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SOME LOCAL SPECTRAL PROPERTIES OF T AND S WITH $AT - SA = 0$

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ABSTRACT. Let T and S be bounded linear operators on Banach spaces $\mathcal{X}$ and $\mathcal{Y}$, respectively. A linear map $A : \mathcal{X} \to \mathcal{Y}$ is said to be an intertwiner if $AT - SA = 0$. In this paper we study the relation between local spectral properties of T and S on the assumption of $AT - SA = 0$. We give some example of intertwiner with T and S.

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1. Preliminaries

Let $\mathcal{X}$ and $\mathcal{Y}$ be Banach spaces over the complex plane $\mathbb{C}$. Let $\mathcal{L}(\mathcal{X}, \mathcal{Y})$ denote the space of all bounded linear operators from $\mathcal{X}$ to $\mathcal{Y}$. And let $\mathcal{L}(\mathcal{X})$ be the Banach algebra of all bounded linear operators on $\mathcal{X}$. For a given $T \in \mathcal{L}(\mathcal{X})$, let $\sigma(T)$, $\sigma_p(T)$ and $\rho(T)$ denote the spectrum, the point spectrum and the resolvent set of T, respectively. The *local resolvent set* $\rho_T(x)$ of T at the point $x \in \mathcal{X}$ is defined as the union of all open subsets U of $\mathbb{C}$ for which there is an analytic function $f : U \to \mathcal{X}$ which satisfies

$$(T - \lambda)f(\lambda) = x \quad \text{for all} \quad \lambda \in U.$$

The *local spectrum* $\sigma_T(x)$ of T at x is then defined as

$$\sigma_T(x) = \mathbb{C} \setminus \rho_T(x).$$

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Clearly, the local resolvent set $\rho_T(x)$ is open, and the local spectrum $\sigma_T(x)$ is closed. For each $x \in \mathcal{X}$, the function $f(\lambda) : \rho(T) \to \mathcal{X}$ defined by

$$f(\lambda) = (T - \lambda)^{-1}x$$

is analytic on $\rho(T)$ and satisfies

$$(T - \lambda)f(\lambda) = x \quad \text{for all} \quad \lambda \in \rho(T).$$

Hence the resolvent set $\rho(T)$ is always subset of $\rho_T(x)$ and hence $\sigma_T(x)$ is always subset of $\sigma(T)$.

The analytic solutions occurring in the definition of the local resolvent set may be thought of as local extensions of the function

$$(T - \lambda)^{-1}x : \rho(T) \to \mathcal{X}.$$

There is no uniqueness implied. Thus we need the following definition.

An operator $T \in L(\mathcal{X})$ is said to have the *single-valued extension property*, abbreviated SVEP, if for every open set $U \subseteq \mathbb{C}$, the only analytic solution $f : U \to \mathcal{X}$ of the equation

$$(T - \lambda)f(\lambda) = 0 \quad \text{for all} \quad \lambda \in U$$

is the zero function on U. Hence if T has the SVEP, then for each $x \in \mathcal{X}$ there is the maximal analytic extension of $(T - \lambda)^{-1}x$ on $\rho_T(x)$.

For a closed subset F of $\mathbb{C}$,

$$\mathcal{X}_T(F) = \{x \in X : \sigma_T(x) \subseteq F\}$$

is said to be an *analytic spectral subspace* of T. It is easy to see that $\mathcal{X}_T(F)$ is a T−invariant linear subspace of $\mathcal{X}$ and also hyperinvariant for T, while generally not closed. Analytic spectral subspaces date back to early work of E. Bishop [4] and have been fundamental in the recent progress of local spectral theory, for instance in connection with functional models and invariant subspaces and also in the theory of spectral inclusions for operators on Banach spaces [13].

It is well known that T has the SVEP if and only if $\mathcal{X}_T(\phi) = \{0\}$, and this is the case if and only if $\mathcal{X}_T(\phi)$ is closed. Moreover, if T does not have SVEP then there exists some non-zero $x \in \mathcal{X}$ for which $\sigma_T(x)$ is empty.

