Intuitionistic fuzzy strong hyper K-subalgebras in Hyper K-algebras

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Abstract

Intuitionistic fuzzifications of (strong) hyper K-subalgebras in hyper K-algebras are discussed, and related properties are investigated. Relations between intuitionistic fuzzy hyper K-subalgebras and intuitionistic fuzzy strong hyper K-subalgebras are provided.

Key words: Intuitionistic fuzzy (strong) hyper K-subalgebra.

1. Introduction

The study of BCK-algebras was initiated by K. Iséki in 1966 as a generalization of the concept of set-theoretic difference and propositional calculus. Since then many researches worked in this area. The hyperstructure theory (called also multialgebras) is introduced in 1934 by F. Marty [12] at the 8th congress of Scandinavian Mathematiciens. Around the 40's, several authors worked on hypergroups, especially in France and in the United States, but also in Italy, Russia, Japan and Iran. Hyperstructures have many applications to several sectors of both pure and applied sciences. Recently in [11] Y. B. Jun et al. introduced and studied hyperBCK-algebra which is a generalization of a BCK-algebra. In [1] and [11] R. A. Borzooei et al. constructed the hyper K-algebras, and studied (weak) implicative hyper K-ideals in hyper K-algebras. In [9] and [10] Y. B. Jun et al. studied the fuzzy (implicative) hyper K-ideals in hyper K-algebras. Y. B. Jun et al. [8] introduced the notion of fuzzy (weak) implicative hyper K-ideals, and investigated related properties. They gave relations among fuzzy weak implicative hyper K-ideals, fuzzy implicative hyper K-ideals, and fuzzy hyper K-ideals. In [4], R. A. Borzooei and Y. B. Jun studied intuitionistic fuzzy hyper BCKideals of hyper BCK-algebras. In [2], R. A. Borzooei and Y. B. Jun discussed intuitionistic fuzzifications of (weak) implicative hyper K-ideals in hyper K-algebras. They gave relations among intuitionistic fuzzy hyper K-ideals, intuitionistic fuzzy weak hyper K-ideals, intuitionistic fuzzy implicative hyper K-ideals and intuitionistic fuzzy weak implicative hyper K-ideals. They provided conditions for an intuitionistic fuzzy hyper K-ideal to be an intuitionistic fuzzy implicative hyper K-ideal, and also discussed conditions for an intuitionistic fuzzy weak hyper K-ideal to be an intuitionistic fuzzy weak implicative hyper K-ideal. In this paper we consider the intuitionistic fuzzifications of (strong) hyper K-subalgebras in hyper K-algebras. We give relations between intuitionistic fuzzy hyper K-subalgebras and intuitionistic fuzzy strong hyper K-subalgebras.

2. Preliminaries

We include some elementary aspects of hyper K-algebras that are necessary for this paper, and for more details we refer to [3] and [13]. Let H be a non-empty set endowed with a hyper operation " \circ ", that is, \circ is a function from $H \times H$ to $\mathcal{P}^*(H) = \mathcal{P}(H) \setminus \{\emptyset\}$. For two subsets A and B of H, denote by $A \circ B$ the set $\bigcup_{a \in A, b \in B} a \circ b$.

By a hyper I-algebra we mean a non-empty set H endowed with a hyper operation " \circ " and a constant 0 satisfying the following axioms:

(H1)
$$(x \circ z) \circ (y \circ z) \prec x \circ y$$
,

(H2)
$$(x \circ y) \circ z = (x \circ z) \circ y$$
,

(H3) $x \prec x$,

(H4)
$$x \prec y$$
 and $y \prec x$ imply $x = y$

for all $x,y,z\in H$, where $x\prec y$ is defined by $0\in x\circ y$ and for every $A,B\subseteq H,A\prec B$ is defined by $\exists a\in A$ and $\exists b\in B$ such that $a\prec b$. If a hyper I-algebra $(H,\circ,0)$ satisfies an additional condition:

(H5) $0 \prec x$ for all $x \in H$,

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In a hyper I-algebra H, the following hold (see [3, Proposition 3.4]):

- (a1) $(A \circ B) \circ C = (A \circ C) \circ B$.
- (a2) $x \circ (x \circ y) \prec y$.
- (a3) $x \circ y \prec z \Leftrightarrow x \circ z \prec y$.
- (a4) $A \circ B \prec C \Leftrightarrow A \circ C \prec B$.
- (a5) $(x \circ z) \circ (x \circ y) \prec y \circ z$.
- (a6) $(A \circ C) \circ (B \circ C) \prec A \circ B$.
- (a7) $A \circ (A \circ B) \prec B$.
- (a8) $A \prec A$.
- (a9) $A \subseteq B$ implies $A \prec B$.

for all $x, y, z \in H$ and for all nonempty subsets A, B and C of H.

