infinite sequence" of shots (*ibid*., p.259)? It is wrong to draw this conclusion. The man will not be dead until some of the shots (in fact, infinitely many of them) are fired and their bullets pass through his heart.

To see this, it is useful to be clear about when the man (call him *Sam*) will be dead in the situation. Let t be an instant in time, and t_0, t_1, t_2, etc. be the instants that are after t by 1 minute, ½ minute, 1/4 minute, etc., respectively. Then Shot-1 ensures that Sam will be dead at t_0 (and thereafter); Shot-2 that he will be dead at t_1 (and thereafter); Shot-3 that he will be dead at t_2 (and thereafter); etc.[4] So he will be dead at any instant after t. But he will continue to be alive until t, which is the last instant at which he will be alive.[5] Now, take any instant after t (e.g., t_0 or t_1) and consider why he will be dead at that instant. The obvious reason is that he will have been hit by many shots (e.g., Shot-1, Shot-2, etc.) by that instant: the bullets of those shots will have pierced his heart by then. So his death at the instant is caused by those shots (or the bullets piercing his heart or the wounds made by the bullets). This does not mean that Sam's death is caused by *one* of those shots: none of them causes his death, as Benardete correctly points out, because none of them is the first (so none of the bullets passes through Sam's heart while he is still alive). But many shots can collectively cause Sam to be dead while none of them does so individually, just as many people (e.g., Russell and Whitehead) can write a book (e.g., *Principia Mathematica*) together while none of them writes it alone.[6]

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4) We may assume that Shot-1, for example, is fired at t_2 with its bullet reaching one side of Sam's heart at t_1 and the other side at t_0, and that he will be dead by the instant at which the bullet reaches the other side of his heart (e.g., t_0).

5) Here we assume that Sam is alive, say, one minute before t, and that there is nothing but the shots (e.g., lightning) that might kill him until t.

6) Hawthorne reaches essentially the same conclusion: "the fusion of the bullets kills the person without any bullet doing so" (2000, p. 627). I think that he invokes the *fusion* of the bullets as an idle detour. And the reason that he gives for the conclusion is faulty. Stipulating that "if a metal object penetrates 1/4 inch into the heart, then the person dies at that very

We can then see that Sam's death is not a before-effect. He will be dead at any instant after *t*, but the death at that instant is not temporally prior to its *causes*: the shots whose bullets will have pierced his heart by that instant (or the *bullets* or the *wounds*), taken together.[7] And he will be alive at *t*, which is the last instant before any of the shots are fired. So he will not be dead before the *shots* are fired. As Benardete correctly points out, "the man must be already dead before *each shot*" (1964, p. 159;

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moment", he argues that "At the point at which the fusion penetrates 1/4 inch, no bullet will have so penetrated" (*ibid*., p. 627). But there is no such instant in the Shooting Situation: at *t*, which is prior to the time of any of the shots, even the fusion of the bullets will *not* have entered the heart; and at any instant after *t*, infinitely many bullets will have reached 1/4 inch into the heart. I think that Hawthorne confuses Benardete's shooting situation (where all the bullets will be fired after *t*) with another situation, one in which the bullets, all fired *before t*, will have reached into the heart by ½x1/4 inch, 3/4x1/4 inch, etc. at *t*. Incidentally, his analysis is objectionable even as an analysis of the latter situation. He suggests that the man will be dead at *t* by saying "the fusion of the bullets will, upon penetrating the heart to 1/4 inch, kill the person" (*ibid*., p. 627). I do not think that this conclusion *logically* or *mathematically* follows from the description of the effects of individual bullets – although human *biology* would probably yield the verdict that the man cannot survive infinitely many bullets inside his heart, and thus will be dead even before *t*, even before the fusion penetrates 1/4 inch into the heart.

7) In formulating the notion of before-effect, Benardete considers only cases in which there is one event that is *the* cause of another. So it is necessary to extend the notion to apply it to cases of collective causation. In such cases, we can say that the effect of some events, which collectively cause it, is a *before-effect* of these events, if it is temporally prior to any of the events. (And when some events collectively cause another, I shall say that the former events are *causes* of the latter, and that each one of them is *a cause* thereof. The shots in the Shooting Situation, for example, are causes of Sam's being dead at t_0.)

my italics) because each shot is preceded by infinitely many other shots. But this means no more than that none of the shots causes his death individually.[8)]

Benardete's first puzzle can be resolved in the same way. Peal-1 ensures that someone (call him *Paul*) will be deaf at t_0 (and thereafter) – by reaching the outer surfaces of his eardrums at t_1 and the inner surfaces at t_0. Similarly, Peal-2 ensures that he will be deaf at t_1 (and thereafter); Peal-3, that he will be deaf at t_2 (and thereafter); etc. So he must be deaf at any instant after t. His deafness at any such instant is due to infinitely many peals (taken together), namely, those that will have passed through his eardrums by that instant. The last instant before any of the peals will have passed through his eardrums is t, but he will not be

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8) We cannot explain why he will be dead by explaining why he will be dead at the *first* instant of his death, because there is no such instant. Benardete seems to suggest that this is puzzling; he says Sam "cannot be said to have died at any moment of time whatsoever!" (*ibid*., p. 260). I do not think that this is puzzling at all. Dying, unlike being dead, is a process, and our talk of the time when one dies, which usually applies to *periods* of time, does not apply well to *instants* of time. While a period of time when one dies includes both instants of one's life and instants of one's death, one cannot be both dead and alive at a single instant. In fact, one cannot have both the earliest instant of death and the last instant of life (so, incidentally, there can be no smallest period when one can be said to die). Either of the two instants, if any such exists, might be considered the instant at which one dies. So we might say that one dies at an instant, if this is either the last instant of his life or the earliest instant of his death. We might then say that Sam will die at t, although he will still be alive at that instant. Or we might say, as Benardete does, that there is no instant at which he will die – while there is the last instant at which he will be alive, and also a time (i.e., period) during which he will die. (See Benacerraf [1962, p. 775] for a related discussion on the notion of disappearance.)

deaf at that instant. As Benardete says, "There simply was no first peal", and so Paul "must be in a state of deafness prior to *each peal*" (1964, p.259; my italics). But this does not mean that he must be deaf prior to the *peals*, which collectively cause him to be deaf.[9)]

It is useful to note that the two puzzles have spatial analogues, puzzles of *action at a distance*. To consider a situation that one might use to pose such a puzzle, let o, p, q_0, q_1, q_2, etc. be

