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Flow Characteristics of Gaseous Leak flows in Narrow Cracks

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Key Words : Leak flow rate, Friction factor, Narrow crack

ABSTRACT

The prediction for gaseous leak flows through a narrow crack is important for a leak-before-break (LBB) analysis. Therefore, the methodology to obtain the flow characteristics of gaseous leak flow in a narrow crack for the wide range by using the product of friction factor and Reynolds number correlations (f Re) for a micro-channel is developed and presented. The correlation applied here was proposed by the previous study. The fourth-order Runge-Kutta method was employed to integrate the nonlinear ordinary differential equation for the pressure and the regular-Falsi method was also employed to find the inlet Mach number. A narrow crack whose opening displacement ranges from 10 to 100μm with a crack length in the range from 2 to 200mm was chosen for sample prediction. The present results are compared with both numerical simulation results and available experimental measurements. The results are in excellent agreement with them. The leak flow rate can be approximately predicted by using proposed methodology.

1. Introduction

Prediction of leak flow rates is important for a variety of industries that use a leak-before-break (LBB) safety assessment as a method of non-destructive testing to assure product reliability and performance. In general, leaks from pressure vessels occur due to pressure-driven convection and permeation or diffusion. Gaseous leaks are mainly induced by pressure-driven convection than permeation[1].

Since the experimental work by Button et al.[2] who measured nitrogen flow rates through rough parallel cracks, many experimental and numerical investigations on leak flows driven by pressure difference in cracked pressure vessels have been undertaken. Most of the published research effort to measure and calculate leak rates has focused on high pressure fluids leaking through relatively wide open cracks in thick walled vessels, e. g., Matsumoto et al.[3], Yano et al.[4].

The studies on the leak flow rates through gaskets have been also investigated, e. g., Geoffroy and Prat[5], Marchand and Derenne[6]. Schefer et al.[1] investigated characterization of leak from compressed hydrogen dispensing systems and related components and developed the equations for the calculation of leak flow rates in various leak regimes of both pressure-driven convection and permeation through metals.

Some investigations on the leak flow through narrow cracks in low pressure and thin walled structures have been performed. Clarke et al.[7] and

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Bagshaw et al.[8] measured the leak flow rates of water and air through cracks. Their experimental results were compared with two-dimensional computational fluid models. Beck et al.[9] developed explicit equations for leak rates through narrow cracks with an idealized zig-zag channel in laminar flow regime in which the compressible effect of gaseous flows was not considered. The leak flow rates of air and water obtained by this model were compared with those of CFD and experiments (Clarke et al,[7] and Bagshaw et al.[8]). Chivers[10] investigated the influence of surface roughness on fluid flow through cracks based on a friction factor. It was mentioned that the use of computational fluid dynamics to study turbulent flow through cracks in future analyses could significantly reduce the uncertainties with fluid friction.

In available literatures with smaller cracks, the leak flow rates have been investigated in which the opening crack displacement is more than 100μm. However, the leak flow rates of cracks with the opening displacement less than 100μm have not been investigated yet except experimental work of Button et al.[2]. Button et al.[2] found friction factor of gas flow through smooth cracks in laminar flow regime is higher than 96/Re. But this discrepancy is not well explained.

As mentioned above, with smaller crack less than 100μm, the f·Re also deviates from the conventional value since gas expands due to the large pressure variation. Therefore, the leak flow rates can not be estimated. This is the motivation of the present study to develop a methodology to predict leak flow rates of gaseous flow through narrow cracks whose opening displacement ranges from 10 to 100μm. Fortunately, the f·Re correlation that can be applicable to estimate leak flows through narrow cracks less than 100μm was proposed by Asako et al.[11]. They found that f·Re is a function of Mach number in gaseous microchannel flows. In the present study, the f·Re correlation is employed since the gaseous flow through a micro-channel was considered as a suitable gaseous leak flow model.

