Game Theory for Routing Modeling in Communication Networks - A Survey

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Abstract: **In this work, we review the routing models that use game theoretical methodologies. A very common assumption in the analysis and development of networking algorithms is the full cooperation of the participating nodes. Most of the analytical tools are based on this assumption. However, the reality may differ considerably. The existence of multiple domains belonging to different authorities or even the selfishness of the nodes themselves could result in a performance that significantly deviates from the expected one. Even though it is known to be extensively used in the fields of economics and biology, game theory has attracted the interest of researchers in the field of communication networking as well. Nowadays, game theory is used for the analysis and modeling of protocols in several layers, routing included. This review aims at providing an elucidation of the terminology and principles behind game theory and the most popular and recent routing models. The examined networks are both the traditional networks where latency is of paramount importance and the emerging ad hoc and sensor networks, where energy is the main concern.**

Index Terms: **Ad hoc networks, Bayesian games, game theory, Nash equilibrium, network routing, price of anarchy, routing modeling, sensor networks.**

I. INTRODUCTION

Although game theoretical concepts (like decision making) have been discussed before, John von Neumann and Oskar Morgenstern established game theory as a separate field of science when they published their book [1] in 1944. Since then great strides have been made in this area, mainly in the field of economics and biology. However, game theory can also be applied to many fields of science, where decision makers have conflicting interests. Thus, it comes as no surprise to read papers related to networking that adopt game theoretical concepts to analyze a protocol's performance or propose a solution that corresponds to a Nash equilibrium (NE) set of strategies.

Even if the works were initially limited to conventional networks, the recent development of wireless networking motivated researches to seek for answers using the tools provided by game theory. A review of some of these attempts can be found in [2]. Convinced that the application of game theory could reveal new concepts regarding networking protocols, in this work we review the most important models proposed thus far for modeling routing in communication networks in general. However, we decided to cast more light on routing in networks with dynamic topologies and energy constraints, as they introduce many new issues that have not been faced before, such as frequent topology changes and energy-aware routing.

There are some reviews that partially refer to routing modeling via game theory. In [3], the authors provide an overview of the applications of game theory to a vast majority of problems related to the operation of ad-hoc networks. They provide a layered perspective, by reporting the progress of applying game theoretical concepts to the various levels of the protocols stack (physical layer, media access control (MAC) layer, network and transport layer). Since they do not focus particularly on the network layer issues, the presentation of the related work in this field is neither extensive nor detailed. Yoo and Agrawal in [4] provide a review of reputation-based, credit-based and game-theory schemes for routing in mobile ad-hoc networks (MANETs), focusing on the effectiveness of the techniques in avoiding selfish behavior. In addition, Mandalas *et al.* [5] present the most popular cooperation enforcement mechanisms.

This work attempts to achieve two objectives at the same time. The first one is to cover the several approaches used to model routing under the framework of game theory. Thus, it tries to be as detailed as needed to provide an understanding of the philosophy behind each approach. The other one is to provide a "tutorial" on these models, which usually requires abstracting the main ideas and conclusions. Hence, the reader will find both model description using equations and results descriptions that are usually presented without any proof. The interested reader can refer to the papers themselves for a more holistic view.

The rest of the paper comprises three sections. The first one (Section II) is an introduction to the most basic game theoretical terminology, that will be used thereafter. The second part (Section III) is a review of the routing models mostly proposed for conventional networks, although some of them may be used in wireless networks as well. Then, the third part (Section IV) covers the case of ad-hoc and sensor networks in particular, followed by the conclusions.

II. GAME THEORY FUNDAMENTALS

In this section we introduce some basic definitions and theorems of game theory, so that the reader gets familiarized with its terminology and way of thinking. Although the feminine pronoun is preferred in many game theoretical texts, we will use the

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masculine pronoun when referring to a general decision maker or game player.

A. Terminology

Game theory could be defined as "*the study of mathematical models of conflict and cooperation between intelligent rational decision makers*" [6]. The people or the entities (decision makers in general) that play the game are called the *players*. The players take part to the game by performing particular *actions* (α_i) or *moves*. The set of player i's possible actions is called the *action space* A_i of player i. An *action profile* **a** is a vector whose ith element is the action α_i of player i, while $\mathbf{a}_{-i}$ denotes the vector of all players' actions except i. Each player has *preferences* for the action profiles. For example, a player may prefer the action profile **a** to another action profile **b**. In order to represent this preference, the *payoff function* is used (it is also called *utility function*). Since a player is affected not only by its own actions, but also by the actions of the other players as well, a utility function u_i assigns a real value to each action profile of the game. The utility function should fulfill some axioms, but in general, it should assign a larger value to an action profile that is preferred over another one. Thus

$$u_i(\mathbf{a}) > u_i(\mathbf{b}), \qquad \text{if } \mathbf{a} \text{ is preferred over } \mathbf{b}. \tag{1}$$

A very critical assumption in game theory is that a player will always act towards the maximization of its own utility. This is called the *rationality assumption*. The actions that a player performs are also called *strategies* of each player. A player is assumed to select those strategies that would result in the highest benefit to him.

Strategies are characterized as *pure strategies* (s) if they are clearly defined choices of actions, while *mixed strategies* (σ) use probabilistic distributions over the pure strategies. The corresponding spaces for the aforementioned strategies are the *pure strategy space* S_i and the *mixed strategy space* Σ_i respectively. Now, we can define the game in its normal form as a three-tuple $< \mathcal{N}, S, \mathbf{u} >$, where $\mathcal{N}$ is the set of players, S is the pure strategy space of the game and **u** is the vector of the utility functions of the players.

In many cases examined in the framework of game theory, the actions of single selfish players are considered. These types of games are called *non-cooperative games*. On the other hand, if a fraction of the players cooperates and forms coalitions, the games are called *cooperation games* and *coalition games*.

If the summation of the utilities of all players is zero in every outcome of the game, then it is called a *zero-sum* game. If not, the game is called *non-zero-sum*.

In a *static* game, the players make their decisions simultaneously at the beginning of the game. A game that is played only once is called a *simultaneous* game. In a *dynamic* or *sequential* game, the players interact with each other, as they do not decide simultaneously, but they follow a sequence. If the interactions are repeated in time, the game is called *repeated*, and each interaction corresponds to a *stage* of the game. In this case the players have the opportunity to modify their strategies over time. *Evolutionary game theory* was developed in order to further analyze dynamic games. Depending on its duration, a repeated game can be *finite* (the game stops after a number of stages) or *infinite*. In a finite game of T stages, the total utility is a function of the strategies followed in each stage of the game:

$$u_i = \sum_{t=1}^{T} u_i(t, s). \tag{2}$$

In an infinite game, the above definition would result in an infinite utility. Thus, the *discounting* technique is used, where the total utility is computed as

$$u_i = \sum_{t=1}^{\infty} \delta^{t-1} u_i(t, s). \tag{3}$$

The parameter δ is called the discount factor and reduces the effect of future payoffs on the total utility.

When the players of the game have full knowledge of the other players' previous moves, they play a game with *perfect information*, in contrast to a game with *imperfect information*, where the full knowledge of the other players' moves is impossible. Furthermore, when the utility functions and the possible strategies are known to all players, then a game with *complete information* is played. On the contrary, a game with *incomplete information* is a game where some users are not fully aware of the played game. Thus, there can exist games with complete but imperfect information, i.e., a game where the decisions are made simultaneously by all players and no information about the other players' moves is possible (imperfect information) but the utility functions are known to every player (complete information). On the other hand, a game where the players' moves are known to everyone else but the utility functions are hidden is a game with incomplete but perfect information. Games with imperfect information are also referred as *Bayesian games*, because the players update their beliefs using the *Bayes' rule* known from probability theory.

Symmetric games are the games where the set of actions is the same for all players and have the same utility functions. What matters is only the strategies played and not who is playing them. In an *asymmetric* game the same set of strategies results in different payoffs for a particular player, depending on which strategy was followed by him.

B. Equilibria

One of the objectives of the theory is to analyze and predict the effect of different strategies. There are strategies, for example, that result in a state of the game where no player has any incentive to deviate from it. This and similar situations are significant operating points of the game and are called *equilibria*. The most well known is the NE.

A NE is a set of strategies where each player has no incentive to deviate, in other words, given the strategies of all other players, if he changes his strategy he can only decrease his utility. More specifically, if s_i is an arbitrary action of player i and s_{-i} is the set of actions of all other players, then the action profile $s^* = (s_i^*, s_{-i}^*)$ constitutes a NE if, for every player i,

$$u_i(s_i^*, s_{-i}^*) \geq u_i(s_i, s_{-i}^*), \ \forall s_i \in S_i. \tag{4}$$

Table 1. The utilities of players 1 (P1) and 2 (P2) in the PD.

P1 \ P2	Confess	Defect
Confess	(5,5)	(0,10)
Defect	(10,0)	(1,1)

The operating point that corresponds to a NE is also referred as *Nash equilibrium point* (NEP). If the strategies are mixed, then the utility function refers to the expected payoff, which is computed based on the probability distribution functions of the players over the pure strategies and the payoff for each pure strategy. Hence, the utility of the mixed strategy $\sigma = (\sigma_i, \sigma_{-i})$ is computed as follows:

$$u_i(\sigma) = \sum_{s_i \in S_i} \sigma_i(s_i) u_i(s_i, \sigma_{-i}). \quad (5)$$

Now, we can define the NE for mixed strategies. Assuming that σ_i is an arbitrary probability function of the pure strategies of player i, the mixed strategy profile $\sigma^* = (\sigma_i^*, \sigma_{-i}^*)$ constitutes a NE if, for every player i,

$$u_i(\sigma_i^*, \sigma_{-i}^*) \geq u_i(\sigma_i, \sigma_{-i}^*), \ \forall \sigma_i \in \Sigma_i. \quad (6)$$

The NE specifies the strategies that will be followed by rational players in a game. If it exists and is unique, it actually provides us with the strategies that will definitely be followed by rational players. Thus, we are able to know the result of the game and the strategies that will be followed before even the game is played. Nevertheless, although the NE corresponds to a stable state of a game, this does not mean that it is the optimal operating point. For this reason, the concept of *Pareto optimal* (PO) point is used, defined as the set of strategies where a player cannot be better off unless he decreases the utility of another player. The PO point corresponds to the social optimum operating point, because all players achieve the maximum payoff without reducing the payoff of any other player. Even if the PO point is different from the NE in general, there are cases where the same strategy profile is both a PO and a NE point at the same time. For the system designers, this coexistence is an objective that they try to achieve.

C. The Prisoner's Dilemma Game

A very popular example in game theory textbooks is the so-called *Prisoner's dilemma* (PD). The game has two players: Two suspects who are accused of a crime, but the prosecution authority does not have enough evidence to charge accusations. Thus, it proposes to each one of them a reduction of the penalty is he confesses. In particular, if only one of them confesses, then he will go free, while the other one will go to prison for 10 years. However, if none of them confesses, both of them will get 1 year of imprisonment. Finally, if they both confess, both of them will be charged with 5 years of imprisonment. Table 1 summarizes the utilities of the two players in each case. Each player has two possible actions, *confess* or *defect*. The form (x, y) is used to indicate that the payoff of the player 1 is x and the payoff of the player 2 is y.

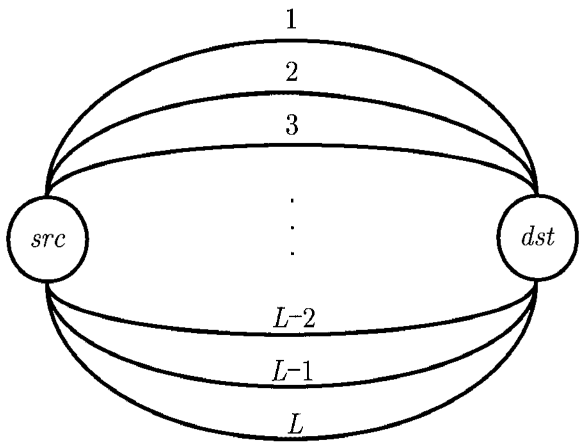

Fig. 1. The simple network of a common source-destination pair, connected via L parallel links of variable capacity.