2. Examples of intertwiners with T and S

For $S \in \mathcal{L}(X)$ and $S \in \mathcal{L}(\mathcal{Y})$ we define the operator $C(S, T)$ on the Banach space $\mathcal{L}(\mathcal{X}, \mathcal{Y})$ of all bounded linear operators from $\mathcal{X}$ to $\mathcal{Y}$ by

$$C(S, T)A = SA - AT \quad \text{for} \quad A \in \mathcal{L}(\mathcal{X}, \mathcal{Y}).$$

For a natural number $n \in \mathbb{N}$, define $C(S,T)^n$ to be the n-th composition of the operator $C(S,T)$, That is,

$$C(S,T)^n A = C(S,T)^{n-1}(SA - AT)$$
$$= \sum_{k=0}^{n} \binom{n}{k} (-1)^k S^{n-k} A T^k.$$

In particular, if the operator T and S commute, if $n \in \mathbb{N}$ is given, and if I denotes the identity operator on $\mathcal{X}$, then the identity $C(S,T)^n I = 0$ holds if and only if $S = T + N$ for some nilpotent operator N of order at most n.

Define the space $\mathcal{I}(S,T)$ as follow:

$$\mathcal{I}(S,T) = \{A : \mathcal{X} \to \mathcal{Y} \mid A \text{ is a linear map such that } C(S,T)^n A = 0 \text{ for some } n \in \mathbb{N}\}.$$

A linear operator $A : \mathcal{X} \to \mathcal{Y}$ is said to be a *intertwiner(or intertwining linear operator)* with T and S if $A \in \mathcal{I}(S,T)$. The space $\mathcal{I}(S,T)$ contains many significant classes of operators.

Example 1. Let $\mathcal{A}$ and $\mathcal{B}$ be complex Banach algebras. And let $\theta : \mathcal{A} \to \mathcal{B}$ be an algebra homomorphism. Then for each $a \in \mathcal{A}$

$$\theta(a)\theta(x) - \theta(ax) = 0 \quad \text{for all} \quad x \in \mathcal{A}.$$

Hence $\theta \in \mathcal{I}(S_{\theta(a)}, T_a)$ for each $a \in \mathcal{A}$, in the sense that $T_a : \mathcal{A} \to \mathcal{A}$ and $S_\theta(a) : \mathcal{A} \to \mathcal{A}$ is the left multiplication operators by a and $\theta(a)$, respectively.

Example 2. Let $\mathcal{A}$ be a complex Banach algebra and let $\mathcal{M}$ be a complex Banach $\mathcal{A}$−module, for which $am = ma$ for all $a \in \mathcal{A}$ and $m \in \mathcal{M}$. Also, let $D : \mathcal{A} \to \mathcal{M}$ be a module derivation, in the sense that the differentiation rule

$$D(xy) = xDy + D(x)y \quad \text{for all} \quad x, y \in \mathcal{A}.$$

A routine calculation shows that

$$C(S_a, T_a)^2 D = 0 \quad \text{for all} \quad a \in \mathcal{A},$$

where $T_a : \mathcal{A} \to \mathcal{A}$ and $S_a : \mathcal{M} \to \mathcal{M}$ denote the left multiplication operators by a, respectively. Hence $D \in \mathcal{I}(S_a, T_a)$ for each $a \in \mathcal{A}$.

Example 3. Let $A : \mathcal{X} \to \mathcal{Y}$ and $B : \mathcal{Y} \to \mathcal{X}$ be bounded linear operators. Then $A \in \mathcal{I}(\lambda I - AB, \lambda I - BA)$ and $B \in \mathcal{I}(\lambda I - BA, \lambda I - AB)$ for every complex number $\lambda \in \mathbb{C}$. In particular, $A \in \mathcal{I}(AB, BA)$ and $B \in \mathcal{I}(BA, AB)$ since $BA \in \mathcal{L}(\mathcal{X})$ and $AB \in \mathcal{L}(\mathcal{Y})$.

Example 4. Let $T \in \mathcal{L}(\mathcal{H})$ be a bounded operator on a Hilbert space $\mathcal{H}$ and $U|T|$ be the polar decomposition of T, where $|T| = (TT^*)^{\frac{1}{2}}$ and U is the appropriate partial isometry. The generalized Aluthge transform associated with T and $s, t \geq 0$ is defined by

$$T(s,t) = |T|^s U |T|^t.$$

In the case $s = t = \frac{1}{2}$, the operator

$$\widetilde{T} = |T|^{\frac{1}{2}} U |T|^{\frac{1}{2}}$$

is called the Aluthge transform of T. It is easy to see that

$$|T|^s U |T|^{t-r} \in \mathcal{I}(T(s,t), T(s+r, t-r))$$

and

$$|T|^r \in \mathcal{I}(T(s+r, t-r), T(s,t))$$

for all $0 \leq r \leq t$.

