A Method of Hysteresis Modeling and Traction Control for a Piezoelectric Actuator

Baek-Ju Sung†, Eun-Woong Lee* and Jae-Gyu Lee**

Abstract – The dynamic model and displacement control of piezoelectric actuators, which are commercially available materials for managing extremely small displacements in the range of sub-nanometers, are presented. Piezoceramics have electromechanical characteristics that transduce energy between the electrical and mechanical domains. However, they have hysteresis between the input voltage and output displacement, and this behavior is very demanding and complicated. In this paper, we propose a method of designing the control algorithm, and present the dynamic modeling equations that represent the hysteretic behavior between input voltage and output displacement. For this process, the piezoelectric actuator is treated as a second-order linear dynamic system and system constants are determined by the system identification method. Also, a classical PID controller is designed and used to regulate the output displacement of the actuator. To evaluate the performance of the proposed method, numerical simulation results are presented.

Keywords: Hysteresis modeling, Piezoelectric actuator, PID control, System identification

1. Introduction

Piezoelectric actuators are electromechanical devices that transduce energy between the electrical and mechanical domains, and they are commercially available materials for managing extremely small displacements in the range of sub-nanometers. Due to their simplicity, high stiffness, low wear and tear, and fast response, they are used for high precision mechanical and electrical engineering applications. However, piezoelectric actuators suffer from hysteretic behavior inherent to nonlinear dielectric materials, and they have hysteresis between the input voltage and output displacement [1].

A well-known description of piezoelectric ceramic was published in 1987 by a standards committee of the IEEE [2]. This description contains linearized constitutive relations, and is generally used to describe the linear modeling of piezoceramics [3]. However, this description is unsuitable for the representation of hysteretic behavior or dynamic motion of the actuator. Many researchers have applied these linearized constitutive relations to formulate the piezoelectric actuator. Croft and McAllister presented the linear dynamic modeling results [4], Ge and Jouaneh treated the hysteresis modeling problem [5], and Leigh and Zimmerman took an iterative algorithm to describe the nonlinear behavior of the actuator [6].

Hysteretic behaviors of the piezoelectric actuator include the relationship between the stress and strain in the elastic-plastic deformation of a material, and the relationship between magnetic field strength and flux density in a hard magnetic material. And mechanically, those behaviors are similar to the motion of the combination of an ideal spring of which an analogy was formulated by the mathematician and physicist James C. Maxwell in the mid 1800s [7]. Several researchers utilized the Maxwell slip model to describe the nonlinear hysteretic behavior and electromechanical phenomenon of piezoceramics. Goldfarb and Celanovic proposed a modeling method for piezoceramics that utilizes the Maxwell slip model to represent the hysteretic nonlinearities inherent to piezoceramics [8]. Helen and Ridha treated the modeling problem for piezoceramics when subjecting variable mechanical load disturbance conditions [9]. Dynamic modeling and close loop position control system design problems were investigated by Paul [10].

On the other hand, the differential equation describing the hysteretic phenomenon was introduced by Han [11], theoretically, in which a partial differential equation (PDE) as a mechanical model is derived and analyzed, followed by consideration of the total model of a piezo-actuated positioning mechanism for the case of voltage and charge steering.

In this paper, we propose a dynamic modeling and controller design method of a piezoelectric actuator. To

† Corresponding Author: System Engineering Research Division, Korea Institutive of Machinery & Material, Korea. (sbj682@kimm.re.kr)
* Dept. of Electrical Engineering, Chungnam National University, Korea. (ewlee@cnu.ac.kr)
** Production/R&D General Division, Tata Daewoo Commercial Vehicle Company Limited, Korea. (jglee3@tata-daewoo.com)
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determine the mathematical model of the system, nonlinear characteristic response between input voltage and output displacement is described by a second-order linear dynamic system, and system constants are determined by the system identification method. Furthermore, a classical PID controller is designed and used to regulate the output displacement of the actuator. The PID controller gain parameters, which can minimize system error between reference position and output position, are selected by optimization technique. To evaluate the performance of the proposed method, numerical simulation results are presented and discussed.

