구조손상 탐색을 위한 부 집합 선택에 의한 정규화 방법

Regularization Method by Subset Selection for Structural Damage Detection

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요 지

본 논문에서는 구조손상 탐색을 위해 매개변수 부 집합 선택에 의한 새로운 정규화 방법을 제안하였다. Residual function을 위해 동적 residual force 벡터를 이용하였다. 과거에는 Residual function으로서 기본 동적 특성치(고유치와 고유모드)를 이용하여 단일구조손상은 탐색할 수 있었지만 다중구조손상 위치를 탐색하기에는 한계가 있었을 뿐 아니라 고유모드와 고유치의 상이한 기여도 때문에 가중치를 적용해야 하는 어려움이 있었다. 본 논문에서 제안된 방법은 고유모드의 불완전한 계측을 보완하기 위하여 모델 확장법을 적용하였다. 제안된 구조손상 탐색법은 다중구조손상 위치를 동시에 찾아 낼 수 있는 장점을 가지고 있다. 2차원 평면 트러스 구조를 이용하여 제안된 방법의 효용성을 검증하였다.

핵심용어 : 정규화, 부집합선택, 손상도 검색, 구조건전성모니터링, 모형 업데이트

Abstract

In this paper, a new regularization method by parameter subset selection method is proposed based on the residual force vector for damage localization. Although subset selection using the fundamental modal characteristics as a residual function has been successful in detecting a single damage location, this method seems to have limited capabilities in the detection of multiple damage locations and typically requires cumbersome weighting values. The method is presented herein and considers cases in which damage detection must be achieved using incomplete measurements of the structural responses. Model expansion is incorporated to deal with this challenge. The unique advantage of employing the new regularization method is that it can reliably identify multiple damage locations. Through an illustrative example, the proposed damage detection method is demonstrated to be a reliable tool for identifying multiple damage locations for a planar truss structure.

Keywords : *regularization, subset selection, damage detection, structural health monitoring, and model updating*

1. Introduction

The recent collapse of the I-35W bridge in Minneapolis highlights the importance of continuous structural health monitoring(SHM) of our infrastructure for public safety. One of the motivations in SHM is damage diagnosis. Damage inspection is an exhaustive and somewhat inconsistent process that requires numerous hours of hard labor. Aging, corrosion, and fatigue are all factors that result in damage in structures, in some cases making them serviceably unsafe. Recent advances in sensor technologies combined with enhanced optimization techniques provide the tools for localization and quantification of damage in civil engineering structures for improved prioritization of maintenance needs and prevention of catastrophic consequences.

One of the common approaches to achieve damage

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detection is to use measured low frequency vibration response data. Extensive surveys on vibration based damage detection and their challenges have been presented by Doebling et al.(1998) and Friswell et al.(1997). Numerous investigations have been conducted to detect damage in large-scale structures using vibration data such as the frequency response function and modal data (natural frequency, damping ratio, and mode shapes). The underlying motivation in vibration based damage detection is that certain dynamic residual function between unhealthy and healthy structure is a nonlinear function of damage state variables.

Two classes of approaches have been investigated to locate damage in structures, including 1) optimal matrix update methods and 2) parameter based finite model updating and regularization. The optimal matrix update methods has focused on applying a minimization to the property perturbation to obtain a solution to the residual modal force equation subject to constraints such as symmetry, positive definiteness, and sparsity. In the context of damage detection, the perturbation is usually described in terms of the stiffness properties. As an objective function, the majority of early work in the optimal matrix update methods used a minimum norm of the global stiffness matrix(Baruch et al., 1978; Berman et al., 1983). Although this method achieves a solution by perturbing the global stiffness matrix as little as possible, all of the elements in the matrices may be altered. Therefore, the effect of any damage would likely be spread out over all degrees of freedom, making it very difficult to locate damage. In 1994, Kaouk, et al. tried to resolve the problem by using a minimum rank of the global stiffness matrix. In 1996, Doebling et al. extended this method using elemental stiffness parameters while constraining the connectivity of the global stiffness matrix. However, this technique can not ensure changes in the stiffness that might be associated with damage in a particular region.