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9) In his further discussion of the puzzles, Benardete suggests essentially the same solutions as mine: Sam's death, he says, "was certainly *caused* by the infinite fusillade of shots", and Paul's hearing "remains unimpaired at any assigned instant prior to *all* of" the shots (*ibid*., p. 260; original italics). But he calls these solutions "placebos" (*ibid*., p. 261), and presents a further puzzle by in effect drawing a contradiction from the description of the peals of gong situation by considering whether any of the peals would have been *heard*. Assuming that "each peal is so very loud that, upon hearing it, anyone is struck deaf", he argues, "At the end of the minute we shall be completely deaf . . . but we shall not have heard a single peal! For at most we could have heard only *one* of the peals" (*ibid*., p. 259; original italics). Then he argues: "Not having heard any of the peals, our hearing must have been retained unimpaired, in which case we should have heard all the peals" (*ibid*., p. 261). He might be right to hold that those in the situation "shall not have heard a single peal" (*ibid*., p. 259), but this does not mean that their "hearing must have been retained unimpaired" (*ibid*., p. 261). Just as a lightning bolt might kill someone without giving him time to feel pain, so might a peal or some peals strike one's eardrums so fast as to damage them without giving them time to send proper signals to one's brain to enable one to hear a sound. So one might become deaf by the peals without hearing any one of them, because it is the *damage to the eardrums* done by a peal (or peals), not the *hearing* of a peal (or peals), that causes one to be deaf. (Now, we can see that it is misleading to begin with "each peal is so very loud that, *upon hearing it*, anyone is struck deaf" [*ibid*., p. 259; my italics].)

points in a straight line, *L*, and suppose that *o* is 1 mile to the left of *p* and q_0, q_1, q_2, etc. to the right of *p* by 1 mile, ½ mile, 1/4 mile, etc. Then the situation can be described as follows:

> *The Wall Situation*: A particle is moving (continuously) in *L* from left to right. (It is at point *o* at the present instant, *s*, one minute before *t*, having passed all the points to the left of *o*.) There will be no impediment to its rectilinear motion until it reaches point *p* at instant *t*. But there is an infinite series of permanent, immovable, and impenetrable walls to the right of *p*: Wall-1, Wall-2, etc. The walls, which are perpendicular to *L*, occupy adjacent segments thereof: Wall-1 occupies $[q_1, q_0)$, Wall-2 $[q_2, q_1)$, Wall-3 $[q_3, q_2)$; etc.[10]

In this situation, the particle will be arrested at point *p* at *t*; that is, it reaches the point at *t*, but cannot pass beyond it.[11] If so, why will it be arrested at *p*? What can arrest it at that point? None of the walls can arrest it at *p*. The particle is not in contact with any of them when it reaches *p*; each one of them is at some distance from *p* (Wall-1 by ½ mile, Wall-2 by 1/4 mile, etc.). If so, must we conclude that the arrest of the particle, if it has a cause, will be due to a sort of action at a distance (One of the walls will somehow arrest it without contacting it!)?[12]

Surely, it is wrong to draw this conclusion. The right conclusion to draw is that the particle will be arrested at *p* by *the walls*, which *collectively* block its advance at that point

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10) $[q_1, q_0)$, for example, is a half-open interval in *L* that includes q_1 and any point between q_1 and q_0, but not q_0.

11) This leaves it open whether it stays at *p* or gets deflected (or destroyed) after *t*.

12) Thanks are due to Adele Mercier, who drew my attention to this puzzle.

although none of them does so individually. Because there are infinitely many walls between *p* and any point to its right (e.g., q_1), the particle cannot reach any point to the right of *p* without penetrating into or moving the walls in between; but it can never do so because the walls are permanent, immovable, and impenetrable. And the arrest of the particle by the walls is not an action at a distance. The walls, taken together, occupy the open interval (p, q_0) in *L*,[13] and the distance from *p* to this interval is 0 (roughly, there is no gap between them). So they are collectively in contact with the particle at instant *t* (when it reaches *p*), although none of them is individually in contact with it at that instant.[14]

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13) The open interval (p, q_0) is an interval in *L* that includes any point between *p* and q_0, but neither *p* nor q_0.

14) So the Wall Situation yields a counterexample to the following assumption about contact (or, in general, distance):

> (C) If some things (e.g., the walls) are (collectively) in contact with something (e.g., a particle), then one of the former must be in contact with the latter.
>
> (Similarly, the Shooting Situation is a counterexample to the temporal analogue of (C).) The assumption holds as far as finitely many things are concerned: if finitely many things (e.g., the first three walls) are in contact with something, one of them must be in contact with it. But this is not generalizable to cases in which infinitely many things are involved. The reason that the walls in the Wall Situation are in contact with the particle at *t* is not that there is one point in the walls whose distance from the particle is 0 (*p* is the only such point), but that they include some points (e.g., q_0, q_1, q_2, etc.) that form a sequence that gets, so to speak, arbitrarily close to *p* (i.e., *p* is the limit of the sequence). This does not make any of the walls contact the particle, because none of them contains all the points in such a sequence.

This solution to the puzzle of action at a distance, we have seen, applies *mutatis mutandis* to Benardete's two puzzles of before-effect. An event can be caused by infinitely many events none of which causes it individually, just as a particle can be arrested by infinitely many walls none of which arrests it individually. The particle's arrest by the walls in the Wall Situation is not an action at a distance, because there is no spatial gap between the walls, taken together, and the particle at the instant of its arrest (i.e., that after which it will make no further progress). Similarly, Sam's death in the Shooting Situation is not a before-effect, because there is no temporal gap between the instants of the shots, taken together, and the last instant of his life (i.e., that after which he will no longer be alive).[15]

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[15] Hawthorne considers a situation similar to the Wall Situation, and gives an analysis of the situation that is essentially the same as mine except that it makes an idle detour through "the fusion of the walls" (2000, p. 626). But he gives too much weight to the falsity of a variant of Principle (C), discussed in note :

> If the *fusion* of some things is in contact with something, then one of the former things must be in contact with the latter thing,

> which he calls "The Contact Principle" (*ibid*., p. 626). He holds that we might be puzzled by the situation because "we are not clear about how to relate fusions to contact" (*ibid*., p. 626). And he applies the same reasoning to Benardete's Shooting Situation, which results in a wrong solution: "By parity of reasoning, we can now say that . . . . At the point at which the fusion penetrates [the heart], no bullet will have so penetrated. So we should say that the fusion of the bullets kills the person . . ." (*ibid*., p. 627). There is no such instant in the Shooting Situation: even the fusion of the bullets will not have penetrated the heart at *t*, while infinitely many

Now, it might be useful to note that there are various ways in which many events cause something collectively. In some cases of collective causation, none of the many causal events would have sufficed to have the same effect without the other events. For example, a person who might have survived the failure of either of his two kidneys might be dead by the simultaneous failure of both kidneys. In some cases, however, many events that might have sufficed to cause something individually might have the same effect only collectively. Suppose that two or more stones any one of which is sufficient to break a window glass hit it simultaneously, which results in the breakage of the window glass. In this case, it would be wrong to attribute the breakage to any one of those stones (or, even less plausibly, to conclude that the breakage is an uncaused event). Surely, the breakage is caused by the stones – collectively. Similar analysis applies to the situations that Benardete considers to present his first two puzzles: in the Shooting Situation, for example, the infinitely many shots (collectively) cause Sam to be dead, although any one of them would have sufficed to have the same effect without the other shots.[16]

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bullets will have individually done so at any instant after *t* (see note for more on this). On my analysis, it is not the spatial principle (C) itself but its temporal analogue that is violated in the Shooting Situation.