2. Piezoelectric Actuator

Piezoceramics transduce energy between electrical and mechanical domains. The application of a force or stress results in the development of a charge in the material, conversely, the application of a charge to the same material will result in a charge in mechanical dimensions or strain. These are known as direct piezoelectric effect and indirect piezoelectric effect, respectively.

For the purpose of sensing or measuring in the range of a sub-nanometer, the multi layer piezoelectric stack actuator is generally used. A typical piezoelectric stack actuator is formed by assembling several wafer elements in series mechanically and connecting the electrodes, as illustrated in Fig. 1.

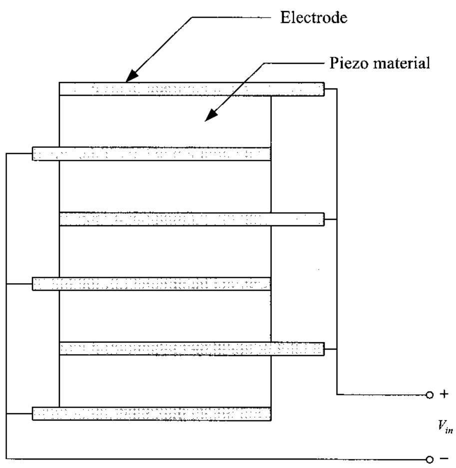

Fig. 1. Schematic of the piezoelectric actuator.

The linear constitutive relations of the piezoelectric actuator are expressed in (1) and (2), respectively [2].

$$S = s^E T + dE \tag{1}$$

$$D = dT + \varepsilon^T E \tag{2}$$

Where, S represents the mechanical strain, s^E is the elastic compliance when subjected to a constant electric field vector E, T represents the mechanical stress, D is electric displacement vector, d is piezoelectric material constant, and ε^{T} is the permittivity measured at a constant stress. All variables are tensor. These equations couple the electrical and mechanical variables, and represent the mechanical strain and electrical displacement exhibited by a piezoelectric ceramic, which are both linearly affected by the mechanical stress and electrical field to which the ceramic is subjected.

Piezoceramic is a dielectric material that suffers from hysteretic behavior inherent to nonlinear dielectric materials. This characteristic is the electrical behavior and affects both micro adjustment and control. Piezoelectric actuators have hysteresis between the input voltage and output displacement, as illustrated in Fig. 2. Hysteretic behavior is based on crystalline polarization effects and molecular friction that exists between charge and voltage as indicated in Fig. 3.

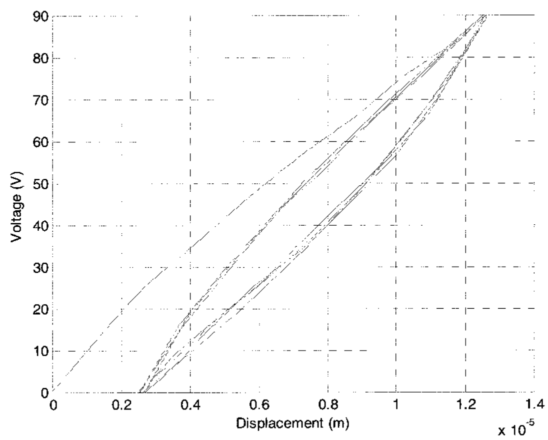

Fig. 2. Hysteretic behavior between input voltage and output displacement.

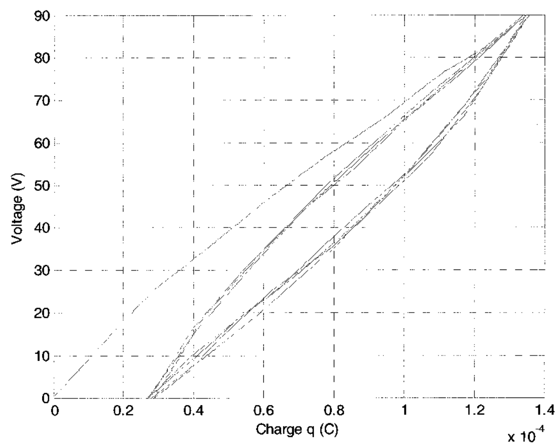

Fig. 3. Nonlinear characteristic between voltage and charge for the piezoelectric actuator.