How should the suspects act? If each one of them thinks selfishly, then the best strategy for him is to confess, no matter what the decision of the other suspect will be. Thus, both of them confess and so the (confess, confess) strategy is the NE of the game and each player is imprisoned for 5 years. However, as it is obvious, this is not the best the two suspects could achieve. If both of them defect, then they would be imprisoned for only 1 year. This is the PO point, which is the social optimum. The PD is a non-cooperative game with imperfect information that can be easily extended to a multi-player or a repeated game and it is the basis for many models used to analyze the performance of routing protocols in telecommunication networks.

III. ROUTING IN COMMUNICATION NETWORKS

Motivated by the achievements of game theory in other fields of science and the increasing interest in networking, many researchers attempted to apply the concepts of this theory to communications and particularly network routing. Next, we review the most important relevant works.

A. Classic Approaches and Nash Equilibria

One of the first papers that applied game theory to the problem of routing was [7]. Like in many other works that will be presented, the model comprises a set $\mathcal{N} = \{1, 2, \cdots, N\}$ of selfish users (the players) that share a set $\mathcal{L} = \{1, 2, \cdots, L\}$ of parallel communication links that connect a common source node to a common destination node, as Fig. 1 indicates. This model, although a simple one, is extensively used in all the related works, mainly because it captures the basic concepts of selfishness in a network and it can be extended to the general case of a real network with many nodes and links. The capacity of each link is denoted by c_ℓ. Every user i has a throughput demand which is an ergodic process with mean value r^i. The action of each player i refers to deciding the fraction f_ℓ^i of the total throughput that will be sent through each link ℓ, so that $\sum_{\ell \in \mathcal{L}} f_\ell^i = r^i$. Similarly, on each link the total flow will be the sum of the flows each user chooses to pass via link ℓ and $f_\ell = \sum_{i \in \mathcal{N}} f_\ell^i$. Let $\mathbf{f}_\ell = (f_\ell^1, f_\ell^2, \cdots, f_\ell^N)$ be the vector of the flows of all users through link ℓ and $\mathbf{f}^i = \left(f_1^i, f_2^i, \cdots, f_L^i\right)$ be the vector of the flows user i decides to pass via all links. Finally, the system flow configuration $\mathbf{f}$ is the vector of the flow configurations of all users: $\mathbf{f} = \left(\mathbf{f}^1, \mathbf{f}^2, \cdots, \mathbf{f}^N\right)$.

Each user i measures its performance using a cost function $C^i(\mathbf{f}) = \sum_{\ell \in \mathcal{L}} C_\ell^i(\mathbf{f}_\ell)$, where C_ℓ^i refers to the cost of user i when choosing to pass a fraction of its flow via link ℓ. The cost is always finite and the aim of each user is to minimize it. Alternatively, one could think of the utility of each user as the negative of this cost, hence the maximization of the utility function corresponds to cost minimization. Thus, users decide on the flow that they will send over each link, such that they face the minimum possible cost.

Since more than one user exist, the cost function of a node depends not only on its decision regarding its own flow, but on the flow configuration of all the other players as well. This means that the nodes compete with each other in order to achieve the minimum cost, meaning that the game is non-cooperative. A possible NE corresponds to a flow configuration where no node would have any incentive to change its flow configuration and achieve lower cost.

Assuming that the sum of users' demands is always lower than the sum of link capacities, it can be proved that a NEP always exists but its uniqueness is still a question. The authors prove several related theorems by taking into account the properties of the cost function. For example, if the cost function C_ℓ^i of each user i is an increasing function of both the i's flow over link ℓ and the total flow f_ℓ over this link, i.e., $C_\ell^i(\mathbf{f}_\ell) = g\left(f_\ell^i, f_\ell\right)$, then the NEP is unique. Two special cases of such a cost function are considered. In the first one, the cost function takes the form $C_\ell^i = f_\ell^i T_\ell(f_\ell)$, while in the second one the undefined term T_ℓ takes the form:

$$T_\ell(f_\ell) = \begin{cases} 1/(c_\ell - f_\ell), & \text{if } f_\ell < c_\ell \\ \infty, & \text{if } f_\ell \geq c_\ell. \end{cases} \tag{7}$$

This definition of T_ℓ is in fact the average delay in an M/M/1 system, thus the cost for a particular user is proportional to the flow the user assigns to each link and inversely proportional to the unused bandwidth of this link. If the link capacity is exceeded, the cost tends to infinity, indicating that another link could be used so that the link capacity is never exceeded.

An extension to the general case of a network comprising more nodes is also provided. More specifically, the network $\mathcal{G} = \{\mathcal{V}, \mathcal{L}\}$ refers to a network with a set of vertices $\mathcal{V}$ and a set of links $\mathcal{L}$. A link is also referred to as (u, v), where u is the starting point and v is the ending point of the link. Each user has a different source-destination pair (src^i, dst^i) and splits its traffic demand r^i among the available paths, each path comprising a set of links. The authors prove that the existence of the NEP is also guaranteed in this case, however, the uniqueness cannot be easily preserved, since more strict properties should hold for the cost functions. They also state that the general case of a network is very hard to analyze, compared to the simple case of the two nodes.

Based on the above model for the general network, Altman *et al.* in [8] provide the necessary conditions in order for the NE to be unique. In specific, for a cost function of the form $C_\ell^i = f_\ell^i T_\ell(f_\ell)$, where $T_\ell = a_\ell f_\ell^{p(\ell)} + b_\ell$, the NE is unique if $0 < p(\ell) = p < p^* = (3N-1)/(N-1)$, for $N \geq 2$. a_ℓ and b_ℓ are two positive parameters that may depend on ℓ. If additionally $b_\ell = 0$ and all users have the same source and destination, the resulting NE is globally optimal and the link flows of different users are proportional to their total traffic.

In contrast to previous works that consider the cost function in a multiplicative way, the authors in [9] assume that the cost function is an additive combination of the objectives of routing, namely the maximization of throughput and the reduction of the delay. So, the utility function C^i of an arbitrary user i follows (8):

$$C^i(F) = \sum_{\ell \in \mathcal{L}} \alpha_\ell^i f_\ell^i + \sum_{\ell \in \mathcal{L}} \beta_\ell^i \frac{f_\ell^i}{c_\ell - f_\ell^i}. \tag{8}$$

In the analysis of non-cooperative networks, two kinds of objective functions were used: The net benefit and the power. The first one measures the user's satisfaction by subtracting the cost from the benefit (benefit-cost), while the second one uses their ratio (benefit/cost).

The first part corresponds to the total flow for user i (the benefit of the user) and the second one corresponds to the total congestion (the cost the user has to pay) for an M/M/1 queue model. Thus, the cost function is said to be of the *benefit-cost* or *net benefit* form, a very common category of cost functions[1]. The parameters α_ℓ^i and β_ℓ^i are the weights each user assigns to each link for the two parts of the expression and their ratio $\gamma_\ell^i = \alpha_\ell^i / \beta_\ell^i$ is the ith *tradeoff* parameter for *link* ℓ.

Using a model similar to that of [7], the authors conclude that a unique NE exists that satisfies the following equality (for user i and link ℓ):

$$f_\ell^{i^*} = (c_\ell - f_\ell^*)^2 \gamma_\ell^i - (c_\ell - f_\ell^*) \tag{9}$$

where

$$f_\ell^* = c_\ell - \frac{(N-1) - \sqrt{(N-1)^2 - 4c_\ell \gamma_\ell}}{2\gamma_\ell} \tag{10}$$

and $\gamma_\ell = \sum_{i \in N} \gamma_\ell^i$. This NE is feasible if

$$c_L \geq \frac{1}{\gamma_\ell^i}\left(\frac{\gamma_\ell}{\gamma_\ell^i} - (N-1)\right), \ \forall i \in \mathcal{N} \tag{11}$$

where $c_L = \max_{\ell \in \mathcal{L}}\{c_\ell\}$. An important conclusion of the above analysis is that the larger the tradeoff parameter of a user, the greater the allocated flow in the network. Hence, a user with high throughput requirements should choose a large tradeoff value, while a user with low throughput requirements could use smaller values of γ_ℓ^i. What is more, if the users increase the tradeoff value that corresponds to a single link, the flow on that link will increase. If all users use the same tradeoff values for all links, then it is proved that the unutilized bandwidth of high-capacity links is greater than the unutilized bandwidth of low-capacity links.

[1] Another common form is the power form, where the ratio of the benefit to the cost is used.

B. Quality of Service

The quality of service (QoS) is taken into consideration in [10], where the authors consider the case of multiple streams with QoS constraints over a multipath network in general, as a continuation of their work in [11]. Streams are to be shared between two available links: One with a high-quality metric and the other with a low-quality metric. The players are the streams and each player's strategy is to select the percentage of its stream that will follow the high quality metric link, denoted as p_i. The utility of each stream U^i is a function of the goodput of each link (T_L for the low-quality and T_H for that high quality links respectively) and the bit rate of the stream R_i. Hence, $U^i = p_i R_i T_H + (1 - p_i) R_i T_L$. Assuming that packet losses are only possible due to unsatisfied QoS constraints, the authors argue that a NE is feasible and formulate an expression providing the player's strategies in the equilibrium. They show that for cases of practical interest the requirements of the NE are satisfied.

C. Min-Max Games

Consider a game with two players, A and B. If player B chooses the strategy that results in the worst case for player A, then the rational player A will still try to maximize its payoff. The *minimax value* is the maximum utility of player A under the worst-case conditions set by his opponent. A similar problem is investigated by Yamaoka and Sakai in [12], where the authors use the minimax principle to model and evaluate the performance of a packet-switched network. The network consists of relay nodes that are connected with each other via links. Inside each node there are two priority queues functioning according to a FIFO policy; a high-priority queue and a low-priority one. Two players are considered; the "packet" and the "network." Each packet aims at keeping its qualities (e.g., delay) at the best possible level. On the other hand, the network aims to prevent packet losses and level off the qualities of the packets. Sometimes the two players have common interests while other times their interests are in conflict. Hence, a non-zero-sum game is formed, where the minimax principle can be applied. The packet player decides the link it has to go next (usually trying to follow the path with the minimum number of hops). The network player decides if a received packet will be enqueued to the high or the low priority queue. A packet has two parameters: The permissible delay time W and the passage delay time t. The quality Q of a packet is computed as $Q = 1 - t/W$. The closer to 0, the lower a packet's quality. The authors use computer simulations to show that the distributed control based on the above model performs better in terms of packet loss rate.

D. Bottleneck Games

An interesting view on routing modeling can be found in [13], where the authors formulate a *bottleneck* game. In conventional approaches to routing games, the performance is considered additive (the sum of link cost functions). In bottleneck games, the bottleneck (or min-max) objectives are only characterized by the worst part (e.g., link). Their target is to study the performance of a network where users route their traffic selfishly, aiming to optimize the performance of their bottlenecks. This gives rise to a non-cooperative game.

The players of the model are the users. Each user i has a specific throughput demand γ_i and may use multiple paths from source src^i towards the destination dst^i, and on each path $p \in P^{(src^i, dst^i)}$ flows f_p^i amount of traffic. The sum of f_p^i for all paths is the throughput demand of the node and the strategy of a node is to decide the flows over each path. Each link ℓ of the network graph is associated with a performance function q_ℓ which depends on the total flow f_ℓ over this link. The *bottleneck of a user* is defined as the performance of the worst link of all users' flows. Finally, the aim of each user is to minimize its bottleneck. Despite the discontinuity of the strategy space, the authors prove that, for both the splittable and the unsplittable traffic case, at least one NE exists. However, it was shown that these equilibria can be very inefficient. In order to improve these inefficiencies, additional routing rules need to be applied to the network. As an example, for the splittable traffic, the NE is optimal if the selected routes contain the minimum number of bottlenecks.