16) A further complication arises in cases that involve infinitely many collective causes. Such cases (e.g., the Wall Situation and the Shooting Situation), as we have seen (see note), might be counterexamples to Principle (C) and its temporal analogue, but there are no counterexamples to these principles involving only finitely many collective causes.

This, we have seen, yields immediate solutions to the two puzzles. But it is not as straightforward to apply the notion of collective causation to the other puzzle, the paradox of gods. To do so, we have yet to address the question "What can, individually or collectively, arrest the man in the situation described in [*a*]?" After addressing this question, as we shall see, we can resolve the paradox of gods as well by applying the notion of collective causation.

2. The Paradox of Gods

To present the situation that Benardete imagines to formulate the paradox of gods, let *o*, *p*, q_0, q_1, q_2, etc. be points in a straight line, *L*, related as in the Wall Situation: *o* is 1 mile to the left of *p*, and q_0, q_1, q_2, etc. to the right of *p* by 1 mile, ½ mile, 1/4 mile, etc. Then the situation can be described as follows:

> *The God Situation*: A man, Guy, decides to move in *L* from *p* to q_0. But there is an infinite sequence of gods waiting to throw up permanent, immovable, and impenetrable walls in adjacent segments of *L* to the right of *p* before he reaches the segments. God-1 intends to throw up a wall occupying $[q_1, q_0)$ if and only if he reaches q_2; God-2 one occupying $[q_2, q_1)$ if and only if he reaches q_3; etc.[17]

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[17] In presenting the paradox of gods (see [a]-[c], quoted in the beginning of this paper), Benardete does not specify where the gods plan to throw up walls. He does so in a note: "The first god vows: if and when the man

Benardete argues that in this situation, "this sequence of mere *intentions* (assuming the contrary-to-fact conditional that each god would succeed in executing his intention if given the opportunity) logically entails the consequence that the man will be arrested" at point p (*ibid*., p. 259f; original italics). He suggests that this is paradoxical or puzzling because the walls that might arrest Guy will never be thrown up because he will not reach any point beyond p.

But Benardete's formulation of the puzzle has some inessential infelicities. Guy's arrest at p falls far short of a *logical* or *mathematical* consequence of the intentions and abilities of the gods. Just as Guy might change his mind (he might cancel his plan to move to q_0), so might the gods change their minds: thinking that Guy would be arrested before q_1, for example, God-1 might cancel his plan to throw up a wall if Guy reaches q_2. If so, all or most of the gods might cancel their plans and Guy might be able to pass beyond p.[18]

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travels ½ mile, I will throw down a wall 1/8 mile ahead of him. The second god vows: if and when the man travels 1/4 mile, I will throw down a wall 1/16 mile ahead of him, &c." (1964, p. 260, no. 1). Like Priest (1999), I make an inessential change to the scheme of placement of walls. The point in either scheme is that the walls will be thrown up before the man reaches the points where the left edges of the walls are to be placed (if the man reaches certain points to their left).

18) Seemingly to forestall this problem, Benardete adds the condition that the intention of God-2, for example, is "unknown" to God-1 (1964, p. 259). This removes one reason that the gods might change their minds. But it fails to address the main point of the objection: there is no reason to take *never changing their minds* to be an essential, let alone logical, feature of gods.

Some might attempt to avoid this problem (and the like) by adding more conditions on the intentions of the gods, their abilities, etc.: God-1 never changes the plan, etc. Such attempts, I think, are beside the point. To present the puzzle, Benardete must assume that the situation satisfies conditions that do not pertain to the intentions or abilities of gods, such as the following:[19)]

> (a) A permanent, immovable, and impenetrable wall that occupies $[q_1, q_0)$ will be thrown up before Guy reaches q_1, if and only if Guy reaches q_2.

To get these conditions satisfied, the intentions of gods (together with their abilities) must bring about *physical changes* as soon as Guy decides to move *whether or not any walls will turn out to be thrown up*. And once such conditions are formulated, there is no need to appeal to their divine origin to present the puzzle. So it is useful to recast the God Situation with no reference to the intentions or abilities of gods or humans. We can do so as follows:

> *The Device Situation*: A particle, M, is moving (continuously) in L from left to right. (It is at point o at the present instant, s,

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19) Benardete would agree. In his further discussion of the paradox of gods, he first suggests: "Actually, there is no before-effect . . . . The cause is the sum-total of the gods' intentions. All of these intentions temporally precede the effect" (*ibid*., p. 260). But then he says, "It is not, after all, the combined intentions of the gods *as such* which block the man's progress at A. It is rather the following sum-total of hypothetical facts, namely (1) if the man travels 1/4 mile beyond A, then he will be blocked from further progress . . . &c." (*ibid*., p. 260).

> one minute before t, having passed all the points to the left of o.) There will be no impediment to its rectilinear motion until it reaches point p at instant t. But there is an infinite series of unfailing devices that can arrest the particle between p and q_0 by throwing up permanent, immovable, and impenetrable walls occupying adjacent segments of L before M reaches the segments. Device-1 will throw up a wall occupying $[q_1, q_0)$, if and only if M reaches q_2; Device-2 one occupying $[q_2, q_1)$, if and only if M reaches q_3; etc.[20]

Because of the way Device-1 works, for example, this situation satisfies a counterpart of (a):

> (a*) A permanent, immovable, and impenetrable wall that occupies $[q_1, q_0)$ will be thrown up before M reaches q_1, if and only if M reaches q_2.

We can formulate some other conditions satisfied by the Device Situation, and show that those conditions, together with (a*), entail that M cannot reach beyond p.

Priest (1999) takes this procedure to turn Benardete's third puzzle into a paradox. He argues that a situation similar to the Device Situation is a *contradictory* but *possible* situation, a physically possible situation that satisfies a contradiction. For further discussion of the puzzle, it is useful to examine Priest's variant thereof.

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20) Some say that different gods have installed the devices (Benardete); others that one omnipotent demon has installed all of them (Priest).

3. Priest's Paradox

Priest imagines a situation similar to the Device Situation and elaborates on it by specifying five conditions on the motion of the particle, *M*, that the situation satisfies. To do so, he assigns real numbers (e.g., -1, 0, 1) to points in *L* (e.g., *o*, *p*, q_0), and refers to the points using numerals: 'point 1' and 'point -1', for example, abbreviate 'the point 1 mile to the right of *p* in *L*' and 'the point 1 mile to the left of *p* in *L*' (and refer to *o* and q_0), respectively. He states the conditions in an elementary language that has variables ranging over real numbers (e.g., '*x*', '*y*') and three non-logical predicates: '*R*(*x*)', '*B*(*x*)', '*y*<*x*', which abbreviate '*M* reaches point *x*', 'a barrier is created at point *x* while *M* is to its left', and 'point *y* is to the left of point *x*', respectively. Here are the five conditions stated in the elementary language (Priest 1999, p. 1f):

(1) [*R*(*x*) & *y*<*x*] 6 *R*(*y*).
(2) [*B*(*y*) & *y*<*x*] 6-*R*(*x*).
(3) -□*y*[*y*<*x* & *B*(*y*)] 6 *R*(*x*).
(4) *x*#0 6-*B*(*x*).
(5) 0<*x* 6 [*B*(*x*): *R*(*x*/2)].