A linear relationship between charge and displacement is exhibited as shown in Fig. 4. But, charge control is not as convenient as voltage control. Charge control is very demanding, costly, uncommon, and offers limited sensitivity compared with voltage control. So, voltage control is commercially used in the control method. Hysteretic behavior greatly influences the accuracy of the high precision system. Therefore, treating the modeling and control problem of hysteretic phenomenon is very important for the piezoelectric actuator.

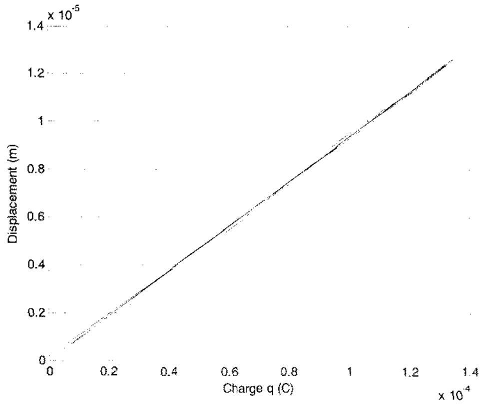

Fig. 4. A linear relationship between output displacement and input charge.

3. System Modeling

Piezoceramic is a dielectric material and it has capacitive behavior. For the purposes of controller design, the general problem of actuator behavior is a hysteretic characteristic exhibited between voltage and displacement as shown in Fig. 2. Examples of such behavior include the relationship between the stress and strain in the elastic-plastic deformation of a material, and the relationship between magnetic field strength and flux density in hard magnetic material. This hysteretic behavior is the result of energy storage and dissipation phenomenon in the ceramic.

Prior to displacement control of the actuator, we reduced the model. If the input voltage V is applied to the piezoelectric actuator, the output displacement X will be generated, and their relation can be linearized as a second-order linear dynamic system in state-space domain. The relationship between V and X is given as (3).

$$\begin{aligned} \dot{X} &= AX + BV \\ Y &= CX \end{aligned} \tag{3}$$

Where, A, B, and C are system constants. To determine the system constants, we utilized the system identification method. The aim of system identification is to construct models from data. System identification deals with the problem of building mathematical models of dynamic systems based on observed data collected from the system. This is a basic scientific methodology and since dynamic models of systems are used in almost all disciplines, system identification has a very broad application area.

Fig. 5 shows the procedure of system identification. Firstly, design experiment and collect experimental data. Secondly, choose the model structure which is a description of the system. State-space models are common representations of the dynamic model and system constants can be reconstructed from the measured input-output data. The order of the state-space model relates to the number of delayed inputs and outputs used in the corresponding linear difference equation. The state-space representation is written as (4).

$$\begin{aligned} x(t+1) &= Ax(t) + Bu(t) \\ y(t) &= Cx(t) + Du(t) \end{aligned} \tag{4}$$

Where, x(t) is the vector of state variables. The model order is the dimension of this vector. u(t) is input variables, and y(t) is output variables. A, B, C, and D are system constants, and if there is no direct influence from u(t) to y(t), D is zero. Thirdly, determine the best model in the model structure using the identification method. And finally, conduct the model validation [12].

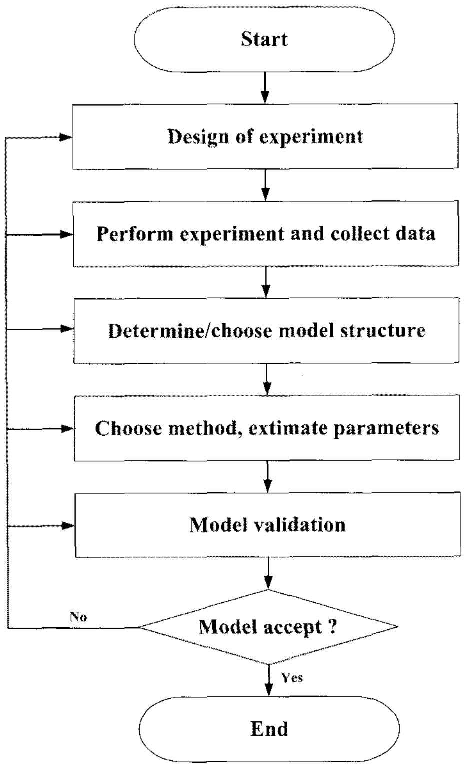

Fig. 5. The procedure of system identification

Fig. 6 shows the endpoint displacement of the piezoelectric actuator to a 90 volt 100 Hz triangle-wave voltage input. We find system constants using the system identification method from the experimental data. Matlab® Toolbox System ID was used to find the system constants [12].