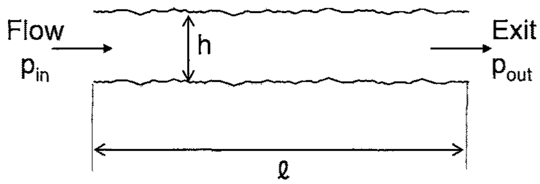


Fig. 1 Flow model in a narrow crack

2. FORMULATION

2.1. Leak flow model and conservation equation

The flow through a narrow crack is modeled as a parallel plate channel as shown in Fig. 1 with the inlet pressure, p_{in} to its upstream section. Gas leakage from pressure vessels typically occurs through a narrow crack that is much greater than the mean free path of the gas molecules. Therefore, during leakage, gas expands along the crack length due to pressure fall and the density change should be considered. Therefore, a compressible flow based continuum model can be used to obtain the leak flow characteristics. The flow is assumed to be steady and laminar. The fluid is assumed to be an ideal gas. The equations of the continuum and state for the ideal gas are expressed by

$$\bar{\rho}\bar{u} = \rho_{in} u_{in} \quad (1)$$

$$\bar{p} = \bar{\rho} R \bar{T} \quad (2)$$

Leak flow rate, $\dot{M}$ and leak flow rate per unit depth, $\dot{m}$ are expressed as

$$\dot{M} = \bar{\rho}\bar{u}A, \quad \dot{m} = \bar{\rho}\bar{u}h \quad (3)$$

where A is the cross-sectional area of the crack, h is the crack opening displacement, $\bar{u}$, $\bar{\rho}$, and $\bar{T}$ are the average velocity, density, and temperature at an arbitrary cross section, respectively.

Reynolds number and Mach number that determines the degree of compressibility are also defined as

$$Re = \frac{2\dot{m}}{\mu} = \frac{\bar{u}D_h}{\mu/\rho}, \quad Ma = \frac{\bar{u}}{a} = \frac{\bar{u}}{\sqrt{\gamma RT}} \tag{4}$$

where D_h is the hydraulic diameter (D_h=2h).
Note that the Re is constant along the crack length but the Ma varies along the crack length. From the above equations, Eq. (2) can be rewritten as

$$\bar{p} = \frac{u_{in}}{\bar{u}}\rho_{in}R\bar{T} = \frac{u_{in}}{\bar{u}}p_{in} = \frac{Ma_{in}}{Ma}p_{in} \tag{5}$$

2.2. Differential equations for pressure distributions

The pressure distribution along crack length can be calculated from the friction factor based on the Darcy's definition. The Darcy friction factor is defined as

$$f = \frac{-2D_h}{\bar{\rho}\bar{u}^2}\left(\frac{d\bar{p}}{dx}\right) \tag{6}$$

In the previous work (Asako et al.[11]), it was found the product of friction factor and Reynolds number (f·Re), Poiseuille number for very small channel is a function of Mach number and the f·Re correlation for the parallel plate channel is expressed as:

$$f \cdot Re = 96 - 4.55Ma + 274.8Ma^2 \tag{7}$$

From Eqs. (6) and (7), the following differential equation for the pressure in the crack is obtained :

$$\frac{d\bar{p}}{dx} = \frac{-\bar{\rho}\bar{u}^2}{2D_h Re}(96 - 4.55Ma + 274.8Ma^2) \tag{8}$$

Eq. (8) can be rewritten by Eqs (2) and (4) as

$$\frac{d\bar{p}}{dx} = \frac{-\mu a Ma_{in}p_{in}}{2D_h}\frac{1}{\bar{p}}\left(96 - 4.55\frac{Ma_{in}p_{in}}{\bar{p}} + 274.8\frac{Ma_{in}^2 p_{in}^2}{\bar{p}^2}\right) \tag{9}$$

If the pressure and Mach number at the inlet are given, Eq. (9) can be solved. However, in a case where the inlet and outlet pressures are specified, the inlet Mach number is unknown. In such a case, the pressure distribution and the leak flow rate can be obtained by solving Eq. (9) iteratively with guessing the inlet Mach number. Such a problem is called as the two-point boundary value problem.

