An Improved Dynamic Programming Approach to Economic Power Dispatch with Generator Constraints and Transmission Losses

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Abstract – This paper presents an improved dynamic programming (IDP) approach to solve the economic power dispatch problem including transmission losses in power systems. A detailed mathematical derivation of recursive dynamic programming approach for the economic power dispatch problem with transmission losses is presented. The transmission losses are augmented with the objective function using price factor. The generalized expression for optimal scheduling of thermal generating units derived in this article can be implemented for the solution of the economic power dispatch problem of a large-scale system. Six-unit, fifteen-unit, and forty-unit sample systems with non-linear characteristics of the generator, such as ramp-rate limits and prohibited operating zones are considered to illustrate the effectiveness of the proposed method. The proposed method results have been compared with the results of genetic algorithm and particle swarm optimization methods reported in the literature. Test results show that the proposed IDP approach can obtain a higher quality solution with better performance.

Keywords: Dynamic programming, Economic power dispatch, Optimization, Prohibited operating zones, Ramp-rate constraints

1. Introduction

The main objective of the economic dispatch problem is to determine the optimal combination of power outputs for committed generating units, which minimizes the total fuel cost while satisfying load demand and operating constraints. This makes the economic power problem a large-scale non-linear constrained optimization problem. Traditional methods such as Lambda-iteration method, the base point and participation factors methods and the gradient method [1-4] are well known for the economic dispatch of generators. In these numerical methods, an essential assumption is that the whole of the generating unit operating range is available for operation. Conventional techniques offer good results but when the search space is non-linear and it has discontinuities they become very complicated with a slow convergence ratio and not always seeking the optimal solution.

In a practical system, the generating units have prohibited operating zones between their minimum and maximum generation limits and the operating range of online units are restricted by their ramp-rate limits due to physical operational limitations. Unit operation in prohibited operating zones may cause amplification of vibrations in shaft bearings, which should be avoided in practice. The prohibited operating zones of a unit divide the operating range between its minimum to maximum generation limits into several disjoint convex sub-regions. Hence, conventional methods cannot be directly applied to solve the economic dispatch problem with prohibited operating zones. Several methods have been reported for the solution of the economic power dispatch problem with prohibited operating zones. The dynamic programming approach [5, 6] is one of the most widely employed methods for the solution of the nonconvex economic power dispatch problem. Unlike the Lambda iteration approach, the dynamic programming method has no restrictions on generator cost function and performs a direct search of solution space. However, for a practical sized system, the fine step size and the large unit number often cause the 'curse of dimensionality' problem or local optimality in the dynamic programming solution process.

Lee et al. [7] decomposed the nonconvex decision space into a small number of subsets such that each of the associated dispatch problems, if feasible, is solved through the conventional Lagrangian relaxation approach. This approach requires fairly extensive computational time when a system owns more units that have prohibited operating zones. Ref. [8] defined a small advantageous set of decision spaces with respect to the system demand and then utilized the iterative method to find the feasible

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optimal solution. This method may not be applicable if the problem contains too many nonlinear constraints for large-scale nonconvex systems.

The stochastic search algorithms such as genetic algorithm (GA) [9], evolutionary programming (EP) [10], [11], simulated annealing (SA) [12], tabu search algorithm (TSA) [13], and particle swarm optimization (PSO) [14, 15], may prove to be effective in solving nonlinear ED problems without any restriction on the shape of the cost curves. Although these heuristic methods do not always guarantee discovering the globally optimal solution in finite time, they often provide a reasonable solution. Further, the stochastic searching algorithms take a longer time for convergence. Neural network [16, 17] models were applied to the economic power dispatch problem. These methods also required tremendous amounts of time for training the network.

This paper presents a novel mathematical approach based on the recursive dynamic programming approach for the solution of the economic power dispatch problem with non-linear constraints. A generalized expression derived in this paper directly expresses the optimal generations in terms of cost and loss coefficients for the given load demand, which can be easily implemented for the system consisting of a great number of generators. The transmission losses are blended with the actual fuel cost through the price factor. The proposed IDP method converged to a superior optimal solution with lesser number of iterations. Test results are provided to illustrate the performance of the proposed IDP algorithm.

2. Problem Formulation

The economic power dispatch problem with ramp-rate limits and prohibited operating zones can be formulated as [14]:

$$Minimize\ F_t = \sum_{i=1}^{n} F_i(P_i) = \sum_{i=1}^{n} a_i P_i^2 + b_i P_i + c_i \ \$/\text{h} \quad (1)$$

where i denotes index of units; F_i, Fuel cost function of unit i; a_i, b_i, and c_i are cost coefficients of generator i; n is the number of generators committed to the operating system; P_i is the power generated by the ith unit, subject to

(i) the power balance constraints:

$$\sum_{i=1}^{n} P_i = P_D + P_L \quad (2)$$

where P_D is the system load demand and P_L is the transmission loss which can be found through the use of B-matrix loss coefficients.

(ii) generating capacity constraints:

$$P_i^{\min} \le P_i \le P_i^{\max}, \quad i = 1,2,\ldots\ldots,n \quad (3)$$

where $P_i^{\min}$ and $P_i^{\max}$ are the minimum and maximum power outputs of the ith unit.