Fig. 7 represents the time history of endpoint displacement X for the linear model. It is indicating that the linear model is well matched with the experimental data. Fig. 8 shows input voltage versus endpoint displacement, which describes the hysteretic behavior between the input voltage and output displacement of the piezoelectric actuator. The system constants A, B, and C that are calculated by the system identification method are summarized in Table 1.

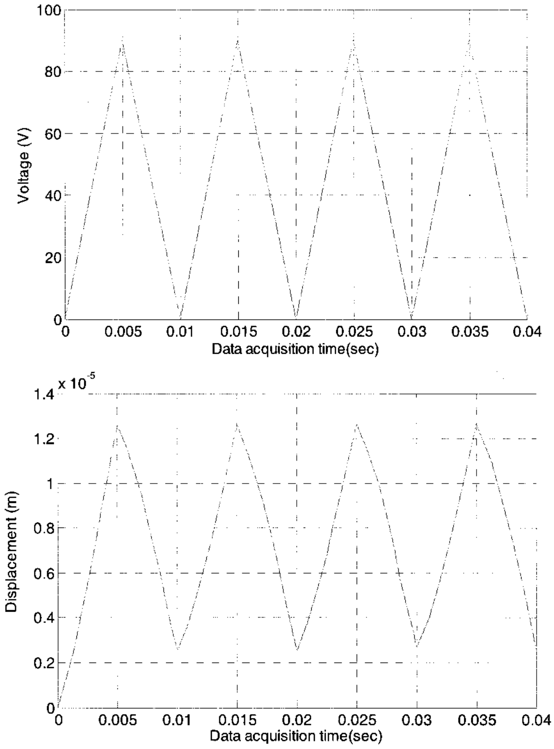

Fig. 6. Endpoint displacement of piezoelectric actuator for a 90 volt 100 Hz triangle-wave voltage input.

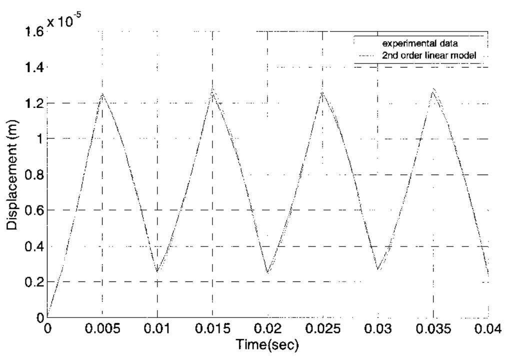

Fig. 7. Measured and linear model endpoint displacement of the piezoelectric actuator.

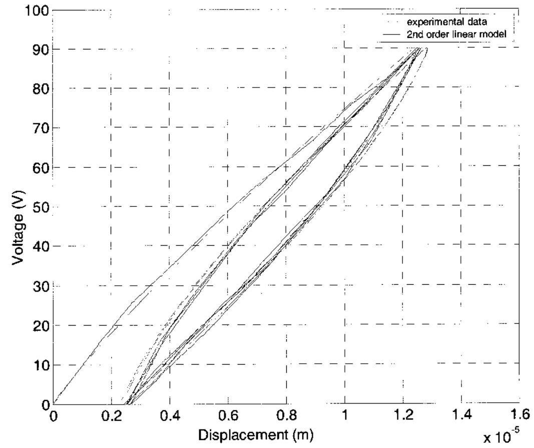

Fig. 8. Hysteresis between input voltage and output displacement for the linear model.

Table 1. System constants

A	B	C
$\begin{bmatrix} 0 & 1 \\ -1.9512\times10^{7} & -57018 \end{bmatrix}$	$\begin{bmatrix} 0.00591 \\ -333.55 \end{bmatrix}$	$\begin{bmatrix} 1 & 0 \end{bmatrix}$

4. Controller Design

For displacement control of the piezoelectric actuator, this paper utilized the classical PID controller and second-order linear piezoelectric model. Fig. 9 presents a control system block diagram used for the displacement control that includes a PID controller, voltage converter, and piezoelectric actuator model.