Model updating approaches attempt to quantify damage within a structure by adjusting the parameters associated with the finite elements in an analytical model(called the identification model) that represents the behavior of the structure. Thus, structural damage detection problems are recast into optimization problems where a set of damage parameters in the analytical model are adjusted to achieve the maximum correlation with the experimental observations. Numerous correlation based metrics have been proposed based on modal quantities, such as natural frequencies and mode shapes. For multiple damage locations, Messina *et al.*(1996; 1998) suggested a series of correlation-based techniques, DLAC(Damage Location Assurance Criterion) and MDLAC(Multiple Damage Location Assurance Criterion). Recently, Koh et al.(2007) investigated practical aspects of the damage detection methods through exploiting various correlations in terms of natural frequencies and mode shapes. However, because all of the parameters in a large-scale baseline finite element model are likely to be changed, the damage detection often becomes an underdetermined problem that has a non-uniqueness issue.

Researchers have suggested several methods to deal with this challenge, often by reducing the number of parameters or expanding the measured data as part of the efforts for damage detection. One approach is to use conventional regularization techniques such as the Tikhonov regularization method or the Lagrange multiplier method. Although the regularization technique applies extra constraints to the ill-posed parameter estimation problem to ensure a unique solution(Fregolent et al., 1996; Rothwell et al., 1989), minimum norm type solutions tend to spread the identified damage over a large number of parameters, making damage location unclear. The other approach is to select a subset of damage parameters that best represents the damage locations by solving a combinatorial optimization problem. In 1997, Friswell, et al. suggested a parameter subset selection method for locating damages using eigen-sensitivity, measuring the differences in natural frequencies between the damaged and undamaged state of a structure. In 2003, Titurus *et al.* extended the procedure provided by Friswell *et al.* by utilizing the mode shape differences due to damage and its sensitivity. Although the subset selection method of utilizing fundamental modal characteristics as a residual function was successful in detecting a single damaged location, this method is limited when attempting to detect multiple damages and typically requires cumbersome weighting because of the difference in units. Recently, damage functions have been used to overcome the limitation by smoothing distribution of the model properties(Song et al., 2007). However, accuracy in approximating continuous distribution of physical properties is dependent on the coarseness of damaged elements and the layout of FE model(Teughels et al., 2003). Therefore, a more effective technique for damage detection is still sought.

In this paper, a new regularization method by parameter subset selection method using the residual force vector is proposed to achieve detection of multiple damaged regions in a structure. Unlike the method proposed by Titurus et al.(2003), the residual force vector is used as a nonlinear function of damage parameters instead of fundamental modal characteristics. The focus herein is on the detection and location of damage and a second stage employing one of several possible optimization approaches for model updating would be employed next to determine the extent of damage(Yun et al., 2007). The impact of having incomplete measurements is considered within the approach.

2. Theoretical Background for Structural Damage Detection

In this section, the proposed damage detection method is described in detail. Because there is a limitation on the number of sensors that can reasonably be used, it is difficult to measure all DOFs in the mode shapes of the reference model. Therefore, the incomplete set of measured modes should be expanded to the size of the finite element model for damage detection. For the expansion process, system equivalent reduction expansion process(SEREP) is used in this paper while subset selection of damage parameters is utilized for damage localization. However, a new residual based on the residual force vector is employed to improve the performance of the damage localization process. The following sections explain SEREP method, problem formulation using residual force vector and regularization by parameter subset selection.

2.1 Model Expansion

In practical modal tests, it is usually challenging to obtain the complete measured modes with a limited number of sensors and it is also difficult to measure rotational DOFs. Within this section, the SEREP method is presented. This method uses theoretical eigenvectors to produce a transformation matrix from the DOFs in the measured mode shapes to all of the DOFs in the analytical model to be updated. The equation for an undamped analytical model can be expressed as

$$\begin{bmatrix} \mathbf{K}_{rr} & \mathbf{K}_{rt} \\ \mathbf{K}_{tr} & \mathbf{K}_{tt} \end{bmatrix} \begin{Bmatrix} \boldsymbol{\varphi}_{rj} \\ \boldsymbol{\varphi}_{tj} \end{Bmatrix} - \lambda_j \begin{bmatrix} \mathbf{M}_{rr} & \mathbf{M}_{rt} \\ \mathbf{M}_{tr} & \mathbf{M}_{tt} \end{bmatrix} \begin{Bmatrix} \boldsymbol{\varphi}_{rj} \\ \boldsymbol{\varphi}_{tj} \end{Bmatrix} = \begin{Bmatrix} \mathbf{0} \\ \mathbf{0} \end{Bmatrix} \quad (1)$$