(iii) the additional constraints for units with prohibited operating zones are:

$$\begin{aligned} &P_i^{\min} \le P_i \le P_{i,1}^{l} \\ &P_{i,j-1}^{u} \le P_i \le P_{i,j}^{l}, \quad j = 2,3,\ldots\ldots,m_i \\ &P_{i,m_i}^{u} \le P_i \le P_i^{\max} \end{aligned} \quad (4)$$

where j is the number of prohibited zones of unit i. l and u denote the lower bound and upper bound of the prohibited zone of the generator.

(iv) ramp-rate constraints:

$$\max\ (P_i^{\min}, P_i^0 - DR_i) \le P_i \le \min\ (P_i^{\max}, P_i^0 + UR_i) \quad (5)$$

where P_i is the current output power, and P_i^0 is the previous output power. UR_i is the up-ramp limit of the ith generator (MW/time-period), and DR_i is the down-ramp-limit of the ith generator (MW/time-period). The transmission losses are represented by:

$$P_L = \sum_{i=1}^{n}\sum_{j=1}^{n} P_i B_{ij} P_j + \sum_{i=1}^{n} B_{0i} P_i + B_{00} \quad (6)$$

The total transmission loss of the system is given by:

$$P_L = P_{L1} + P_{L2} + \ldots\ldots\ldots + P_{Ln} \quad (7)$$

where

$$P_{Li} = P_i B_{i1} P_1 + P_i B_{i2} P_2 + \ldots\ldots\ldots + P_i B_{in} P_n + B_{0i} P_i + (B_{00}/n), \quad \text{for } i = 1,2,\ldots\ldots,n$$

The modified loss formula for the first generator can be written as:

$$\begin{aligned} P_{L1} = {} & P_1 B_{11} P_1 + P_1^2 B_{12}(P_2/P_1) + P_1^2 B_{13}(P_3/P_1) + \ldots\ldots\ldots \\ & + P_1^2 B_{1n}(P_n/P_1) + B_{01} P_1 + (B_{00}/n) \end{aligned}$$

$$= P_1^2 B_{11}[1+(B_{12}/B_{11})(P_2/P_1)+(B_{13}/B_{11})(P_3/P_1)+\dots\dots\dots$$
$$+(B_{1n}/B_{11})(P_n/P_1)]+B_{01}P_1+(B_{00}/n)$$
$$= P_1^2 B_{11}[\phi_{11}+\phi_{12}+\phi_{13}+\dots\dots\dots+\phi_{1n}]+B_{01}P_1+(B_{00}/n)$$
$$= d_1P_1^2+e_1P_1+f_1 \tag{8}$$

where

$$d_1 = B_{11}[\phi_{11}+\phi_{12}+\phi_{13}+\dots\dots\dots+\phi_{1n}])$$
$$e_1 = B_{01}$$
$$f_1 = (B_{00}/n)$$

In general, the modified loss coefficients of the ith generator can be expressed as:

$$d_i = B_{ii}\left(1+\sum_{\substack{j=1\\j\neq i}}^{n}(B_{ij}/B_{ii})(P_j/P_i)\right) \tag{9}$$

$$e_i = B_{0i} \tag{10}$$

$$f_i = B_{00}/n \tag{11}$$

The cost of transmission losses in between plants is accounted with the actual fuel cost by using a price factor g. The price factor g of each generator is the ratio between the fuel costs at its maximum power output to its maximum power output:

$$g_i = \frac{(a_iP_{i\max}^2+b_iP_{i\max}+c_i)}{P_{i\max}} \tag{12}$$

The modified form of the cost equation of the n-generator system is given by:

$$F_t = \sum_{i=1}^{n} a_iP_i^2+b_iP_i+c_i+g_i(d_iP_i^2+e_iP_i+f_i) \text{ \$/h} \tag{13}$$

The analytical nature of the above problem formulation leads to the high possibility of an accurate solution for the economic power dispatch problem including transmission losses.

3. An Improved Dynamic Programming Approach for Economic Power Dispatch

The economic dispatch problem can be solved through a conventional dynamic programming approach. Conventional dynamic programming has the following disadvantage; the computational requirements of the conventional dynamic programming based method depend on the size of the discrete capacity used. With a capacity step of one MW, which is the usual accuracy required in the economic dispatch calculation, the number of states at each stage is quite large for even a small system. Increase in the number of states at each stage is the curse of dimensionality in the literature of dynamic programming. In contrast to this, a generalized dynamic programming based recursive expression derived in this article directly expresses the optimal generation of each unit for the specified load demand by using fuel and loss coefficients. The proposed IDP approach simplifies the task of evaluating the optimal generation schedule of thermal generating units in the economic dispatch problem of a power system.

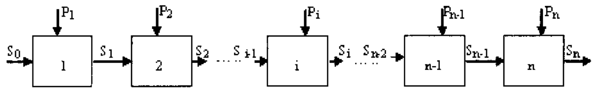

Fig. 1. Multistage decision problem

The dynamic programming technique represents a multistage decision problem as a sequence of single-stage decision problems. As applied to dynamic programming, a multistage decision process is one in which a number of single-stage processes are connected in series so that the output of one stage is the input of the succeeding stage. Thus, an n variable problem is represented as a sequence of n single-variable problems that are solved successively. The computational advantage is that instead of n-variable optimization, single variable problem optimization is required. A serial multistage decision process applied economic dispatch of thermal units is represented schematically as shown in Fig. 1.

