Exact Bit Error Probability of Orthogonal Space-Time Block Codes with Quadrature Amplitude Modulation

Sang-Hyo Kim, Jae-Dong Yang, and Jong-Seon No

Abstract: **In this paper, the performance of generic orthogonal space-time block codes (OSTBCs) introduced by Alamouti [2], Tarokh [3], and Su and Xia [11] is analyzed. We first define one-dimensional component symbol error function (ODSEF) from the exact expression of the pairwise error probability of an OSTBC. Utilizing the ODSEF and the bit error probability (BEP) expression for quadrature amplitude modulation (QAM) introduced by Cho and Yoon [9], the exact closed-form expressions for the BEP of linear OSTBCs with QAM in quasi-static Rayleigh fading channel are derived. We also derive the exact closed-form of the BEP for some OSTBCs which have at least one message symbol transmitted with unequal power via all transmit antennas.**

Index Terms: **Bit error probability (BEP), orthogonal space-time block codes (OSTBCs), pairwise error probability (PEP), quadrature amplitude modulation (QAM), space-time block codes, space-time codes.**

I. INTRODUCTION

Multiple input multiple output (MIMO) systems with space-time codes which were introduced by Tarokh, Seshadri, and Calderbank [1], significantly outperform single transmit antenna systems due to their transmit diversity. Alamouti proposed a simple transmit diversity scheme with two transmit antennas which employs the 2×2 complex orthogonal design [2]. Tarokh, Jafarkhani, and Calderbank [3] generalized the Alamouti's scheme to space-time block codes from orthogonal designs which we call orthogonal space time block codes (OSTBCs). The OSTBCs have the advantage that they guarantee full diversity and low decoding complexity linear with respect to the number of the message symbols in a codeword matrix.

As parts of some efforts to analyze the nature of space-time codes, Simon [4] and Taricco and Biglieri [5] independently worked on the exact expression for the pairwise error probability (PEP). Lu, Wang, Kumar, and Chugg [6] also derived the exact PEP and the bit error probability (BEP) of BPSK and QPSK for some OSTBCs. Using the results, the exact symbol error probability (SEP) of OSTBCs with square quadrature amplitude modulations (QAMs) was derived [7]. Recently, Raju, Annavajjala, and Chockalingam [8] analyzed the BEP of OSTBCs by approximating log likelihood ratios (LLR).

Cho and Yoon [9] derived the general BEP expression of rectangular QAM in AWGN channel. But, the exact closed-form expression for BEP of OSTBCs with QAM has not been reported so far. In this paper, we define the one-dimensional symbol error function (ODSEF) for each symbol in an OSTBC as given in [10], which can be obtained from the exact PEP expression [4], [5]. We also define terms "homogeneous" and "nonhomogeneous" which distinguish the power allocation uniformity of a certain message symbol over given transmit antennas. It is shown that, for all homogeneous and some nonhomogeneous OSTBCs, the ODSEF can be expressed in closed-form. Then, using the general BEP expression for QAM [9] and the ODSEF, we derive the exact closed-form expression for the BEP of all homogeneous [1], [2] and some known nonhomogeneous OSTBCs [11].

II. SYSTEM MODEL

Let L_t be the number of transmit antennas and L_r the number of receive antennas of MIMO system. Let $\mathbf{X} = [x_{n,i}]$ be the $N \times L_t$ codeword matrix of an OSTBC, where $x_{n,i}$ is the symbol transmitted from the ith transmit antenna at the nth time. We consider the quasi-static Rayleigh fading channel, where fading is assumed to be constant over the duration of each codeword matrix and independent. Let $\alpha_{i,j}$ be the channel coefficient from the ith transmit antenna to the jth receive antenna and $\mathbf{A} = [\alpha_{i,j}]$ be the channel matrix. $\alpha_{i,j}$'s are independent complex Gaussian random variables with zero mean and unit variance. Then, the $N \times L_r$ received matrix $\mathbf{Y} = [y_{n,j}]$ is given by