Let S be a subset of a hyper K-algebra $(H, \circ, 0)$ containing 0. If S is a hyper K-algebra with respect to the hyper operation " \circ " on H, we say that S is a hyper K-subalgebra of $(H, \circ, 0)$ (see [3]).

Lemma 2.1. [3] Let S be a nonempty subset of H. Then S is a hyper K-subalgebra of H if and only if $x \circ y \subseteq S$ for all $x, y \in S$.

3. Intuitionistic fuzzy strong hyper K-subalgebras

In what follows let H denote a hyper K-algebra unless otherwise specified. An *intuitionistic fuzzy set* (IFS, for short) in H is an expression α given by

$$\alpha = \{ \langle x, \mu_{\alpha}(x), \gamma_{\alpha}(x) \rangle \mid x \in H \}$$

where the functions $\mu_{\alpha}: H \to [0,1]$ and $\gamma_{\alpha}: H \to [0,1]$ denote the degree of membership (namely $\mu_{\alpha}(x)$) and the degree of nonmembership (namely $\gamma_{\alpha}(x)$) of each element $x \in H$ to α , respectively, and

$$0 \le \mu_{\alpha}(x) + \gamma_{\alpha}(x) \le 1$$

for all $x\in H$. For the sake of simplicity, we shall use the notation $\alpha=\langle H,\mu_{\alpha},\gamma_{\alpha}\rangle$ instead of $\alpha=\{\langle x,\mu_{\alpha}(x),\gamma_{\alpha}(x)\rangle\mid x\in H\}$. Let $\alpha=\langle H,\mu_{\alpha},\gamma_{\alpha}\rangle$ be an IFS in H and let $m,n\in[0,1]$ with $m+n\leq 1$. Then the IFS $C_{(m,n)}$ in H is defined by $C_{(m,n)}(x)=(m,n)$, i.e., $\mu_{C_{(m,n)}}(x)=m$ and $\gamma_{C_{(m,n)}}(x)=n$ for all $x\in H$. The representation " $\alpha(x)\geq (m,n)$ " means that $\mu_{\alpha}(x)\geq m$ and $\gamma_{\alpha}(x)\leq n$. Then the set $H_{\alpha}^{(m,n)}:=\{x\in H\mid \alpha(x)\geq C_{(m,n)}(x)\}=\{x\in H\mid \mu_{\alpha}(x)\geq m,\gamma_{\alpha}(x)\leq n\}$ is called an *intuitionistic level set* of α in H.

Definition 3.1. [6] An IFS $\alpha = \langle H, \mu_{\alpha}, \gamma_{\alpha} \rangle$ in H is called an *intuitionistic fuzzy hyper K-subalgebra* of H if it satisfies:

$$\inf_{\substack{a \in x \circ y \\ b \in x \circ y}} \mu_{\alpha}(a) \ge \min\{\mu_{\alpha}(x), \mu_{\alpha}(y)\},$$

$$\sup_{b \in x \circ y} \gamma_{\alpha}(b) \le \max\{\gamma_{\alpha}(x), \gamma_{\alpha}(y)\}$$
(1)

for all $x, y \in H$.

Lemma 3.2. [6] An IFS $\alpha = \langle H, \mu_{\alpha}, \gamma_{\alpha} \rangle$ in H is an intuitionistic fuzzy hyper K-subalgebra of H if and only if the nonempty intuitionistic level set $H_{\alpha}^{(m,n)}$ is a hyper K-subalgebra of H for all $m, n \in [0,1]$ with $m+n \leq 1$.