In the proposed approach, the objective of a multistage decision process is to find the optimal generation of committed generating units P_1, P_2, ... , P_n so as to minimize the objective function F_t. In a serial multistage decision process, the individual stages are connected head to tail with no recycling. For the ith stage, the input state vector is denoted by S_{i-1} and the output state vector is denoted as S_i. The output from stage i-1 is equal to the input to stage i. Therefore the output of each stage is given by:

$S_1 = P_1 = S_2 - P_2$, $S_2 = P_1 + P_2 = S_3 - P_3, \cdots,$
$S_n = P_1+P_2+P_3 \dots P_n$

The output of the first stage in a multistage decision problem is given by:

$$S_1=P_1=S_2-P_2 \tag{14}$$

Therefore,

$$f_1'(S_1) = \min(a_1P_1^2 + b_1P_1 + c_1 + g_1(d_1P_1^2 + e_1P_1 + f_1))$$
$$f_1'(S_1) = \min((a_1 + g_1d_1)P_1^2 + (b_1 + g_1e_1)P_1 + c_1 + g_1f_1) \quad (15)$$

For cost minimization, using dynamic programming, a recursive expression is presented as:

$$f_2'(S_2) = \min((a_2 + g_2d_2)P_2^2 + (b_2 + g_2e_2)P_2 + (c_2 + g_2f_2) + f_1'(S_1))$$
$$f_2'(S_2) = \min((a_2 + g_2d_2)P_2^2 + (b_2 + g_2e_2)P_2 + (c_2 + g_2f_2) + (a_1 + g_1d_1)(S_2 - P_2)^2 + (b_1 + g_1e_1)(S_2 - P_2) + c_1 + g_1f_1)) \quad (16)$$

The optimum generation of a second generator is obtained by differentiating Equation (16) with respect to P2 and equating to zero:

$$2(a_2 + g_2d_2)P_2 + (b_2 + g_2e_2) + 2(a_1 + g_1d_1)P_2 - 2(a_1 + g_1d_1)S_2 - (b_1 + g_1e_1) = 0$$

The expression for optimal generation of a second generator is given by:

$$P_2 = \frac{2(a_1 + g_1d_1)S_2 + (b_1 + g_1e_1) - (b_2 + g_2e_2)}{2(a_1 + g_1d_1) + 2(a_2 + g_2d_2)} \quad (17)$$

From Equation (14):

$$P_1 = S_2 - P_2 \quad (18)$$

Substituting Equation (17) in Equation (18) gives the expression for P_1:

$$P_1 = \frac{2(a_2 + g_2d_2)S_2 - (b_1 + g_1e_1) + (b_2 + g_2e_2)}{2(a_1 + g_1d_1) + 2(a_2 + g_2d_2)} \quad (19)$$

Substituting Equations (17) & (19) in Equation (16) and then simplifying gives:

$$f_2'(S_2) = \min S_2^2\left(\frac{1}{\frac{1}{a_1 + g_1d_1} + \frac{1}{a_2 + g_2d_2}}\right) + S_2\left(\frac{\frac{b_1 + g_1e_1}{a_1 + g_1d_1} + \frac{b_2 + g_2e_2}{a_2 + g_2d_2}}{\frac{1}{a_1 + g_1d_1} + \frac{1}{a_2 + g_2d_2}}\right) + \frac{K^2(a_1 + g_1d_1 + a_2 + g_2d_2)}{(2(a_1 + g_1d_1) + 2(a_2 + g_2d_2))^2} + \frac{K((b_2 + g_2e_2) - (b_1 + g_1e_1))}{2(a_1 + g_1d_1) + 2(a_2 + g_2d_2)} + c_1 + g_1f_1 + c_2 + g_2f_2 \quad (20)$$

where $K = b_1 - b_2$. Substituting $S_2 = S_3 - P_3$ in Equation (20):

$$f_2'(S_2) = \min (S_3 - P_3)^2\left(\frac{1}{\frac{1}{a_1 + g_1d_1} + \frac{1}{a_2 + g_2d_2}}\right) + (S_3 - P_3)^* \left(\frac{\frac{b_1 + g_1e_1}{a_1 + g_1d_1} + \frac{b_2 + g_2e_2}{a_2 + g_2d_2}}{\frac{1}{a_1 + g_1d_1} + \frac{1}{a_2 + g_2d_2}}\right) + \frac{K^2(a_1 + g_1d_1 + a_2 + g_2d_2)}{(2(a_1 + g_1d_1) + 2(a_2 + g_2d_2))^2} + \frac{K((b_2 + g_2e_2) - (b_1 + g_1e_1))}{2(a_1 + g_1d_1) + 2(a_2 + g_2d_2)} + c_1 + g_1f_1 + c_2 + g_2f_2 \quad (21)$$

The objective $f_3'(S_3)$ is given by:

$$f_3'(S_3) = \min((a_3 + g_3d_3)P_3^2 + (b_3 + g_3e_3)P_3 + c_3 + g_3f_3 + f_2'(S_2)) \quad (22)$$

Differentiating Equation (22) with respect to P_3 and equating to zero gives:

$$P_3 = \frac{2\alpha_2S_3 + \beta_2 - (b_3 + g_3e_3)}{2\alpha_2 + 2(a_3 + g_3d_3)} \quad (23)$$

where

$$\alpha_2 = \frac{1}{\frac{1}{(a_1 + g_1d_1)} + \frac{1}{(a_2 + g_2d_2)}} \quad \text{and} \quad \beta_2 = \left(\frac{b_1 + g_1e_1}{a_1 + g_1d_1} + \frac{b_2 + g_2e_2}{a_2 + g_2d_2}\right)\alpha_2$$

