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INTUITIONISTIC FUZZY FILTERS OF ORDERED SEMIGROUPS

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ABSTRACT. The notion of intuitionistic fuzzy filters in ordered semigroups is introduced and relation between intuitionistic fuzzy filters and intuitionistic fuzzy prime ideals is investegated. The notion of intuitionistic fuzzy bi-ideal subsets and intuitionistic fuzzy bi-filters are provided and relation between intuitionistic fuzzy bi-filters and intuitionistic fuzzy prime bi-ideal subsets is established. The concept of intuitionistic fuzzy right filters(left filters) is given and their relation with intuitionistic fuzzy prime right (left) ideals is discussed.

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1. Introduction

The theory of fuzzy sets proposed by Zadeh [28], has achieved a great success in various fields. Out of several higher order fuzzy sets, intuitionistic fuzzy sets introduced by Atanassov [2], have been found to be highly useful to deal with vagueness. Gau and Buehrer [12] presented the concept of vague sets. But, Burillo and Bustince showed that the notion of vague sets coincides with that of intuitionistic fuzzy sets. Szmidt and Kacprzyk [26] proposed a non-probablistic-type entropy measure for intuitionistic fuzzy sets. De et al. [9] studied the Sanchez's approach for medical dignosis and extended this concept with the notion of intuitionistic fuzzy set theory. There are many applications of intuitionistic fuzzy set theory in mathematics. Davvaz et al. [8] applied the concept of this notion to H_v-modules. They introduced the concept of intuitionistic fuzzy H_v-submodules of an H_v-module. Dudek et al. [10] considered the intuitionistic fuzzification of the concept of sub-hyperquasigroups of a hyperquasigroup.

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Several authors studied intuitionistic fuzzification of subrings/ideals of a ring [1], [5]. In particular, Jun used intuitionistic fuzzification of bi-ideals in ordered semigroups and introduced several basic properties of this notion with many other characterizations of ordered semigroups [13]. The concept of fuzzy sets in ordered semigroups was first given by Kehayopulu and Tsingelis [18]. In [20], they studied several fuzzy analogous results for ordered semigroup. In [18], the concept of fuzzy sets in ordered groupoids/ordered semigroups is given, and it is shown that a fuzzy set f of an ordered groupoid/ordered semigroup S is a fuzzy filter of S if and only if the complement f' of f is a prime fuzzy ideal of S. In this paper we follow the ideas of fuzzy filters given in [18], [24] for ordered semigroups and define the intuitionistic fuzzy analogous results for ordered semigroups. We establish a relation between intuitionistic fuzzy filters and intuitionistic fuzzy prime ideals of ordered semigroups. We study the relation between the intuitionistic fuzzy bi-filters and intuitionistic fuzzy prime bi-ideal subsets of ordered semigroups. We provide an intuitionistic fuzzification link between one-sided filters and one-sided prime ideals of ordered semigroups.

2. Basic concepts in ordered semigroups

An ordered semigroup is an ordered set S at the same time a semigroup such that

$$a, b \in S, a \leq b \Longrightarrow xa \leq xb \text{ and } ax \leq bx \text{ for all } x \in S.$$

Let $(S, ., \leq)$ be an ordered semigroup. For $A \subseteq S$, we denote

$$(A] := \{t \in S | t \leq h \text{ for some } h \in A\}.$$

For $A, B \subseteq S$, we denote,

$$AB := \{ab | a \in A, b \in B\}.$$

Let $(S, ., \leq)$ be an ordered semigroup, $\emptyset \neq A \subseteq S$, A is called a *subsemigroup* of S if $A^2 \subseteq A$.

Definition 1 ([15]). Let $(S, ., \leq)$ be an ordered semigroup. $\emptyset \neq A \subseteq S$ is called a right (resp. left) ideal of S if:

(1) $AS \subseteq A$ (resp. $SA \subseteq A$) and

(2) If $a \in A$ and $S \ni b \leq a$, then $b \in A$.

If A is both a right and a left ideal of S, then it is called an *ideal* of S.

