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Estimating Reliability and Distribution of Ratio in two independent Different Variates

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Abstract

We consider densities of quotient Y/X and ratio X/(X+Y), and estimation of reliability P(Y<X) when X and Y are independent p-dimensional Rayleigh random variable and uniform random variable, respectively, and also consider the quotient and ratio densities, and the estimation problem when X and Y are two different independent p-dimensional Rayleigh random variable and Rayleigh random variable, respectively.

Keywords: P-dimensional Rayleigh; Rayleigh distribution; Reliability.

1. Introduction

The Rayleigh distribution was derived by Lord Rayleigh(1919) in connection with a study of acoustical problems. a p-dimensional Rayleigh distribution has been known as the distribution of distance X from the origin to the point $(Y_1, Y_2, \ldots, Y_p)$ in a p-dimensional Euclidean space, where the components $Y_i's$ are independent random variables, each of which is distributed as normal distribution with mean 0 and variance σ^2. This p-dimensional Rayleigh
σ distribution is positively skewed as increases or p increases.

The special cases in which p=1, 2, and 3 are important in various scientific applications. The one dimensional Rayleigh distribution is sometimes known as the folded normal, the folded Gaussian, or the half normal distribution. The two dimensional case may be not only the most important of all Rayleigh distribution, and it's also a special case of the Weibull distribution.

Balakrishnan(1989) studied approximate MLE of the scale parameter of the

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Rayleigh distribution with censoring. Ali and Woo(2005b) studied inference on reliability in a p-dimensional Rayleigh distribution. Many authors have considered various aspects of a uniform distributions(see Johnson et al(1974)).

McCool(1991) considered inference on reliability P(X<Y) in the Weibull case. Ali & Woo(2005a) considered inference on reliability P(Y<X) when X and Y are two independent Levy distributions. And in recent, Kim(2006) and Woo(2006 & 2007)) studied inference on the reliability in a truncated Rayleigh distribution and a half-triangle distribution.

In this paper, we derive the quotient Y/X and the ratio X/(X+Y)- distributions of two independent p-dimensional Rayleigh random variable and uniform random variable. and consider point and interval estimations of the reliability P(Y<X) when X and Y are independent p-dimensional Rayleigh random variable and uniform random variable, respectively, and in the second case when X and Y are independent p-dimensional Rayleigh random variable and Rayleigh random variable, respectively.

2. Uniform and p-dimensional Rayleigh

In this section, we consider the case for the estimation of P(Y<X) when (X, Y) is a pair of uniform and p-dimensional Rayleigh random variables, respectively. As an application of this case X, representing time to sustain temperature, is a uniform random variable and Y, representing time to sustain life of an electric tube, is p-dimensional Rayleigh random variable.

2.1. Quotient and reliability

Let X and Y be two independent p-dimensional Rayleigh random variable (Ali and Woo(2005b)) and a uniform random variable each having the following density:

$$f_X(x) = \frac{1}{(2\sigma^2)^{p/2}\Gamma(p/2)} 2x^{p-1} e^{-\frac{x^2}{2\sigma^2}} . x > 0$$
$$f_Y(y) = \frac{1}{\theta}, \quad \text{if} \quad 0 < y < \theta \ , \tag{2.1}$$

where $\theta > 0$, $\sigma > 0$, and p is a known positive integer, and $\Gamma(a)$ is the gamma function.

Let W=Y/X. Then, from the quotient density in Rohatgi(1976, p.141) and the formula 3.381(1) in Gradshteyn and Ryzhik(1965, p.317), the density of W=Y/X is given by:

$$f_W(w) = \frac{\sqrt{2}\,\sigma}{\Gamma(p/2)\theta} \cdot \gamma(\frac{p+1}{2}, \frac{\theta^2}{2\sigma^2 w^2}), \quad \text{if } 0 < w < \infty, \qquad (2.2)$$

where $\gamma(\alpha, x)$ is the incomplete gamma function.

By transformation of variable, $x \equiv \theta^2/(2\sigma^2 w^2)$ in the density (2.2), and the formula 13.42 in Oberhettinger(1974, p.144) and the formula 15.1.20 in Abramowitz and Stegun (1970, p.556), integration of the function $f_W(x)$ over $(0, \infty)$ in (2.2) is obvious 1.