Definition 3.3. An IFS $\alpha = \langle H, \mu_{\alpha}, \gamma_{\alpha} \rangle$ in H is called an *intuitionistic fuzzy strong hyper K-subalgebra* of H if it satisfies:

$$(\forall x, y \in H) \left(\inf_{a \in x \circ y} \mu_{\alpha}(a) \ge \mu_{\alpha}(x), \sup_{b \in x \circ y} \gamma_{\alpha}(b) \le \gamma_{\alpha}(x) \right). \tag{2}$$

Example 3.4. Let $H = \{0, a, b\}$ be a hyper K-algebra with the following Cayley table:

$$\begin{array}{c|cccc} \circ & 0 & a & b \\ \hline 0 & \{0\} & \{0\} & \{0\} \\ a & \{a\} & \{0,a\} & \{0,a\} \\ b & \{b\} & \{a,b\} & \{0,a,b\} \\ \end{array}$$

Define an IFS $\alpha = \langle H, \mu_{\alpha}, \gamma_{\alpha} \rangle$ in H by

$$\alpha = \big\langle H, \big(\tfrac{0}{0.6}, \tfrac{a}{0.6}, \tfrac{b}{0.2}\big), \big(\tfrac{0}{0.08}, \tfrac{a}{0.08}, \tfrac{b}{0.7}\big) \big\rangle.$$

Then $\alpha = \langle H, \mu_{\alpha}, \gamma_{\alpha} \rangle$ is an intuitionistic fuzzy hyper K-subalgebra of H (see [6]). It can be easily check that $\alpha = \langle H, \mu_{\alpha}, \gamma_{\alpha} \rangle$ is also an intuitionistic fuzzy strong hyper K-subalgebra of H.

Proposition 3.5. Every intuitionistic fuzzy strong hyper K-subalgebra $\alpha = \langle H, \mu_{\alpha}, \gamma_{\alpha} \rangle$ satisfies the following assertions:

$$(\forall x \in H) (\mu_{\alpha}(0) \ge \mu_{\alpha}(x) \& \gamma_{\alpha}(0) \le \gamma_{\alpha}(x)).$$

Proof. Since $x \prec x$ for all $x \in H$, we have $0 \in x \circ x$. Hence $\mu_{\alpha}(0) \geq \inf_{z \in x \circ x} \mu_{\alpha}(z) \geq \mu_{\alpha}(x)$ and $\gamma_{\alpha}(0) \leq \sup_{z \in x \circ x} \gamma_{\alpha}(z) \leq \gamma_{\alpha}(x)$. This completes the proof. \square

Theorem 3.6. Every intuitionistic fuzzy strong hyper K-subalgebra is an intuitionistic fuzzy hyper K-subalgebra.

Proof. Let $\alpha = \langle H, \mu_{\alpha}, \gamma_{\alpha} \rangle$ be an intuitionistic fuzzy strong hyper K-subalgebra of H and let $x, y \in H$. Then

$$\inf_{z \in x \circ u} \mu_{\alpha}(z) \ge \mu_{\alpha}(x) \ge \min\{\mu_{\alpha}(x), \mu_{\alpha}(y)\},\$$

$$\sup_{z \in x \circ y} \gamma_{\alpha}(z) \le \gamma_{\alpha}(x) \le \max\{\gamma_{\alpha}(x), \gamma_{\alpha}(y)\}\$$

for all $x, y \in H$. Hence $\alpha = \langle H, \mu_{\alpha}, \gamma_{\alpha} \rangle$ is an intuitionistic fuzzy hyper K-subalgebra of H.

The converse of Theorem 3.6 is not true as seen in the following example.

Example 3.7. Let $H = \{0, a, b\}$ be a hyper K-algebra with the following Cayley table:

$$\begin{array}{c|cccc} \circ & 0 & a & b \\ \hline 0 & \{0\} & \{0,a,b\} & \{0,a,b\} \\ a & \{a\} & \{0,a,b\} & \{0,a,b\} \\ b & \{b\} & \{a,b\} & \{0,a,b\} \end{array}$$

Define an IFS $\alpha = \langle H, \mu_{\alpha}, \gamma_{\alpha} \rangle$ in H by

$$\alpha = \big\langle H, \big(\tfrac{0}{0.9}, \tfrac{a}{0.3}, \tfrac{b}{0.3}\big), \big(\tfrac{0}{0.09}, \tfrac{a}{0.7}, \tfrac{b}{0.7}\big) \big\rangle.$$

Then $\alpha = \langle H, \mu_{\alpha}, \gamma_{\alpha} \rangle$ is an intuitionistic fuzzy hyper Ksubalgebra of H. But it is not an intuitionistic fuzzy strong hyper K-subalgebra of H since $\inf_{z\in 0\circ a}\mu_{\alpha}(z)=0.3<0.9=\mu_{\alpha}(0)$ and/or $\sup_{z\in 0\circ a}\gamma_{\alpha}(z)=0.7>0.09=\gamma_{\alpha}(0).$

Combining Lemma 3.2 and Theorem 3.6, we have

Theorem 3.8. If an IFS $\alpha = \langle H, \mu_{\alpha}, \gamma_{\alpha} \rangle$ in H is an intuitionistic fuzzy strong hyper K-subalgebra of H, then the nonempty intuitionistic level set $H_{\alpha}^{(m,n)}$ is a hyper Ksubalgebra of H for all $m, n \in [0, 1]$ with $m + n \le 1$.