E. Price of Anarchy

A detailed analysis of the efficiency of the NEs was initially made by Koutsoupias and Papadimitriou [14], who used the model of non-cooperative game for the routing traffic problem and investigated the performance loss due to lack of cooperation among users. Considering the simple model of L parallel links connecting a source-destination pair, they assume that all links have the same capacity and the players assign their traffic to the links. The pure strategy for the users would be to select a single link to pass their traffic through. A mixed strategy would correspond to a probability distribution on the set of links, which would indicate the frequency with which each link is selected. Hence, the traffic is assumed unsplittable. Let us assume that a user i selects a link ℓ with probability p_ℓ^i. Let M^ℓ denote the expected load on link ℓ and L_ℓ denote the initial load on that link. Then

$$M_\ell = L_\ell + \sum_{i \in \mathcal{N}} p_\ell^i f^i \tag{12}$$

where f^i is the amount of traffic for user i.

The cost for user i when its traffic f^i is assigned to link ℓ is

$$C_\ell^i = L_\ell + f^i + \sum_{j \neq i} p_\ell^j f^j = M_\ell + \left(1 - p_\ell^i\right) f^i \tag{13}$$

and each user's target is to minimize its total cost by selecting the appropriate distribution over the links.

The main objective of the work was to compute the *worst-case ratio*. It is defined as the ratio of the worst NE to the global optimum solution, usually in terms of latency. This ratio is also known as *price of anarchy* and it has been the center of interest for many works as yet. The authors proved in that work that for two equal capacity parallel links the worst-case ratio is $3/2$, no matter what the number of users is. If the two links are not the same, this ratio reaches the golden ratio ϕ.

Following the previous work, Roughgarden and Tardos in [15] consider the problem of assigning traffic to the links of the network, so that the aggregated latency is minimized. Specifically, they study the effect of lack of coordination among the nodes of the network, meaning that every node acts selfishly and selects the link with the minimum latency from its source towards its destination. They consider only pure strategies, compared to [14] where mixed strategies are investigated. The model adopted assumes a set of K source-destination pairs (src_k, dst_k). A set of possible paths $\mathcal{P}_k$ is associated with each such pair, and $\mathcal{P} = \cup_k \mathcal{P}_k$. A flow $f : \mathcal{P} \rightarrow \mathcal{R}^+$ is a function that assigns a positive real number to each path and the total flow f_e via edge e is the sum of flows of all paths where e is involved : $f_e = \sum_{P:e \in \mathcal{P}} f_P$. Each (src_k, dst_k) pair is associated with a traffic rate r_k and a flow f is said to be feasible if for all k

$$\sum_{P \in \mathcal{P}_k} f_P = r_k. \tag{14}$$

The latency l_e of each edge e is a non-negative, non-decreasing and continuous function of the flow on that edge. Then, the latency l_P on a path P given a flow f is denoted by $l_P(f) = \sum_{e \in P} l_e(f_e)$. Finally, the cost of a flow f is defined as

$$C(f) = \sum_{P \in \mathcal{P}} l_P(f) f_P. \tag{15}$$

A selfish behavior of a user would mean that it selects the minimum latency path to route its traffic. A flow is in NE if for every source-destination pair the selected path leads to the minimum latency and any other path would result in a higher latency. It is shown that when a feasible flow is in NE, then the cost can be expressed as

$$C(f) = \sum_{k=1}^{K} L_k(f) r_k. \tag{16}$$

On the other hand, in order to find the optimal flow that minimizes the total latency, a non-linear programming problem is formulated. The objective is to minimize the expression $\sum_{e \in \mathcal{E}} c_e l_e$, where $\mathcal{E}$ is the set of edges of the graph representing the network and $c_e = l_e(f_e) f_e$. Assuming only linear latency functions l_e, then the price of anarchy is proved to be $4/3$. Roughgarden also proved in [16] that the price of anarchy is not dependent on the network topology and the worst possible ratio may occur even in very simple networks.

F. Price of Routing

In their work [17], Awerbuch *et al.* study the price of anarchy in the case of unsplittable flows. An arbitrary player k assigns all its traffic to only one path. Using the same terminology as previously, in the case of pure strategies, since the flow of each user is unsplittable, then $\mathcal{P}_k$ has only one member P_k and its entire traffic passes via this path and thus via the edges that form the path. Thus,

$$f_e = \sum_{P:e \in \mathcal{P}} f_P = \sum_{k:e \in P_k} f_{P_k} = \sum_{k:e \in \mathcal{P}_k} r_k. \tag{17}$$

The latency for user k that uses path P' instead of path P is defined as:

$$c_{P',k} = \sum_{(e \in P') \wedge (e \in P)} l_e(f_e) + \sum_{(e \in P') \wedge (e \notin P)} l_e(f_e + r_k). \tag{18}$$

For mixed strategies, a set of random variables $\{X_{P,k}\}$ is defined, which indicates if user k uses the path P. By definition $Pr\{X_{P,k} = 1\} = p_{P,k}$. Another set of random variables $\{X_{e,k}\}$ indicates if edge e is used by user k. It is defined that $X_{e,k} = \sum_{p|e \in P} X_{P,k}$ and $Pr\{X_{e,k} = 1\} = p_{e,k}$. Then, the total flow on edge e is $f_e = \sum_k X_{e,k} r_k$. So, the expected latency of user k for using path P is computed as

$$\begin{aligned} c_{P,k} &= E\left\{\sum_{e \in P} l_e(f_e) | X_{P,k} = 1\right\} \\ &= \sum_{e \in P} E\{l_e(f_e + (1 - X_{P,k}) r_k)\}. \end{aligned} \tag{19}$$

In general, the expected cost $C(S)$ for a given system S of pure or mixed strategies is defined as the expected total latency incurred by S: $C(S) = \sum_{e \in E} l_e(f_e) f_e$, and finally the coordination ratio (or price of anarchy) is defined as

$$R = \max_S \frac{C(S)}{C(S^*)} \tag{20}$$

where the maximum is computed over all NEs strategies S and S^* denotes the optimal system of pure strategies. Note that the cost is the influence of both the latency and the traffic load of each user.

The analysis shows that for linear latency functions, the price of anarchy is approximately 2.618 for the weighted and 2.5 for the unweighted (meaning that all users require the same amount of bandwidth) traffic. What is more, for polynomial latency functions of degree d the worst-case coordination ratio is $d^{\Theta(d)}$ for pure and mixed strategies.

G. Price of Stability

While price of anarchy considers the worst NEP, the ratio of the best NE to the global optimum is also a significant metric of NEP efficiency. It was first defined in [18] and later named as *price of stability* in [19], which studies the network design games with fair cost allocation. This term is useful because it can show how close to the global optimum a NE can reach, and therefore if the corresponding strategies are proposed to the players, no player will have an incentive to deviate (which stabilizes the system), while the achieved payoffs will be close to the global optimal ones.

H. Bounded Flow Demands

In [20], the authors consider the problem of flow control over single and multiple links. In contrast to previous works, in this case the authors insert bounds on the flow demands of each user. In particular, the players are the users of the set $\mathcal{N} = \{1, \cdots, N\}$ who wish to send their flows over L parallel links. User i's flow

f^i is chosen from the interval $[m^i, M^i]$, where m^i and M^i are the lower and the upper bound of the flow of the corresponding user and it is a private knowledge only, meaning that no user has any information about the bounds of the other users. This flow is split over all available links and $\mathbf{f}^i = \{f_1^i, f_2^i, \cdots, f_L^i\}$ is a vector containing the fraction of user i's flow over each one of the L links. The flow profile for all users is $\mathbf{f} = (f^1, f^2, \cdots, f^N)$.

For the case of a single flow, omitting the subscripts that refer to the links, the benefit for user i managing to send flow f^i over the single link is defined as

$$B^i\left(f^i\right) = \left(f^i\right)^{\beta^i} \tag{21}$$

where β^i is a weighting factor which in general is different for each user. The cost function is a function of the congestion on the link:

$$\eta^i(\mathbf{f}) = \begin{cases} 1/\left(c-F\right), & F < c \\ \infty, & F \geq c \end{cases} \tag{22}$$

where F is the total flow over the link and c represents the link capacity, known a priori by all users, and should satisfy all users' minimum demands: $c > \sum_{j=1}^{N} m^j$. The authors adopt the power form for the utility function, thus the utility for each user i is computed as

$$U^i(\mathbf{f}) = \frac{B^i(f^i)}{\eta^i(\mathbf{f})} = \begin{cases} \left(f^i\right)^{\beta^i}(c-F), & F < c \\ \infty, & F \geq c. \end{cases} \tag{23}$$

For this single link case it is proved that a NE exists and is unique. Furthermore, the authors show that the flows f^* at NEP, under the assumption that $\beta^i = \beta$, $\forall i \in \mathcal{N}$, are given by the following expression

$$f^{*i} = \begin{cases} M^i, & \text{if } M^i < P^* \\ m^i, & \text{if } m^i > P^* \\ P^*, & \text{otherwise} \end{cases} \tag{24}$$

where $P^* = \beta(c - \sum_{j=1}^{N} f^{*j})$ is a critical point and an algorithm is provided for its computation.

In the case of multiple flows, they first define the available capacity for user i as $c_\ell^i = c_\ell - \sum_{j \neq i} f_\ell^j$ and then compute the fraction of c_ℓ^i used by each user on every link:

$$\mathbf{r}^i = \{r_1^i, r_2^i, \cdots, r_L^i\}, \quad r_\ell^i = f_\ell^i / c_\ell^i. \tag{25}$$

A normalized vector is produced by dividing with the sum of all fractions

$$\hat{\mathbf{r}}^i = \{\hat{r}_1^i, \hat{r}_2^i, \cdots, \hat{r}_L^i\}, \quad \hat{r}_\ell^i = \frac{r_\ell^i}{\sum_\ell r_\ell^i}. \tag{26}$$

The cost function is now defined as

$$\eta^i(\mathbf{f}) = \begin{cases} \{R(\mathbf{r}^i)(c-F)\}^{-1}, & F < c \\ \infty, & F \geq c \end{cases} \tag{27}$$

and the authors choose the function $R(\mathbf{r}^i)$ to be defined as

$$R\left(\hat{r}_\ell^i, \hat{r}_2^i, \cdots, \hat{r}_L^i\right) = 1 - \sum_{\ell=1}^{L} \hat{r}_\ell^i \log\left(\hat{r}_\ell^i\right) \tag{28}$$

and thus the $R - 1$ is an entropy function. Finally, the utility function is defined as

$$U^i(\mathbf{f}) = \frac{\sum_{\ell=1}^{L} f_\ell^i}{\eta^i(\mathbf{f})}. \tag{29}$$

It is shown that the NEP is unique and the equilibrium flows $\widetilde{f}_\ell^i$ are computed using the equilibrium flows of the single-link case f^{*i} as $\widetilde{f}_\ell^i = (c_\ell/c) f^{*i}$.

I. Combined Routing and Flow Control

Altman *et al.* in [21] consider the combined problem of routing and flow control in a network of parallel links. Starting from the case of a single user, they let $\lambda = \{\lambda_1, \lambda_2, \cdots, \lambda_L\}$ be the vector of the throughput experienced by the user on every link ℓ. Using the expression for the delay on an M/M/1 queue, the average delay experienced by the user when splitting its traffic over the L links is

$$d(\lambda) = \left(1/\sum_{\ell=1}^{L} \lambda_\ell\right) \sum_{\ell=1}^{L} \frac{\lambda_\ell}{c_\ell - \lambda_\ell}. \tag{30}$$

The tradeoff function is defined as the ratio of the perceived throughput to experienced delay:

$$U(\lambda) = \frac{\left(\sum_{\ell=1}^{L} \lambda_\ell\right)^\beta}{d(\lambda)} = \left(\sum_{\ell=1}^{L} \lambda_\ell\right)^{\beta+1} \sum_{\ell=1}^{L} \frac{\lambda_\ell}{c_\ell - \lambda_\ell}. \tag{31}$$

Using the two previous equations and with the help of the logarithm of $U(\lambda)$ the authors prove that for a single user, an optimum solution exists and it implies that the links with the largest capacity are preferred, while the links with lower capacity may be assigned with zero traffic.