A PID controller improves the transient response and steady-state error between reference position X^* and output position X. The converter transduces the control command from reference displacement signal to voltage signal. The piezoelectric actuator model is a second-order linear dynamic model and output displacement X is controlled by the feed-back control system. Transfer functions for each subsystem are defined as (5) through (7), respectively.

$$\frac{X_c}{\varepsilon}(s) = K_P + \frac{K_I}{s} + K_D s \tag{5}$$

$$\frac{V_{in}}{X_c}(s) = T \tag{6}$$

$$X(s) = C(sI - A)B \tag{7}$$

Where, X_c is PID controller, and K_P, K_I, and K_D are PID controller gains. T is voltage to displacement converting ratio, and A, B, and C are system constants which are summarized in Table 1.

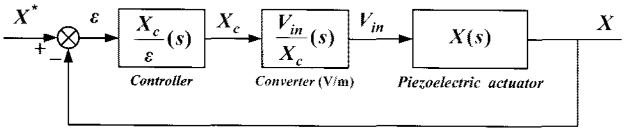

Fig. 9. Close loop displacement control system block diagram.

The range of PID controller gains, which guarantees the stability of the system, is determined by *Routh-Hurwitz criterion* using the equivalent close-loop transfer function of Fig. 9 [13]. Since the selecting range of gains is very extensive, it is difficult to find the PID controller gains that minimize the system error. In this paper, we utilized the optimization technique to determine the controller gains. Firstly, determine the range of PID gains, which guarantees the stability of the system, through the *Routh-Hurwitz criterion* using the equivalent close-loop transfer function of Fig. 9. Then, define the cost function J expressed by (7).

$$J = \int_0^{\infty} (X - X^*)^2 dt \tag{8}$$

Cost function J is composed of reference position X^* and output position X. The optimization process aims to minimize the cost function J by suitable combination of PID gains which means that the transient response and steady-state error between reference position X^* and output position X are minimized. The schematic of the optimization procedure is shown in Fig. 10.

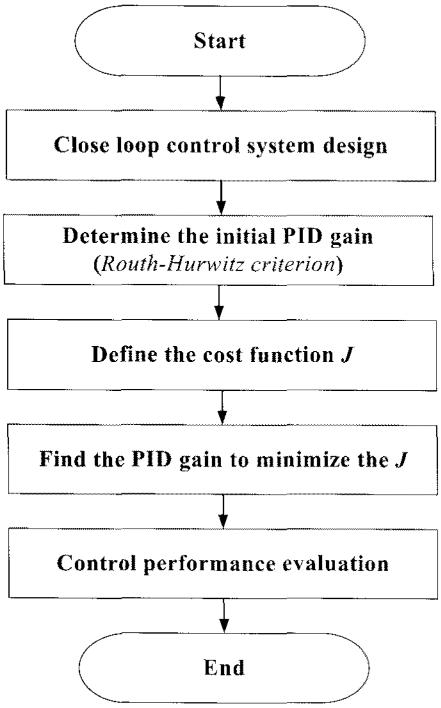

Fig. 10. The schematic of optimization procedure.

Fig. 11 provides the system response for the reference displacement. Endpoint displacement of the piezoelectric actuator is well matched with reference displacement via optimized gains rather then the nominal gains which are just satisfied with the *Routh-Hurwitz criterion*. The PID controller gains used for simulation are summarized in Table 2. Matlab® Optimization Toolbox was used to determine the PID controller gains [13].

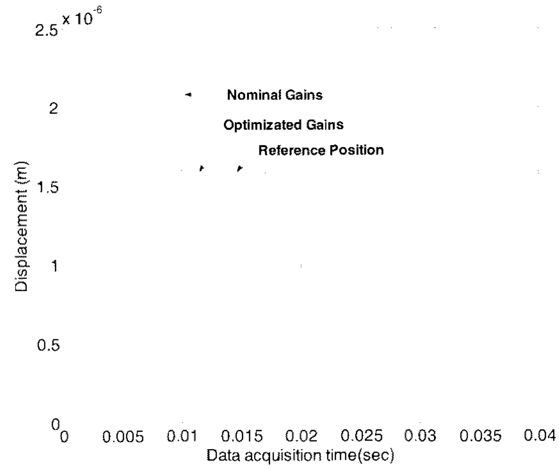

Fig. 11. System response for the nominal and optimized gains.