The general expression for the optimal generation of the ith generator for a given load demand can be expressed as:

$$P_i = \frac{2\alpha_{i-1}S_i + \beta_{i-1} - (b_i + g_ie_i)}{2\alpha_{i-1} + 2(a_i + g_id_i)} \quad for \;\; i = 2,3,\ldots\ldots n \quad (24)$$

where

$$\alpha_{i-1} = \frac{1}{\sum_{k=1}^{i-1}\frac{1}{(a_k + g_kd_k)}} \quad \text{and} \quad \beta_{i-1} = \left(\sum_{k=1}^{i-1}\frac{b_k + g_ke_k}{a_k + g_kd_k}\right)\alpha_{i-1}$$

In the backward recursion development, the optimum generation scheduling for the given load demand is obtained by proceeding from the last stage of the multistage decision process where the output of S_n=P_D+P_L is specified. The optimal generation of each unit for the given power demand can be effortlessly computed using Equation (24).

The overall procedure of the proposed methodology can be summarized in an algorithmic form as

(i) Read the system data.

(ii) Initialize the recursive process for a given demand neglecting the transmission losses. Calculate the optimum generation of each generating unit for the given load demand using the generalized recursive equation:

$$P_i = \frac{2\alpha_{i-1}S_i + \beta_{i-1} - b_i}{2\alpha_{i-1} + 2a_i} \quad for \;\; i = 2,3,\ldots\ldots n \quad (25)$$

where $\alpha_{i-1} = \frac{1}{\sum_{k=1}^{i-1}\frac{1}{a_k}}$ and $\beta_{i-1} = \left(\sum_{k=1}^{i-1}\frac{b_k}{a_k}\right)\alpha_{i-1}$

The loss coefficients d_i and e_i in Equation (24) should be neglected while calculating the optimum generation schedule of committed units neglecting transmission losses.

(iii) Using the optimal generation schedule, calculate the total transmission loss of the system.

(iv) Obtain the modified form of self-coefficients through Equation (8).

(v) Find the modified cost equation for each unit using price factor and modified loss coefficients for inclusion of the transmission loss.

(vi) Calculate the total generation required using relation:

$$\sum_{i=1}^{n} P_i = (Demand + Losses) = P_{Dnew} \qquad (26)$$

(vii) Using generalized recursive Equation (24), calculate the generation of each unit for new demand P_{Dnew}.

(viii) Determine the total transmission loss of the system using the new generation schedule.

(ix) Check $\sum_{i=1}^{n} P_i - (Demand + Losses) \le \varepsilon \qquad (27)$

If this equation is satisfied stop the above procedure, otherwise proceed to Step (iv).

4. Numerical Simulation Results and Discussion

The proposed dynamic programming approach has been tested on three sample test systems consisting of six, fifteen, and forty generating units including ramp-rate limits and prohibited operating zones. The simulations were carried out on a Pentium III Processor. The software was developed using the MATLAB 6.5. The data employed for the six and fifteen-unit systems can be found from [14, 15]. During normal operation of the system, the loss coefficients B with the 100-MVA base is taken from [14, 15] and B loss coefficients matrix for the sample test systems are given in Appendix-1.

4.1 Six-Unit Test System

A six-thermal unit sample system with ramp-rate limits and prohibited zones of generating units is used to demonstrate the performance of the proposed method. The fuel cost function of each unit is a quadratic function of the generator real power output. The cost coefficients, generation limits, ramp-rate constraints and prohibited zones of the generating units are given in Tables 1 and 2. The load demand of the system is 1263 MW. The operating regions of the units having prohibited operating zones are separated into isolated sub-regions. The operating regions of the unit having prohibited operating zones, after incorporating ramp-rate constraints, of the six-unit sample system are given by

For unit-1 (320-350), (380-500)
For unit-2 (80-90), (110-140), (160-200)
For unit-3 (100-150), (170-210), (240-265)
For unit-4 (60-80), (90-110), (120-150)
For unit-5 (110-140), (150-200)
For unit-6 (50-75), (85-100), (105-120)

Table 1. Generating unit capacity and coefficients for 6-unit system

Unit	P_i^{min}	P_i^{max}	a_i($/MW^2)	b_i($/MW)	c_i ($)
1	100	500	0.0070	7.0	240
2	50	200	0.0095	10.0	200
3	80	300	0.0090	8.5	220
4	50	150	0.0090	11.0	200
5	50	200	0.0080	10.5	220
6	50	120	0.0075	12.0	190

Table 2. Ramp-rate limits and prohibited operating zones of generating units for 6-unit system

Unit	P_i^0	UR_i (MW/h)	DR_i (MW/h)	Prohibited zones (MW)
1	440	80	120	[210-240][350 - 380]
2	170	50	90	[90 - 110][140 - 160]
3	200	65	100	[150-170][210 - 240]
4	150	50	90	[80 - 90][110 - 120]
5	190	50	90	[90 - 110][140 - 150]
6	110	50	90	[75 - 85][100 - 105]

The unit can only be dispatched to operate in one of the operating zones, resulting in multiple decision spaces for the economic dispatch problem. A decision space may be feasible or infeasible with respect to the system demand. The feasible optimal dispatch will reside in one of the feasible spaces. The feasible region of the thermal unit, which does not have prohibited operating zones, will be its full effective operating range. In the proposed approach, the optimal dispatch of generating units, neglecting prohibited operating zones, is obtained for the given load demand. If the desired optimal generation level of each unit is not located in a prohibited zone, the optimal feasible solution for the economic dispatch problem is obtained and no further action is needed.