where 'r' indicates the retained DOFs, 't' denotes the truncated DOFs, K and M are the stiffness and mass matrices, respectively, and $\boldsymbol{\phi}_j$ indicates the jth mode shape vector. The theoretical eigenvectors are constructed from the analytical model in equation (1). The resulting transformation matrix is given as

$$\mathrm{T} = \begin{bmatrix} \boldsymbol{\Phi}_{rm} \\ \boldsymbol{\Phi}_{tm} \end{bmatrix} \boldsymbol{\Phi}_{rm}^{*} \quad (2)$$

where 'r' corresponds to the number of sensors on tested models, 'm' indicates the number of measured modes from the testing, $\boldsymbol{\Phi}_{rm}$ is the mode shape matrix corresponding to the retained DOFs in the analytical model, and $\boldsymbol{\Phi}_{tm}$ is the mode shape matrix corresponding to the truncated DOFs in the analytical model. $\boldsymbol{\Phi}_{rm}^{*}$ in equation (3) represents the

Moore-Penrose pseudo-inverse of this matrix given by

$$\Phi^{+}_{rm} = \left(\Phi^{T}_{rm}\Phi_{rm}\right)^{-1}\Phi^{T}_{rm} \tag{3}$$

It is noteworthy to mention that the number of measured modes(m) is usually smaller than the number of retained DOFs(r) in the analytical model. When this occurs inherent errors are present from the use of the pseudo-inverse which has a least squares error. Additional sources of error in the measured modes include noise and measurement errors which also propagate through the expansion process. However, when m is equal to r, the SEREP expansion process does not introduce additional errors. The measured mode shape matrix can thus be expanded using

$$\left[\Phi_{expanded}\right]_{neq\times m} = \left[T\right]_{neq\times r}\left[\Phi_{measured}\right]_{r\times m} \tag{4}$$

where 'neq' indicates the total number of DOFs in the analytical model.

2.2 Problem Formulation using Residual Force Vector

Herein it is assumed that structural damage can be modeled as a reduction in Young's modulus(E) for a selected number of finite elements. Although damage is inherently nonlinear, this assumption is valid because we are typically considering measured responses due to small vibration levels pre- and post-damage which are both linear situations. Thus, the global stiffness matrix of the finite element model can be expressed as the summation of damaged and undamaged elemental stiffness matrices, where the local element stiffness is multiplied by a reduction factor as

$$[\mathbf{K}] = \sum_{i=1}^{nelem}\left(1-\theta_i\right)[\mathbf{k}]_i \tag{5}$$

where 'nelem' is the total number of elements in the analytical model, θ_i is the damage parameter, which equals zero for healthy state and unity for complete damage state, and $[\mathbf{K}]$ is the structural stiffness matrix. $[\mathbf{K}]$ is assembled from the elemental stiffness matrix, $[\mathbf{k}]$. In this paper, the parameter subset selection method is adopted for damage localization using the residual force vector to enhance localization performance. The problem is theoretically formulated here. First, the residual function is expressed as

$$J = \left\|\mathbf{z}_m - \mathbf{z}(\boldsymbol{\theta})\right\|^2 \tag{6}$$

where $\mathbf{z}_m$ is the vector consisting of measured data and $\mathbf{z}(\boldsymbol{\theta})$ consists of the same quantities as a function of damage parameters, $\boldsymbol{\theta}$. Because the residual function is a nonlinear function of the damage parameters, it is a combinatorial optimization problem in which $\boldsymbol{\theta}$ is sought by minimizing this function. By taking the Taylor series of $\mathbf{z}_m$, the residual function can be described as

$$\mathbf{z}_m = \mathbf{z}(0) + \mathbf{S}\boldsymbol{\theta} + \frac{\mathbf{S}'}{2!}\boldsymbol{\theta}^2 + \frac{\mathbf{S}''}{3!}\boldsymbol{\theta}^3 + \dots \tag{7}$$

where $\mathbf{S}$ is the sensitivity matrix that contains the first-order derivatives of the measured quantities with respect to the damage parameters and $\mathbf{z}(0)$ is the measured data from the healthy structure. Neglecting the higher order terms in equation (7), the linearized equation can be rewritten as

$$\mathbf{S}\boldsymbol{\theta} = \mathbf{b} \tag{8}$$

where $b = z_d - z_u$ is the difference in the measured values between the damaged and undamaged state. $\mathbf{S}$ and $\mathbf{b}$ are calculated in terms of the residual force vector. The ith residual force vector for an undamaged structure is given by

$$\mathbf{R}_i = \mathbf{K}\boldsymbol{\varphi}_i - \lambda_i\mathbf{M}\boldsymbol{\varphi}_i \tag{9}$$

where $\mathbf{K}$ and $\mathbf{M}$ are the stiffness and mass matrices of the structure, respectively. If the analytical model to be updated truly represents the healthy state of the physical structure, the residual function is expected to be close to zero. In practical applications, the calibration process to determine an analytical model of the healthy structure can be accomplished by minimizing the modeling error, for example, by choosing accurate finite element models and accurate boundary conditions. However, for physical structures some modeling errors will always remain.