Table 2. PID controller gains

	K_P	K_I	K_D
Nominal gain	2	8	5
Optimized gain	10.3147	6.0×10^3	2.4951

The close loop displacement control system, which includes the modeling equation of the piezoelectric actuator and a PID controller, is applied for numerical simulation. The example problem taken in this paper is a commercially available multilayer piezoelectric actuator.

The input voltage range is 0 to 180 volts, and the corresponding output displacement range is 0 to 20 microns. The mechanical stiffness is 5×10^6 N/m^2, electrical capacitance is 1.4μf, mechanical resonant frequency is 69 KHz, overall length is 20 mm, and the Young's modulus is 4.4×10^{10} N/m^2. Voltage to displacement converting ratio T and mass m are determined by observation of experimental data. The specifications for the piezoelectric actuator are summarized in Table 3.

The validity of the designed control system is examined by simulating the close loop displacement control system. Two types of pwm signal are used as the desired position of the piezoelectric actuator to evaluate the proposed method. Fig. 12 and Fig. 13 show the time history of uncontrolled and controlled endpoint displacement, for a

Table 3. Specification for the piezoelectric actuator

Parameters	Values
Maximum displacement [μm]	17.4 ± 2.0
Rated displacement [μm]	11.6 ± 2.0
Generated force [N]	850
Resonance frequency [kHz]	69
Capacitance [μF]	1.4
Insulation resistance [MΩ]	10
Overall length [mm]	20
Mass [kg]	0.004
Voltage to displacement converting ratio [V/m]	6.173×10^6

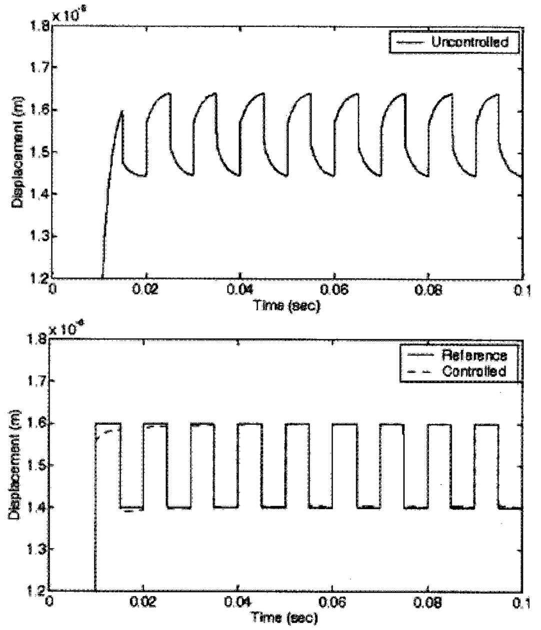

Fig. 12. Close loop response for 100 Hz, 1.5 μm pwm signal.

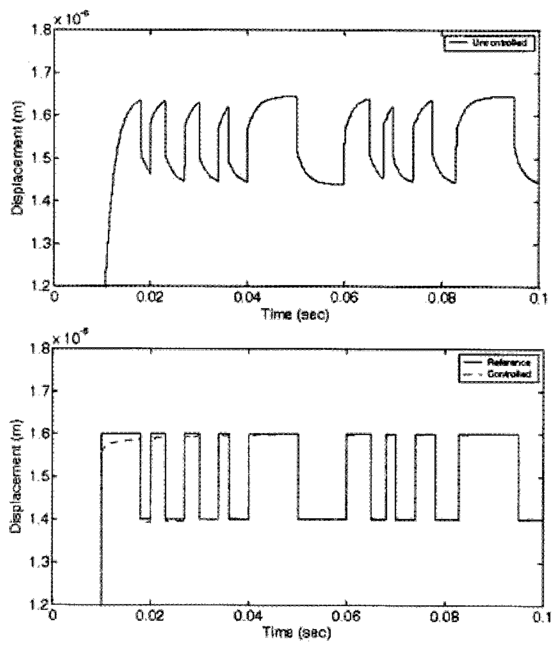

Fig. 13. Close loop response for duty cycle pwm signal.