From the density (2.2), the formula 13.42 in Oberhettinger(1974, p.144), and the formula 15.1.20 in Abramowitz and Stegun (1970, p.556), we can obtain the k-th moment of W=Y/X when X and Y are independent p-dimensional Rayleigh random variable and uniform random variable, respectively.

$$E(W^k) = \frac{2^{k/2}\Gamma((p-k)/2)}{(k+1)\Gamma(p/2)}(\theta/\sigma)^k, \quad \text{if } p > k \ .$$

From the density (2.2) and the formulas 3.381(1)&(3) in Gradshteyn and Ryzhik(1965, p.317), we can obtain the reliability P(Y<X) as following:

<Fact 1> When two independent random variables X and Y have the density (2.1), then

$$R \equiv P(Y < X) = \frac{1}{\Gamma(p/2)}[\frac{\sqrt{2}}{\rho} \cdot \gamma(\frac{p+1}{2}, \frac{1}{2}\rho^2) + \Gamma(\frac{p}{2}, \frac{1}{2}\rho^2)] \ ,$$

which the reliability is a function of $\rho \equiv \theta/\sigma$, .

Proof. By definition of the incomplete gamma function,

since $\frac{d}{dx}\gamma(\alpha, x) = x^{\alpha-1} \cdot e^{-x}$ and $\frac{d}{dx}\Gamma(\alpha, x) = -x^{\alpha-1} \cdot e^{-x}$, and hence

$$\frac{d}{d\rho}P(Y < X) = -\frac{\sqrt{2}}{\rho^2} \cdot \gamma(\frac{p+1}{2}, \frac{1}{2}\rho^2) < 0 \ .$$

Therefore, we can obtain it.

2.2. Estimating reliability P(Y<X)

Here we consider estimation on the reliability P(Y<X) when X and Y are independent p-dimensional Rayleigh random variable and uniform random variable, respectively having the densities (2.1). Because R=P(Y<X) is a monotone function ρ of in Fact 2, an inference on the reliability is equivalent to an inference on (see McCool(1991)).

Hence we only consider estimation on $\rho \equiv \theta/\sigma$ when the σ and θ are parameters in (2.1) instead of estimating R=P(Y<X) when p is a known positive integer.

Assume $X_1, X_2, \ldots, X_m$ and $Y_1, Y_2, \ldots, Y_n$ be two independent samples from X and Y with the densities in (2.1), respectively. When the shape parameter p in the density (2.1) is known, from the formula 3.381(4) in Gradshteyn and Ryzhik(1965, p.317), we can obtain the following Fact 2:

<Fact 2> If $X_1, X_2, \ldots, X_m$ are a sample drawn from a p-dimensional Rayleigh distribution with density (2.1), then we get the followings easily:

(a) $Z \equiv \sum_{i=1}^{m} X_i^2$ follows a gamma random variable with a shape parameter pm/2 and scale parameter $2\sigma^2$. (b) $E(1/Z^{1/2}) = \Gamma(\frac{pm-1}{2})/(\sqrt{2}\,\Gamma(\frac{pm}{2})\sigma)$, if $pm > 1$.
(c) $E(1/Z) = 1/((pm-2)\sigma^2)$. if $pm > 2$.

When the shape parameter p is known, the MLE $\hat{\sigma}$ of σ and the MLE $\hat{\theta}$ of θ are:

$$\hat{\sigma} = \sqrt{\frac{1}{pm} \cdot \sum_{i=1}^{m} X_i^2}, \quad \hat{\theta} = Y_{(n)} \ . \quad \text{if } p \text{ is known positive,}$$

where $Y_{(n)} = \max(Y_1, Y_2, \ldots, Y_n)$.

Hence the MLE $\hat{\rho}$ of ρ is $\hat{\rho} = \frac{\hat{\theta}}{\hat{\sigma}} = \sqrt{pm} \cdot Y_{(n)} / (\sum_{i=1}^{m} X_i^2)^{1/2}$.