2.3. Calculation methodology

Attention will now be focused on the calculation of the Eq. (9). In order to integrate the differential equation, the forth-order Runge-Kutta method was employed for the numerical integration of the system of a nonlinear ordinary differential equation, Eq. (9). This method was utilized because of its high accuracy and convenience involving two-point boundary value problems. Runge-Kutta method requires both the inlet pressure and the inlet Mach number. However, the inlet Mach number is unknown. In order to determine the inlet Mach number appropriately, the regular-Falsi method that is very practical for finding the inlet Mach number was employed. The outlet pressure is fixed at atmospheric condition. The procedure for the calculations is as follows[12]:

1. Prior to the calculations, give computational parameters such as h, , ain also p_{in}, p_{out}.
2. Guess the range of the inlet Mach number. $Ma_{in,A}$ and $Ma_{in,B}$ represent the upper and lower limits of the range, respectively.
3. Solve Eq. (9) for $Ma_{in,A}$ and $Ma_{in,B}$ by the forth-order Runge-Kutta integration scheme, respectively as shown in Fig. 2.
4. Using regular-Falsi method, determine the new guess value of the inlet $Ma_{in,C}$ by

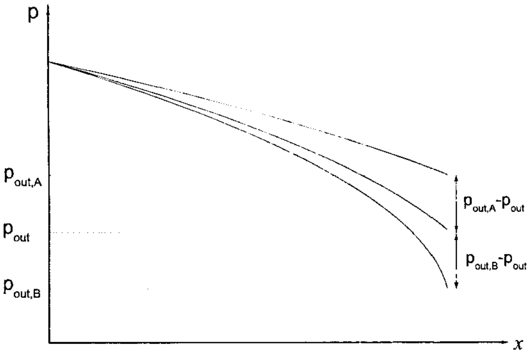


Fig. 2 Pressure distributions as a function of x

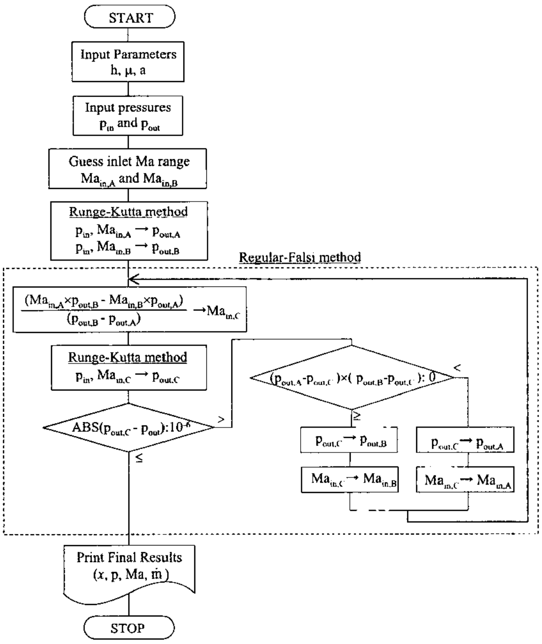


Fig. 3 Flow chart of Runge-Kutta and regular-Falsi method

$$Ma_{in,C} = \frac{Ma_{in,A}\cdot p_{out,B} - Ma_{in,B}\cdot p_{out,A}}{p_{out,B} - p_{out,A}}$$

where, $p_{out,A}$ is the outlet pressure for $Ma_{in,A}$ and $p_{out,B}$ is the outlet pressure for $Ma_{in,B}$.

5. Solve Eq. (9) for $Ma_{in,C}$ by the forth-order Runge-Kutta integration scheme.
6. Repeat step 4 until the difference between $p_{out,C}$ calculated by step5 and the given p_{out} (p_{out}=100 kPa) converges to within a convergence criterion, 10^{-6}.
7. Print x, p Ma and $\dot{m}$.

Fig. 3 illustrates the flow chart of the solution process.