If an optimal generation of a unit falls in a prohibited operating zone, the feasible optimal level would most likely be located in one of the adjacent feasible operating regions, that is, the operating region above or below the prohibited operating zone. For a unit whose desired generation level is located in a feasible operating region, its feasible optimal level may be re-dispatched to any of the two neighboring operating zones or may remain in the same operating region. The feasible optimal generation schedules for all these possible combinations of decision spaces for the specified load demand have been obtained through the proposed algorithm. The global optimal solution will be the least operating cost among the solution associated with all decision spaces.

To demonstrate the superiority of the proposed improved dynamic programming approach, results are compared with various techniques reported in the literature [14]. The optimal solution obtained through the proposed method has been compared with the results obtained through evolutionary and behavioral random search algorithms such as genetic algorithm and particle swarm optimization. The generation schedule of committed thermal units is summarized in Table 3. For the six-unit system, the number of iterations taken by the proposed analytical method with tolerance of $\varepsilon = 0.00001$ is five. The solutions using IDP satisfies the system constraints, such as ramp-rate limits and prohibited zones of thermal generating units.

The IDP algorithm provides a minimum generation cost of 15450 \$/h. It is observed from Table 3 that the optimal cost of generation obtained using IDP for a six-unit sample system is less than the GA method but equal to the PSO method, although the computation time is negligible for the proposed mathematical approach. From the comparison, it is clear that the proposed method is able to find the optimum solution for the economic power dispatch problem including prohibited operating zones and ramp-rate constraints.

An overview of particle swarm optimization and genetic algorithm is given in Appendix-2.

Table 3. Economic dispatch results for 6-unit system

Unit power output (MW)	Proposed IDP method	PSO method [14]	GA method [14]
P_1	450.9555	447.4970	474.8066
P_2	173.0184	173.3221	178.6363
P_3	263.6370	263.4745	262.2089
P_4	138.0655	139.0594	134.2826
P_5	164.9937	165.4761	151.9039
P_6	85.3094	87.1280	74.1812
Total output	1275.98	1276.01	1276.03
Loss (MW)	12.9794	12.9584	13.0217
Total generation cost (\$/h)	15450	15450	15459

4.2 Fifteen-Unit Test System

The sample system has 15 thermal units and the characteristics of thermal units are given in Tables 4 and 5. The ramp-rate limits restrict the operating range of the committed units for adjusting their output power between two operating periods. The generation may increase or decrease in accordance with ramp-rate limits. Thus, only the prohibited operating zones within the effective generation limits need be considered. In this sample system, four units have prohibited zones and the remaining nine units can operate within their effective operating limits.

Table 4. Generating unit data for 15-unit system

Unit	P_i^{min}	P_i^{max}	a_i	b_i	c_i	UR_i	DR_i	P_i^0
1	150	455	0.000299	10.1	671	80	120	400
2	150	455	0.000183	10.2	574	80	120	360
3	20	130	0.001126	8.8	374	130	130	105
4	20	130	0.001126	8.8	374	130	130	100
5	150	470	0.000205	10.4	461	80	120	190
6	135	460	0.000301	10.1	630	80	120	400
7	135	465	0.000364	9.8	548	80	120	350
8	60	300	0.000338	11.2	227	65	100	95
9	25	162	0.000807	11.2	173	60	100	105
10	25	160	0.001203	10.7	175	60	100	110
11	20	80	0.003586	10.2	186	80	80	60
12	20	80	0.005513	9.9	230	80	80	40
13	25	85	0.000371	13.1	225	80	80	30
14	15	55	0.001929	12.1	309	55	55	20
15	15	55	0.004447	12.4	323	55	55	20

Table 5. Prohibited zones of generating units for 15-unit system

Unit	Prohibited zones (MW)
2	[185 - 225][305 - 335][420 - 450]
5	[180 - 200][305 - 335][390 - 420]
6	[230 - 255][365 - 395][430 - 455]
12	[30 - 40][55 - 65]

The load demand of the system is 2630MW. The units 2, 5, 6 and 12 have prohibited operating zones. The operating regions of these units after including ramp-rate constraints are

For unit-2 (240-305), (335-420)
For unit-5 (150-180), (200-270)
For unit-6 (280-365), (395-430), (455-460)
For unit-12 (20-30), (40-55), (65-80)

Table 6 summarizes the results of economic power dispatch of the fifteen-unit sample system through the proposed method in comparison with the results of PSO and GA methods reported in [14]. The optimal combination of power generations obtained through the proposed approach minimizes the total generation cost and satisfies the system constraints such as power balance constraint, ramp-rate limits, and prohibited zones.