Taking the derivative of the residual force vector with respect to the jth damage parameter, sensitivities of the residual force vector can be expressed as

$$\frac{\partial \mathbf{R}_i}{\partial \theta_j}=\frac{\partial \mathbf{K}}{\partial \theta_j}\boldsymbol{\varphi}_i+\mathbf{K}\frac{\partial \boldsymbol{\varphi}_i}{\partial \theta_j}-\frac{\partial \lambda_i}{\partial \theta_j}\mathbf{M}\boldsymbol{\varphi}_i-\lambda_i\mathbf{M}\frac{\partial \boldsymbol{\varphi}_i}{\partial \theta_j} \tag{10}$$

It should also be noted that the mass matrix is assumed to not change. Thus, the sensitivity of eigenvalues and mode shapes are calculated with the following equations [17].

$$\frac{\partial \lambda_i}{\partial \theta_j}=\boldsymbol{\varphi}_i^T\left(\frac{\partial \mathbf{K}}{\partial \theta_j}-\lambda_i\frac{\partial \mathbf{M}}{\partial \theta_j}\right)\boldsymbol{\varphi}_i$$

$$\frac{\partial \boldsymbol{\varphi}_i}{\partial \theta_j}=\sum_{k=1,k\neq i}^{n}\left[\frac{\boldsymbol{\varphi}_i^T\left(\partial \mathbf{K}/\partial \theta_j-\lambda_i\,\partial \mathbf{M}/\partial \theta_j\right)\boldsymbol{\varphi}_k}{(\lambda_k-\lambda_i)}\right]\boldsymbol{\varphi}_k$$

$$-\frac{1}{2}\boldsymbol{\varphi}_i^T\frac{\partial \mathbf{M}}{\partial \theta_j}\boldsymbol{\varphi}_i \tag{11}$$

However, because the mass matrix is constant, the second term in the sensitivity of the mode shapes will vanish. Therefore, the sensitivity matrix **S** in terms of the residual force vectors is

$$\mathbf{S}=\begin{bmatrix}\frac{\partial \mathbf{R}_1}{\partial \theta_1} & \frac{\partial \mathbf{R}_1}{\partial \theta_2} & \cdot & \frac{\partial \mathbf{R}_1}{\partial \theta_p}\\ \frac{\partial \mathbf{R}_2}{\partial \theta_1} & \frac{\partial \mathbf{R}_2}{\partial \theta_2} & \cdot & \frac{\partial \mathbf{R}_2}{\partial \theta_p}\\ \cdot & \cdot & \cdot & \cdot\\ \frac{\partial \mathbf{R}_m}{\partial \theta_1} & \frac{\partial \mathbf{R}_m}{\partial \theta_2} & \cdot & \frac{\partial \mathbf{R}_m}{\partial \theta_p}\end{bmatrix}\in R^{(neq\times m)x(p)} \tag{12}$$

where 'neq', 'm', and 'p' are, respectively, the number of DOFs, the number of measured modes, and the number of damage parameters. The **b** vector is also calculated by

$$\mathbf{b}_i=\mathbf{R}_{di}-\mathbf{R}_{ui}$$
$$\text{where }\ \mathbf{R}_{di}=\mathbf{K}\boldsymbol{\varphi}_{di}-\lambda_{di}\mathbf{M}\boldsymbol{\varphi}_{di} \tag{13}$$

where λ_{di} and $\boldsymbol{\varphi}_{di}$ are the measured eigenvalues and mode shapes of the damaged structure, respectively. Because the residual force vector for an undamaged structure, $\mathbf{R}_{ui}$ will be approximately equal to the zero vector as explained previously, residual values of the b vector originate from $\mathbf{R}_{di}$. Therefore, a linearized subset selection problem is posed as in equation(8). If the damage parameters are defined for 'p' finite elements and 'm' modes are obtained using modal testing, then **S** is an (neq ×m)×p matrix and **b** is a vector of length p. Then the given problem is to find the subset of parameters that minimizes the residuals within these equations. In the problem statement, it is implicit that the goal is to identify the minimum number of nonzero damage parameters to produce a sufficiently small residual. Because the number of rows of the **S** matrix is greater than the number of columns, the problem is ill-posed in an over-determined sense. Therefore, the parameter subset selection method is described for purposes of regularization in the following section.