Priest renders them into English as follows:[21]

(1*) The object *M* "cannot reach any point unless it traverses every point to its left."

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[21] All the quotations in (1*)-(5*) are from Priest (1999, p. 2).

> (2*) "If a barrier is created between where the object is and some point to its right [before the object reaches the point at which the barrier is created], the object will never reach there."
> (3*) The object M "will reach any point to its right unless some barrier prevents it."[22]
> (4*) "The area before x=0 [i.e., point 0] is barrier free."
> (5*) "to the right of x=0 [i.e., point 0], a barrier will be created at point x [before M reaches the point] if, but only if, the object reaches point $x/2$."

Now, (1)-(5) are inconsistent as Priest shows.[23] They imply both of the following:

> (a) -□$x[0<x$ & $R(x)]$. (M reaches no point to the right of p.)
> (b) □$x[0<x$ 6 $R(x)]$. (M reaches any point to the right of p.)

First, (1), (2), and (5) imply (a).[24] Assume, for *reductio*, that M reaches a point, point x, to the right of p. Then M reaches point $x/4$, which is to the left of point x (by (1)), and a barrier is created at point $x/2$ while M is to its left (by (5)). So M does not reach point x (by (2)), which contradicts the assumption. Second, (a) together with (3)-(5) implies (b). If M reaches no

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22) Note that (3*) is weaker than (3), which amounts to 'M will reach any point unless some barrier prevents it.' Given (1) or (1*), however, (3*) implies (3). Note also that (2) and (3) logically imply (1) although (2*) and (3*) do not imply (1*).

23) They are not *logically* but *mathematically* inconsistent; they logically imply contradictions *given some basic truths about real numbers* (e.g., '□$x[0<x$ 6 $0<x/2$ & $x/2<x)]$').

24) Benardete can be seen to draw (a) from (1), (2), and a variant of (5): '□$n[R(1/2^n)$: $B(1/2^{n-1})]$', where 'n' is a variable ranging over natural numbers. See the discussion in section 4.

point to the right of p, no point to the right of p will have a barrier while M is to its left (by (5)). So no point whatsoever will do so (by (4)). If so, M reaches any point, including any point to the right of p (by (3)). Now, (a)-(b) imply contradictions, such as '$R(1)$ &-$R(1)$', which states that M both reaches and does not reach point 1 (i.e., q_0).[25)]

What does this mean? I think that it just means that (1)-(5) are *incompatible*, that is, that there is no possible situation (or possible world) that satisfies all of them. (In particular, I think that the Device Situation does not satisfy (3), as I argue below.) Priest argues to the contrary. He argues that they are *compatible*, though inconsistent, and concludes that there is a possible situation that satisfies the contradictions that they imply:

> ... (1) to (3) are features of continuous motion, and are true in this world. (4) and (5) are not. ... There are no parts of the world where passing certain spots brings barriers spontaneously into existence. But there appears to be no reason why there should not be areas like this. For example ... a demon ... [who] is bound by the laws of physics, and so ... cannot suspend *the laws of motion* (1) to (3) ... might 'mine' an area of space in accordance with (4) and (5). ... In such a world, motion produces contradiction. Otherwise put: in our world, motion could be contradictory. (1999, p.2; my italics)

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25) Note that (2)-(4) are inconsistent, as Laraudogoitia (2000) points out. For (2)-(3) logically imply (1). (2*)-(5*), unlike (1*)-(5*), are consistent (for example, M might reach all and only the points to the right of point 1), but they are inconsistent assuming a condition satisfied by the Device Situation:

> (0) M is moving (continuously) in L from left to right, and M reaches point -1 at s, having reached any point to its left before s.

In this passage, he holds:[26]

> *A*. (1)-(3) are *physically necessary* (i.e., they are satisfied by any *physically possible situation*, any possible situation that does not violate any laws of physics).[27]
>
> *B*. The conjunction of (4) and (5)[28] is *physically possible* (i.e., there is a *physically possible* situation that satisfies it).[29]

These theses imply that (1)-(5) are *physically compatible*, that is, there is a physically possible situation that satisfies all of them. So he concludes that some contradictions on motion (e.g., '*R*(1) & -*R*(1)') are physically possible.

I do not think that this is a good argument. What is wrong with it? Some might attempt to fault it by challenging Thesis *B*.[30] I agree with them that both Benardete's scenario of an infinity of gods and Priest's scenario of a single demon fall short of establishing the thesis. For it is one thing to say that we might in some sense *imagine* or *conceive* an area mined as stated by (4) and (5), as Benardete or Priest thinks, quite another to say

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26) I say of a sentence that it is *physically necessary* (*possible* or *impossible*), if it states a physical necessity (possibility or impossibility). This terminology, which I adopt for convenience of exposition, is not a reflection of my view on modality.

27) Some might take Priest to hold merely that (1)-(3) are "true in this world" (*ibid*., p. 2), but this, even together with Thesis *B*, does not yield the desired conclusion (see note).

28) The conjunction of (4)-(5) is logically equivalent to '$B(x)$: [$0<x$ & $R(x/2)$]'.

29) This is equivalent to '(4) and (5) are physically compatible.'

30) See, e.g., Laraudogoitia, who argues that "the demon [that Priest imagines] is trying to do something impossible" because there is no (physically) possible situation that satisfies the conjunction of (4)-(5) (2000, p. 155). See also Yablo (2000) and Shackel (2005).

that the existence of such an area is compatible with all the laws of physics, known or not. For example, perpetual motion (of the first kind) is not (and never was) physically possible, if contemporary physics is to be trusted, because it violates the law of conservation of energy; nevertheless, we can in some sense imagine or conceive perpetual motion (at least we could do so a few hundred years ago). But to show that their cases for Thesis *B* fail is not to show that the thesis is false. I think that those who reply to Priest's argument by rejecting it are led to do so by assuming the other thesis, Thesis *A*. Assuming this, they might take the argument to yield a *reductio* of Thesis *B*. But Thesis *A* has a serious problem.[31)]

To assess Thesis *A*, it is useful to focus on (2) and (3) because they logically imply (1).[32)] He says that "(2) just spells out the meaning of 'barrier'", which he uses interchangeably with 'wall' (*ibid*., p. 2).[33)] So it is reasonable to take him to use

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31) Note that I do not categorically assert that the conjunction of (4) and (5) is physically possible; there might be some laws of physics, known or not, that rule it out. But I do not think that one can discover such laws, which are not logical or mathematical truths, by examining Priest's argument. See also note .

32) Priest, recall, holds that (1)-(3) simply state "features of continuous motion" (*ibid*., p. 2). Regarding (1), he says that it just "says that [*M*'s] motion is continuous" (*ibid*., p. 2). This is not quite right; it rather amounts to '*M* moves continuously from left to right', which is not physically necessary. But we need not dwell on this issue at this point. For (1) logically follows from (2)-(3), and (3) has a similar problem (see note below).