Table 1 Crack opening displacement, length, p_{in}, p_{out} and Ma_{in}

h (μm)	ℓ (mm)	p_{in} (kPa)	p_{out} (kPa)	Ma_{in}
10	2	150~600	100	0.026~0.156
10	10	150~1200	100	0.005~0.072
10	20	150~1600	100	0.003~0.050
100	20	120~200	100	0.104~0.309
100	100	120~350	100	0.024~0.173
100	200	120~400	100	0.011~0.111

3. RESULTS AND DISCUSSIONS

The calculations were performed for two crack opening displacements using Runge-Kutta and regular-Falsi methods to obtain leak flow rates. Air of R=287 J·(kg·K)$^{-1}$, γ=1.4, μ=1.862x10^{5} Pa·s at 300K was assumed for the working fluid. The crack opening displacement ranges from 10 to 100μm. The inlet pressure, p_{in} also ranges from 120 to 1600 kPa. The outlet pressure is fixed at atmospheric condition. The crack opening displacement, p_{in}, p_{out} and corresponding Ma_{in} for sample calculations are listed in Table 1. The inlet Mach number ranges from 0.003 to 0.309.

3.1. Pressure distribution and Mach number

The pressure distribution along the crack length for the crack opening displacement of 10μm is plotted in Fig. 4. The corresponding Mach number is also plotted in the figure. The figure is the typical variations of pressure and Mach number for compressible flow. The pressure gradient becomes steep near the outlet as the inlet pressure increases. The Mach number increases approaching to the outlet due to the acceleration of flow. Therefore, with the smaller crack opening displacement, the compressibility effect is significant.

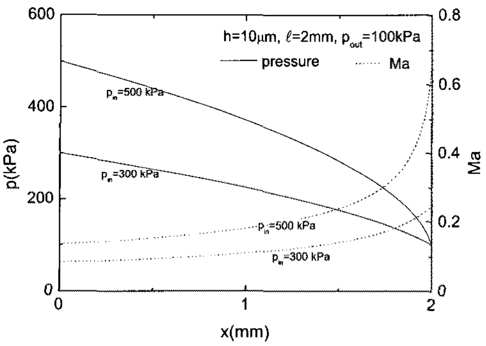


Fig. 4 Pressure distribution and Mach number

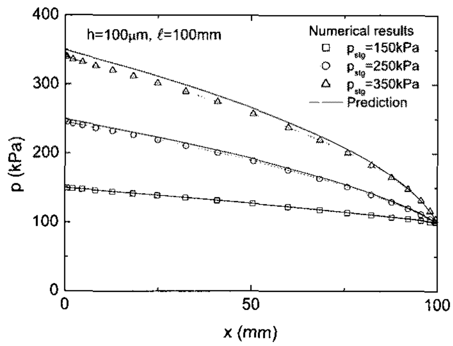


Fig. 5 Pressure distribution as a function of x

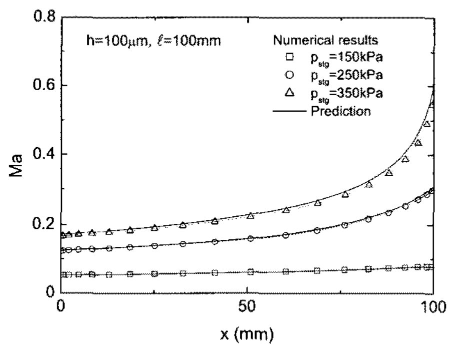
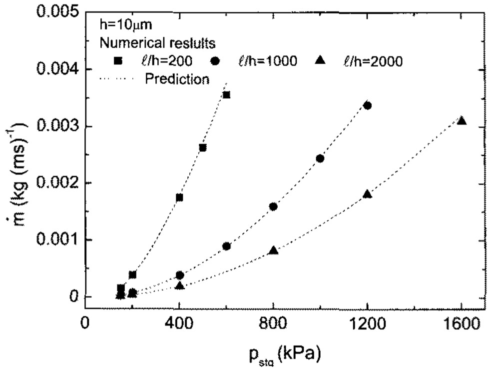
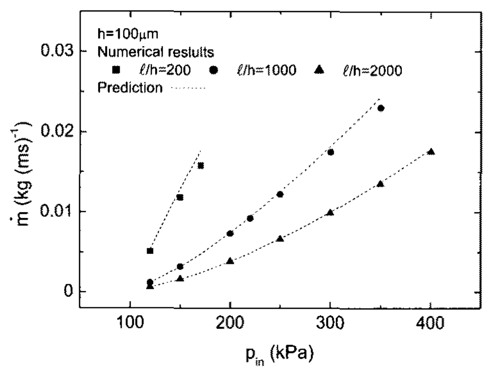