In the case of Titurus et al.'s(2003) method, which utilizes modal parameters, the residual vector **b** and sensitivity matrix **S** are expressed as

$$\mathbf{S}=\begin{bmatrix}S_{\lambda,1,1} & S_{\lambda,1,1} & \cdot & S_{\lambda,1,p}\\ \cdot & \cdot & \cdot & \cdot\\ S_{\lambda,m,1} & S_{\lambda,m,2} & \cdot & S_{\lambda,m,p}\\ S_{\varphi,1,1} & S_{\varphi,1,2} & \cdot & S_{\varphi,1,p}\\ \cdot & \cdot & \cdot & \cdot\\ S_{\varphi,m,1} & S_{\varphi,m,2} & \cdot & S_{\varphi,m,p}\end{bmatrix}\in R^{((m+1)\times neq)x(p)};$$

$$\mathbf{b}=\begin{Bmatrix}\lambda_{d1}-\lambda_{u1}\\ \cdot\\ \lambda_{dm}-\lambda_{um}\\ \varphi_{d1}-\varphi_{d1}\\ \cdot\\ \varphi_{dm}-\varphi_{dm}\end{Bmatrix} \quad (14)$$

Also, the sensitivity values are expressed as

$$S_{\lambda,i,j}=\left.\frac{d\lambda_i}{d\theta_j}\right|_{\theta=0} \text{ and } S_{\phi,i,j}=\left.\frac{d\varphi_i}{d\theta_j}\right|_{\theta=0}. \quad (15)$$

If the highest order of nonlinearity of eigenfrequencies and eigenmodes, with respect to the damage parameters, are respectively q and r, the highest order of nonlinearity in the residual vector **b** from equation(14) is either q or r. However, in the proposed method the highest nonlinear order in the residual vector **b**(equation(13)) is the max(qr, r) consistently for all rows. This observation implies that changes in the residual function due to damage are more pronounced in the proposed method than in the original residual function in equation(14). From the additional third and fourth terms in equation(10), the sensitivity of the proposed method is larger than the sensitivity in equation(15). More importantly, when comparing the size of the sensitivity matrices **S** for the two cases, the proposed method is less over-determined than Titurus et al's method. Thereby, the proposed method is advantageous to implement over previous methods. These advantages are also evidenced by numerical examples presented subsequently in Section 3.

2.3 Regularization through Parameter Subset Selection Method

In the identification of damage parameters, sub-optimal problems are often sequentially formulated using the forward selection approach (Lallement et al., 1990). In each sub-optimal problem, one damage parameter is selected out of the remaining damage parameters. The difference of measured data, **b**, in equation(13) can be viewed as a linear combination of a set of column vectors within the S=[a_1 a_2 ⋯ a_p] matrix using

$$\mathbf{b}=\sum_{j=1}^{p}\theta_j\mathbf{a}_j \quad (16)$$

where p indicates the number of damage parameters. Overall, the main task of forward selection is to select a column vector in the **S** matrix that best represents the residual vector **b**, that is, yields the minimum value of the resulting residual function

$$J_j=\left\|\mathbf{b}-\mathbf{a}_j\,{}^{e}\theta_j\right\|^2 \quad (17)$$

where the summation rule with respect to j is not applied here and the least square estimate ${}^{e}\theta_j$ of the jth parameter can be obtained by taking a derivative of $J=\|b-a_j\theta_j\|^2$ with respect to θ_j as

$${}^{e}\theta_j=\frac{\mathbf{a}_j^T\mathbf{b}}{\mathbf{a}_j^T\mathbf{a}_j} \quad (18)$$