33) Priest uses the two words interchangeably. See his English expression for the condition stated by '*B*(*x*)': 'a barrier (wall) is created at point *x* whilst [*M*] is to its left' (1999, p. 1).

'barrier' and 'wall' to mean what I mean by 'permanent, immovable and impenetrable wall'. On this reading, (2) is analytic and thus physically necessary. But (3) cannot be taken to be physically necessary.

Priest argues that it is physically necessary as follows:

> (3) is a version of Newton's first law of motion. An object will travel in a straight line unless *impeded* by something. In particular, [the object *M*] will reach any point to its right unless some *barrier* prevents it. (1999, p. 2; my italics)

It is reasonable to take the second sentence in this passage to state a consequence of Newton's first law of motion, the law of inertia: an object, by the law, must travel in a straight line (with the same velocity) unless there is a *force* acting on it; so it will travel in a straight line (in the same direction) unless there is some *impediment*, namely, *force against* its rectilinear motion. The sentence (so understood) can be seen to imply the following:

> (6) *M* will reach any point in *L* to its right, if there is no *impediment* to its motion (given that *M* moves from left to right in *L*).

But this does not imply (3) or (3*), which we might take to amount to the following:[34]

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[34] Strictly speaking, (3) or (3*) amounts to '*M* will reach any point in *L* (to its right) if there is no *barrier* set up at a point in between before *M* reaches the point', but it is hard to take Priest to hold that this is physically necessary. I take his claim to be that the conditional 'If *M* moves in *L* from left to right, then *M* will reach any point to its right in *L*

(3a) *M* will reach any point in *L* to its right, if there is no *barrier* that is set up at a point in between before *M* reaches the point (given that *M* moves in *L* from left to right).

Barriers (or *walls*) set up in front of a moving object can exert *impediments* to (i.e., *forces against*) its motion, but there might be forces of other kinds, forces other than those exerted by walls, that can impede the motion. So (3a) might fail to hold without violating the law of inertia.

Surely, (3a) follows from (6) given additional assumptions, such as the following:

(7) There is no force against *M*'s rectilinear motion in *L* except those exerted by *barriers*.

But this does not mean that (3a) is physically necessary unless (7) is physically necessary. And (7) cannot be taken to be true, let alone physically necessary. There must be, in the actual world, some other forces (e.g., the gravitational forces exerted by remote bodies) against *M*'s rectilinear motion. Some might object that to derive (3a) from (6), it is sufficient to make a weaker assumption:

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if there is no *barrier* set up at a point in between before *M* reaches the point' is physically necessary, and it is this claim that I challenge. (Accordingly, when Priest claims that the conjunction of (4) and (5) is physically possible [Thesis *B*], I take him to mean that the conjunction of (4), (5), *and* '*M* moves (continuously) in *L* from left to right, and *M* reaches point -1 at *s*, having reached any point to its left before *s*' [see condition (0) in note] is so). In the subsequent discussion, incidentally, I do not distinguish (3) and (3*). I mention only (3), but the points made on (3) apply to (3*) as well.

> The vector sum of all the forces that act on *M* except those exerted by barriers is null or at least does not amount to an impediment to *M*'s motion.

Although this might be physically possible or even true, it cannot be taken to be physically necessary. So one cannot use it to establish the physical necessity of (3a).

Now, can we rescue Priest's argument by taking him to use 'barrier' to mean what I mean by 'impediment', i.e., any force against rectilinear motion? On this reading, (3) or (3a) is equivalent to (6) and, thus, physically necessary. But the reading makes (2) fail to be physically necessary, let alone analytic: a force acting against *M*'s motion at a point might not be strong enough to arrest it at that point. To restore the physical necessity of (2), which Priest says "just spells out the meaning of 'barrier'" (1999, p. 2), we would have to take 'barrier' to mean *insurmountable impediment* and '$B(x)$' to abbreviate 'a force strong enough to block *M*'s motion at point *x* (if *M* reaches point *x*) is created at point *x* while *M* is to its left'. This reading does not help to rescue the argument, either, because it fails (3a). Its antecedent (for point *x*), on the reading, states that no *insurmountable* impediment is created at any point to the left of point *x* (while *M* is to its left), but *M* might fail to reach a point that satisfies this condition. Suppose that there is an infinite series of forces that act against *M*'s motion at point ½, point 3/4, point 7/8, etc., and that they halve the speed of *M* at those points. Then *M* cannot reach point 1. So (3a) fails to be physically necessary on this reading as well.[35]

Some might try to reformulate Priest's argument by replacing the thesis that (3) is physically *necessary* with weaker theses, such as the following:

(i) (3) is true.
(ii) (3) is physically possible.

Priest holds (i), it seems, because he thinks that (3) is physically necessary. This reason for holding (i) fails, as we have seen. And I do not think that (i) is a plausible thesis: there is not likely to be any object in the actual world that moves in a perfectly straight line (unless it encounters a wall). Still, there might turn out to be such an object in a remote region of space where the outside forces are balanced, which makes (3), considered a statement about that object, true. And even if there is no such object in the actual world, there seem to be no laws of physics that preclude situations that satisfy (3), such as those in which the only object in the world is a particle moving inertially. So I think that (ii) is a plausible thesis. But this does not help Priest. To argue that the contradiction '*R*(1) & -*R*(1)', for example, is physically possible because it follows from (1)-(5), he needs to assume that (3)-(5) are physically *compatible*. But this does not follow from (ii) together with Thesis *B*, the

35) To avoid this problem, some might consider taking '*B*(*x*)' to abbreviate 'a force strong enough to block *M*'s motion at point *x* is created at point *x* (while *M* is to its left) whereas there is no force against *M*'s motion to the left of point *x*.' (3), on this reading, implies '*M* reaches point *x*, if there *is* a force against *M*'s motion at every point to the left of point *x*', which clearly fails to be physically necessary.

thesis that the conjunction of (4)-(5) is physically possible. Two sentences that are both physically possible (e.g., 'There is a wall at *p*' and its negation) might fail to be physically compatible. Moreover, it does not help to assume the stronger, if less plausible, thesis (i). Two sentences one of which is true and the other physically possible (e.g., 'There is a wall at *p*' and its negation) might not be physically compatible, either.[36)]

Can one reformulate Priest's argument by holding that (3)-(5) are physically compatible?[37)] This is subject to an immediate objection: a contradiction results from (3)-(5) together with (2), which is an analytic truth; so (3)-(5) cannot be compatible, let alone physically so. Priest, who holds dialetheism, the view that some contradictions are true (and thus physically possible),[38)] would dispute an assumption in this objection: any sentences that yield a contradiction are incompatible. But even Priest would agree that if some sentences yield a contradiction, this is strong, if defeasible, evidence that they are incompatible. Otherwise, he might as well have held that 'There is a wall at *p*' and its

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36) This, I think, is the reason that Priest holds that (1)-(3) are not just true, but physically necessary (see the earlier note).