$$\mathbf{Y} = \sqrt{\frac{\overline{\rho}}{E_m}}\mathbf{X}\mathbf{A} + \mathbf{W}$$

where $y_{n,j}$ is the received symbol of the jth receive antenna at the nth time, $\overline{\rho}$ the average signal to noise ratio (SNR), E_m the average energy transmitted from the L_t transmit antennas during a symbol period, and $\mathbf{W} = [w_{n,j}]$ the noise matrix which consists of independent, identically distributed (i.i.d.) complex Gaussian random variables with zero mean and unit variance. The perfect channel estimation is assumed.

In this paper, the square and rectangular linear OSTBCs are considered, where the codeword matrices have columnwise orthogonality. Let s be the message vector of length L_s given by $\mathrm{s} = (s_1, s_2, \cdots, s_{L_s})$, which is encoded into the codeword matrix $\mathbf{X}$. Let b_s be the number of bits per message symbol s_k. Let $\mathcal{C}(\mathrm{s})$ be a mapping from an L_s-tuple complex message vector to an $N \times L_t$ codeword matrix. For linear OSTBCs, each element in a codeword matrix is a linear combination of the message symbols s_k and their complex conjugates.

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Various linear complex OSTBCs have been introduced in [2], [3], and [11] which are listed as

$$\mathcal{C}_1 = \begin{pmatrix} s_1 & s_2 \\ -s_2^* & s_1^* \end{pmatrix}, \quad (1)$$
$$L_s = 2, N = L_t = 2, \text{code rate} = 1,$$

$$\mathcal{C}_2 = \begin{pmatrix} s_1 & s_2 & \frac{s_3}{\sqrt{2}} \\ -s_2^* & s_1^* & \frac{s_3}{\sqrt{2}} \\ \frac{s_3^*}{\sqrt{2}} & \frac{s_3^*}{\sqrt{2}} & \frac{(-s_1-s_1^*+s_2-s_2^*)}{2} \\ \frac{s_3^*}{\sqrt{2}} & -\frac{s_3^*}{\sqrt{2}} & \frac{(s_2+s_2^*+s_1-s_1^*)}{2} \end{pmatrix}, \quad (2)$$
$$L_s = 3, N = 4, L_t = 3, \text{code rate} = 3/4,$$

$$\mathcal{C}_3 = \begin{pmatrix} s_1 & s_2 & s_3 & 0 \\ -s_2^* & s_1^* & 0 & s_3 \\ s_3^* & 0 & -s_1^* & s_2 \\ 0 & s_3^* & -s_2^* & -s_1 \end{pmatrix}, \quad (3)$$
$$L_s = 3, N = 4, L_t = 4, \text{code rate} = 3/4,$$

$$\mathcal{C}_4 = \begin{pmatrix} s_1 & s_2 & s_3 & 0 & s_4 \\ -s_2^* & s_1^* & 0 & s_3 & s_5 \\ s_3^* & 0 & -s_1^* & s_2 & s_6 \\ 0 & s_3^* & -s_2^* & -s_1 & s_7 \\ s_4^* & 0 & 0 & -s_7^* & -s_1^* \\ 0 & s_4^* & 0 & s_6^* & -s_2^* \\ 0 & 0 & s_4^* & s_5^* & -s_3^* \\ 0 & -s_5^* & -s_6^* & 0 & s_1 \\ s_5^* & 0 & s_7^* & 0 & s_2 \\ -s_6^* & -s_7^* & 0 & 0 & s_3 \\ s_7 & -s_6 & -s_5 & s_4 & 0 \end{pmatrix}, \quad (4)$$
$$L_s = 7, N = 11, L_t = 5, \text{code rate} = 7/11.$$

It is obvious that they are columnwise orthogonal.