Finding the minimum value of the residual function $J=\|b-a_j\theta_j\|^2$ is equivalent to finding a vector $\mathbf{a}_j$ that forms the minimum angle with the vector **b**. This procedure seeks the best basis vector $\mathbf{a}_j$ that is closest to the damage residual vector **b**. If the first basis vector $\mathbf{a}_{j1}$ and its corresponding damage parameter θ_{j1} are found, Gram-Schmidt orthogonalization is generally performed on the remaining column vectors to ensure a well-conditioned sub-matrix of **S**. A vector orthogonal to the first vector is produced by taking the original second vector and projecting out the component of the vector that lies along the first vector. This task is accomplished through the following equations

$$\mathbf{a}_j\leftarrow\mathbf{a}_j-\alpha_j\mathbf{a}_{j1} \text{ and } \mathbf{b}\leftarrow\mathbf{b}-\mathbf{a}_{j1}\,{}^{e}\theta_{j1}$$
$$\text{where } \alpha_j=\mathbf{a}_{j1}^T\mathbf{a}_j/\mathbf{a}_{j1}^T\mathbf{a}_{j1} \quad (19)$$

where α_j is the component of the vector $\mathbf{a}_j$ that lies along the first vector and ${}^{e}\theta_{j1}$ can be calculated by equation(18). After orthogonalization, the residual function in equation(17) for each parameter j is calculated and the minimum value is chosen as follows

$$\min\left(\left\{J_2, J_3, \ldots, J_p\right\}\right) \rightarrow \mathbf{a}_{j2} \quad \text{and} \quad \theta_{j2} \tag{20}$$

Equivalently, the minimum angle between orthgonalized column vector $\mathbf{a}_j$ and residual vector $\mathbf{b}$ can be sought. Therefore, the angle can be described as

$$\phi = \cos^{-1}\left(\sqrt{\frac{\left(\mathbf{a}_j^T\mathbf{b}\right)^2}{\left(\mathbf{a}_j^T\mathbf{a}_j\right)\left(\mathbf{b}^T\mathbf{b}\right)}}\right) \tag{21}$$

This iterative procedure is continued to identify m damage parameters while retaining the parameters chosen in the previous steps. When a total of m damage parameters are selected in the subset, the residual sum of squares is defined as

$$RSS_m = \left\| b - \sum_{i=1}^{m} \mathbf{a}_{j_i} \, {}^e\theta_{j_i} \right\|^2 \tag{22}$$

where ${}^e\theta_j$ is the least squares estimator for the jth parameter, equation(18). Efroymson(1960) suggested a stepwise regression algorithm which provides a basis by which to decide whether a new parameter should be included in the subset. If $[\theta_1, \theta_2, \cdots, \theta_m]$ are already selected as damage parameters and a new parameter θ_{m+1} is chosen for evaluation, then the F-to-enter statistic can be expressed as

$$F_a = \frac{RSS_m - RSS_{m+1}}{RSS_{m+1}/(n-m-1)} \tag{23}$$

where RSS indicates the residual sum of squares and n is the number of total parameters. If the F_a value is greater than a predetermined value(F_{in}), the parameter is included in the subset. If the criterion is not met, the parameter is excluded. However, this test can also be performed to remove a selected parameter from the subset by utilizing the F-to-remove statistic which is expressed by

$$F_r = \frac{RSS_{m-1} - RSS_m}{RSS_m/(n-m)} \tag{24}$$

If the F_r value is less than a predetermined value (F_{out}), the parameter is removed from the subset. In 1996, Miller mathematically proved that this addition and removal process terminates with a finite number of different subsets. Therefore, Miller introduced the following objective function

$$L(s) = RSS_m \prod_{i=1}^{m}\left(1 + \frac{F}{n-i}\right) \tag{25}$$

where F is any value such that $F_{out} \le F \le F_{in}$ and s represents a subset of variables. Efroymson's algorithm is viewed as a heuristic algorithm that minimizes the objective function. Thus, a finite sequence of values for the objective function will decrease at each step and ultimately terminate the process. Within this study, the F-to-enter statistic is only utilized for convenience.

3. Illustrative Example

In this section, an example is provided to demonstrate the performance of the proposed method. The first two examples consider simple beam structures for examination of the capabilities of the proposed method. The effects of measurement noise on the performance of the proposed method are also investigated. Comparisons to the original method using fundamental modal quantities are also presented in the first example. In the second example, the transformation error through SEREP modal expansion due to the limited number of sensors is considered. The third example considers a more complex frame structure.