37) Priest suggested this in reply to an earlier version of this paper (private correspondence). In the reply, he argues that (3)-(5) are physically compatible because an omnipotent demon can mine an area according to them. This is not a good reason for holding it. Because the demon is assumed not to violate any laws of physics, we have no reason to think that the demon can mine an area according to (3)-(5) unless we have an independent reason to accept their physical compatibility.

38) His main argument for dialetheism lies in his analysis of semantic paradoxes; he holds that both the liar sentence and its negation, for example, are true. See, e.g., Priest (2006).

negation are physically compatible, if inconsistent, to conclude that some contradictions are physically possible.

Some might argue that there is a strong reason for holding the physical compatibility of (3)-(5) that overrides the contrary evidence given by the proof that they are inconsistent with the analytic truth (2). To do so, they might hold that we have a strong intuition that the Device Situation (or the God Situation) is a physically possible situation. We might have such an intuition, and the situation might indeed be physically possible. If so, however, there is no good reason to think that it satisfies (3). The description given for the situation includes sentences that fairly directly imply (4)-(5) (or the like[39]). By contrast, they include no statement whatsoever on whether there are any things other than barriers or walls that might block *M*'s motion. Some might take this omission in the description to implicate that there are no such things. But it is wrong to do so, if the putatively implicit statement, (3), conflicts with the explicit statements. And the inconsistency of (3)-(5) with the analytic truth (2) is a good reason to think that (3) conflicts with (4)-(5).

Moreover, this gives a good reason to conclude that the Device Situation, if physically possible, must fail to satisfy (3). More precisely, any physically possible situations that satisfy the description of the Device Situation must fail to satisfy (3) because they satisfy (4)-(5) (or the like) as well as (2). Although such situations, being physically possible, must satisfy

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[39] The description does not include (5), but includes a related sentence, (5a), which is stated in section 4. And (5a), like (5), is inconsistent with (1)-(4).

consequences of the law of inertia (e.g., (3a)) as well, (3) is not one of its consequences. So we have no good reason to hold that (3)-(5) are compatible. Their inconsistency with the analytic truth (2), by contrast, gives a strong reason for holding that they are not compatible. So I conclude that they are not even compatible, let alone physically so.

4. Accumulation of Forces

We can now turn to Benardete's third puzzle, the paradox of gods. To attack the puzzle, it is useful to consider the Device Situation (or the like) rather than the God Situation. By doing so, as we have seen, we can remove epicycles arising from the interactions of intentions, abilities, and activities of humans or gods to focus on the relevant conditions and their consequences.

The Device Situation satisfies three of Priest's five conditions:

(1) $[R(x)$ & $y<x]$ 6 $R(y)$. (*M* can reach a point in *L* only if it reaches every point in *L* to its left.)[40]

(2) $[B(y)$ & $y<x]$ 6 $-R(x)$. (If a barrier is set up at a point in *L* while *M* is to its left, *M* will never reach any point in *L* to the right of the point.)

(4) x#0 6 $-B(x)$. (No barrier will ever be set up at *p* or a point

[40] (1) is not quite a consequence of the description of the Device Situation (*M* might leave *L* at *p*, but return to *L* at q_0 after taking a continuous path outside *L*), but we may assume that the situation satisfies (1) by taking '$R(x)$' to abbreviate '*M* reaches point *x* without ever leaving *L*'. Note also that I use 'barrier' henceforth as a shorthand for 'permanent, immovable, and impenetrable wall'.

in *L* to its left while *M* is to its left.)

And it satisfies a variant of (5):[41]

(5a) $R(1/2^n)$: $B(1/2^{n-1})$. (A barrier will be set up at point $1/2^{n-1}$ while *M* is to its left, if and only if *M* reaches point $1/2^n$.)

where '*n*' is a variable ranging over natural numbers. Now, (1), (2), and (5a) imply the following:[42]

(a) -□*x*[0<*x* & *R*(*x*)]. (*M* reaches no point to the right of *p*.)

On the other hand, the situation satisfies the following conditions as well:[43]

(0) *M* is moving (continuously) in *L* from left to right, and *M* reaches point -1 at instant *s*, having reached any point in *L* to its left before *s*.

(4a) There is no impediment to *M*'s rectilinear motion until it reaches point *p* at *t*.

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41) (5a), like (5), is inconsistent with (1)-(4). So the Device Situation does not satisfy (3), unless it is an impossible situation. It is implicit in the description of the Device Situation that no barrier will be set up except those set up by the devices. So we may take it to satisfy a condition stronger than (5a):

(5a*) $B(x)$: ∃$n[x=1/2^{n-1}$ & $R(1/2^n)]$.

(Note that this implies (4) as well as (5a).)

42) To show this, it is necessary to appeal to the mathematical truth '∀*x*[0<*x* 6 ∃$n(1/2^n<x)$].'

43) Notice that the description of the Device Situation includes explicit statements of conditions (0) and (4a). (4a) is stronger than (4), which, even together with the other three conditions (i.e., (1), (2), and (5a)), does not imply that *M* reaches *p*.

And these imply the following:

(c) *x*#0 6 *R*(*x*). (*M* reaches *p* and any point to its left.)

So *M* will be arrested at point *p* (in the situation); it will reach *p* (by (c)), but no point in *L* to its right (by (a)). But no barrier will ever be set up (while *M* is to its left)[44] because *M* will never reach beyond *p*. If so, why will *M* be arrested? What can cause its change in motion?

Note that the conclusion that *M* will be arrested at *p* in the Device Situation does not rest on the assumption that the situation is *physically possible*. To pose a puzzle by raising questions about the cause (or causes) of the arrest, however, it is necessary to assume this. Without the assumption, one can draw no further conclusion about its causes. There would remain various possibilities: *M* might stop or change its course at *p* spontaneously without encountering any impediment; the devices might arrest *M* without interacting with it;[45] some strange force

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44) To draw this conclusion, it is necessary to appeal to (5a*), '*B*(*x*) : ☙ *n*[*x*=$1/2^{n-1}$ & *R*($1/2^n$)]', which is stronger than (5a), but we may ignore the difference between (5a) and (5a*) because we cannot take the arrest of *M* at *p* to be necessarily due to the violation of (5a*).

45) In discussing his variant of the God Situation, a situation comparable to the Device Situation, Hawthorne concludes that "changeless causation" (or causation that involves no interaction) must take place in the situation (2000, p. 630). He holds that considering the situation leads to the conclusion that "By suitably combining things that need to change in order to produce a result, we can generate a fusion that can produce that result without undergoing change", which he holds is "a big metaphysical surprise" (*ibid*., p. 630). This analysis would make the situation (an analogue of the Device Situation) a *physically impossible*, if metaphysically

might arise to arrest it; etc. Most, if not all, of such possibilities might strike one as strange. But presenting such a strange possibility does not necessarily result in posing a paradox or puzzle. A situation that extends the Device Situation by having *M* stop without encountering any impediment, for example, would strike one as strange or even mysterious because it would be an unfamiliar situation that violates some laws of physics. But it is one thing to describe a strange situation not familiar to us that turns out to be physically impossible, quite another to present a puzzle. Otherwise, one could pose a puzzle simply by describing a situation as one in which *M* both reaches and does not reach *p*. So Benardete's presentation of the paradox of gods must rest on the implicit assumption that the situation under consideration is physically possible.