All column vectors in an OSTBC do not always have the same magnitude. The squared magnitude of the ith column in a linear OSTBC $\mathcal{C}(\mathbf{s})$ corresponds to the total energy transmitted from the ith transmit antenna during N symbol times, that is,

$$\sum_{k=1}^{L_s} g_{k,i}|s_k|^2 \quad (5)$$

where $g_{k,i}$ denotes the multiplicity of $|s_k|^2$ in the squared magnitude of the ith column of $\mathcal{C}(\mathbf{s})$. Using (5) and the columnwise orthogonality of the linear OSTBCs, we have

$$\mathcal{C}(\mathbf{s})^H\mathcal{C}(\mathbf{s}) = \sum_{k=1}^{L_s} \text{diag}\{g_{k,1}, g_{k,2}, \cdots, g_{k,L_t}\}|s_k|^2 \quad (6)$$

where $(\cdot)^H$ denotes the Hermitian operator and diag$\{\}$ the diagonal matrix.

The linear OSTBCs can be classified according to the regularity of the values of $g_{k,i}$. A symbol s_k in an OSTBC is called *homogeneous* if nonzero values of $g_{k,i}$ are constant and otherwise, *nonhomogeneous*. We call an OSTBC *homogeneous* if all symbols in the code are homogeneous and otherwise, *nonhomogeneous*. Most known OSTBCs [2], [3], [12] are homogeneous codes, while Su and Xia [11] introduced two nonhomogeneous codes, and one of them is $\mathcal{C}_4$ in (4).

Let E_s be the average symbol energy of s_k. Thus the average energy E_m transmitted from all the L_t transmit antennas during a symbol period is given as

$$E_m = \frac{1}{N}\sum_{i=1}^{L_t}\sum_{k=1}^{L_s} g_{k,i}E_s.$$

In this paper, we consider only QAMs with Gray map [9]. A rectangular M-ary QAM, $M = I \times J$, is composed of I-ASK for the real part of s_k and J-ASK for the imaginary part. Let b_s be the number of bits which are modulated to a symbol s_k. Then we have $M = 2^{b_s}$. Let $I = 2^{b_{s,1}}$ and $J = 2^{b_{s,2}}$. The minimum distance among constellations is assumed to be $2d$. For a given average symbol energy E_s, we have $d = \sqrt{3E_s/(I^2 + J^2 - 2)}$.

III. ONE-DIMENSIONAL SYMBOL ERROR FUNCTION

A. Symbol Decoding of OSTBCs

Using the columnwise orthogonality, the linear OSTBCs can be decoded symbol by symbol [3], [13]. In this section the symbol by symbol maximum likelihood decoding is reviewed. Let $\mathbf{Y} = [\mathbf{y}_1, \mathbf{y}_2, \cdots, \mathbf{y}_{L_r}]$ be a received signal matrix corresponding to the transmission of $\mathbf{X}$ through the known channel $\mathbf{A}$, where $\mathbf{y}_j = (y_{1,j}, y_{2,j}, \cdots, y_{N,j})^T$. The maximum-likelihood decoder chooses the codeword matrix $\hat{\mathbf{X}} = [\hat{x}_{n,i}]$ which minimizes the decision metric given by

$$D = \sum_{n=1}^{N}\sum_{j=1}^{L_r}\left|y_{n,j} - \sqrt{\overline{\gamma}}\sum_{i=1}^{L_t}\alpha_{i,j}\hat{x}_{n,i}\right|^2 \quad (7)$$

where $\overline{\gamma} = \overline{\rho}/E_m$. Let $\mathbf{A}' = [\mathbf{a}_1, \mathbf{a}_2, \cdots, \mathbf{a}_{L_r}] = \sqrt{\overline{\gamma}}\mathbf{A}$, where $\mathbf{a}_j = (\sqrt{\overline{\gamma}}\alpha_{1,j}, \sqrt{\overline{\gamma}}\alpha_{2,j}, \cdots, \sqrt{\overline{\gamma}}\alpha_{L_t,j})^T$. Using the columnwise orthogonality of OSTBCs, the decision metric in (7) can be rewritten as