100 Hz, 1.5 μm pwm signal and duty cycle pwm signal, respectively. The simulation result indicates that controlled endpoint displacement is well matched with the desired position. Fig. 14 provides the difference between reference and controlled endpoint displacement.

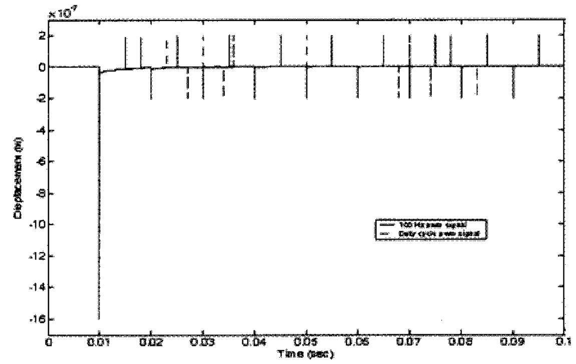

Fig. 14. Difference between reference and controlled endpoint displacement.

5. Conclusion

Dynamic modeling and a controller design method for a piezoelectric actuator were presented. In this paper, the piezoelectric actuator system is linearized as a second-order linear dynamic model, and the system identification method was used to determine the system constants. A classical PID controller was designed and used to regulate the output displacement, and the PID controller gains were selected by optimization technique. Numerical simulation results demonstrate that the relation of input voltage and output displacement is clearly modeled and furthermore that endpoint displacement can be controlled.

First, the modeling result indicates that the system identification method successfully constructs a linear model from experimental data and hysteretic behavior between input voltage and output displacement is clearly modeled by the proposed method.

Second, we determined the PID controller gains by using the optimization technique for minimizing the system error. Fig. 11 shows that the transient response and steady-state error is improved by a PID controller.

Third, the numerical simulation was executed for the close loop displacement control system. Two types of pwm signal are used as the desired position of the actuator, and Fig. 12 and Fig. 13 show that endpoint displacement was well matched with the desired position.

Fourth, the proposed method well satisfied the control performance. We will continue this study for better completion and will introduce more detailed results in the next paper including control performance validation by experimental test.

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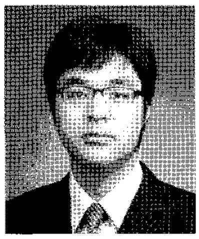
Baek-Ju Sung
He received his Ph.D. in Electrical Engineering from Chungnam National University in 2006, and his M.S and B.S. degrees in Electrical Engineering from Busan National University in 1992 and 1990, respectively. He currently works at the Korea Institute of Machinery & Materials.

Eun-Woong Lee
He received his B.S., M.S., and Ph.D. degrees in Electrical Engineering from Hanyang University, Korea, in 1971, 1974, and 1983, respectively. From Mar. 1971 to Dec. 1976, he was a faculty member of Daejon Junior Technical College. Since 1976, he has been a Professor in the Department of Electrical Engineering, and has served as Dean of the Engineering College, Chungnam National University, Daejon, Korea. From 1982 to 1983 and from 1985 to 1986, he was a Visiting Professor at the Department of Electrical Engineering, McGill University, Montreal, Canada. From 1999 to 2002, he was Vice President and Acting President of "The Korean Federation of Teachers Associations". Now he serves as President of the "Korean Institute of Electrical Engineering (KIEE)" and Vice President of both the "Korea Electric Engineers Association" and the "Korea Electric Association". His interests are electrical machines, special purpose motors, and power quality. Dr. Lee has been a Senior Member of IEEE, and is a Life Member of KIEE.



Jae-Gyu Lee
He received his B.S. and M.S. degrees in Aerospace Engineering from Chonbuk National University in 2004 and 2006, respectively. From Feb. 2006 to Apr. 2008, he worked at the Korea Institute of Machinery & Materials where he was in charge of dynamic modeling and control of mechanical systems. He currently works at the Tata Daewoo Commercial Vehicle Company Limited. His research interests are system dynamics and control, orbital mechanics, and computer-aided engineering.