Let $\mathcal{H}$ be a Hilbert space over the complex plane $\mathbb{C}$ with the inner product $\langle \cdot, \cdot \rangle$. And $\mathcal{L}(\mathcal{H})$ denotes the C^*-algebra of bounded linear operators on a Hilbert space $\mathcal{H}$. And let T^* denote the adjoint of T. The operator $T \in \mathcal{L}(\mathcal{H})$ is said to be *hyponormal* if its self commutator $[T^*, T] = T^*T - TT^*$ is positive, that is

$$\langle [T^*, T]x, x \rangle \ \geq \ 0,$$

or equivalently

$$\|T^*x\| \leq \|Tx\|$$

for every $x \in \mathcal{H}$. And $T \in \mathcal{L}(\mathcal{H})$ is said to be a *cohyponormal operator* if T^* is hyponormal, equivalently, $T^*T \leq TT^*$.

The following example is the main theorem of [13].

Example 5. Let $T \in \mathcal{L}(\mathcal{K})$ be a cohyponormal operator on a Hilbert space $\mathcal{K}$, and let $S \in \mathcal{L}(\mathcal{H})$ be a hyponormal operator on a Hilbert space $\mathcal{H}$. If $A : \mathcal{K} \to \mathcal{H}$ is a bounded linear operator then $A \in \mathcal{I}(S,T)$ if and only if $AT = SA$.

3. Some local spectral properties of T and S with $AT - SA = 0$

For an arbitrary operator $T \in \mathcal{L}(\mathcal{X})$, we define the *analytic residuum*, denoted by $\mathcal{S}(T)$, as the open set of points $\lambda \in \mathbb{C}$ for which there exists a non-zero analytic function $f : U \to \mathcal{X}$ on some open neighborhood U of λ so that

$$(T - \mu)f(\mu) = 0 \quad \text{for all} \quad \mu \in U.$$

Evidently, $\mathcal{S}(T)$ is a subset of the interior of the point spectrum of T. Moreover, T has the SVEP if and only if $\mathcal{S}(T) = \emptyset$.

For a bounded linear operator $T \in \mathcal{L}(\mathcal{X})$, let $\sigma_{sur}(T)$ denote the *surjectivity spectrum* of T. That is,

$$\sigma_{sur}(T) = \{\lambda \in \mathbb{C} : (T - \lambda)\mathcal{X} \neq \mathcal{X}\}.$$

It is well known that $\sigma(T) = \sigma_{sur}(T) \cup \mathcal{S}(T)$.

Proposition 1. *Let $T \in \mathcal{L}(\mathcal{X})$ and $S \in \mathcal{L}(\mathcal{Y})$. If $A \in \mathcal{I}(S,T)$ is continuous then the analytic residuum of T is contained in the analytic residuum of S.*

Proof. Suppose that $C(S,T)^n(A) = 0$ for some $n \in \mathbb{N}$. Let $\lambda \in \mathcal{S}(T)$. Then there is an open neighborhood U and a non-zero analytic function $f : U \to \mathcal{X}$ satisfying $(T - \mu)f(\mu) = 0$ on U. Define $g : U \to \mathcal{Y}$ by

$$g(\mu) = \sum_{k=0}^{n-1}(-1)^k C(S,T)^k(A)\frac{f^{(k)}(\mu)}{k!} \quad \text{for all} \quad \mu \in U.$$

Then g is well defined and non zero analytic on U. By the definition of the commutator it is clear that

$$(S - \mu)C(S,T)^k(A) = C(S,T)^{k+1}(A) + C(S,T)^k(A)(T - \mu)$$

for all $k \in \mathbb{N}$ and $\mu \in \mathbb{C}$. Since $(T - \mu)f(\mu) = 0$ for any $\mu \in U$, if we differentiate this equation k-times, we have

$$(T - \mu)f^{(k)}(\mu) = kf^{(k-1)}(\mu) \quad \text{for all} \quad \mu \in U \quad \text{and} \quad k \in \mathbb{N}.$$

Therefore, for each $\mu \in U$ we have,

$$\begin{aligned}(S-\mu)g(\mu) &= \sum_{k=0}^{n-1}(-1)^k(S-\mu)C(S,T)^k(A)\frac{f^{(k)}(\mu)}{k!}\\ &= \sum_{k=0}^{n-1}(-1)^k(C(S,T)^{k+1}(A)+C(S,T)^k(A)(T-\mu))\frac{f^{(k)}(\mu)}{k!}\\ &= A(T-\mu)f(\mu)\\ &= 0.\end{aligned}$$

Hence $\lambda \in \mathcal{S}(S)$. This completes the proof. □

Corollary 2. *Let $S \in \mathcal{L}(\mathcal{X},\mathcal{Y})$ and $R \in \mathcal{L}(\mathcal{Y},\mathcal{X})$. Then we have,*

$$\mathcal{S}(\lambda I - RS) = \mathcal{S}(\lambda I - SR) \quad \textit{for all} \quad \lambda \in \mathbb{C}.$$

In particular, $\mathcal{S}(RS) = \mathcal{S}(SR)$.