From the results in Fact 2 and the greatest order statistics $Y_{(n)}$ has the density, $f_{Y_{(n)}}(y) = \frac{n}{\theta^n} y^{n-1}$, if $0 < y < \theta$,

we can obtain the expectation and variance of $\hat{\rho}$:

$$E(\hat{\rho}) = \sqrt{\frac{pm}{2}} \frac{n \cdot \Gamma(\frac{pm-1}{2})}{(n+1)\Gamma(\frac{pm}{2})} \cdot \rho, \quad \text{if } pm > 1 \ . \tag{2.3}$$

$$Var(\hat{\rho}) = pmn \ [\frac{1}{(pm-2)(n+2)} - \frac{n \cdot \Gamma^2(\frac{pm-1}{2})}{2(n+1)^2 \Gamma^2(\frac{pm}{2})}] \cdot \rho^2, \quad \text{if } pm > 2 \ . \tag{2.4}$$

From the expectation (2.3), an unbiased estimator $\tilde{\rho}$ of ρ is defined by:

$$\tilde{\rho} = \sqrt{2} \frac{(n+1) \cdot \Gamma(\frac{pm}{2})}{n \cdot \Gamma(\frac{pm-1}{2})} \cdot \frac{Y_{(n)}}{(\sum_{i=1}^{m} X_i^2)^{1/2}} \ .$$

Hence, from the results in Fact 2, we can derive the variance of $\tilde{\rho}$:

$$Var(\tilde{\rho}) = [\frac{2(n+1)^2 \cdot \Gamma^2(\frac{pm}{2})}{n(n+2)(pm-2)\Gamma^2(\frac{pm-1}{2})} - 1] \cdot \rho^2, \quad \text{if } pm > 2 \ , \qquad (2,5)$$

From the results (2.3), (2.4), and (2.5), Table 1 shows mean squares errors(MSE) of the MLE $\hat{\rho}$ and an unbiased estimator $\tilde{\rho}$ of $\rho \equiv \theta/\sigma$:

<Table 1> Mean square errors of the MLE $\hat{\rho}$ and an unbiased estimator $\tilde{\rho}$

m	n	$p=1$		$p=3$		$p=4$	
		$\hat{\rho}$	$\tilde{\rho}$				
10	10	0.07126	0.07319	0.02759	0.02649	0.02401	0.02169
	20	0.07213	0.06674	0.01993	0.02031	0.01551	0.01555
	30	0.07435	0.06543	0.01885	0.01907	0.01409	0.01430
20	10	0.03582	0.03672	0.02076	0.01706	0.01925	0.01482
	20	0.02998	0.03049	0.01145	0.01095	0.00955	0.00872
	30	0.02961	0.02922	0.00972	0.00970	0.00767	0.00748
30	10	0.02758	0.02649	0.01876	0.01408	0.01781	0.01262
	20	0.01993	0.02033	0.00893	0.00798	0.00773	0.00653
	30	0.01885	0.01907	0.00700	0.00674	0.00570	0.00529

where unit: ρ^2 .

From Table 1, we observe the followings:

<Fact 3> (a) The unbiased estimator $\tilde{\rho}$ performs better MSE than the MLE $\hat{\rho}$ in a sense of MSE, when (i) $p=3,4$, (m=(20, 30), n=(10, 20, 30)). and (ii) p=1,(m =10, n=(20, 30)) and (m=20, n=30).

(b) The MLE $\hat{\rho}$ performs better than the unbiased estimator $\tilde{\rho}$ in a sense of MSE, when (i) $p=3,4$, (m=10, n=(20, 30)). (ii) $p=1$, (m=20, n=(10, 20)) and (m=30, n=(20, 30)).

From Fact 3, since the unbiased estimator $\tilde{\rho}$ and the MLE $\hat{\rho}$ don't dominate each other generally, we need to recommend a bias estimator $\hat{\hat{\rho}}$ of ρ which has a minimum MSE, as we choose a constant "c" such that

$$E((\ c \cdot\ Y_{(n)}/(\sum_{i=1}^{m} X_i^2)^{1/2} - \rho)^2) \text{ is minimized.}$$

From the results (2.3) and (2.4), we can obtain a bias estimator $\hat{\hat{\rho}}$ having a minimum MSE among all estimators of ρ as the following:

$$\hat{\hat{\rho}} = \frac{(pm-2)(n+2)\Gamma((pm-1)/2)}{\sqrt{2}\,(n+1)\Gamma(pm/2)} \cdot \frac{Y_{(n)}}{(\sum_{i=1}^{m} X_i^2)^{1/2}}.$$

Therefore, the estimator $\hat{\hat{\rho}}$ performs better than an unbiased estimator $\tilde{\rho}$ and the MLE $\hat{\rho}$ in a sense of MSE.