In the same work, two other cases regarding the form of utility function are examined. In the first one, $U_1(\lambda)$ is defined as

$$U_1(\lambda) = \sum_{\ell=1}^{L} \lambda_\ell^\beta \left(c_\ell - \lambda_\ell\right), \quad \beta \in (0,1) \tag{32}$$

which is actually the sum of utility functions on each link, each one corresponding to the product of the throughput and the delay on the link. In this case, the optimal solution has the form $\lambda_\ell = (\beta/(\beta+1))c_\ell$, which is much simpler than that for $U(\lambda)$. Note, though, that in this case all flows are assigned non-zero traffic, as the utility function lacks a global knowledge of the overall throughput and delay. The second utility function proposed is

$$U_2(\lambda) = \left(\sum_{\ell=1}^{L} \lambda_\ell\right)^\beta \left(\bar{c} - \sum_{\ell=1}^{L} \lambda_\ell\right) \tag{33}$$

where $\bar{c}$ is the total capacity of all links. In this case a set of optimal solutions λ_ℓ is possible provided that the following expression is satisfied

$$\sum_{\ell=1}^{L} \lambda_\ell = \frac{\beta}{\beta+1} \bar{c}. \tag{34}$$

For multiple users, similarly to (31), each one's utility is defined as

$$U^i(\lambda) = \begin{cases} \left(\sum_{\ell=1}^{L} \lambda_\ell^i\right)^{\beta+1} \Big/ \sum_{\ell=1}^{L} \frac{\lambda_\ell^i}{c_\ell - \lambda_\ell^i}, & \text{if } \lambda_\ell^i > 0 \\ 0, & \text{if } \lambda_\ell^i = 0 \end{cases} \quad (35)$$

or in its logarithmic form:

$$L^i(\lambda) = (\beta+1)\log\left(\sum_{\ell=1}^{L} \lambda_\ell^i\right) - \log\left(\sum_{\ell=1}^{L} \frac{\lambda_\ell^i}{c_\ell - \lambda_\ell^i}\right). \quad (36)$$

Since, the examination of NEs is difficult for the general case, the authors investigate the equilibrium when the number of users tends towards infinity. This is the so-called *asymptotic NE*, which is defined below:

For N player game with N arbitrary large, the set of flows $\{\lambda_\ell^i\}$ constitute an asymptotic NE if for all $i \in \mathcal{N}$

$$\lim_{N\to\infty} L^i\left(\lambda_\ell^{i*}, \{\lambda_\ell^{j*}\}_{j\neq i}\right) = \lim_{N\to\infty} \max_{\{\lambda_\ell^i\}_{i\in\mathcal{N}}} L^i\left(\lambda_\ell^i, \{\lambda_\ell^{j*}\}_{j\neq i}\right). \quad (37)$$

The aforementioned equilibrium flows are said to constitute a $O(1/N)$ NE with exponent κ if there exists a non-positive scalar κ, independent of N, such that for all $i \in \mathcal{N}$

$$L_i\left(\lambda_\ell^{i*}, \{\lambda_\ell^{j*}\}_{j\neq i}\right) = \max_{\{\lambda_\ell^i\}_{i\in\mathcal{N}}} L_i\left(\lambda_\ell^i, \{\lambda_\ell^{j*}\}_{j\neq i}\right) + \frac{\kappa}{N} + O(1/N). \quad (38)$$

For the symmetric case where $\lambda_\ell^i = \lambda_\ell/N$ these flows are proved to constitute a potential asymptotic and $O(1/N)$ NE with $\kappa < 0$.

J. Partially Optimal Routing

In very large networks it is very probable that more than one administrative domain will coexist. Within each domain, the corresponding authority minimizes the cost. However, in order to route traffic, it should pass via other domains. Thus, although optimal routing takes place inside domains, selfish behavior is observed when routing packets across the domains. This is the problem of partially optimal routing, investigated in [22]. Except for the minimization problem for the entire network, the authors consider a minimization problem for each domain separately. If these two problems have the same equilibria, then the optimization inside each domain results in the optimization in the entire network. Unfortunately, this is not the case. A partially optimal routing may worsen the performance of the entire network, under some circumstances. The inefficiency caused by partially optimal routing is also examined and it is proved that for a class of latency functions, the partially optimal solution is no worse than 25% of the global optimal one. In the case of multiple entry points in each domain, the partially optimal routing can be arbitrarily bad even for simple latency functions.

K. Repeated Games and Evolutionary Game Theory

In classical one-shot games the players do not have the opportunity to learn the behavior of the other players nor can they alter their behavior during the game. However, in the real world the players will be able to communicate with each other, form coalitions and in general modify their behavior as the game is repeated, so as to maximize their benefit. Thus, a non-cooperative dynamic repeated game should be used in order to model these situations. While all aforementioned works assume one-shot games, in their work [23] La and Anantharam use dynamic repeated games to model routing. As usually, they consider a two-node network of L parallel paths where a set of $\mathcal{N}$ users wish to send their flows via the available links. In this work, the authors think of the users as *network access providers*, not individual users. Without loss of generality, they assume that the paths are ordered in decreasing capacity c_ℓ and users are ordered with decreasing average traffic rate r_i. They follow the common practice of the discount factor $\delta \in (0, 1)$ to model the finite game. In each stage the game is basically described like in [7]: The flow f_i of a player i is split over the links so that $f_i = \sum_{\ell\in\mathcal{L}} f_\ell^i$, and the total flow f_ℓ over a link ℓ is $f_\ell = \sum_{i\in\mathcal{N}} f_\ell^i$. The flow profile $f = \{f_1, f_2, \cdots, f_N\}$ is the vector of the flows of each player. The objective of each user is to minimize the cost $J_k(f)$, which depends not only on its decision on how to split the traffic over the link, but also on all other players' decisions. A flow f^* is a NEP if

$$\begin{aligned} J_i(f^*) &= J(f_1^*, \cdots, f_{i-1}^*, f_i^*, f_{i+1}^*, \cdots, f_N^*) \\ &= \min_{f_i\in F_i} J(f_1^*, \cdots, f_{i-1}^*, f_i, f_{i+1}^*, \cdots, f_N^*). \end{aligned} \quad (39)$$

In order to continue the analysis, the authors consider a particular class of cost functions with the following properties:

- $J_i(f) = \sum_{\ell\in\mathcal{L}} J_\ell^i(f_\ell)$.
- J_ℓ^i is continuous.
- $J_\ell^i(f_\ell) = J_\ell^i(f_\ell^i, f_\ell) = f_\ell^i T_\ell(f_\ell)$.
- $T_\ell(f_\ell) = T(c_\ell - f_\ell)$.
- $T_\ell(f_\ell)$ is positive, strictly increasing, convex and continuously differentiable.
- $T_\ell(f_\ell) \to \infty$ as $f_\ell \to c_\ell$.

These conditions are sufficient to guarantee the existence of a NE. Unfortunately this equilibrium does not correspond to the system-wide optimal flow f' that minimizes the total cost $C = \sum_{i\in\mathcal{N}} J_i(f)$. Nevertheless, the theory of dynamic games implies that this optimal point can be reached in the case of repeated games as a NE. For this reason, the reservation cost v_i of a user i is defined as follows:

$$\underline{v}_i = \max_{f_{-i}\in F_{-i}} \left(\min_{f_i\in F_i} J_i(f_i, f_{-i})\right). \quad (40)$$

Also, let the NEP of the one stage game correspond to flow f^* and a cost J_i^* for each user i. Then, the authors prove that in the repeated game case, if the discount factor δ is sufficiently close to 1, then there is a NEP that achieves the minimum total cost C'. What is more, it can be proven that there exists a system flow $\widehat{f}$ that achieves the global-wide optimum cost C' and a cost for each user that is either smaller or equal to the cost that corresponds to the NEP.

The authors also investigate the case of general networks with only one source-destination pair. They managed to prove that in this case too, a NEP exists for the repeated game where the minimum cost C' is achieved, provided that the discount factor is sufficiently close to 1.

Fischer and Vöcking in [24] investigate the evolution of simple routing games in time. Towards a more realistic model, they assume that the game is repeatedly played against random opponents. Based on their observations as time passes by, the players have the opportunity to optimize their behavior. According to their model, for any set of commodities $k \in \mathcal{K} = \{1, \cdots, K\}$, a fraction of r_k agents wishes to send an equivalent amount of traffic from the same source src_k to the same destination dst_k via the available set of paths $\mathcal{P}_k$. If the agents are selfish, then each one's objective is to minimize its own latency. We may assume that an agent may consider revising its routing strategy from time to time (following a Poisson process). In this case, it will consider changing the current path and using a new one, preferably with a lower latency. If it finds such a path, it might transit to it with probability proportional to the latency gain. Letting the number of agents tend towards infinity and replacing the random variables representing the change of the population shares in one step with their expected values, then we can derive the following differential equation:

$$\dot{f}(P) = \lambda_k f_P \left(\bar{\ell}_k - l_P\right) \text{for } k \in \mathcal{K}, P \in \mathcal{P}_k \qquad (41)$$

where f_P and l_P are the flow and the latency of path P, $\bar{l}_k$ denotes the average latency of commodity k and $\dot{f}$ indicates the derivative of flow f with respect to time. Finally, λ_k is a factor used to ensure that probabilities do not exceed 1 and do not influence the solution orbit of the system of differential equations. The above equation is also known as *replicator dynamics* and has been studied in the context of evolutionary game theory. The authors generalize this concept to rerouting dynamics, where each agent becomes active at Poisson rates and performs the following functions:

- Sampling – It picks a path P with probability σ_P. Although a simple computation of this probability would be $\sigma_P = 1/m$, where m is the number of active paths, in replicator dynamics $\sigma_P = f_P$, which means that the probability of sampling a path is proportional to the fraction of agents using this path.
- Migration – An agent migrates from path P to path Q with probability $\mu(l_P, l_Q)$. For replicator dynamics $\mu(l_P, l_Q) = \max\{(l_P - l_Q)\,\lambda, 0\}$, where λ is a parameter used to upper bound the probability, so that it does not exceed 1.

Hence, letting r_{PQ} be the rate at which the agent migrates from path P to path Q, we can write the following expressions:

$$\begin{aligned} r_{PQ} &= f_P \cdot \sigma_Q \cdot \mu(l_P, l_Q) \\ \dot{f}_P &= \sum_{Q \in \mathcal{P}_k} \left(r_{QP} - r_{PQ}\right). \end{aligned} \qquad (42)$$

A solution to this system of differential equations (one equation for each commodity) exists only if the involved functions l_e, σ and μ are Lipschitz continuous, as stated by the Picard-Lindelöf theorem. It would be interesting if NEs are global attractors of the solutions of the aforementioned system of equations. In this case, the evolution of the game will converge to the NEs. Thus, the concept of *evolutionary stability* is needed.

Assuming that at the initial state no link is unused and that a single commodity exists, then a flow vector f is evolutionary stable iff

(a) It is a NE, and

(b) for all best replies $\tilde{f}$ to f and $\tilde{f} \neq f$, $\tilde{f}\ell(\tilde{f}) > f\ell(\tilde{f})$.

A best reply $\tilde{f}$ to a flow vector f is a flow vector that uses minimum latency path with respect to the latency induced by f. A flow vector f is *essentially evolutionary stable* if condition (b) above holds and f differs from any best response $\tilde{f}$ for at least one edge. It is proved that NEs are essentially evolutionary stable for single commodity networks and that in terms of edge flows, all replicator dynamics of the form of (42) converge to a NE. Moreover, the authors investigate the effect of stale information and calculate bound of the update intervals.

IV. GAME THEORY IN AD HOC AND SENSOR NETWORK ROUTING

In the previous section we presented the routing models for general networks, which refer to conventional networks in most cases. Nevertheless, these models could be used to study ad-hoc and sensor networks as well. A MANET could be considered as a general network $\mathcal{G}(\mathcal{V}, \mathcal{L})$, where $\mathcal{V}$ is the set of vertices (the nodes) and $\mathcal{L}$ is the set of links that connect the nodes. The first thing that should be defined for a game is the players. In all previous cases, the players were users that desired to send their traffic from a source to a destination. Their strategies were the paths that the traffic would follow, whether or not the traffic was split among them. In order to achieve this, the usage of source routing was silently assumed. Thus, the role of the nodes of the network is to just route the packets as specified by them. Their role was passive.

In the case of ad-hoc and sensor networks, though, it would not be appropriate to define the players in a similar way. The reason is that in these networks the role of each node is of paramount importance. What is more, since the nodes are devices with limited power sources, energy consumption is considered as the most critical parameter. This is rational since the network connectivity is based upon the existence of nodes. The energy depletion of some of them could result in network partition, a situation that should be avoided at all costs.