Table 6. Economic dispatch results for 15-unit system

Unit power output (MW)	Proposed IDP method	PSO method [14]	GA method [14]
P_1	455.0000	439.1162	415.3108
P_2	420.0000	407.9727	359.7206
P_3	130.0000	119.6324	104.4250
P_4	130.0000	129.9925	74.9853
P_5	270.0000	151.0681	380.2844
P_6	460.0000	459.9978	426.7902
P_7	430.0000	425.5601	341.3164
P_8	60.0000	98.5699	124.7867
P_9	25.0000	113.4936	133.1445
P_{10}	63.0411	101.1142	89.2567
P_{11}	80.0000	33.9116	60.0572
P_{12}	80.0000	79.9583	49.9998
P_{13}	25.0000	25.0042	38.7713
P_{14}	15.0000	41.4140	41.9425
P_{15}	15.0000	35.6140	22.6445
Total output	2658.04	2662.4	2668.4
Loss (MW)	27.9777	32.4306	38.2782
Total generation cost ($/h)	32590	32858	33113

The total fuel generation cost of the IDP method is 32590 $/h and that of PSO and GA [14] are 32858 $/h and 33113 $/h, respectively. The number iterations required for the convergence for the fifteen-unit system with a demand of 2630 MW is four. It is evident from the test results that the proposed mathematical approach is capable of obtaining high quality solutions for the economic power problem including generator constraints and transmission losses.

The computation time of the proposed IDP method is compared with PSO and GA methods to validate the computation efficiency. The comparison for 6-unit and 15-unit sample systems is listed in Table 7. The computation time taken by the IDP algorithm for six and fifteen unit sample systems is 0.01 and 0.02 seconds, respectively. From the comparison, it is clear that the CPU time required by the proposed analytical method to arrive at the final solution is less compared to GA and PSO methods.

The drawback of stochastic search algorithms such as PSO and GA are the lack of guarantee of convergence in finite time and the large number of problem specific parameters that is required. Further PSO and GA methods will typically identify a different solution each time they are applied. This is due to the probabilistic nature of the algorithms. Because of the analytical nature of the proposed method, it finds a better solution and the same result is found every time when applied to economic dispatch problems.

Table 7. Comparison of computation efficiency of proposed IDP method with PSO and GA methods

Example	Method	CPU time (sec.)	CPU time per iteration (sec.)
6-unit	IDP	0.05	0.01
	PSO	14.89	0.07
	GA	42.07	0.21
15-unit	IDP	0.09	0.02
	PSO	26.70	0.13
	GA	49.30	0.25

4.3 Forty-Unit Test System

This test system comprises 40 generating units. The feasibility of the proposed IDP approach for the large-scale economic dispatch problem is illustrated on a forty thermal units system considering ramp-rate limits and prohibited operating zones. The system data is taken from [18]. The required power demand to be met by all forty units is 7000 MW. The proposed IDP approach is applied to determine the optimal generation schedule of committed units that minimizes the total generation cost. The optimal power output of a 40-unit system for the load demand (7000MW) is given in Table 8. The solution satisfies all the system constraints and the total generation cost calculated from the optimal generation schedule is 100767.6861 $/h. The computational time taken by the analytical approach is 0.143 seconds. From the simulation results, it is clear that the developed IDP approach obtains an optimal solution for large-scale economic dispatch problems with lesser computational time.

An improved dynamic programming approach for economic power dispatch discussed in this paper has the following salient features:

1. The simplified generalized expression derived in this paper directly gives optimal generation scheduling of thermal units for the given load demand without need of iterative steps (neglecting transmission losses). Hence, the computation of the total generation cost becomes an easier task.

Table 8. Economic dispatch results for 40-unit system

Unit	Generation (MW)	Unit	Generation (MW)
P_1	40.5439	P_{21}	456.6654
P_2	60.0000	P_{22}	460.0000
P_3	140.4525	P_{23}	460.0000
P_4	24.0000	P_{24}	460.0000
P_5	26.0000	P_{25}	460.0000
P_6	115.0000	P_{26}	460.0000
P_7	110.0000	P_{27}	460.0000
P_8	217.0000	P_{28}	10.0000
P_9	265.0000	P_{29}	10.0000
P_{10}	130.0000	P_{30}	10.0000
P_{11}	205.0000	P_{31}	20.0000
P_{12}	205.0000	P_{32}	20.0000
P_{13}	125.0000	P_{33}	20.0000
P_{14}	132.0895	P_{34}	20.0000
P_{15}	125.0000	P_{35}	18.0000
P_{16}	125.0000	P_{36}	18.0000
P_{17}	125.0000	P_{37}	20.0000
P_{18}	456.6654	P_{38}	25.0000
P_{19}	458.9178	P_{39}	25.0000
P_{20}	456.6654	P_{40}	25.0000

2. The proposed approach needs less number of iterations for convergence after incorporating transmission losses in the economic power dispatch problem.
3. The proposed IDP approach can be implemented for large-scale systems.
4. The proposed approach gives the optimal solution with less computational effort.

5. Conclusion

A novel mathematical approach based on recursive dynamic programming to the economic power dispatch problem with the generator constraints and transmission losses has been presented. The superiority of the proposed IDP method has been demonstrated with three economic power dispatch problems including ramp-rate limits, prohibited operating zones, and transmission losses. The simulation results show that the proposed method can offer a more cost effective production cost with negligible computational time than those obtained from PSO and GA methods. The advantage of the IDP method is its ability in finding high quality solutions reliably with fast convergence characteristics. It will give the same optimal solution for different trials and it can be easily implemented for the system consisting of a greater number of generating units.