Now we consider an interval estimator of ρ when the parameter p in the p-dimensional Rayleigh density (2.1) is known. From independence of X′s and Y′s, Fact 2(a), the density of $Y_{(n)}$, the quotient density in Rohatgi(1975, p.141), and the formula 3.381(1) in Gradshteyn and Ryzhik(1965, p.317), $Q \equiv \rho \cdot (\sum_{i=1}^{m} X_i^2\ )^{1/2} / Y_{(n)}$ is a pivot quantity having the following density as:

$$f_Q(x) = \frac{n \cdot 2^{n/2}}{\Gamma(\frac{pm}{2})} \cdot x^{-n-1} \cdot \gamma(\frac{pm+n}{2}, \frac{x^2}{2}\ ), \quad \text{if } x > 0\ , \tag{2.6}$$

where $p > 0$ is a known integer.

<Remark 1> Using the formula 13.42 in Oberhettinger(1974, p.144) and the formula 15.1.20 in Abramowitz and Stegun (1970, p.556), the integration of $f_Q(x)$ over$(0, \infty)$ in (2.6) is one.

For given $0 < p_i < 1,\ i = 1,2,\ 0 < 1 - p_1 - p_2 < 1$, there exist $l(p_1)$ and $u(p_2)$ such that

$$\int_0^{l(p_1)} f_Q(x)dx = p_1, \ \int_{u(p_2)}^{\infty} f_Q(x)dx = p_2 \quad . \tag{2.7}$$

From the upper and lower bounds in (2.7) and $Q \equiv \rho \cdot (\sum_{i=1}^{m} X_i^2\ )^{1/2} / Y_{(n)}$,

ρ an $(1-p_1-p_2)100\%$ confidence interval of is given by:

$$\left(\frac{Y_{(n)}}{(\sum_{i=1}^{m} X_i^2\ )^{1/2}} \cdot l(p_1), \frac{Y_{(n)}}{(\sum_{i=1}^{m} X_i^2\ )^{1/2}} \cdot u(p_2)\right)\ .$$

<Remark 2> For a given $0 < p_i < 1,\ i = 1,2$, the points $l(p_1)$ and $u(p_2)$ in the integral (2.7) can be evaluated numerically by the iterating method in numerical analysis.

$\hat{\rho}$ While, since the MLE is a consistent estimator of from the results (2.3) & ρ (2.4), an asymptotic confidence interval of is given as the following:

For a given $0 < \gamma < 1$,

$$\hat{\rho} \pm z_{\gamma/2} \cdot \hat{\rho} \sqrt{pmn\left[\frac{1}{(pm-2)(n+2)} - \frac{n \cdot \Gamma^2(\frac{pm-1}{2})}{2(n+1)^2 \cdot \Gamma^2(\frac{pm}{2})}\right]}$$

ρ is an $(1-\gamma)100\%$ asymptotic confidence interval of ,

where $\int_u^\infty \varphi(t)dt = \gamma/2,\ u \equiv z_{\gamma/2} \cdot \varphi(t)$ is the standard normal density and

$\hat{\rho} = \sqrt{pm} \cdot Y_{(n)} / (\sum_{i=1}^{m} X_i^2)^{1/2}$.

2.3 Ratio of two random variables

Here we consider distribution of the ratio R=X/(X+Y) when two independent random variables X and Y have the density (2.1).

Since R=1/(1+Y/X) =1/(1+W) and the density of W is (2.2), the density of R is given by:

$$f_R(r) = \frac{\sqrt{2}\,\sigma}{\Gamma(p/2)\theta} \cdot \gamma\left(\frac{p+1}{2}, \frac{\theta^2}{2\sigma^2} \cdot \frac{r^2}{(1-r)^2}\right) \cdot \frac{1}{r^2}, \quad \text{if } 0 < r < 1 . \qquad (2.8)$$

Although it's difficult for us to find the k-th moment of R explicitly, since we know that the k-th moment of R exists. the following numerical moments of the ratio R=X/(X+Y) are evaluated approximately by computer computations (a partition of r is $\Delta r = 10^{-7}$) when $\sigma = 1$ in (2.8), which values of computations are stabilized by $\Delta r = 10^{-7}$.