Fig. 6 Mach number as a function of x



(a) h=10μm



(b) h=100μm

Fig. 7 Leak flow rate as a function of pin

3.2. Comparison with numerical results

In order to compare with the two-dimensional CFD results, the numerical simulations based on the arbitrary-Lagrangian-Eulerian method were performed with isothermal flow condition for all cases of Table 1. The channel geometry is a parallel plate channel. The 2D plane is divided into quadrilateral cells: 200 along the crack length and 20 through the crack opening displacement. Additional details of the numerical methodology are documented by Amdsen et al.[13] and Asako et al.[11]

The pressure distributions obtained by both Eq. (9) and the numerical simulations are plotted along the crack length in Fig. 5. The corresponding Mach number is plotted in Fig. 6. The results are for the crack opening displacement of 100μm and the crack length of 100mm. The pressure distributions and Mach numbers obtained by Eq. (9) almost coincides with that of the numerical result. As can be seen in

Fig. 6, the inlet Mach number ranges from 0.004 to 0.276. The maximum difference in the inlet Mach number between Eq. (9) and numerical results is 10%. However, the values obtained by the numerical result are lower than those obtained by Eq. (9) since the entrance loss increases as the inlet pressure increases.

The leak flow rates obtained by the numerical simulation for all cases of Table 1 are plotted as a function of inlet pressure in Fig. 7 (a) and (b). Those predicted by Eq. (3) are also plotted in the figure. Both results have similar trends. The leak flow rate increases with increasing the inlet pressure. The leak flow rate predicted by the present work coincides with that of the numerical simulation within 10%. The leak flow rates of predictions are higher than those of the numerical results since f·Re correlation proposed by Asako et al.[11] is obtained for quasi-fully developed region where the flow accelerates neglecting the entrance loss.

3.3. Comparison with experimental data

The experimental leak flow rate data used in the present work have been obtained by Turner et al.[14]. They measured local pressure of nitrogen, helium, and air flow along micro channels with a rectangular cross-section. The micro-channel were etched into silicon wafers and capped with smooth glass. The channel length was about 27mm and the hydraulic diameter ranges from 4 to 100μm. Five pressure taps were located along the channel with equal spacing of 5mm and additional two pressure taps were at the inlet and outlet ports. The details of the experiment were well documented in their paper (Turner et al.[14]). There seems few investigations on the gaseous leak flow with cracks less than 100μm. Therefore, in the present work, the gas flow through a micro-channel[14] was assumed to be a suitable leak flow model.

The leak flow rates were obtained from the pressure data for nitrogen flows through the micro-channel. The dimensions of the channels and the range of inlet and outlet pressure, corresponding leak flow rates are listed in Table 2. To facilitate presentation of experimental data, Eq. (9) obtained for parallel plate crack was arranged with Poiseuille number of the incompressible flow, $f{\cdot}Re_{incomp}$ determined from the channel geometry of the cross section (e. g., shah and London,[15]) as:

Table 2 Channel dimensions, pin, and leak flow rate

h (μm)	w (μm)	ℓ (mm)	D_h (μm)	p_{in} (kPa)	$\dot{M}$ (kg s 1)
5.21	1056	26.69	10.2	155-713	2.08×10 7 ~ 7.49×10 7
12.29	1061	26.82	24.3	274~697	1.32×10 7 ~ 9.60×10 7
21.43	1059	26.59	42.01	153~757	1.37×10 7 ~ 5.35×10 6
50.1	994	26.87	95.38	147~313	1.42×10 6 ~ 9.34×10 6

$$\frac{dp}{dx}=\frac{-\mu a Ma_{in}p_{in}}{2D_h}\frac{1}{p}\left(f\cdot Re_{incomp}-4.55\frac{Ma_{in}p_{in}}{p}+274.8\frac{Ma_{in}^2p_{in}^2}{p_{in}^2}\right) \quad (10)$$