For all these reasons, the investigation of ad-hoc and sensor networks using game theory resulted in two great changes in the methodology used so far: The players are usually the nodes themselves and the utility function is highly related to the energy consumption as well. In many cases, the term *forwarding game* is used to specify a game where the nodes decide whether to forward a packet for another node or not. Since each node wishes to preserve its energy in order to be able to send as much traffic as possible, forwarding a packet for another node is not rational, at least at first glance. In the rest of this section we present the most significant proposed models in order to examine the cooperation level of such networks and identify their equilibria.

A. The Packet Forwarding Game

The first work that applied game theory to ad-hoc networks in order to study packet forwarding was [25]. In this paper, the authors propose a policy that can be followed by the nodes and results in a NE and a PO operating point at the same time. Their contribution, though, is the proposal of a model based on game theoretic terminology, in order to study the performance of the network. We will describe this model extensively, as many later proposals are based on it.

The model assumes a population of N energy-constrained nodes (not all nodes participate in the network simultaneously) divided into K energy classes with each class containing n_i nodes. Assuming that E_i and L_i represent the energy and the expected lifetime of each class, then their ratio $\rho_i = E_i/L_i$ represents the average power constraint of each energy class. A source initiates a session towards a destination node but it relies on the intermediate nodes to forward traffic. Thus, it sends a request to all the intermediate nodes and expects from them to reply with a positive or a negative acknowledgment. Only if all nodes respond positively, the session starts and the traffic flows via these nodes. The time is considered slotted, with each slot lasting for a session. A session is said to be of type j, if at least one node belongs to class j and the rest of the nodes belong to higher energy classes.

Let $A_h^j(k)$ be the number of relay requests generated by node h for type j sessions until time k that have been accepted, and $B_h^j(k)$ the total number of relay requests made by node h for type j sessions until time k. Hence, the variable $\phi_h^{(j)}(k) = A_h^j(k)/B_h^j(k)$ is the ratio of the accepted relay requests made by node h for type j sessions until time k. In a similar way they define $\psi_h^{(j)}(k)$ as the ratio of relay requests for type j sessions that node h itself accepted until time k. The authors also define the normalized acceptance ratio (NAR) as

$$\text{NAR} = \lim_{k\to\infty} \phi_h^j(k). \tag{43}$$

We can think of NAR as an indication of the throughput experienced by each user for each session type.

The game players are the nodes and their actions correspond to accepting or refusing the relay requests. Whenever a request is accepted by all intermediate nodes, the payoff is considered equal to unity. The utility of node h for type j sessions is equal to

$$U_h^j = \lim_{k\to\infty} \left(A_h^j(k)/k\right). \tag{44}$$

Assuming that the probability that node h is a source in a type j session is p_h^j, then the total utility of node h is

$$U_h = \sum_j p_h^j U_h^j. \tag{45}$$

The nodes are considered selfish and their only objective is to maximize their total utility.

In order to investigate feasible operation regions, they assume that each node uses a simple probabilistic policy for accepting relay requests. Let τ_{ij} denote the probability that a node in class i accepts a relay request for a session of type j. In order to keep the model simple, the authors consider this probability as constant and independent of any parameter, such as the source of the session or the history. The authors prove that if the nodes are rational (i.e., self-interested) then $\tau_{ij} = \tau_{jj}$, which means that no matter what is the energy class that a node belongs to, it accepts or rejects relay requests as if it was of the same class as the session itself. A set of equations is provided that can be used to derive the optimal value of τ_j.

Although a PO point is feasible, this does not mean that the game will result in this point. As mentioned earlier, the above strategy is a stationary one. A node has a fixed probability for accepting relay requests. Therefore, a node could deviate and deny all the requests. This node would spend its energy only in sessions initiated by it, while other nodes would forward its traffic. Thus, a node has an incentive to deny all requests, no matter how the other nodes behave. In consequence, the "always deny" strategy is a dominating strategy and in the end no traffic will be exchanged at all. Hence, stationary strategies are not appropriate in this case. For this reason, the authors propose a behavioral strategy, which is based on the past behaviors of the nodes, called generous tit-for-tat (GTFT). GTFT was previously proposed [26] as a strategy that results in cooperation in the non-cooperative repeated PD game. In TFT strategy, each node cooperates at the beginning and then mimics the other node's action. In GTFT, each node mimics the other node's action in the previous game; however, it is generous from time to time by cooperating.

In the case of ad-hoc networks, each node maintains two variables, $\phi_h^j(k)$ and $\psi_h^j(k)$ for each session. Note that no explicit information is stored for each node in particular, but only two general variables that correspond to the past history as experienced by each node. The proposed decision algorithm m-GTFT (m for multiple relays) is the following:

- If $\psi_h^{(j)}(k) > \tau_{jj}$ or $\phi_h^{(j)}(k) > L_{ij}\psi_h^{(j)}(k) - \epsilon$ then *reject*,
- else *accept*.

Therefore, node h rejects a request for a type j session in two cases. In the first case ($\psi_h^{(j)}(k) > \tau_{jj}$) node h has relayed more traffic for session j than it should. In the second case ($\phi_h^{(j)}(k) > L_{ij}\psi_h^{(j)}(k) - \epsilon$), the amount of traffic that other nodes relayed in favor of node h is less than the amount of traffic that node h has relayed for other nodes of this type of session. The parameter ϵ is a small positive number, thus node h is a little generous and relays traffic for others even if they have not relayed the same amount of traffic for the benefit of h. Finally, the parameter L_{ij} is inserted because when more than one relay is used, a node plays the role of a relay for more time that it is a source of traffic. It is proved that both GTFT and m-GTFT constitute NEs that converge towards the PO operating points.

Although the results of the above work are very interesting, the assumptions made are very strict. Nodes are assumed to send a positive or negative acknowledgement to the source of the packet, a methodology that is not reliable from the cooperation point-of-view. What is more, the topology remains constant during the transmission of the packet from the source to the destination and all packet transmissions cost the same amount of energy. These assumptions can be considered unrealistic. Furthermore, the dynamic nature of the routing decisions, as long as the unreliability of the information provided by the other nodes are not taken into account. Nevertheless, the contribution is of major importance, as the authors provided a means of investigating the performance of selfish nodes participating in the forwarding game.

In [27], Urpi *et al.* attempt to describe a more general framework than the one previously presented. Again, the nodes constitute the players of the game and are divided into K energy classes. They are assigned with a parameter that represents how critical the energy consumption is for them. Unlike [25], the energy class of each node is not known to the rest of the nodes. Thus, players have only beliefs (i.e., distributions) about the other players' energy classes (a game with imperfect information).

A node h is assumed to maintain the following information at the beginning of frame k (time is slotted):

- The set of its neighbors $N_h(k)$.
- Its remaining energy $B_h(k)$.
- The number of packets that the node generated and that have to be sent to its neighbor b $T_h^b(k)$.
- The number of packets neighbor b forwarded for the node during the previous frame $F_b^h(k-1)$.
- The number of packets that the node received from neighbor b as a final destination $R_h^b(k-1)$.
- The number of packets that the node received from neighbor b as a final destination, and the neighbor was the source of the packet $\widetilde{R}_h^b(k-1)$.

The action of each player in the game is to decide how many packets originated by himself will be transmitted to each one of its neighbors during the current frame and the number of packets he will forward for each one of his neighbors. The payoff of a node is a combination of two parameters. The first one is the ratio of the packets the neighbors forwarded for the node or received as a final destination and the number of packets sent by the node (w_k). The second one is the ratio of the sent packets to the packets that the node desired to sent (g_k), both calculated in each time frame. Each player's energy class $e(i)$ is considered secret and is not known to the other players. The dynamic nature of the ad-hoc network is taken into account as well by using a discount parameter δ. The higher the mobility of the network, the smaller the parameter δ. All these parameters are used in the following expression:

$$\alpha_{e(i)} W_i(t_k) + (1-\alpha_{e(i)}) G_i(t_k) \tag{46}$$

where

$$W_i(t_k) = \begin{cases} w(k), & \text{if } S_i(t_{k-1}) + F_i(t_{k-1}) > 0 \\ 0, & \text{otherwise} \end{cases} \tag{47}$$

$$G_i(t_k) = \begin{cases} g(t_k), & \text{if } \sum_{j\in N_i(t_k)} T_i^j(t_k) \\ 0, & \text{otherwise.} \end{cases} \tag{48}$$

The authors use the above model to prove some very interesting theorems. First of all, the intuitive conclusion that it is not possible to force a node to forward more packets than it sends (on average at least) without a cooperation enforcement mechanism. They also show that it is possible to enforce a cooperative strategy only for low-mobility networks or networks exchanging huge amount of traffic during each frame, if non-cooperation is punished from other nodes by not forwarding packets for this node for some time. They conclude that a cooperative strategy can be fruitful only if a significant percentage of the nodes adopt it. Finally, they propose a strategy for a simple two player game with two energy classes where initially every node forwards all packets and then cooperates only if the other node cooperates too, otherwise it punishes the other node by not forwarding packets for it, until it starts cooperating.

A difference worthy of note between this model and the one in [25] is that in this case nodes prevent themselves from transmitting a packet if the probability that it will not reach the destination is high. Another significant difference is that no global knowledge is assumed. All parameters depend on information that can be locally collected and easily evaluated, leading to a more realistic model.

B. Dynamic Bayesian Games

In order to extend the above models to form dynamic stage games with incomplete information, Nurmi [28] proposed a new one where the decisions are not taken simultaneously, as in the previous cases. This discrete time model demands from each node to keep track of all packets it has sent in each time slot to each one of its neighbors. Again, each node has a belief about the other nodes' energy classes. A source uses the beliefs about the forwarders' energy classes to decide if it will send a packet. Similarly, a relay node uses its belief about the source's energy class to decide if it will forward the packet or not. However, these beliefs are not arbitrary; they are dependent on the packets sent by each node.

What is interesting in this model is that the strategy of each player (the number of packets that are sent) is not fixed or predetermined according to an algorithm, but it is a probability, whose distribution depends on the history of the game, the energy classes of the nodes and the actions of other nodes. The information collected by each node about the other nodes' status increases with time and the beliefs are updated using the Bayes' rule. The subgame perfect and perfect bayesian equilibria can be used to analyze the model under specific functions. The author argues that his proposal can be used to analyze routing algorithms, as the utility functions and the probability distributions addressed in the model are not specified.

More specifically, the time is considered discrete, with each time step denoted by t_k, $k = 1, 2, \cdots$. The players of the game are the nodes, forming the set $\mathcal{N} = \{1, 2, \cdots, N\}$. For a node $i \in \mathcal{N}$, its set of neighboring nodes (nodes that are within its communication range) is denoted as Γ_i. Each node has an energy class $\theta_i(t_k)$, which corresponds to its energy level at each time step. Node i generates $g_i(t_k)$ messages at time step t_k, but the messages-packets that are actually sent to the network are denoted by $s_i(t_k) \leq g_i(t_k)$. Every node has an action history $h_i(t_k) = \{s_i(t_1), s_i(t_2), \cdots, s_i(t_{k-1})\}$, a vector containing the sent messages at all previous time steps. A message is broadcasted by its source to the entire neighborhood and each neighbor individually decides whether to forward the message or not. We denote the number of packets neighbor j forwarded for node i at time step t_k by $f_j^i(t_k)$. The number of packets that neighbor j forwarded for source i at every time step forms a vector $h_j^i(t_k) = (f_j^i(t_1), f_j^i(t_2), \cdots, f_j^i(t_{k-1}))$ and the history profile is the vector $\overline{h}_j^i(t_k) = (h_i(t_k), h_j^i(t_k))$. The source node does not send all its traffic. It decides how many packets to send according to its belief about the energy classes of its neighbors, which is rational since if all its neighbors have depleted their energy, no packet will be forwarded and thus the source consumes energy unnecessarily. However, provided that the energy classes of the nodes are private information and hidden from the rest of the players, every node i has a belief about another node j energy class, which is represented by a probability distribution μ_i^j function:

$$\mu_i^j(t_k) = p\left(\theta_j^i(t_k)|\theta_i(t_k), \overline{h}_j^i(t_k)\right) \tag{49}$$

where $\theta_j^i(t_k)$ is the energy class of neighbor j forwarding packets for node i at time step t_k.