<Table 2> Mean, variance, skewness, and kurtosis of the density of the ratio X/(X+Y) when X and Y are independent p-dimensional Rayleigh random variable and uniform random variable each having the following density (2.1), and when $\sigma = 1$ in (2.8).

p	θ	mean	variance	skewness	kurtosis
1	1/2	0.68900	0.05338	-0.95739	3.25682
	1	0.56862	0.06180	-0,33882	2.27932
	4	0.31783	0.05314	0.89417	3.17899
3	1/2	0.84991	0.01073	-1.1567	5.29242
	1	0.75211	0.02157	-0.55110	3.20185
	4	0.48150	0.04317	0.55549	2.59460
5	1/2	0.88885	0.00508	-0.81295	4.66846
	1	0.80707	0.01247	-0.40397	3.36207
	4	0.55033	0.03618	0.48070	2.43958

From Table 2, we observe the followings:

The density (2.8, $\sigma=1$) of the ratio is skewed to the right when p=1,2,3 and $\theta=4$, for other cases the density is skewed to the left. And the density (2.8, $\sigma=1$) of the ratio has kurtosis which is greater than 3 when p=1,2,3, and $\theta=1/2$.

3. p-dimensional Rayleigh and Rayleigh

In this section, we consider the case for the estimation of P(Y<X) when (X, Y) is a pair of p-dimensional Rayleigh and Rayleigh random variables, respectively. As an application of the case X, representing life time of an electric machine, is a Rayleigh random variable and Y, representing life time of a machine powered by battery, is a p-dimensional Rayleigh random variable.

3.1 Quotient and reliability

When two independent random variables X and Y have the following densities (3.1).

$$f_X(x)=\frac{1}{(2\sigma^2)^{p/2}\Gamma(p/2)}2x^{p-1}e^{-\frac{x^2}{2\sigma^2}}. \quad x>0 \ ,$$

$$f_Y(y)=\frac{2}{\beta^2}\cdot y\cdot e^{-\frac{y^2}{\beta^2}}, \quad \text{if} \quad 0<y<\infty \ , \tag{3.1}$$

where $\sigma>0$, $\beta>0$ and p is a known positive integer, and $\Gamma(a)$ is the gamma function.

Let U=Y/X. Then, from the quotient density in Rohatgi(1976, p.141) and the formula 2.19 in Oberhettinger(1974, p.15), the density of U=Y/X is given by:

$$f_U(u)=p(\frac{\beta^2}{2\sigma^2})^{p/2}(u^2+\frac{\beta^2}{2\sigma^2})^{-\frac{p}{2}-1}\cdot u, \quad \text{if} \quad 0<u<\infty \ . \tag{3.2}$$

When two independendnet random variables X and Y have the densities (3.1). from the quotient density (3.2) of U, the k-th moment of U=Y/X is obtained by:

$$E(U^k)=\frac{\Gamma(1+k/2)\Gamma((p-k)/2)}{\Gamma(p/2)}(\frac{\beta^2}{2\sigma^2})^{k/2}, \quad \text{if } p>k \ .$$

Next we can obtain the reliability P(Y<X) when X and Y are independent p-dimensional Rayleigh random variable and Rayleigh random variable having the densities (3.1). From the densities (3.1) and the formula 3.381(4) in Gradshteyn and Ryzhik(1965, p.317), we can obtain the reliability P(Y<X) as following:

<Fact 4> When X and Y are independent p-dimensional Rayleigh random

variable and Rayleigh random variable having the following densities (3.1), the reliability is given by;

$$R \equiv P(Y < X) = 1 - \left(1 + \frac{2}{\eta}\right)^{-\frac{p}{2}}, \quad where \ \eta \equiv \frac{\beta^2}{\sigma^2} \ ,$$

which the reiliability is a monotone function of η.

3.2 Estimating reliability P(Y<X)

From Fact 4, because R=P(Y<X) is a monotone function of η , inference on the reliability is equivalent to inference on η (see McCool(1991)).

In this section we only consider estimation on $\eta = \beta^2/\sigma^2$ when the σ^2 and β^2 are parameters in the densities (3.1), instead of estimating R=P(Y<X) when p is a known positive integer.

Assume $X_1, X_2, \ldots, X_m$ and $Y_1, Y_2, \ldots, Y_n$ be two independent samples from X and Y each having density (3.1). When the parameter p in the density (3.1) is known, the MLE $\widehat{\sigma^2}$ of σ^2 and the MLE $\widehat{\beta^2}$ of β^2 are given by:

$$\widehat{\sigma^2} = \frac{1}{pm} \cdot \sum_{i=1}^{m} X_i^2, \quad \widehat{\beta^2} = \frac{1}{n} \sum_{j=1}^{n} Y_j^2 \ . \quad \text{if } p \text{ is known positive.}$$

And hence the MLE $\hat{\eta}$ of η is $\hat{\eta} = \dfrac{\widehat{\beta^2}}{\widehat{\sigma^2}} = pm \sum_{j=1}^{n} Y_j^2 / (n \sum_{i=1}^{m} X_i^2)$.