Then, for the all cases in Table 2, the solutions to Eq. (10) by the forth-order Runge-Kutta integration scheme and regular-Falsi methods were obtained. In the case of h = 50.1μm, the obtained pressure distributions and corresponding Mach

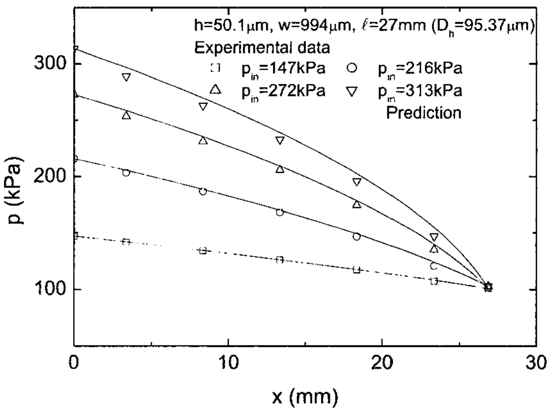


Fig. 8 Pressure distribution as a function of x

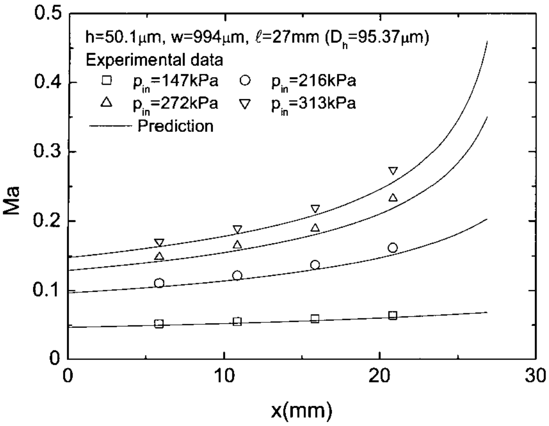


Fig. 9 Mach number as a function of x

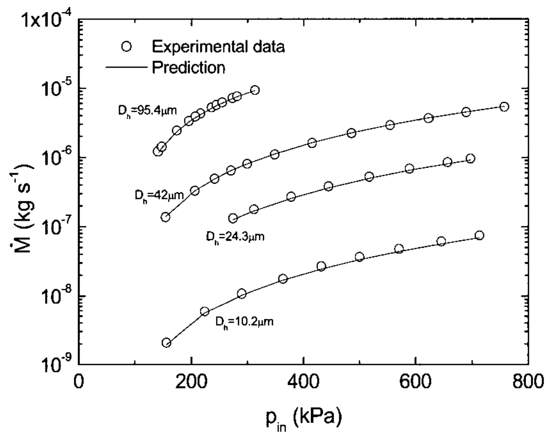


Fig. 10 Leak flow rate as a function of pin

number are plotted along crack length in Figs. 8 and 9. Those of experimental data are also plotted in the figures. The present results coincide well with the experimental data. The leak flow rates predicted by Eq. (3) for all cases in Table 2 are also plotted as a function of inlet pressure in Fig. 10 with experimental results. As can be seen in the figure, the predicted leak flow rates coincide well with the experimental data within 3%.

4. CONCLUDING REMARKS

A differential equation for the pressure of crack length dependence is developed to predict leak flow rate in narrow crack. The calculations were preformed using the forth-order Runge-Kutta and regular-Falsi algorithms. The following conclusions are reached.

(1) The compressibility effect becomes important as the crack opening displacement becomes small.
(2) The leak flow rate is predicted using the pressure and Mach number obtained by the differential equation for pressure.
(3) The leak flow rate predicted by the present work coincides well with the experimental data. The following equation is developed by $f \cdot Re_{incomp}$ for the crack geometry of the crack cross section.

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