Similarly, the decisions of the neighboring nodes depend on their beliefs about the energy class of the source, which is a probability distribution function $\phi_j^i(t_k)$ as well. These probabilities depend on the number of packets send by the source, hence

$$\phi_j^i(t_k) = p\left(\theta_i(t_k)|\theta_j^i(t_k), \overline{h}_j^i(t_k), s_i(t_k)\right). \tag{50}$$

Hence, the joint belief system for nodes i and j can be represented as $\overline{\mu}_j^i(t_k) = \left(\mu_i^j(t_k), \phi_j^i(t_k)\right)$. Each sender-forwarder pair play a Bayesian game, so each node participates in more than one such game at the same time. What is more, a node can be a sender in a game and a forwarder in another one. The actions of the players are defined as probabilities $p_x(a_x|\overline{h}_j^i(t_k), \theta_x)$, where a_x represents the action, which is the number of packets sent or forwarded. The vector $\overline{u}_j^i = (u_i, u_j^i)$ consists of the utility u_i of the sender i and the utility of node j forwarding packets for node i u_j^i. At the end of each time step, the belief system is updated using the Bayes's rule as follows:

$$\mu_i^j(t_{k+1}) = \frac{p(h_j^i(t_k), f_j^i(t_k)|\theta_j^i(t_k))p(\theta_j^i(t_k))}{p(h_j^i(t_k), f_j^i(t_k))}. \tag{51}$$

The concept of Bayesian equilibrium and sequential equilibrium can be used to analyze the game and find its equilibria. If the restriction of the strategies to only one stage (one time step) constitutes a NE, then the game is *subgame perfect*. Now, if the players' actions, restricted to one stage, are optimal given the beliefs of the players at the beginning of the stage, then these actions form a *perfect Bayesian equilibrium* (PBE). It is proved that this model admits a PBE. The same author proposes in [29] a more specific set of strategies for the forwarder and the sender that are proved to converge to a sequential equilibrium and that at least one such equilibrium exists. The difference is that the forwarders are assumed to lack beliefs about the energy class of the source.

C. Topology Aware Approaches

The forwarding game has also been addressed in [30], where the authors attempt to answer the question whether cooperation may arise from the network on its own or an incentive mechanism is required, by taking the topology into account. The players are the nodes and each node h selects a strategy for each time instance, i.e., the probability $p_h(k)$ of forwarding packets for other nodes. The players do not distinguish between different nodes and their decision on the forwarding probability is applied to every route that includes the node. The payoff of each player is the sum of the gain of delivering packets to destinations and the loss of forwarding packets for other players. Nodes update their information in every time slot according to their experienced throughput in the previous one and choose their cooperation level for every slot. An infinite number of different strategies can be defined within the model, for example the "always defect," "always cooperate," TFT or GTFT.

A metamodel is introduced to formalize the evolution of the cooperation levels of the nodes. The relationship between the nodes is represented by a dependency graph and a machine with inputs and outputs is assigned to each vertex of this graph (corresponding to nodes). The internal of each machine consists of a multiplication gate that multiplies the inputs. This product is passed to a gate that implements the strategy function of the node the gate corresponds to. The result is the output, which is passed to the other nodes as input via the links of the dependency graph. In this way, the entire network is modeled as an automaton with discrete states that correspond to the cooperation levels of the nodes. Letting the automaton operate in time provides us with the evolution of the states of the machines in time, which is the cooperation level of the nodes.

After this new model introduction, the authors focus on finding possible NEs. In order to model a finite game, an infinite game with a discount factor δ is used. The final payoff is computed as a weighted sum of the payoff in each time slot. The weight is the discount factor to the power of the time slot index, so that the most important values are the older ones, as indicated in (3). Using the game model and the metamodel for stationary networks, they confirm that "always defect" is a NE, as previously stated. However, they prove that another interesting NE may exist. Assuming that node h is a relay node on a path r from a source src to a destination dst, then all nodes play TFT is a NE if:

- There is always a dependency loop between h and the src of every route r where h is a forwarder.
- The maximum forwarding cost for node h on every route r where it is a forwarder must be on average smaller than its possible future benefit.

A similar equilibrium exists in the case a node is the source of more than one routes. Although theoretically cooperation may arise in a stationary network, the authors argue that in practice this is rarely the case. Thus, for stationary networks a cooperation enforcement mechanism is almost always necessary.

Using the same methodology as in [30], the same authors study the mobile case in [31], using the benefit-cost utility function and a topology that constantly changes without considering the energy levels of the nodes. They use $\beta_i(k)$ to denote the number of packets that were originated by node i and were successfully sent to the destination until time k and $\gamma_i(k)$ to denote the number of packets node i forwarded for other nodes. Their ratio $\rho_i(k) = \beta_i(k)/\gamma_i(k)$ is called *interaction ratio* at step k. The utility function is defined as:

$$U_i(k) = \frac{B\beta_i(k) - C\gamma_i(k)}{k} \tag{52}$$

where B and C are two constants representing the benefit of successfully sending a packet and the forwarding cost respectively, and each node's objective is to maximize the expected value of this cost as the time tends to infinity. The strategy of each player is to forward packets when the interaction ratio is above a specified threshold κ_i, otherwise the node refuses to cooperate. Varying the threshold κ_i, different strategies could be modeled. In general, κ_i is considered a function of the average number of forwarders per source-destination pair. If a player belongs to more routing paths than the average number of relays per path, then generosity is required. Since this condition happens more often when mobility is low, the main conclusion of this work is that the higher the mobility level, the easier the emergence of cooperation in a mobile ad-hoc network, or the less generosity is required from the players.

This result, though, seems contradictory with the results in [27], where the authors claim that cooperation enforcement is easier in low-mobility networks. They conclude that when using only local information, some time is needed in order to successfully punish a node for misbehaving. And this can happen only when nodes do not move very fast and the neighborhood remains constant for some time. To explain this disagreement, we need to pay more attention to the models and especially to the strategies. In [27] the authors assume that every node maintains for each of its neighbors the number of packets forwarded for the neighbor's sake and the number of packets the neighbor forwarded for the node. What is more, a misbehaving node is punished by its neighbors by not forwarding its packets for some time. Thus, if the mobility is low, the node has enough time to punish a misbehaving neighbor. On the other hand, in [31] the authors assume that each player maintains information about the history of the game; however, players do not distinguish between the nodes. A node needs to be generous if it is a forwarder to more routing paths than the average number of forwarders per path. The higher the mobility, the less this situation lasts. Concluding, the two works are not contradictory. They reveal that the strategy space of the game has a great influence in the conditions required for cooperation.

D. Minimax Games

Using a different approach, the authors in [32] propose the modeling of an ad hoc network routing as a two-person zero-sum minimax game between the set of nodes and the network. In a minimax game, the players' target is to maximize the guaranteed minimum gain. According to their proposal, all nodes in the network constitute a single player (the *set-of-routers*) that runs the routing protocol or technique. This player's move consists in sending all the routing messages specified by the routing protocol. The second player (the *network*) changes the network topology by deciding which link between nodes will be up and active. The set-of-routers wins the game if all nodes maintain a correct view of the network when the game ends. On the other hand, the network wins if the nodes are mistaken about the network status and are unable to obtain the correct one. The cost function is a lexicographic ordering of the following measures:

- The final state's inconsistency of the network topology compared to the actual state of the network
- The amount of traffic the routes used.

The network tries to maximize the cost function while the set-of-routers tries to minimize it.

In order to evaluate the model, the authors implemented three different routing techniques, the link-state routing, the reverse-path forwarding (RPF) routing and the distance vector routing, for a 7 node network. The target of the modeling was twofold. The first goal was to reveal if a protocol is sound or not. The term soundness refers to the routers having or being able to obtain a correct view of the network topology. The second goal was to analyze the protocol's performance, namely the speed of convergence and the overhead induced.

Using a custom tool they developed, the authors first analyzed soundness. They found that RPF was always sound while the distance vector technique suffers from the well-known count-to-infinity problem, even for the split horizon variant. Regarding overhead, they confirmed the common belief that flooding needs more overhead that RPF. In terms of convergence, it was found that link state RPF performs better than distance vector and link state flooding.

E. Analysis of Reputation Enforcement Mechanisms

In [33], the authors attempt to analyze the CORE protocol (or any other history-based protocol) by means of game theory analysis tools. CORE [34] is a reputation-based cooperation enforcement mechanism. Every node monitors its neighbors' behavior and rates it. Only nodes whose reputation is greater than a predefined threshold are served, while the other nodes are gradually isolated unless they alter their behavior and start cooperating. Two approaches are used; a cooperative and a non-cooperative one.

The authors propose to model the CORE algorithm as a mechanism that introduces identical ERC types in every node in the network and to study a static PD game with ERC preferences. In ERC [35] theory the utility of an agent is not only based on its own absolute payoff, but also on the relative payoff with respect to the total payoff of all agents. So, a node's utility function comprises two parts. The first one is the absolute payoff and the second one is the relative payoff, computed with respect to the payoff of all other nodes in the network. When the number of cooperating nodes is k, the payoff to a non-cooperating node is $B(k)$, while an additional cost $C(k)$ is subtracted when the node cooperates, hence its payoff is $B(k) - C(k)$. The following assumptions are made:

- $B(k+1) - B(k) < C(k+1)$ (or $B(k+1) - C(k+1) < B(k)$).
- $NB(k+1) - (k+1)C(k+1) \geq NB(k) - kC(k)$.
- $B(k+1) - C(k+1) \geq B(k) - C(k)$.

The first one specifies that playing cooperatively reduces the absolute payoff, regardless of the number of cooperating nodes. Hence, this assumption alone does not provide an incentive for cooperation. So, the two other additional assumptions are needed. The second one denotes that the higher the number of cooperating nodes, the greater the accumulated payoff for all users, which is a desired result from the social point of view. Finally, the third assumption specifies that from the point of view of a single node, the more nodes cooperate in the network, the larger its payoff. The analysis of a one-shot N-person PD game shows that 1) there is at least one equilibrium where everybody defects and 2) there is a NE where at least half of the nodes cooperate.

Considering a cooperation game, the authors prove that there is a unique equilibrium as long as at least one node's payoff is the result of its absolute payoff. This means that if the ERC preferences are introduced, the cooperation effort does not change. Thus, CORE assures that a coalition size of at least half of the nodes exists. A more realistic modeling of CORE demands the usage of non-cooperative game theory. Based on a non-cooperative iterated PD game model, it was proven that TFT strategy is in fact a special case of the CORE strategy. Furthermore, CORE is robust against imperfect knowledge about the other nodes' moves.

Milan *et al.* investigate in [36] the effect of packet collisions on the cooperation level achieved by reputation based mechanisms. According to the model, node's i probability of dropping a packet at time k is $p_i^{(k)}$, its payoff for playing the strategy $s_i^{(k)}$ is $u_i = \beta p_i^{(k)} - \alpha p_{-i}^{(k)}$ and the discounted payoff of user i is

$$U_i = \sum_{k \geq 0} \delta^k u_i^{(k)}, \quad 0 < \delta < 1. \tag{53}$$

In order to model packet collisions (due to the mac layer operation), the iterated PD with noise model is used [37]. The authors prove that for a linear network, the TFT strategy is not enough for the mutual cooperation to emerge. On the other hand, using GTFT, mutual cooperation is a subgame perfect equilibrium if $(\beta/\alpha)/(1-\lambda)^2 < \delta < 1$, where λ is the generosity factor. In other words, the higher the traffic in the network, the more far-sighed the nodes have to be, in order to achieve cooperation. This is equivalent of requiring each packet to have a sufficiently high value with respect to the transmission cost. For a more realistic topology, the above expression is transformed to the following:

$$\frac{\beta}{\alpha} \frac{1}{(1-\lambda)^{2n}} < \delta < 1 \tag{54}$$

where n is the number of potentially colliding neighbors. Additionally, they propose two other strategies, the one-step trigger (OT) and the grim-trigger (GT). Both of them achieve cooperation with less strict requirements than GTFT. However, this result comes at the expense of the potential serious performance degradation.