3.1 Fourteen-Bay Planar Truss Structure

In this example, a 14 bays planar truss structure is selected to demonstrate the performance of the two-stage damage detection method. The truss structure is modeled by 53 truss elements with 28

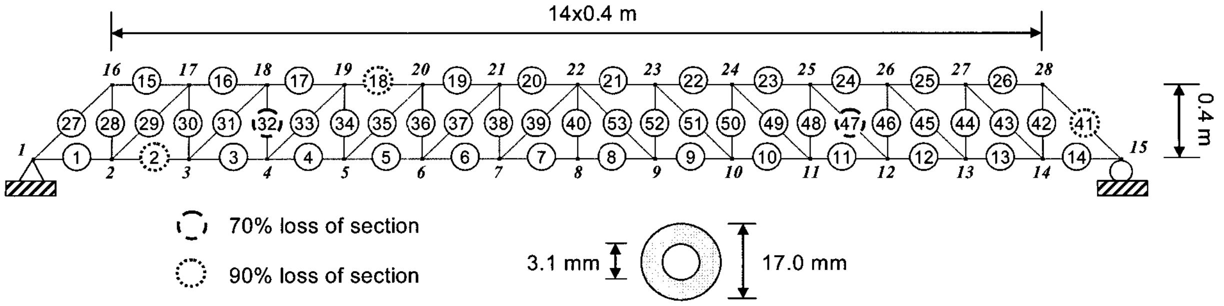
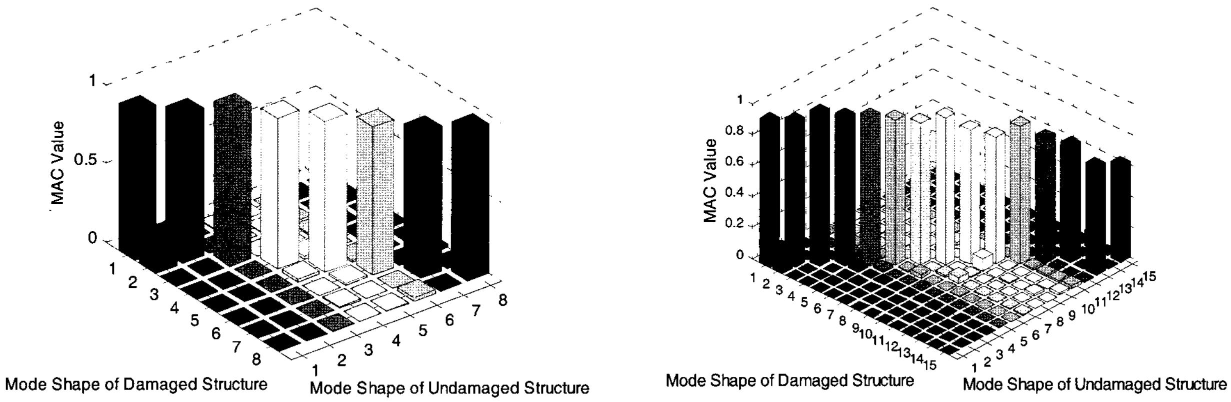