$$D = \sum_{j=1}^{L_r}\|\mathbf{y}_j - \hat{\mathbf{X}}\mathbf{a}_j\|^2 = \sum_{k=1}^{L_s} H_k\left|\frac{b_k}{H_k} - \hat{s}_k\right|^2 + C \quad (8)$$

where the positive real number H_k is given by

$$H_k = \overline{\gamma}\sum_{j=1}^{L_r}\sum_{i=1}^{L_t} g_{k,i}|\alpha_{i,j}|^2 \quad (9)$$

and b_k is a function of the received signals $y_{n,j}$ and the known channel coefficients $\alpha_{i,j}$. It was rigorously derived in [13]. Let $b_k' = b_k/H_k$, which is dependent on $y_{n,j}$ and $\alpha_{i,j}$. The decision metric D then reduces to

$$D = \sum_{k=1}^{L_s} H_k\left|b_k' - \hat{s}_k\right|^2 + C = \sum_{k=1}^{L_s} H_k D_k + C \quad (10)$$

where the decision metric D_k for each symbol is defined as

$$D_k = |b'_k - \hat{s}_k|^2. \quad (11)$$

Since the H_k's and C are not dependent on the message symbols $\hat{s}_k$, the maximum-likelihood decision metric D can be minimized by minimizing each symbol decision metric D_k, independently, which means that it is possible to decode each symbol, independently. The decision metric D_k can easily be proved to be written as

$$D_k = |b'_k - \hat{s}_k|^2 = |s_k + \frac{e_k}{H_k} - \hat{s_k}|^2 \quad (12)$$

where e_k is a function of the noise terms $w_{n,j}$'s and channel coefficients $\alpha_{i,j}$'s. Each b'_k is only dependent on the corresponding symbol s_k so that the codes can be decoded in symbol by symbol manner.

B. One-Dimensional Symbol Error Function

Equation (12) shows that the decision boundary is fixed for any s_k and H_k, where maximum likelihood decoding is performed symbolwisely. Thus, we have an analogy between the detection of QAM in AWGN channel [9] and that in this transmit diversity channel except for the different noise characteristics.

In the decoding of $\mathcal{C}(\mathbf{s})$, the error probability for s_k is determined by the statistics of $b'_k = s_k + n_k$, where $n_k = e_k/H_k$ in (12). Instead of the complementary error function, we use ODSEF which is symbol error function regarding the statistics of n_k. The ODSEF is defined for each s_k in OSTBC $\mathcal{C}(\mathbf{s})$ as

$$\mathcal{Q}_k(l\sqrt{\overline{\gamma}}) = \frac{1}{2}Pr(|\,b'_k - s_k| > l).$$

The ODSEF can be represented as the PEP of two codewords such that two corresponding message vectors are different in only one element. Let $\mathbf{x}$ and $\mathbf{y}$ be two message vectors of length L_s, in which all corresponding elements are equal except $x_k \neq y_k$ and suppose $|x_k - y_k| = 2l$. From the exact expression for the PEP of space-time codes [4], the ODSEF is given by

$$\begin{aligned}\mathcal{Q}_k(l\sqrt{\overline{\gamma}}) &= Pr(\mathcal{C}(\mathbf{x}) \to \mathcal{C}(\mathbf{y})) \\ &= \frac{1}{\pi}\int_0^{\pi/2} \prod_{i=1}^{L_t} \left[1 + \frac{g_{k,i}}{\sin^2\theta}\overline{\gamma}\, l^2\right]^{-L_r} d\theta\end{aligned}$$

where $Pr(\mathcal{C}(\mathbf{x}) \to \mathcal{C}(\mathbf{y}))$ is the probability of erroneous decoding to $\mathcal{C}(\mathbf{y})$ when $\mathcal{C}(\mathbf{x})$ is transmitted.