Based on the previous works, Altman *et al.* propose in [38] a less aggressive punishment policy than in [25]. The model assumes that a fixed forwarding probability γ_j is selected by a node for all packets, independently of their source. The utility function includes three parts: The reward as a source, the reward as a destination (which was not previously considered) and the loss as a forwarder. Each node is able to compute the equilibrium forwarding probability; however, in the absence of an incentive mechanism, this probability is zero. Thus, a punishment mechanism is proposed. According to it, if a node is found to decrease its forwarding probability, then all other nodes decrease the forwarding probability for the packets whose source is the deviating node. A distributed implementation of the algorithm is also provided. The model lacks general applicability, since the forwarding decisions are assumed independent of the source and the dynamic structure of the routing decisions is not taken into account.

A generalized approach for packet relaying is presented in [39]. The strategy for each node is to decide if it will forward packets or not. A reward mechanism is assumed, that is a node i gets its reward r for enforcing node j to forward packets. Packet forwarding induces a cost of s for the forwarding node. The interaction between the nodes is assumed bidirectional. For a single-stage game, the equilibrium corresponds to the refusal of participation from all nodes. For the repeated game, the authors map the action selection to a state transition process modeled by a Markov chain when the choices are memoryless, i.e., they depend only on the current state to select the next one. Within this setting, the always forward strategy is an equilibrium too. Generalizing the model, a probability p of forwarding a packet is decided by every node. The benefit of a node at time t_n can be expressed as $b_i^{t_n} = r \cdot p_{-i}^{t_n} - s \cdot p_i^{t_n}$. When the game is played repeatedly, the total benefit will be

$$B = \sum_{n=0}^{\infty} \delta^n (r \cdot p_{-i}^{t_n} - s \cdot p_i^{t_n}) \tag{55}$$

where δ is the discount factor.

Using this simple framework, many strategies can be modeled. For instance, the TFT strategy corresponds to a probability

$$p_i^{t_n} = \begin{cases} 1, & n = 0 \\ p_{-i}^{t_{n-1}}, & n > 0. \end{cases} \tag{56}$$

Modeling other strategies in a similar way, the authors provide the necessary conditions that r and s should satisfy, in order for these strategies to be beneficial.

F. Inter-Cluster Routing

Kannan *et al.* [40] formulate a game theoretical model to study inter-cluster routing in sensor networks. In particular, they consider the effects of *path length* and *path energy cost* at the same time. Although the shortest path is generally desirable, it is not the best option from the energy efficiency point of view. The model consists of a set of *leader nodes* (the cluster-heads) and *sensor nodes*. The players are the sensor nodes who should decide to forward or not a packet to another sensor for each pair of leader nodes. Based on the forwarding decisions of the sensor nodes, a path P is formed for every source-destination pair of leader nodes. The payoff of sensor i forwarding the packet to sensor j is defined as $\pi_i = E_j - \xi L(P)$, where E_j is the residual energy of sensor j and $L(P)$ is the length of the path P. ξ is a non-zero constant parameter that represents the portion of the path length cost that will be paid by sensor i. In this case, the NE corresponds to the optimal length-energy constraint (LEC) for the given pair. The authors propose a distributed implementation of a protocol that calculates the optimal LEC.

G. Alternative Utility Functions

Based on the work of [25], the authors of [41] propose a Pareto utility function. They define two parameters, the average amount of throughput per unit of energy expenditure $U_h^j(k) = A_h^j(k)/\rho_j$ and the average amount of relay rate per energy expenditure $C_h^j(k) = B_h^j(k)/\rho_j$. The first one defines a measure of the help provided by others to node h and the second one corresponds to the assistance provided by node h to other nodes. Assuming that the probability of accepting a relay request is τ_j, then the average energy efficiency per slot spent by node h as a source when participating in a type j session is

$$\begin{aligned}\Gamma_{jh}^{(s)} &= \frac{1}{N}\lim_{k\to\infty} U_h^j(k)\\ &= \frac{1}{N}\frac{1}{\rho_j}\sum_{m=1}^{M} q(m) \sum_{p_1,\cdots,p_j} \theta(m;p_1,\cdots,p_j)\tau_j^{p_1+\cdots+p_j} \quad (57)\end{aligned}$$

where $1/N$ is the probability that node h is a source, $q(m)$ denotes the probability that the source requires m relays and $\theta(m;p_1,\cdots,p_j)$ is a probability function conditioned that the type j class has m relay nodes. The number of selected nodes in classes $1,\cdots,j$ are denoted by $p_1,\cdots,p_j$, respectively. Finally, $\tau_j^{p_1+\cdots+p_j}$ is the probability that all relay nodes accept the relay request. In the case of a relay node, the energy efficiency per slot has the following form:

$$\Gamma_{jh}^{(r)} = \frac{1}{N\rho_j}\sum_{m=1}^{M} mq(m) \sum_{p_1,\cdots,p_j} \theta(m-1;p_1,\cdots,p_j)\tau_j^{p_1+\cdots+p_j}. \quad (58)$$

Now, the average energy per slot consumed by a node when participating as a source in all types of sessions is computed by $\Gamma_h^{(s)} = \sum_{j=1}^{K} \rho_j \Gamma_{jh}^{(s)}$ and a similar expression exists for the case of a relay node. The PO probability τ_j^* can be computed if we consider the limitation $0 \le \tau_j^* \le 1$ and the fact that the equation $\Gamma_{jh}^{(s)} + \Gamma_{jh}^{(r)} = \rho_j$ should be true for every $j \in 1,\cdots,K$.

H. Multi-Domain Routing

An interesting point of view is presented in [42]. The authors cope with the problem of packet forwarding in large scale sensor networks, where not all nodes are managed by a single authority, instead multiple authorities are involved, i.e., a multi-domain network. Following the work of [43] where multi-domain networks are examined, the authors investigate the routing problem under the framework of evolutionary game theory. Their examination differs from the classical concept in the sense that the most important part in their model is not the node itself but the class. A class is defined as a set of sensor nodes belonging to the same administration authority and having full cooperation among them. The time is slotted and in the beginning of each slot, every class decides whether to cooperate or not with all other classes. In the case both classes cooperate, each one gets a reward $R = \gamma - \beta$, where γ is the gain of cooperating and β is the cost of forwarding other classes' packets. If both defect, then both get a reward of zero. Now, if only one cooperates, the cooperating class gets $S = -\beta$, as it gains nothing, while the non-cooperating class gains $T = \gamma$, as it has no cost. This description resembles that of PD game, where each player either cooperates or defects. Among all possible strategies, the authors consider only two of them: Always cooperate ($S_1 = \{C,C,C,\cdots\}$), and always defect ($S_2 = \{D,D,D,\cdots\}$). The forwarding game has now the form of the iterative PD game and can be examined in a similar way.

I. Imperfect Information Repeated Games

A parameter that is usually omitted in most of the related works is that of imperfect knowledge. In a real ad hoc or sensor network, the multihop nature of communication combined with a high-error-rate physical channel may frequently result in lack of perfect knowledge. This issue is taken into account in [44]. The authors use a model of two players. One player is the source of a packet and the other is a relay node. Each player has two possible actions, F (forward) and D (drop). Each player is informed about the other player's action via a private signal from the set $\Omega = \{f,d\}$. Let p_f denote the probability that an F function is considered as D and p_e denote the probability that a D action is considered as F by the other player. The cost for packet forwarding is c while the gain for the forwarded packets is g. The expected payoff $u_i(\alpha)$ for player i when the strategy profile is α and the observed actions are ω_i is

$$u_i(\alpha) = \sum_{\omega_i\in\Omega} \widetilde{u}_i(\alpha_i,\omega_i,\alpha_{-i})\text{Prob}\{\omega_i|\alpha_{-i}\} \quad (59)$$

where $\widetilde{u}_i$ is the utility function of player i. For the static game, the NE is (D,D), as previous works have mentioned too, while the best outcome $(g-c, g-c)$ is achieved for the strategy profile (F,F).

Now, when the game is repeated, using the discount factor δ and the private history h_i^t for node i, the expected payoff is given by the following expression

$$u_i(\alpha) = (1-\delta)\sum_{t=0}^{\infty} \delta^t u_i^t\left(\alpha_1^t(h_1^t), \alpha_2^t(h_2^t)\right) \quad (60)$$

where ω_i^t is the observation of player i at time t. If the information of all players would be perfect, then the Folk theorem could be used to find an equilibrium that would enforce a cooperation strategy for a discount factor close to 1. However, this is not possible in the case of imperfect information.

The effect of imperfect information is examined in [45] as well. The players of the game are the nodes that participate in the network. If a_i is the action of player i and P is a random public signal with value p, then the utility $U_i(a,p)$ of player i will depend on both of them and in its simplest form can be expressed as the difference between the benefit $\alpha_i(p)$ corresponding to the packets forwarded from other nodes and the cost $\beta_i(a_i)$ of forwarding packets for other nodes. Letting $F(p;\mathbf{a})$

denote the distribution of parameter p, influenced by the action profile of all nodes' $\mathbf{a}$, the expected utility π_i for node i will be

$$\pi_i = \int_{p\in\Omega} U_i(a_i, p)\, dF(p; \mathbf{a}). \tag{61}$$

Moving from the static game to the repeated game, we denote by $\sigma_i(t)$ the strategy of node i at each stage t of the game and by $\sigma(t)$ the joint strategy vector at stage t. The repeated game payoff for node i is calculated using the discount factor δ:

$$v_i(\sigma) = (1-\delta)\sum_{t=1}^{\infty} \delta^{t-1}\pi_i(\sigma(t)). \tag{62}$$

A strategy $\sigma^* = \{\sigma_i^*, \sigma_{-i}^*\}$ is a *perfect public equilibrium* (PPE) if for all stages after a stage t_0:

$$v_i(\sigma^*) \geq v_i(\sigma_i', \sigma_{-i}^*),\ \forall \sigma_i'. \tag{63}$$

The authors propose a method to reduce the repeated game to a single stage game, so that the analysis becomes easier. They express the expected payoff as

$$\pi_i^w(\mathbf{a}) = (1-\delta)\pi_i(\mathbf{a}) + \delta \int_{p\in\Omega} w_i(p)\, dF(p; \mathbf{a}) \tag{64}$$

where $w_i(p)$ is a continuation payoff function that maps every possible value of the public signal to a payoff real value. Then, the repeated game payoff can be written as

$$v_i(\sigma) = (1-\delta)\pi_i(\mathbf{a}) + \delta \int_{p\in\Omega} w_i(p)\, dF(p; \mathbf{a}). \tag{65}$$

The above expression reveals that the strategy σ is a PPE if the action profile $\mathbf{a}$ is a NE at the first stage of the game. Then, the authors propose the GT strategy, which corresponds to the strategy where a player cooperates provided that the public signal exceeds a threshold value. Thus, when the public signal is below this value, the player switches to the NE of the one-stage game. The authors use the aforementioned analysis to examine this strategy and prove that it is a PPE, under conditions that are further studied by means of simulations. The contribution of this work is that it provides a methodology to model repeated games with incomplete information by using a public signal and study them in a relatively simple way by taking into account the NE of the corresponding single stage game.

J. Multicast Tree Formation

The construction of an optimal energy-constrained reliable query routing (RQR) tree in sensor network is studied in [46]. The sink node broadcasts a query to all sensors and a multicast tree is constructed, which will be used from the sensors to send back to the sink their data matching the query's requirements. Along this path, data aggregation takes place.

The players of the game are the N sensors. Two types of cost are used. The communication cost c_{ij}, which corresponds to the energy needed for node i to reach node j and the latter can fully recover the data, and the participation cost PC_i, which refers to the cost of participating in the routing tree and is a function of the remaining battery life, the traffic flow through the node and the currently consumed processing power. The strategy of node i is a binary vector $s_i = \{s_{i1}, s_{i2}, \cdots, s_{iN}\}$, where $s_{ij} = 1$ ($s_{ij} = 0$) corresponds to sending (not sending) a data packet to node j. Although this is a pure strategy, a mixed strategy can be defined as well. Since usually a node forwards packets to only one of its neighbors, the vector s_i has all its elements equal to zero except one. As a consequence, the strategy profile for all nodes $\mathbf{s}$ represents a tree originated at the sink node.