From the results in Fact 2, we can obtain the expectation and variance of $\hat{\eta}$:

$$E(\hat{\eta}) = \frac{pm}{pm-2} \cdot \eta, \quad \text{if } pm > 2. \tag{3.3}$$

$$Var(\hat{\eta}) = \frac{p^2 m^2 (pm + 2n - 2)}{n(pm-2)^2 (pm-4)} \cdot \eta^2, \quad \text{if } pm > 4 \ , \tag{3.4}$$

$\hat{\eta}$ From (3.3), an unbiased estimator of η is defined as the following:

$$\tilde{\eta} = (pm-2) \sum_{j=1}^{n} Y_j^2 / (n \sum_{i=1}^{m} X_i^2) \ .$$

Hence, from the results in Fact 2, we can obtain the variance of $\hat{\eta}$ as follows:

$$Var(\tilde{\eta}) = \frac{pm + 2n - 2}{n(pm-4)} \cdot \eta^2, \quad \text{if } pm > 4 \ . \tag{3.5}$$

From the results (3.3), (3.4), and (3.5), we can show the following.

<Fact 5> The unbiased estimator $\hat{\eta}$ performs better than the MLE $\hat{\eta}$ in a sense of mean squared error.

When the parameter p in the p-dimensional Rayleigh density (3.1) is known, we also consider an interval estimator of η . From the results in Fact 2, independence of X's and Y's, and definition of F-distribution, we get the following:

$Q \equiv \eta \cdot n \sum_{i=1}^{m} X_i^2 \,/\, (pm \sum_{j=1}^{n} Y_j^2)$ is a pivot quantity having a F-distribution with (pm, 2n)-degree of freedom. And hence, an $(1-\gamma)100\%$ confidence interval of η is given by:

$$\left(\frac{1}{F_{\gamma/2}(2n, pm)} \cdot \frac{pm \sum_{j=1}^{n} Y_j^2}{n \sum_{i=1}^{n} X_i^2},\quad F_{\gamma/2}(pm, 2n) \cdot \frac{pm \sum_{j=1}^{n} Y_j^2}{n \sum_{i=1}^{n} X_i^2}\right) .$$

3.3 Ratio of two random variables

Here we consider distribution of the ratio R=X/(X+Y) when two independent random variables X and Y have the densities (3.1).

Since R=1/(1+Y/X) =1/(1+U) and the density of U is (3.2), the density of R is given by:

$$f_R(r) = p\left(\frac{\beta^2}{2\sigma^2}\right)^{p/2} \cdot \left(\left(\frac{1-r}{r}\right)^2 + \frac{\beta^2}{2\sigma^2}\right)^{-\frac{p}{2}-1} \cdot \frac{1-r}{r^3}, \quad \text{if} \quad 0 < r < 1 \ . \qquad (3.6)$$

Although it's difficult for us to find the k-th moment of R explicitly, but, since we know that the k-th moment of R exists. the following numerical moments of the ratio R=X/(X+Y) are evaluated approximately by computer computations (a partition of r is $\Delta r = 10^{-6}$) when $\beta = 1$ and $\sigma = 1$ in (3.6), which values of computations are stabilized by $\Delta r = 10^{-3}$ to $\Delta r = 10^{-6}$.

<Table 3> Asymptotic mean, variance, skewness, and kurtosis of the density of the ratio X/(X+Y) when X and Y are independent p-dimensional Rayleigh random variable and Rayleigh random variable each having the following density (3.1).

p	mean	variance	skewness	kurtosis
1	0.44344	0,24818	-0,00458	2.17143
2	0.57387	0.36360	-0.29565	2.63685
3	0.63871	0.43312	-0.34813	2.82629
4	0.67901	0.48108	-0.35396	2.88300
5	0.70723	0.51694	-0.35466	2.91382

From Table 3 we observe the following:

When $\beta = 1$ and $\sigma = 1$ in the density of the ratio, the density of the ratio X/(X+Y) is skewed to the left, and its kurtosis is less than 3 in an asymptotic evaluation.

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