Proof. Since $S \in \mathcal{I}(\lambda I - SR, \lambda I - RS)$ is continuous, it follows from Proposition 1 that

$$\mathcal{S}(\lambda I - RS) \subseteq \mathcal{S}(\lambda I - SR) \quad \text{for all} \quad \lambda \in \mathbb{C}.$$

The converse implication follows by interchanging R and S. □

Corollary 3. *Let $S : \mathcal{X} \to \mathcal{Y}$ and $R : \mathcal{Y} \to \mathcal{X}$ be bounded linear operators. Then for each $\lambda \in \mathbb{C}$, $\lambda I - RS$ has the SVEP if and only if $\lambda I - SR$ has the SVEP. In particular, RS has the SVEP if and only if SR has the SVEP.*

An operator T has *finite ascent* if for every $\lambda \in \mathbb{C}$ there is an $n \in \mathbb{N}$ such that $\ker(T-\lambda)^n = \ker(T-\lambda)^{n+1}$, where $\ker(T)$ is the kernel of T.

Proposition 4. *Let $T \in \mathcal{L}(\mathcal{X})$ and $S \in \mathcal{L}(\mathcal{Y})$. Suppose that there is an injective map A with $C(S,T)A = 0$. If S has finite ascent then T has finite ascent.*

Proof. It is clear that

$$A(T-\lambda)^n = (S-\lambda)^n A \quad \text{for all} \quad n \in \mathbb{N} \quad \text{and} \quad \lambda \in \mathbb{C}.$$

Suppose that $\ker(S-\lambda)^m = \ker(S-\lambda)^{m+1}$ for some $m \in \mathbb{N}$. Clearly,

$$\ker(T-\lambda)^m \subseteq \ker(T-\lambda)^{m+1} \quad \text{for all} \quad \lambda \in \mathbb{C}.$$

Let $x \in \ker(T - \lambda)^{m+1}$. Then we have

$$\begin{aligned}(S-\lambda)^{m+1}Ax &= A(T-\lambda)^{m+1}x \\ &= 0.\end{aligned}$$

Therefore, we have $Ax \in \ker(S - \lambda)^{m+1} = \ker(S - \lambda)^m$. And hence

$$\begin{aligned}A(T-\lambda)^{m}x &= (S-\lambda)^{m}Ax \\ &= 0.\end{aligned}$$

Since A is injective, we have $(T - \lambda)^m = 0$. This completes the proposition. □

Corollary 5. *Let $S : \mathcal{X} \to \mathcal{Y}$ and $R : \mathcal{Y} \to \mathcal{X}$ be bounded linear operators. Assume that S and R are injective. For each $\lambda \in \mathbb{C}$, $\lambda I - RS \in \mathcal{L}(\mathcal{X})$ has finite ascent if and only if $\lambda I - SR \in \mathcal{L}(\mathcal{Y})$ has finite ascent.*

Proof. Assume that $\lambda I - RS \in \mathcal{L}(\mathcal{X})$ has finite ascent. Then clearly we have

$$S \in \mathcal{I}(\lambda I - SR, \lambda I - RS).$$

Since S is injective, by Proposition 4, $\lambda I - SR$ has finite ascent. The reverse implication is obtained by symmetry. □

Lemma 6. *Let $T \in \mathcal{L}(\mathcal{X})$ and $\lambda \in \mathbb{C}$. Suppose that $(T - \lambda)^n x = 0$ for some non zero vector $x \in \mathcal{X}$ and $n \in \mathbb{N}$. Then $\lambda \in \sigma_p(T)$.*

Proof. We will prove this lemma by mathematical induction.
(i) For $n = 1$, it is trivial.
(ii) Suppose that this lemma holds for $n = k$.
For $n = k + 1$, let $(T - \lambda)^{k+1}x = 0$ for some non zero $x \in \mathcal{X}$. Then,
case 1. $(T - \lambda)^k x = 0$. Then by the assumption $\lambda \in \sigma_p(T)$.
case 2. $(T - \lambda)^k x \neq 0$. Since $(T - \lambda)(T - \lambda)^k x = 0$, we have $\lambda \in \sigma_p(T)$.
By (i), (ii) this lemma holds for all $n \in \mathbb{N}$. □

Proposition 7. *Let $T \in \mathcal{L}(\mathcal{X})$ and $S \in \mathcal{L}(\mathcal{Y})$. Suppose that there is an injective linear map $A \in \mathcal{I}(S, T)$. Then $\sigma_p(T) \subseteq \sigma_p(S)$.*