Using the Simon's result [4], [14], for a homogeneous symbol in the code, the ODSEF can be derived in the closed-form as

$$\begin{aligned}\mathcal{Q}_k(l\sqrt{\overline{\gamma}}) &= \frac{1}{\pi}\int_0^{\pi/2} \left[1 + \frac{\overline{\gamma}}{\sin^2\theta} g_k l^2\right]^{-L_D L_r} d\theta \\ &= \frac{1}{2}\left\{1 - \sqrt{\frac{g_k\overline{\gamma}\, l^2}{1 + g_k\overline{\gamma}\, l^2}} \sum_{m=0}^{L_D L_r - 1} \binom{2m}{m} \right. \\ &\quad \left. \times \left(\frac{1}{4(1 + g_k\overline{\gamma}\, l^2)}\right)^m \right\} \quad (13)\end{aligned}$$

where $g_{k,i} = g_k$ or 0 and $L_D \leq L_t$ is the number of nonzero $g_{k,i}$, $1 \leq i \leq L_t$, i.e., the diversity order for s_k in the code.

For a nonhomogeneous symbol which has two distinct nonzero $g_{k,i}$'s, the ODSEF can also be derived in the closed-form using Simon's result (see (5A.58) of [14]). Two nonhomogeneous OSTBCs introduced by Su and Xia [11] belong to this case. As an example, the 7/11 OSTBC $\mathcal{C}_4$ in (4) is nonhomogeneous because the symbols s_1, s_2, and s_3 are nonhomogeneous. Applying Simon's result((5A.58) of [14]), we have the closed-form ODSEF for s_1 as

$$\mathcal{Q}_1(l\sqrt{\overline{\gamma}}) = 2^{4L_r-1}\left[\sum_{k=0}^{L_r-1} B_k I_k(2\overline{\gamma}\, l^2) - \frac{1}{2}\sum_{k=0}^{4L_r-1} \frac{C_k I_k(\overline{\gamma}\, l^2)}{2^k}\right]$$

where

$$\begin{aligned}B_k &= \frac{A_k}{\binom{5L_r-1}{k}}, \\ C_k &= \sum_{n=0}^{L_r-1} \frac{\binom{k}{n}A_n}{\binom{5L_r-1}{n}}, \\ A_k &= (-1)^{L_r-1+k}\frac{\binom{L_r-1}{k}}{(L_r-1)!}\prod_{\substack{n=1\\ n\neq k+1}}^{L_r}(5L_r - n), \\ I_k(c) &= 1 - \sqrt{\frac{c}{c+1}}\left[1 + \sum_{n=1}^{k}\frac{(2n-1)!!}{n!2^n(1+c)^n}\right]\end{aligned}$$

where !! denotes the double factorial. For the other nonhomogeneous symbols s_2 and s_3 of $\mathcal{C}_4$ in (4), their ODSEFs have the same form.

IV. BEP OF ORTHOGONAL SPACE TIME BLOCK CODES WITH QAM

M-ary QAM can be considered as a two-dimensional ASK. Consider an M-ary QAM which employs I-ASK and J-ASK in each dimension, where $M = 2^{b_s}$, $I = 2^{b_{s,1}}$, and $J = 2^{b_{s,2}}$. Every symbol s_k in an OSTBC can be transmitted as arbitrary M-QAM. Cho and Yoon derived the general expression for the BEP of two-dimensional ASK in AWGN channel [9]. Using their method [9] and the ODSEF, the exact closed-form expression for the BEP of OSTBCs is derived in this section.