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possible, situation (violating Newton's third law of motion or the principle of conservation of momentum), although it is not clear from his discussion whether he takes it to be such a situation. And he does not explain why the other possibilities of comparable standing mentioned above (e.g., 'arrest' without cause or the rise of a mysterious force), for example, must be ruled out. I see no reason why they must be less compatible with the conditions stated in the description of the situation. He holds that changeless causation is *logically* entailed by the description of the situation: "getting together enough of the mundane things and suitably arranging them is all by itself *logically sufficient* to entail changeless causation" (*ibid*., p. 630; my italics). But he cannot defend this claim without ruling out the other possibilities, such as those mentioned above. Notice that the analysis that I present below lays out another possibility: the blocking devices collectively arrest *M* by interacting with it when it reaches *p*. This possibility, I think, is more compatible with the conditions stated in the description of the Device Situation; it is physically compatible with the conditions as long as these are themselves physically compatible. See also note below.

What can we conclude if we assume that the Device Situation is physically possible?[46] Using this assumption, we can conclude that *M* must be arrested by an *impediment*, some *force* against its rectilinear motion. The situation, to be physically possible, must conform to the law of inertia and satisfy the following:

> (6) *M* will reach any point in *L* to its right, if there is no *impediment* to its motion (given that *M* moves from left to right in *L*).

So it must satisfy the following:

> (4b) There is an impediment to *M*'s rectilinear motion at instant *t*, when it reaches *p*.

If so, what gives rise to the impeding force in the situation?

This question, notice, arises for any physically possible situation that satisfies the above-mentioned conditions (i.e., (0), (1), (2), (4a), and (5a)),[47] including the Device Situation and the God

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[46] Notice that I do not make the categorical assertion that the Device Situation is physically possible (or, more precisely, there is a physically possible situation that satisfies the conditions stated in its description). I do not think that Benardete's paradox or Priest's puzzle or any similar considerations that are mostly philosophical in nature can establish that it is physically impossible, and I doubt that any established laws of physics preclude the situation. But this does not establish that the situation *is* physically possible – the existence of tachyons, particles traveling at superluminal speed, for example, might turn out to be physically impossible, although there might be no known laws of physics that rule it out. So my discussion is neutral on the ultimate status of (5a) or the entire description of the Device Situation.

[47] (4) is not listed because it is implied by (4a).

Situation.[48] In such situations, an impeding force must arise to arrest *M*. Given that no barrier will ever be thrown up (while *M* is to its left) in the situations,[49] it might seem that one cannot explain how the force arises. So Benardete suggests that its rise in such situations would be strange or mysterious. He concludes his presentation of the paradox of gods by saying that the arrest "will be described by the man as [due to] a strange field of force blocking his passage forward" (*ibid*., p. 260).[50] So the

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48) It is not immediate from the description of the God Situation that it is a physically possible situation satisfying the conditions (or their analogues for the man, Guy), as we have seen, but we might take it to be implicit in the description that it is such a situation. And, for convenience of discussion, we may henceforth identify Guy with the single particle *M* to take the God Situation to satisfy the above-mentioned conditions on *M*.

49) So the following must fail in the situation:

(7) There is no force against *M*'s rectilinear motion in *L* except those exerted by *barriers*.

50) Some might take Benardete to mean simply that while there is *no* force that blocks his advance, Guy will have an erroneous feeling of such a force (and finds it to be strange). If so, Benardete would be imagining a physically *impossible* situation to present the puzzle, which we can see by considering a comparable situation that extends the Device Situation by satisfying the additional condition that there is no force against *M*'s motion at *p*. But there is no puzzle to solve, as we have seen, once it is made clear that the described situation might turn out to be physically impossible, e.g., one in which one can be arrested without encountering any impediment. (Incidentally, a situation that fits the literal description of the God Situation but that fails to satisfy the above-mentioned conditions might satisfy the no-force condition without becoming physically impossible: for example, the man might not actually exert his force to advance beyond *p*

paradox of gods boils down to a challenge, the challenge to explain how the impeding force arises in any physically possible situation that satisfies the above-mentioned conditions. We can meet the challenge, I think, by attending to the changes brought about to implement condition (5a), such as installation of infinitely many blocking devices.[51)]

Why would one think that the rise of the impeding force in, e.g., the Device Situation would be strange? Those who do so might assume that there is nothing to the right of point *p*. If this is correct, it would be hard to see how a force that can arrest *M* at the point arises in the situation. But it is wrong to make that

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because he mistakenly feels blocked by a strange force. This, again, shows the advantage of focusing on the conditions stated in the Device Situation.)

51) In presenting his solution to the paradox of gods, Benardete analyzes the God Situation more or less in the way I have done, and offers an explanation of the rise of the impeding force: "the cause of [the] arrest is simply the man's encounter with a field of force, and *this field of force* is simply the physical equivalent of *an omnibus law of nature which is compounded out of an infinite sequence of contrary to fact conditionals*", such as "if the man travels 1/2 mile beyond A, then he will be blocked from further progress" (1964, p. 260; my italics). This explanation confuses issues of physics (or of causes) with those of logic and mathematics (or of consequences). The arrest of the man and the existence of the impeding force, we have seen, are logical or mathematical consequences of (5a) (or, as he puts it, "an infinite sequence of contrary to fact conditionals") – together with some other conditions and laws of physics (e.g., the law of inertia). But this only explains *that* the impending force must arise in the situation. It does not explain *how* it would arise. This is a question about the *cause* of the force that we cannot answer simply by clarifying conditions that imply its *existence*. (Incidentally, Benardete thinks that his solution to the paradox of gods and related solutions to the first two puzzles are "placebos" [*ibid*., p. 261] that do not help to resolve a variant of the first puzzle. See note .)

assumption simply because the situation has no barrier, no *permanent*, *immovable*, *and impenetrable wall*, to the right of *p*. A situation that has no such wall might still have many objects (e.g., ordinary walls) to the right of *p* that can exert enough force to arrest *M* at the point. And the Device Situation, which has devices that implement condition (5a), must have infinitely many objects to the right of *p*.

To see this, consider, for example, Device-1, which implements an instance of (5a):

> (D1) *R*(½) : *B*(1). (A barrier will be created at point 1 while *M* is to its left, if and only if *M* reaches point ½.)