Theorem 3.9. Let S be a subset of H and let $\alpha(S) =$ $\langle H, \mu_{\alpha(S)}, \gamma_{\alpha(S)} \rangle$ be an IFS in H defined by

$$\mu_{\alpha(S)}(x) := \begin{cases} t_1 & \text{if } x \in S, \\ t_2 & \text{otherwise,} \end{cases}$$
 (3)

$$\gamma_{\alpha(S)}(x) := \left\{ \begin{array}{ll} s_1 & \text{if } x \in S, \\ s_2 & \text{otherwise,} \end{array} \right.$$

for all $x \in H$, where $t_1, t_2, s_1, s_2 \in [0, 1]$ with $t_1 > t_2$, $s_1 < s_2, t_i + s_i \le 1$ for i = 1, 2. Then

(i) $\alpha(S) = \langle H, \mu_{\alpha(S)}, \gamma_{\alpha(S)} \rangle$ is an intuitionistic fuzzy hyper K-subalgebra of H if and only if S is a hyper K-subalgebra of H.

Proof. (i) Assume that $\alpha(S) = \langle H, \mu_{\alpha(S)}, \gamma_{\alpha(S)} \rangle$ is an intuitionistic fuzzy hyper K-subalgebra of H and let $x,y\in$ S. Then $\mu_{\alpha(S)}(x) = t_1 = \mu_{\alpha(S)}(y)$ and $\gamma_{\alpha(S)}(x) = s_1 = t_1$ $\gamma_{\alpha(S)}(y)$. For any $a \in x \circ y$, we have

$$\mu_{\alpha(S)}(a) \ge \inf_{u \in \sigma(S)} \mu_{\alpha(S)}(u) \ge \min\{\mu_{\alpha(S)}(x), \, \mu_{\alpha(S)}(y)\} = t_1$$

$$\gamma_{\alpha(S)}(a) \le \sup_{v \in x \circ y} \gamma_{\alpha(S)}(v) \le \max\{\gamma_{\alpha(S)}(x), \gamma_{\alpha(S)}(y)\} = s_1.$$

Thus $\mu_{\alpha(S)}(a) = t_1$ and $\gamma_{\alpha(S)}(a) = s_1$, and so $a \in S$. Therefore $x \circ y \subseteq S$, and S is a hyper K-subalgebra of H by Lemma 2.1. Conversely, suppose that S is a hyper Ksubalgebra of H and let $x, y \in H$. If $x \notin S$ or $y \notin S$, then clearly

$$\inf_{a \in x_{0}} \mu_{\alpha(S)}(a) \ge t_2 = \min\{\mu_{\alpha(S)}(x), \mu_{\alpha(S)}(y)\},$$

$$\sup_{b \in x \circ y} \gamma_{\alpha(S)}(b) \le s_2 = \max\{\gamma_{\alpha(S)}(x), \gamma_{\alpha(S)}(y)\}.$$

Assume that $x \in S$ and $y \in S$. Then $x \circ y \subseteq S$. Thus

$$\inf_{a \in x \circ y} \mu_{\alpha(S)}(a) = t_1 = \min\{\mu_{\alpha(S)}(x), \mu_{\alpha(S)}(y)\},\,$$

$$\sup_{b \in x \circ y} \gamma_{\alpha(S)}(b) = s_1 = \max\{\gamma_{\alpha(S)}(x), \gamma_{\alpha(S)}(y)\}.$$

Therefore $\alpha(S) = \langle H, \mu_{\alpha(S)}, \gamma_{\alpha(S)} \rangle$ is an intuitionistic fuzzy hyper K-subalgebra of \hat{H} .

Corollary 3.10. Let $\alpha(S) = \langle H, \mu_{\alpha(S)}, \gamma_{\alpha(S)} \rangle$ be defined as in Theorem 3.9. If it is an intuitionistic fuzzy strong hyper K-subalgebra of H, then S is a hyper K-subalgebra of H.

Proof. It follows directly from Theorems 3.6 and 3.9.