Appendix-1

B Loss coefficients matrix for six-unit system

$$B_{ij} = \begin{pmatrix} 0.0017 & 0.0012 & 0.0007 & -0.0001 & -0.0005 & -0.0002 \\ 0.0012 & 0.0014 & 0.0009 & 0.0001 & -0.0006 & -0.0001 \\ 0.0007 & 0.0009 & 0.0031 & 0.0000 & -0.0010 & -0.0006 \\ -0.0001 & 0.0001 & 0.0000 & 0.0024 & -0.0006 & -0.0008 \\ -0.0005 & -0.0006 & -0.0010 & -0.0006 & 0.0129 & -0.0002 \\ -0.0002 & -0.0001 & -0.0006 & -0.0008 & -0.0002 & 0.0150 \end{pmatrix}$$

$B_{0i} = 1.0e-03*[-0.3908\ -0.1297\ 0.7047\ 0.0591\ 0.2161\ -0.6635]$

$B_{00} = 0.0056$

B Loss coefficients matrix for fifteen-unit system

$$B_{ij} = \begin{pmatrix} 0.0014 & 0.0012 & 0.0007 & -0.0001 & -0.0003 & -0.0001 & -0.0001 & -0.0001 & -0.0003 & -0.0005 & -0.0003 & -0.0002 & 0.0004 & 0.0003 & -0.0001 \\ 0.0012 & 0.0015 & 0.0013 & 0.0000 & -0.0005 & -0.0002 & 0.0000 & 0.0001 & -0.0002 & -0.0004 & -0.0004 & -0.0000 & 0.0004 & 0.0010 & -0.0002 \\ 0.0007 & 0.0013 & 0.0076 & -0.0001 & -0.0013 & -0.0009 & -0.0001 & 0.0000 & -0.0008 & -0.0012 & -0.0017 & -0.0000 & -0.0026 & 0.0111 & -0.0028 \\ -0.0001 & 0.0000 & -0.0001 & 0.0034 & -0.0007 & -0.0004 & 0.0011 & 0.0050 & 0.0029 & 0.0032 & -0.0011 & -0.0000 & 0.0001 & 0.0001 & -0.0026 \\ -0.0003 & -0.0005 & -0.0013 & -0.0007 & 0.0090 & 0.0014 & -0.0003 & -0.0012 & -0.0010 & -0.0013 & 0.0007 & -0.0002 & -0.0002 & -0.0024 & -0.0003 \\ -0.0001 & -0.0002 & -0.0009 & -0.0004 & 0.0014 & 0.0016 & -0.0000 & -0.0006 & -0.0005 & -0.0008 & 0.0011 & -0.0001 & -0.0002 & -0.0017 & 0.0003 \\ -0.0001 & 0.0000 & -0.0001 & 0.0011 & -0.0003 & -0.0000 & 0.0015 & 0.0017 & 0.0015 & 0.0009 & -0.0005 & 0.0007 & -0.0000 & -0.0002 & -0.0008 \\ -0.0001 & 0.0001 & 0.0000 & 0.0050 & -0.0012 & -0.0006 & 0.0017 & 0.0168 & 0.0082 & 0.0079 & -0.0023 & -0.0036 & 0.0001 & 0.0005 & -0.0078 \\ -0.0003 & -0.0002 & -0.0008 & 0.0029 & -0.0010 & -0.0005 & 0.0015 & 0.0082 & 0.0129 & 0.0116 & -0.0021 & -0.0025 & 0.0007 & -0.0012 & -0.0072 \\ -0.0005 & -0.0004 & -0.0012 & 0.0032 & -0.0013 & -0.0008 & 0.0009 & 0.0079 & 0.0116 & 0.0200 & -0.0027 & -0.0034 & 0.0009 & -0.0011 & -0.0088 \\ -0.0003 & -0.0004 & -0.0017 & -0.0011 & 0.0007 & 0.0011 & -0.0005 & -0.0023 & -0.0021 & -0.0027 & 0.0140 & 0.0001 & 0.0004 & -0.0038 & 0.0168 \\ -0.0002 & -0.0000 & -0.0000 & -0.0000 & -0.0002 & -0.0001 & 0.0007 & -0.0036 & -0.0025 & -0.0034 & 0.0001 & 0.0054 & -0.0001 & -0.0004 & 0.0028 \\ 0.0004 & 0.0004 & -0.0026 & 0.0001 & -0.0002 & -0.0002 & -0.0000 & 0.0001 & 0.0007 & 0.0009 & 0.0004 & -0.0001 & 0.0103 & -0.0101 & 0.0028 \\ 0.0003 & 0.0010 & 0.0111 & 0.0001 & -0.0024 & -0.0017 & -0.0002 & 0.0005 & -0.0012 & -0.0011 & -0.0038 & -0.0004 & -0.0101 & 0.0578 & -0.0094 \\ -0.0001 & -0.0002 & -0.0028 & -0.0026 & -0.0003 & 0.0003 & -0.0008 & -0.0078 & -0.0072 & -0.0088 & 0.0168 & 0.0028 & 0.0028 & -0.0094 & 0.1283 \end{pmatrix}$$

$B_{0i} = (-0.0001\ -0.0002\ 0.0028\ -0.0001\ 0.0001\ -0.0003\ -0.0002\ -0.0002\ 0.0006\ 0.0039\ -0.0017\ -0.000\ -0.0032\ 0.0067\ -0.0064)$

$B_{00} = 0.0055$

Appendix-2

Particle Swarm Optimization

Kennedy and Eberhart introduced the PSO method [19], motivated by social behavior of organisms such as fish schooling and bird flocking. PSO, an optimization tool, provides a population-based search procedure in which individuals called particles change their position with time. In a PSO system, particles fly around in a multidimensional search space. During flight, each particle adjusts its position according to its own experience, and the experience of neighboring particles, making use of the best position encountered by itself and its neighbors. The swarm direction of a particle is defined by the set of particles neighboring the particle and its history experience.