In order to ensure that this condition holds, the device would have to be placed around points 1 and ½. To see this, it is useful to divide the device into two coordinated parts: (i) a *blocker*, which can throw up a barrier at point 1 (i.e., q_0); and (ii) a *detector*, which can detect *M* at point ½ (i.e., q_1) and send a signal to the blocker if and only if *M* reaches the point. Clearly, the blocker is essential to the device. The detector is equally essential: the blocker cannot 'tell' when to throw up a barrier (and when not to) unless it is coordinated with a device that can 'tell' it whether or not *M* has reached q_1. Now, the blocker would have to be located around q_0, close enough to the point to be prepared to build a barrier upon receiving a signal from the detector. For example, it must have the material that can be turned into a barrier close enough to q_0 so that the barrier can be completed within the time it would take for *M* to move from q_1 to q_0. (The simplest way to do so would be to have a very strong wall already in place at q_0 to make it permanent, immovable, and impenetrable with just a small amount of energy transferred by a signal.) And we can be more specific about the placement of the detector. We can imagine various ways in which it might operate: it might place a stationary particle at q_1 that can be sent to the blocker if, and only if, it is hit by M at the point; it might constantly beam a ray of photons to q_1 that can be deflected to the blocker if, and only if, M reaches the point; etc. No matter what the details of its operation are, however, it is hard to see how the device can work properly without extending as far as to q_1 by placing a stationary particle there or constantly beaming a ray thereto, etc. So q_1 must be an outpost, so to speak, of Device-1 occupied at any instant by its vanguard particle. So the device must occupy q_1 as the outpost for its detector while placing its blocker (including the material it uses) around q_0. Similarly, Device-2 must occupy q_2 while placing a part around q_1, Device-3 must occupy q_3 while placing a part around q_2, and so on.

So the devices that implement (5a) in the Device Situation must, at any instant, have infinitely many particles between p and q_0, including (but not limited to) the vanguards placed at their outposts: q_0, q_1, q_2, etc. And those particles must *accumulate toward* point p from the right; that is, there must be infinitely many particles between p and any point to its right (e.g., q_0, q_1, q_2) no matter how close this point might be to p. So at instant t, when M reaches p, it comes to contact the devices although it may not come to contact any one of them. There is no gap between the devices (taken together) and *M* at *t* in the Device Situation, just as there is no gap between the walls (taken together) and *M* at *t* in the Wall Situation.

If so, it is not surprising that the devices can arrest *M* at *p*. They can do so while none of them can individually do so, just as the infinitely many walls in the Wall Situation can arrest *M* at *p* while none of them can individually do so. Because the devices collectively contact *M* at *t*, it is not strange at all that they can collectively exert enough force against *M*'s motion at *t* even though none of them can do so by itself.[52] To reach any point to the right of *p*, *M* must push through infinitely many particles, which get more densely clustered around *p* than the particles forming any wall that we are familiar with are clustered in the wall. So it would be surprising if those particles, taken together, exerted no force against *M*'s motion. The question is not whether they will give rise to an impeding force, but how strong the force will be. And we can use logic, mathematics, and a modicum of physics to answer this question: any physically possible devices that implement (5a) must collectively exert a force strong enough to arrest *M* at *p*. We can invoke the law of inertia to draw this conclusion, just as we can invoke the principle of conservation of energy to draw the conclusion that any physically possible machine must fail to be a perpetual

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52) None of them can by itself exert enough force to arrest *M*, because Device-1, for example, is designed to arrest *M* with its blocker by the time it reaches q_1 (if it ever reaches the point), not directly at q_0 with its detector.

motion machine (of the first kind), a system that produces more energy than it consumes.

Notice that it is wrong to demand an account of one specific mechanism through which the blocking devices give rise to the strong impeding force, just as it is wrong to suppose that there is one specific way in which all the attempts to build a perpetual motion machine must fail. There are various physically possible machines, and they fail to achieve perpetual motion for different mechanical reasons. Similarly, there must be many different configurations that can yield devices that implement (5a), if this is physically possible, and different configurations would result in the impeding force in different ways. But just as the conservation principle imposes constraints on physically possible machines, so does the law of inertia impose constraints on configurations of physically possible devices that collectively implement (5a).

This, we can see, resolves Benardete's paradox of gods. To use the God Situation (or the like) to present a paradox or puzzle, as we have seen, Benardete must assume that it is a physically possible situation.[53] If so, the intentions of the gods under the given circumstances must have brought about changes in the world that result in implementation of (5a) while retaining the other conditions stated in the description of the Device Situation. We can use the conditions, together with the law of inertia, to conclude that Guy (or *M*) will be arrested at *p* by an impeding

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53) Otherwise, there is no reason why Guy cannot reach beyond *p* while nothing blocks his advance, why the gods might not arrest him without interacting with him, or why their mere intentions might not give rise to a strange force in a way that cannot be explained.

force. And we can see that the rise of such a force is a consequence of the infinitely many changes instigated by the gods' intentions under the circumstances. We cannot specify *the* mechanism through which the force arises. But this is not because the rise of the force would be strange or mysterious, but because the specific way in which the changes are made to implement (5a) is not given. The way in which the force arises depends on the way in which those changes are made, and the details of the changes would differ among physically possible situations that satisfy the description of the God Situation (or the like). No matter what the details are, however, the condition of the area to the right of point *p* cannot remain the same after the changes. Only those who fail to reflect on this with undue focus on the mere absence of barriers would conclude that the rise of the impeding force in such situations would be strange.

Reflections on the devices or changes for implementing (5a), we have seen, leads to the conclusion that they must be accompanied by the rise of a force strong enough to arrest the man. This applies not only to devices or changes brought about by humans but also to those brought about by Benardete's gods, who can transcend neither logic nor mathematics nor physics. Even those gods cannot instigate changes in the physical world to implement the condition without ending up incurring a force that can arrest the man.[54] They cannot do so because they, in the

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54) The arrest of *M* at *p* in the God Situation is not intended by any of the gods (nor is it intended by them collectively). So it is a sort of side-effect of their intentions that, under certain conditions, can collectively bring about implementation of (5a). It should not be a surprise that even the gods'

end, cannot overcome the power of collection and accumulation. Many small particles can collectively exert a great force that none of them can individually exert; and a force (or change) of an anticipated scale can result from accumulation of minuscule forces (or changes), especially when infinitely many of them accumulate. Now, these point, taken individually, would be small points to make. But many small points can add up to provide solutions to intriguing puzzles.[55]

actions, individually or collectively, must usually have side-effects. For example, a god might intend to place a particle, *P*, at a place, *A*, where it does not exist at a time *t*, without intending to remove it from a remote place, *B*, where it exists at *t*, and it might well achieve the aim by hitting *P* with another particle, *Q*. In that case, both *P*'s subsequent absence in *B* and its subsequent presence in *A* are effects of the action, but only the latter is its intended effect.

55) So I think it is wrong to hold, as Hawthorne does, that the paradox of gods heralds "a big metaphysical surprise" of "changeless causation" (2000, p. 630). On my analysis of the Device Situation, notice, the devices interact with *M* when it reaches *p*, and go through other changes while arresting it: they come to contact *M* when it reaches *p*, and *M* must react to them with a force equal in magnitude to the force that they exert to *M* (the principle of conservation of momentum), although the reaction, distributed to the infinitely many devices, would probably not be noticeable in any one of them individually.

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