Since reliability is considered as a critical issue, the authors model a node failure by assuming that a sensor i can fail with probability $(1-p_i) \in [0,1)$. Every sensor is interested in sending its data to the sink node via the constructed tree $\mathcal{T}$, however, driven by its selfishness, its objective is to achieve it with the minimum possible cost. Its benefit X_i will be a function of the reliability level R_i of the path from i towards the sink and the expected value g_i of the information that can finally reach the sink. A value v_i, $i \in \{1, 2, \cdots, N\}$ is assigned to the data retrieved by node i and match the query's attributes. Thus, $g_i = g_i(v_1, v_2, \cdots, v_N)$ and $X_i = g_i(v_1, v_2, \cdots, v_N)R_i$. The benefit is modeled via two functions:

$$V_i^I = v_i + \sum_{j\in F_i} V_j \tag{66}$$

$$V_i^{II} = v_i + \sum_{j\in F_i} p_j V_j \tag{67}$$

where F_i denotes node's i parent node in the tree $\mathcal{T}$. Finally, the payoffs are given by

$$\Pi_i(\mathbf{s}) = \begin{cases} X_i - (c_{ij} + PC_i), & \text{if } i \in \mathcal{T} \\ 0, & \text{otherwise.} \end{cases} \tag{68}$$

The difference between the two models is that the second one takes into consideration the reliability of the path followed until it reaches node i. Thus, its choice will be affected not only by the reliability level of its path towards the sink, but also by the reliability of the path followed so far to reach the node. Thus, the second model has some kind of "memory," while the first one is only interested in the data reaching a node, where the decision depends only on the reliability of the ongoing path.

The well known concept of NE in this case corresponds to the optimal RQR, where every node gets the maximum payoff, provided that it does not negatively influence the payoff of any other node, resulting in a suboptimal performance. The first benefit model results in less but more reliable equilibrium paths than model II, because it permits more edges to become part of the routing tree. Although the computation of the equilibrium paths is generally difficult, the authors propose a method to find the optimal paths in polynomial time when geographic routing is used.

K. Auction-Based Approaches

Another popular way to model ad-hoc network routing is by means of auction theory [47]. In [48], the authors propose a mechanism for route discovery and route maintenance (Ad hoc-VCG) based on the Vickrey-Clarke-Groves (VCG) auction [47], where the payment for a good includes the externality the winning bidder imposes to all other bidders. Each time a source has packets for a destination and a new path should be established, an auction takes place. Every node declares its own *cost-of-energy* (the cost assigned by a particular node to every unit of energy spent) c_i and the transmitted power P_i^{emit}. A set of rules is applied for forwarding route requests. The destination node collects all the possible paths and decides which one is the shortest $(SP = \{\sigma_1, \sigma_2, \cdots, \sigma_k\})$. Then, it computes the payment M_{σ_i} to each intermediate node σ_i of the shortest path, using (69):

$$M_i = |SP^{-\sigma_i}| - |SP| + c_{\sigma_i} P^{\min}_{\sigma_i,\sigma_{i+1}} \quad (69)$$

where $|SP|$ is the total cost of the shortest path, $|SP^{-\sigma_i}|$ is the cost of the shortest path if node σ_i is excluded from the set of nodes, c_{σ_i} is the cost-of-energy declared by node σ_i and $P^{\min}_{\sigma_i,\sigma_{i+1}}$ is the minimum power required for node σ_i to reach the next node on the shortest path σ_{i+1}. The authors prove that every node declares its true cost-of-energy and its true emission power, and that the mechanism they propose is truthful in general.

An interesting observation in this model is the *overpayment.* The payment to every intermediate node is always higher than their forwarding cost. The additional amount payed to an intermediate node depends on its contribution to the decrease of the total cost of the path. The authors prove that the ratio of the total overpayment cost over the true cost of the shortest path is upper bounded. However, the drawbacks of their proposal is that the communication sessions should last for long time and the routing paths should not change a lot during a session. Another disadvantage is the excessive overhead required, in order for the destination to obtain information from the entire network.

Extending this work, Wu *et al.* [49] propose the usage of double auctions. The Ad hoc-VCG is suited for single source-destination pairs, in a "one-to-many" concept. However, the authors suggest that it would be more realistic to assume that in real ad hoc networks, a "many-to-many" relation exists, as many source-destination pairs compete for the available routes and resources of the network. Therefore, they propose the usage of modified double auctions. Their philosophy is to be able to submit a bit asynchronously and to permit counteroffers.

Ji *et al.* [44] describe an auction for ad-hoc network routing following a different way. Instead for each particular node to be the bidder, they assume that the bidders are the available routes. The players of the game are the nodes and each node can only be of one of three types during a communication session: Sender, destination, or relay. Nodes are selfish and their intension is to always maximize their payoffs. The source-destination pair on one hand and the nodes on all the possible routes on the other hand constitute a non-cooperative game. Since the private type of a node is not known to the others, the game is with incomplete information. As previously mentioned, the bidders are not the nodes themselves but the available routes. In this way, the sender may exploit path diversity to increase its gain. The authors provide both the static and the dynamic game definition.

A different approach for the auctioneer is proposed by [50]. Every node in the network organizes an auction offering its resources (usually the bandwidth). The senders bid for the resource of the nodes, in order to form the paths towards the destinations. A path will only be available to a sender if it wins the auctions of the intermediate nodes. The bandwidth of each node is divided into pieces and the bidders get the percentage they want at the price of their bid, starting from the sender with the highest bid, until the available bandwidth is exhausted.

In contrary to previous works, the authors of [51] argue that it is very hard to apply VCG pricing to ad hoc networks and inter-domain routing. More specifically, the authors discuss two cases. The first one is the case of resource constraint ad-hoc networks, where the application of VCG auctions with N players requires solving $N + 1$ optimization problems in order to find the amount the traffic sender should pay. However, these optimization problems are usually hard to solve for general topologies. The second case refers to inter-domain routing, where four properties should be applied at the same time: Incentive compatibility, efficiency, individual rationality and budget balance. The authors conclude that it is not possible to achieve all four of them at the same time, for networks without resource constraints. The reason is that the sum of subsidies needed to be payed to the forwarding nodes is higher than the sum of charges payed by the senders, which means that an external resource is always needed to inject money into the network, which is not realistic. As a result, the authors propose to relax one of the previous constraints, namely efficiency.

L. Pricing Game Models

In [52], the pricing game in an ad-hoc network of one source and one destination is studied. A source src wants to send data to destination dst at a rate R_s using intermediate nodes as relays. For a link (i, j), i is the predecessor of j and j is the offspring of i. For an arbitrary node i, its set of predecessors is $\mathcal{P}_i$ and its set of offspring nodes is $\mathcal{O}_i$ and f_{ij} represents the flow on the edge (i, j). A given flow vector f is a *routing* of a session if it satisfies the flow conservation constraints: $\sum_{h\in\mathcal{O}_s} f_{sh} = R_s$, $\sum_{h\in\mathcal{P}_w} f_{hw} = R_s$ and for each relay node i, $\sum_{h\in\mathcal{P}_i} f_{hi} = \sum_{j\in\mathcal{O}_i} f_{ij} = r_i$, where r_i is the incoming flow rate at i. Each link (i, j) is associated with a strictly increasing cost function $\mathcal{D}_{ij}(f_{ij})$ and it is private information to nodes i and j only. The network's total cost is then $\sum_{(i,j)} \mathcal{D}_{ij}(f_{ij})$ and the social optimal routing is the one that minimizes this cost. However, the source and the relays behave selfishly. The pricing game constitutes in each relay i announcing a charging function $P_i^h(f_{hi})$ specifying the payment it demands if node h forwards data to i at rate f_{hi}. The pricing game begins with each relay declaring its bid to its predecessors. Upon reaching the source, the latter decides on an flow allocation that achieves the minimum cost. When a relay receives the payment and the corresponding traffic, it allocates the incoming traffic to its offsprings in a way that minimizes the total payment. The payoff for each relay is the profit obtained by serving the traffic. Using nonlinear charging functions instead of constant unit price the authors claim to lead to a much richer set of results and that the social optimal point always corresponds to an equilibrium. Nevertheless, inefficient equilibria can always exist. What is more, they demonstrate that the price of anarchy due to multihop communication is unbounded.

A model that attempts to incorporate packet loss probability along with imperfect knowledge is proposed in [53]. In this work, the authors are based on the Aoyagi oligopoly game, modifying it to correspond to the characteristics of multihop wireless networks. The game consists of two kinds of phases, collusion and punishment. The game starts with a collusion phase. The players select their prices p_i and privately observe their own demand d_i, which depends on the price profile $p = \{p_1, p_2, \cdots, p_N\}$. At the end of each stage, each player is to make a public report following a particular rule. The rule corresponds to specifying if the demand of every node is higher than a predefined threshold. If the reports are unanimous, the game continues to the next collusion stage. Otherwise, a punishment phase begins. Hence, for each player it is desirable not to enter the punishment phase, but also to maximize its profit. The threshold m^* that maximizes the probability of unanimous reports is the collusive reporting threshold and corresponds to a price profile p^* where all players collaborate. If a set of expressions is satisfied, we can set a threshold that would correspond to an equilibrium price profile, where no deviation would be beneficial. The authors propose to draw an analogy between prices and demands to packet loss probability and received packet count respectively, in order for the model to be exploitable in the wireless multihop networks setting. However, since a global signal is not possible, the reports and punishments are limited in local regions. Relay nodes actually make a mutual agreement on the packet loss probability and the authors prove that the corresponding reports on exceeding their received packet rate are truthful.

M. Flooding Models

Attempting to model flooding in ad hoc networks, Wang *et al.* in [54] define the forwarding dilemma game. The players are the nodes receiving a flooding packet and the game is played each time a node receives such a packet. The available strategies S_i are either to forward the packet ($S_i = 1$) or to drop it ($S_i = 0$). If a node forwards a packet, it bares a cost $f_i(c)$, where c is the forwarding cost and $f(\cdot)$ is a decreasing function. If the packet is finally forwarded by any of the participating nodes, then the gain for an arbitrary node i is G_i. Hence, the utility of a node is

$$U_i(S_i) = \begin{cases} G_i - f(c_i), & \text{if } S_i = 1 \\ G_i, & \text{if } S_i = 0 \text{ and } \exists S_j = 1, j \neq i \\ 0, & S_j = 0, \forall j. \end{cases} \quad (70)$$

The game has N equilibria in pure strategies, each one corresponding to a single node forwarding the packet and the rest $N - 1$ nodes dropping it. In mixed strategies, where a node i forwards the packet with probability p_i, the expected utility will be

$$E\{U_i\} = p_i(G_i - f(c_i)) + G_i\left((1-p_i)\prod_{j\neq i}(1-p_j)\right). \quad (71)$$

Assuming that the costs and gains are the same for all nodes, then the game is symmetric and the equilibrium in mixed strategies corresponds to a probability of forwarding $p = 1 - (C/G)^{1/N-1}$, which is the same for all participating nodes. The authors exploit this result to formulate a mechanism that reduces the overhead of routing protocols when they flood routing packets into the network.

V. CONCLUSIONS

In this work, we attempted to present the most basic proposals for modeling routing in communication networks. In particular, using the conventional networks as a start, we continued by presenting the most popular models that are used in networks with dynamic topology, such as ad-hoc networks, or with energy constrained nodes, such as sensor networks. A general conclusion is that game theory provides the tools for analyzing selfishness and complex interactions between network nodes. This path reveals an aspect of network routing that could not have been analyzed in any other way. We believe that combining these techniques with other methods and analytical tools could result in a totally new perspective of ad-hoc and sensor networks routing.

Despite the results accomplished so far, there is space for more detailed investigation of the effects of selfishness in wireless ad hoc and sensor networks. Combined models of the network and the higher or lower layers could reveal the interactions between them and inspire mechanisms that could be used in order to force cooperation, e.g., by designing more sophisticated MAC protocols. Furthermore, topology changes seem to play a critical role in selfish packet forwarding that has not been investigated in detail yet.

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