Let $P(s_k)$ be the BEP for the symbol s_k. Then, the BEP for the code $\mathcal{C}$ is given by

$$P(\mathcal{C}) = \frac{1}{L_s}\sum_{k=1}^{L_s} P(s_k). \quad (14)$$

Thus, we have to derive $P(s_k)$. An M-QAM symbol s_k should be divided into I-ASK and J-ASK. For $b_{s,1}$ bits in I-ASK, the error probability of the mth bit is derived as

$$\begin{aligned}P_m^I(s_k) =& \frac{2}{I}\sum_{i=0}^{(1-2^{-m})I-1}\left\{(-1)^{\left\lfloor \frac{2^{m-1}i}{I}\right\rfloor}\left(2^{m-1}\right.\right. \\ &\left.\left. - \left\lfloor\frac{2^{m-1}i}{I} + \frac{1}{2}\right\rfloor\right)\mathcal{Q}_k\left((2i+1)\sqrt{\overline{\gamma}}d\right)\right\} \quad (15)\end{aligned}$$

where $\lfloor x \rfloor$ denotes the largest integer not greater than x. Equation (15) is directly derived from (9) in [9] by replacing erfc$(\cdot)$ with $2\mathcal{Q}_k(\cdot)$. Thus, the BEP of I-ASK for s_k is given by

$$P^I(s_k) = \frac{1}{b_{s,1}} \sum_{m=1}^{b_{s,1}} P_m^I(s_k)$$

and the BEP for s_k is derived as

$$P(s_k) = \frac{1}{b_s}\left(P^I(s_k)b_{s,1} + P^J(s_k)b_{s,2}\right).$$

For the homogeneous symbol s_k in (15), $\mathcal{Q}_k(\cdot)$ can be given in the closed-form expression as (13) and then $P(s_k)$ can also be given in the closed-form because it is the summation of ODSEFs.

For square M-QAMs, we have $P(s_k) = P^I(s_k)$, where $M = I \times I$. Here are the examples of the BEPs of square M-QAMs such as QPSK, 16QAM, 64QAM, and 256QAM:

$$\begin{aligned}
P_{QPSK} =& \frac{1}{L_s}\sum_{k=1}^{L_s} \mathcal{Q}_k(\sqrt{\overline{\gamma}}d),\\
P_{16QAM} =& \frac{1}{L_s}\sum_{k=1}^{L_s} \Big\{\frac{3}{4}\mathcal{Q}_k(\sqrt{\overline{\gamma}}d) + \frac{1}{2}\mathcal{Q}_k(\sqrt{\overline{\gamma}}3d)\\
& - \frac{1}{4}\mathcal{Q}_k(\sqrt{\overline{\gamma}}5d)\Big\},\\
P_{64QAM} =& \frac{1}{L_s}\sum_{k=1}^{L_s} \frac{1}{12}\Big\{7\mathcal{Q}_k(\sqrt{\overline{\gamma}}d) + 6\mathcal{Q}_k(\sqrt{\overline{\gamma}}3d)\\
& - \mathcal{Q}_k(\sqrt{\overline{\gamma}}5d) + \mathcal{Q}_k(\sqrt{\overline{\gamma}}9d) - \mathcal{Q}_k(\sqrt{\overline{\gamma}}13d)\Big\},\\
P_{256QAM} =& \frac{1}{L_s}\sum_{k=1}^{L_s} \frac{1}{32}\Big\{15\mathcal{Q}_k(\sqrt{\overline{\gamma}}d) + 14\mathcal{Q}_k(\sqrt{\overline{\gamma}}3d)\\
& -\mathcal{Q}_k(\sqrt{\overline{\gamma}}5d) + 5\mathcal{Q}_k(\sqrt{\overline{\gamma}}9d) + 4\mathcal{Q}_k(\sqrt{\overline{\gamma}}11d)\\
& - 5\mathcal{Q}_k(\sqrt{\overline{\gamma}}13d) - 4\mathcal{Q}_k(\sqrt{\overline{\gamma}}15d) + 5\mathcal{Q}_k(\sqrt{\overline{\gamma}}17d)\\
& + 4\mathcal{Q}_k(\sqrt{\overline{\gamma}}19d) - 3\mathcal{Q}_k(\sqrt{\overline{\gamma}}21d) - 2\mathcal{Q}_k(\sqrt{\overline{\gamma}}23d)\\
& + \mathcal{Q}_k(\sqrt{\overline{\gamma}}25d) - \mathcal{Q}_k(\sqrt{\overline{\gamma}}29d)\Big\}.
\end{aligned}$$