Definition 3.11. [5] Let H_1 and H_2 be hyper K-algebras. A mapping $f: H_1 \to H_2$ is called a weak homomorphism if it satisfies:

(i)
$$f(0) = 0$$
,

(ii)
$$(\forall x, y \in H_1) (f(x \circ y) \subseteq f(x) \circ f(y)).$$

Theorem 3.12. Let $f: H_1 \rightarrow H_2$ be a weak homomorphism of hyper K-algebras. If $\beta = \langle H_2, \mu_\beta, \gamma_\beta \rangle$ is an intuitionistic fuzzy strong hyper K-subalgebra of H_2 , then $\beta_f = \langle H_1, \mu_{\beta_f}, \gamma_{\beta_f} \rangle$ is an intuitionistic fuzzy hyper K-subalgebra of H_1 , where μ_{β_f} and γ_{β_f} are defined by $\mu_{\beta_f}(x) = \mu_{\beta}(f(x))$ and $\gamma_{\beta_f}(x) = \gamma_{\beta}(f(x))$ for all $x \in H_1$.

Proof. For any $x, y \in H_1$, we have

$$\inf_{z \in x \circ y} \mu_{\beta_f}(z) = \inf_{z \in x \circ y} \mu_{\beta}(f(z)) \ge \inf_{f(z) \in f(x \circ y)} \mu_{\beta}(f(z))$$

$$\ge \inf_{f(z) \in f(x) \circ f(y)} \mu_{\beta}(f(z)) \ge \mu_{\beta}(f(x))$$

$$\ge \min\{\mu_{\beta}(f(x)), \mu_{\beta}(f(y))\}$$

$$\ge \min\{\mu_{\beta_f}(x), \mu_{\beta_f}(y)\},$$

$$\gamma_{\alpha(S)}(y). \text{ For any } a \in x \circ y, \text{ we have } \sup_{z \in x \circ y} \gamma_{\beta_f}(z) = \sup_{z \in x \circ y} \gamma_{\beta}(f(z)) \leq \sup_{f(z) \in f(x \circ y)} \gamma_{\beta}(f(z))$$

$$\mu_{\alpha(S)}(a) \geq \inf_{u \in x \circ y} \mu_{\alpha(S)}(u) \geq \min\{\mu_{\alpha(S)}(x), \, \mu_{\alpha(S)}(y)\} = t_1, \qquad \leq \sup_{f(z) \in f(x) \circ f(y)} \gamma_{\beta}(f(z)) \leq \gamma_{\beta}(f(x))$$

$$\gamma_{\alpha(S)}(a) \leq \sup_{v \in x \circ y} \gamma_{\alpha(S)}(v) \leq \max\{\gamma_{\alpha(S)}(x), \, \gamma_{\alpha(S)}(y)\} = s_1.$$

$$\leq \max\{\gamma_{\beta}(f(x)), \, \gamma_{\beta}(f(y))\}$$

$$\leq \max\{\gamma_{\beta_f}(x), \, \gamma_{\beta_f}(y)\}.$$

Hence $\beta_f = \langle H_1, \mu_{\beta_f}, \gamma_{\beta_f} \rangle$ is an intuitionistic fuzzy hyper K-subalgebra of H_1 . Let $(H_1, \circ_1, 0_1)$ and $(H_2, \circ_2, 0_2)$ be hyper K-algebras. Define a hyper operation \circ on the product $H_1 \times H_2$ as follows:

$$(a_1, a_2) \circ (b_1, b_2) = (a_1 \circ_1 b_1, a_2 \circ_2 b_2)$$

for all $(a_1, a_2), (b_1, b_2) \in H_1 \times H_2$, where for $A \subseteq H_1$ and $B \subseteq H_2$ by (A, B) we mean

$$(A,B) = \{(a,b) \mid a \in A, b \in B\}, \ 0 = (0_1, 0_2)$$

and $(a_1,a_2) \prec (b_1,b_2) \Leftrightarrow a_1 \prec b_1,\ a_2 \prec b_2$. Then $(H_1 \times H_2,\circ,0)$ is a hyper K-algebra (see [3]).

Theorem 3.13. If $\alpha_1 = \langle H_1, \mu_{\alpha_1}, \gamma_{\alpha_1} \rangle$ and $\alpha_2 = \langle H_2, \mu_{\alpha_2}, \gamma_{\alpha_2} \rangle$ are intuitionistic fuzzy hyper K-subalgebras of hyper K-algebras $(H_1, \circ_1, 0_1)$ and $(H_2, \circ_2, 0_2)$, respectively, then $\alpha_1 \times \alpha_2 = \langle H_1 \times H_2, \mu_{\alpha_1 \times \alpha_2}, \gamma_{\alpha_1 \times \alpha_2} \rangle$ is an intuitionistic fuzzy hyper K-subalgebra of $(H_1 \times H_2, \circ, 0)$.