Figure 1 Fourteen-Bay Planar Truss Structures and Damage Scenario



(a) 8 Measured Modes (b) 15 Measured Modes

Figure 2 MAC Value between Damaged and Undamaged Structure

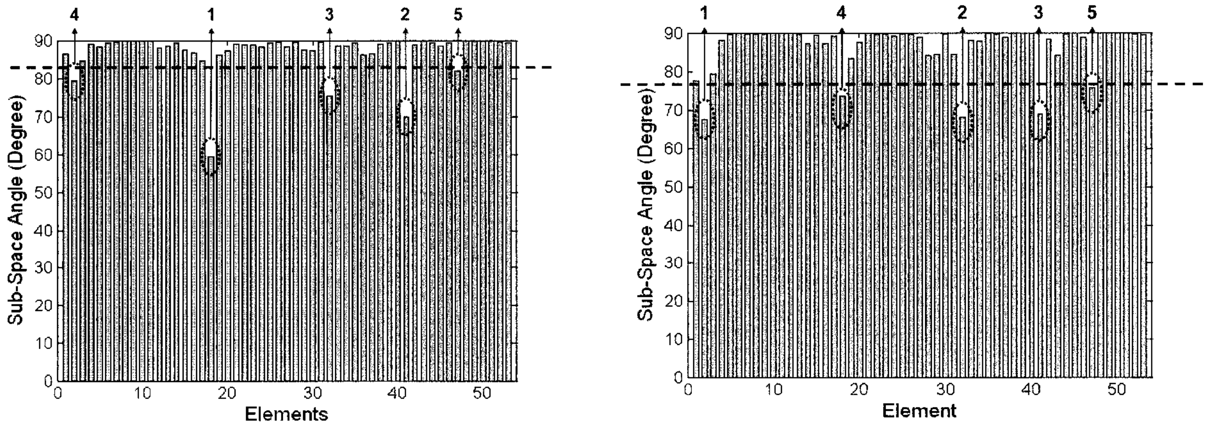
nodes as shown in Figure 1. The total length of the structure is 5.56m with 0.40m in each bay, and the height of structure is 0.40m. The members are steel bars with a tube cross section with an inner diameter of 3.1mm and an outer diameter of 17.0 mm. The physical properties are assumed as: the elastic modulus of the material=1.999×10^5N/mm^2; and the mass density=$7{,}827\times10^{-9}$kg/mm^3: The members are connected with pinned joints. There are two supports at the ends of the structure: a pin support at the left end and a vertical roller support at the right end. The structure totals 53DOFs. Two different cases are tested: the first case in which entire DOFs are measured and the second case in which partial DOFs are measured. So SEREP is used for the second case. To simulate the partially measured degrees, a total of 26 degrees, i.e. vertical DOFs at nodes 2~14 and 16~28 are taken as sensor locations. Damages are incurred to elements 2, 18, 32, 41 and 47, with elements 32 and 47 having a 70% loss of sections, and the rest having a 90% loss of sections.

Figure 2 shows change of mode shapes in terms of MAC values between damaged and undamaged structures. Table 1 shows the change in natural frequencies and mode shapes through the MAC values. Some of the changes in the natural frequencies are shown to be more than 10%. Higher modes are more sensitive to damage than lower modes. The subset selection is applied to locate damaged elements. In the first case, 8 modes are measured and in the second case, 15 modes are measured. Figure 3 shows the sub-space angles for each element. All damaged elements are selected in the first 5 selections both in the two cases. The angle indicates the subspace angle between the orthogonalized column vector and the residual vector. Therefore, the angles of healthy elements are very close to 90 degree while damage elements show relatively small angle.

The numbers depicted in Figure 3 indicate the order of selection of corresponding element. When all DOFs are measured, 8 measured modes were sufficient for detecting all damaged elements.

Table 1Change of First 15 Natural Frequencies and MAC values Before and After Damage

Mode	Natural Frequency (Hz)		Change of N.F. (%)	MAC value
	Damaged	Undamaged		
1	3.7150	4.4500	16.51	0.9337
2	4.5393	5.2146	12.95	0.9316
3	8.9612	9.8177	8.72	0.9822
4	12.5824	13.9797	9.99	0.9572
5	14.8201	15.8300	6.37	0.9552
6	18.9977	20.4747	7.21	0.9407
7	22.5332	24.8147	9.19	0.9182
8	24.8532	26.0186	4.47	0.9499
9	29.8965	32.1394	6.97	0.8688
10	32.3656	34.8162	7.03	0.8165
11	35.0009	37.7979	7.39	0.8853
12	39.9861	43.8854	8.88	0.8024
13	42.3155	44.1181	4.08	0.7501
14	44.5331	49.2666	9.60	0.6147
15	45.7877	50.5778	9.47	0.6229



(a) 8 Measured Modes without SEREP (b) 15 Measured Modes with SEREP

Figure 3 Sub-Space Angle used in Identification of Damaged Elements

However, when partial DOFs are measured, 15 measured modes were required to successfully detect all damaged elements. It implies that considering additional modes are a complementary step to maintain damage detection capability in case of incomplete measurements of the mode shapes.

4. Conclusions and Future Work

In this paper, a new regularization method by parameter subset selection has been proposed for identifying damage within structural systems. For residual function, residual force vector is employed. The method is beneficial because the number of the damaged parameters can be greatly reduced for the subsequent model updating process. For the modal expansion process, system equivalent reduction and expansion process(SEREP) is utilized in this paper. The advantage that this formulation using the residual force vector for damage localization has over the original method is that it can more accurately identify multiple damage locations. This ability can be attributed to the modified residual function and that fact that the sensitivity matrix has a higher order of nonlinearity than the fundamental modal properties. The performance of the proposed method was demonstrated using an illustrative example with multiple damage locations. The proposed method in this paper has proven to be a robust and viable tool in structural health monitoring applications. However, further validation using physical specimens and real world measurement noise will be pursued in future studies to investigate the robustness of this approach.

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