In Fig. 1, we plot the exact BEP and simulated result of Alamouti's scheme with several square QAMs in quasi-static Rayleigh fading channel where $L_r = 1, 2$ and they exactly match. For every homogeneous OSTBC, the closed-form BEP is available. Moreover, the exact closed-form expression for the BEP of all known nonhomogeneous codes [11] can be derived using their ODSEF because they have only two nonzero distinct $g_{k,i}$'s. Fig. 2 shows the BEP performance curves of nonhomogeneous $\mathcal{C}_4$ obtained by simulation and closed-form expression.

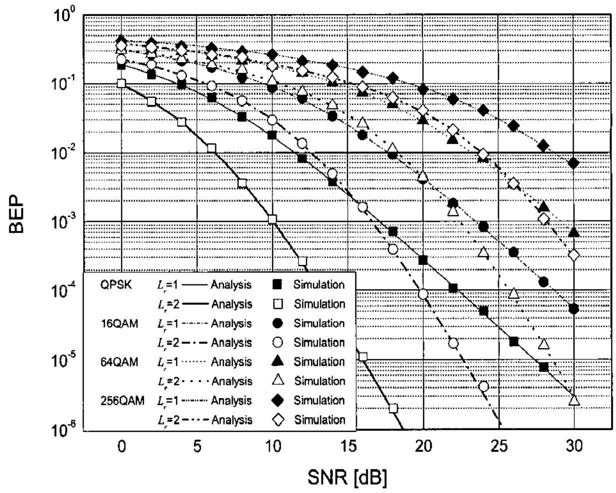

Fig. 1. Exact bit error probability of Alamouti scheme for QPSK, 16QAM, 64QAM, and 256QAM.

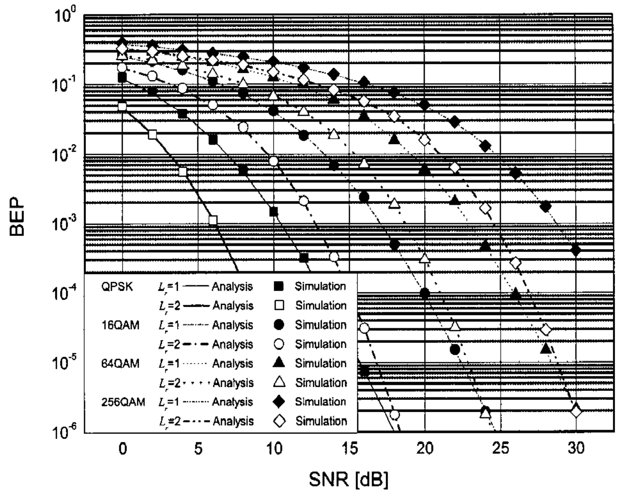

Fig. 2. Exact bit error probability of $\mathcal{C}_4$ for QPSK, 16QAM, 64QAM, and 256QAM.

V. CONCLUSION

In this paper, using the ODSEF, we have derived the exact closed-form expression for the BEP of linear OSTBCs with square and rectangular QAMs. For all the homogeneous OSTBCs, the ODSEFs and the BEP for QAM can be represented in the closed-form. For some known nonhomogeneous OSTBCs introduced by Su and Xia [11], the ODSEF and the BEP for QAM can also be derived in closed-form. Therefore, the exact BEP of all known OSTBCs [1], [2], [11], [12] with QAM can be given in the closed-form expressions.

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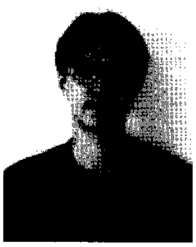

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