Proof. For any $(x_1, x_2), (y_1, y_2) \in H_1 \times H_2$, we have

$$\begin{split} &\inf_{(a_1,a_2)\in(x_1,x_2)\circ(y_1,y_2)} \mu_{\alpha_1\times\alpha_2}(a_1,a_2) \\ &= \inf_{\substack{a_1\in x_1\circ_1y_1\\ a_2\in x_2\circ_2y_2}} \min\{\mu_{\alpha_1}(a_1),\mu_{\alpha_2}(a_2)\} \\ &= \min\{\inf_{\substack{a_1\in x_1\circ_1y_1\\ a_1\in x_1\circ_1y_1}} \mu_{\alpha_1}(a_1),\inf_{\substack{a_2\in x_2\circ_2y_2\\ a_2\in x_2\circ_2y_2}} \mu_{\alpha_2}(a_2)\} \\ &\geq \min\{\min\{\mu_{\alpha_1}(x_1),\mu_{\alpha_1}(y_1)\},\\ \min\{\mu_{\alpha_2}(x_2),\mu_{\alpha_2}(y_2)\}\} \\ &= \min\{\min\{\mu_{\alpha_1}(x_1),\mu_{\alpha_2}(x_2)\},\\ \min\{\mu_{\alpha_1}(y_1),\mu_{\alpha_2}(y_2)\}\} \\ &= \min\{\mu_{\alpha_1\times\alpha_2}(x_1,x_2),\mu_{\alpha_1\times\alpha_2}(y_1,y_2)\}, \end{split}$$

$$\sup_{\substack{(a_1,a_2) \in (x_1,x_2) \circ (y_1,y_2) \\ = \sup_{\substack{\alpha_1 \in x_1 \circ y_1 \\ \alpha_2 \in x_2 \circ 2y_2}} \max \{ \gamma_{\alpha_1}(a_1), \gamma_{\alpha_2}(a_2) \} \\ = \max \{ \sup_{\substack{\alpha_1 \in x_1 \circ y_1 \\ \alpha_2 \in x_2 \circ 2y_2}} \sup \gamma_{\alpha_1}(a_1), \sup_{\substack{\alpha_2 \in x_2 \circ 2y_2 \\ < = \max}} \gamma_{\alpha_2}(a_2) \} \\ \leq \max \{ \max \{ \gamma_{\alpha_1}(x_1), \gamma_{\alpha_1}(y_1) \}, \\ \max \{ \gamma_{\alpha_2}(x_2), \gamma_{\alpha_2}(y_2) \} \} \\ = \max \{ \max \{ \gamma_{\alpha_1}(x_1), \gamma_{\alpha_2}(x_2) \}, \\ \max \{ \gamma_{\alpha_1}(y_1), \gamma_{\alpha_2}(y_2) \} \} \\ = \max \{ \gamma_{\alpha_1 \times \alpha_2}(x_1, x_2), \gamma_{\alpha_1 \times \alpha_2}(y_1, y_2) \}.$$

Hence $\alpha_1 \times \alpha_2 = \langle H_1 \times H_2, \mu_{\alpha_1 \times \alpha_2}, \gamma_{\alpha_1 \times \alpha_2} \rangle$ is an intuitionistic fuzzy hyper K-subalgebra of $(H_1 \times H_2, 0, 0)$.

Corollary 3.14. If $\alpha_1 = \langle H_1, \mu_{\alpha_1}, \gamma_{\alpha_1} \rangle$ and $\alpha_2 = \langle H_2, \mu_{\alpha_2}, \gamma_{\alpha_2} \rangle$ are intuitionistic fuzzy strong hyper K-subalgebras of hyper K-algebras $(H_1, \circ_1, 0_1)$ and $(H_2, \circ_2, 0_2)$, respectively, then $\alpha_1 \times \alpha_2 = \langle H_1 \times H_2, \mu_{\alpha_1 \times \alpha_2}, \gamma_{\alpha_1 \times \alpha_2} \rangle$ is an intuitionistic fuzzy (strong) hyper K-subalgebra of $(H_1 \times H_2, \circ, 0)$.

Proof. Straightforward.

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