Let x and v denote a particle's coordinates (position) and its corresponding flight speed (velocity) in a search space, respectively. Therefore, the ith particle is represented as $x_i = (x_{i1}, x_{i2}, \ldots, x_{id})$ in the d-dimensional space. The best previous position of the ith particle is recorded and represented by the $pbest_i = (pbest_{i1}, pbest_{i2}, \ldots, pbest_{id})$. The index of the best particle among all the particles in the group is represented by the $gbest_d$.

The rate of the velocity for particle i is represented as $v_i = (v_{i1}, v_{i2}, \ldots, v_{id})$. The modified velocity and position of each particle can be calculated using the current velocity and the distance from $pbest_{id}$ to $gbest_d$ as given in the following relations:

$$\begin{aligned} v_{id}^{(k+1)} &= w * v_{id}^{(k)} + c_1 * rand_1 * (pbest_{id} - x_{id}^{(k)}) \\ &\quad + c_2 * rand_2 * (gbest_d - x_{id}^{(k)}), \end{aligned} \tag{28}$$

$$\begin{aligned} x_{id}^{(k+1)} &= x_{id}^{(k)} + v_{id}^{(k+1)}, \\ & i = 1, 2, \ldots, n; \quad d = 1, 2, \ldots, m; \end{aligned} \tag{29}$$

where

- n number of particles in a group;
- m number of members in a particle;
- k pointer of iterations;
- w inertia weighting factor;
- c_1, c_2 acceleration constants;
- $rand_1$, $rand_2$ random numbers between 0 and 1;
- $v_i^{(k)}$ velocity of individual i at iteration k $V_d^{\min} \le v_{id}^k \le V_d^{\max}$;
- $x_i^{(k)}$ current position of particle i at iteration k.

In the above procedure, the parameter V^{max} determined the resolution, or fitness, with regions to be searched between the present position and the target position. If V^{max} is too high, particles might fly past good solutions. If V^{max} is too small, particles may not explore sufficiently beyond local solutions. The constants c_1 and c_2 represent the weighting of the stochastic acceleration terms that pull each particle toward the *pbest* and *gbest* positions. The acceleration constants c_1 and c_2 were often set to be 2.0 according to past experiences. In general, the inertia weight

w is set according to the following equation:

$$w = w_{\max} - \frac{w_{\max} - w_{\min}}{iter_{\max}} \times iter \tag{30}$$

where $iter_{max}$ is the maximum number of iterations and *iter* is the current number of iterations.

The above iterative process on the population will continue until there is no appreciable improvement in the fitness value or predefined maximum number of iterations reached.

Genetic Algorithm

A genetic algorithm is a robust search and optimization algorithm developed by John Holland. GA mimics the evolutionary principles and chromosomal processing in natural genetics to seek solutions from a vast search [20], [21]. A GA is an iterative procedure, and consists of a constant size population of individuals. Each individual is represented by a finite array of symbols, known as a string. Each individual string encodes a possible solution in a given problem space. Every string is assigned a fitness value derived from a performance measure defined by the criteria to be optimized in the problem. The algorithm starts with an initial population of individuals that is generated at random. At every evolutionary step, the population of solutions is modified to a new population by applying three operators similar to natural genetic operators: reproduction, crossover, and mutation.

The reproduction generates a mating pool by selecting good fitness strings from the population. GA employs tournament selection for its simplicity. In a typical tournament selection two strings are randomly chosen from the population, and the fitter of the two is selected for insertion into the mating pool. After the reproduction, the crossover operator is applied to strings of the mating pool. In a single-point crossover operation, a crossover site is chosen at random and all bits to the right of the crossover site are exchanged between two strings. In a two-point crossover operator, two sites along the string are chosen at random and the sub-strings included between these sites are exchanged between the parents. The uniform crossover exchanges every bit between parents with a certain probability. Normally, crossover is not performed on the entire population. A crossover probability of P_c dictates that P_c x 100 percent string in the population are used in the crossover operation and that the best (1 - P_c) x 100 percent of the population are simply copied to the new population.

In addition to the crossover operator, a mutation operator is used to enhance the search in a GA. The mutation operation flips a bit in a string with a very small mutation probability, P_m. Mutation is necessary to maintain diversity in the population. In each generation, the population is evaluated and tested for termination of the algorithm. If the termination criterion is not satisfied, the population is operated upon by the three genetic operators and then re-evaluated. This procedure is continued until the termination criterion is met.

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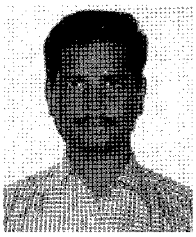

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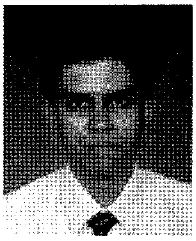

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