Proof. Suppose that $C(S,T)^n(A) = 0$ for some positive integer $n \in \mathbb{N}$. Let $\lambda \in \sigma_p(T)$ and let $x \in \mathcal{X}$ be an eigenvector for the eigenvalue λ. Then we have

$$\begin{aligned} 0 &= C(S,T)^n Ax \\ &= C(S-\lambda, T-\lambda)^n Ax \\ &= \sum_{k=0}^{n} \binom{n}{k} (-1)^k (S-\lambda)^{n-k} A (T-\lambda)^k x \\ &= (S-\lambda)^n Ax. \end{aligned}$$

Since $Ax \neq 0$, by the injectivity of A, therefore by lemma 6 we have, $\lambda \in \sigma_p(S)$. This completes the proof. □

Theorem 8. *Let $T \in \mathcal{L}(\mathcal{X})$ and $S \in \mathcal{L}(\mathcal{Y})$. Suppose that $C(S,T)A = 0$ for some $A \in \mathcal{L}(\mathcal{X},\mathcal{Y})$. Then for every $x \in \mathcal{X}$, we have*
(1) $\sigma_S(Ax) \subseteq \sigma_T(x) \subseteq \sigma_S(Ax) \cup \{0\}$.
(2) If moreover A is bijective, then $\sigma_S(Ax) = \sigma_T(x)$.

Proof. Suppose that $SA = AT$. Let $\lambda \notin \sigma_T(x)$ and let $x(\cdot) : U \to \mathcal{X}$ be an analytic function on an open neighborhood U of λ such that $(T-\mu)x(\mu) = x$ for all $\mu \in U$. Then we have

$$\begin{aligned} Ax &= A(T-\mu)x(\mu) \\ &= (S-\mu)Ax(\mu), \end{aligned}$$

for all $\mu \in U$. And hence $\lambda \notin \sigma_S(Ax)$. Thus $\sigma_S(Ax) \subseteq \sigma_T(x)$ is proved.

To show the second inclusion, let $\lambda \notin \sigma_S(Ax) \cup \{0\}$ and let $y(\cdot) : V \to \mathcal{Y}$ be an analytic function on an open neighborhood V of λ with $0 \notin V$ such that $(S-\mu)y(\mu) = Ax$ for all $\mu \in V$. Then define $z(\cdot) : V \to \mathcal{X}$ by

$$z(\mu) = \frac{1}{\mu}(Ay(\mu) - x).$$

Then clearly $z(\cdot)$ is an analytic function such that $(T-\mu)z(\mu) = x$, and hence $\lambda \notin \sigma_T(x)$. Thus $\sigma_T(x) \subseteq \sigma_S(Ax) \cup \{0\}$ is proved.

Suppose that $0 \in \sigma_S(Ax)$. Then by the first inclusion we have

$$\sigma_S(Ax) = \sigma_T(x).$$

It remains to show that if A is bijective and $0 \notin \sigma_S(Ax)$ then $0 \notin \sigma_T(x)$. Suppose that A is bijective. Let $0 \notin \sigma_S(Ax)$. Then there is an analytic function $f : W \to \mathcal{Y}$ on an open neighborhood W of 0 such that

$$(S-\mu)f(\mu) = Ax \quad \text{for all} \quad \mu \in W.$$

Then define the $z(\cdot) : W \to \mathcal{X}$ by $z(\mu) = A^{-1}f(\mu)$ Then we have

$$\begin{aligned} A(T-\mu)z(\mu) &= (S-\mu)Az(\mu) \\ &= (S-\mu)f(\mu) \\ &= Ax \end{aligned}$$

for all $\mu \in W$. Since A is bijective, we have

$$(T-\mu)z(\mu) = x \quad \text{for all} \quad \mu \in W.$$

Therefore, we have $0 \notin \sigma_T(x)$. This completes the proof. □

As an immediate application of Theorem 8, we obtain the following corollary.

Corollary 9. *Let $S \in \mathcal{L}(\mathcal{X}, \mathcal{Y})$ and $R \in \mathcal{L}(\mathcal{Y}, \mathcal{X})$. Then we have*
(1) $\sigma_{SR}(Sx) \subseteq \sigma_{RS}(x) \subseteq \sigma_{SR}(Sx) \cup \{0\}$ for every $x \in \mathcal{X}$.
(2) If S is bijective then $\sigma_{RS}(x) = \sigma_{SR}(Sx)$ for every $x \in \mathcal{X}$.

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