Definition 2 ([7]). Let $(S, ., \leq)$ be an ordered semigroup. A non-empty subset B of S is called a bi-ideal subset of S if:

(1) $a \in B$, $x \in S$ implies $axa \in B$ and

(2) If $a \in B$, $S \ni b \leq a$ then $b \in B$.

Definition 3 ([16]). Let $(S, ., \leq)$ be an ordered semigroup. A subsemigroup F of an ordered semigroup S is called a filter of S if:

(1) $a, b \in S$, $ab \in F$ implies $a, b \in F$ and

(2) $a \in F$, $S \ni b \geq a$ implies $b \in F$.

Definition 4 ([7]). Let $(S, ., \leq)$ be an ordered semigroup. A subsemigroup F of an ordered semigroup S is called a left (resp. right) filter of S if:

(1) $a, b \in S$, $ab \in F$ implies $b \in F$ (resp. $a \in F$) and
(2) $a \in F$, $S \ni b \geq a$ implies $b \in F$.
F is called a bi-filter of S if F satisfies condition (2) of definition 4 and
$(1)'$ $a, b \in S$, $aba \in F$ implies $a \in F$.

3. Basic concepts of (intuitionistic) fuzzy ideals

Let $(S, ., \leq)$ be an ordered semigroup. By a fuzzy subset f of S, we mean a mapping $f : S \longrightarrow [0, 1]$. If $(S, ., \leq)$ is an ordered semigroup and $A \subseteq S$, the characteristic function f_A of A is a fuzzy subset of S, defined as follows:

$$f_A : S \longrightarrow [0,1] | x \longrightarrow f_A(x) := \begin{cases} 1, & \text{if } x \in A, \\ 0, & \text{if } x \notin A. \end{cases}$$

Definition 5 ([18]). Let S be an ordered semigroup. A fuzzy subset f of S is called a fuzzy left (resp. right) ideal of S if:
(1) $x \leq y \Longrightarrow f(x) \geq f(y)$,
(2) $f(xy) \geq f(y)$ (resp. $f(xy) \geq f(x)$) for all $x, y \in S$.
If f is both a fuzzy left ideal and a fuzzy right ideal of S, then f is called a *fuzzy ideal* of S or a *fuzzy two-sided ideal* of S.
Equivalently, f is called a fuzzy ideal of S if:
(1) $x \leq y \Longrightarrow f(x) \geq f(y)$,
(2) $f(xy) \geq \max\{f(x), f(y)\}$ for all $x, y \in S$.

Definition 6 ([18]). Let $(S, ., \leq)$ be an ordered semigroup. A fuzzy subset f of S is called a fuzzy filter of S if:
(1) $x \leq y \Longrightarrow f(x) \geq f(y)$,
(2) $f(xy) = \min\{f(x), f(y)\}$ for all $x, y \in S$.

Definition 7 ([24]). Let $(S, ., \leq)$ be an ordered semigroup. A fuzzy subset f of S is called a fuzzy left(resp. right) filter of S if:
(1) $x \leq y \Longrightarrow f(x) \geq f(y)$,
(2) $f(xy) \geq \min\{f(x), f(y)\}$ for all $x, y \in S$.
(3) $f(xy) \leq f(y)$ (resp. $f(xy) \leq f(x)$) for all $x, y \in S$.

Definition 8 ([24]). Let $(S, ., \leq)$ be an ordered semigroup. A fuzzy subset f of S is called a fuzzy bi-filter of S if:
(1) $x \leq y \Longrightarrow f(x) \geq f(y)$,
(2) $f(xy) \geq \min\{f(x), f(y)\}$ for all $x, y \in S$.
(3) $f(xyx) \leq f(x)$ for all $x, y \in S$.

Definition 9 ([24]). Let $(S, ., \leq)$ be an ordered semigroup. A fuzzy subset f of S is called a fuzzy bi-ideal subset of S if:
(1) $x \leq y \Longrightarrow f(x) \geq f(y)$.
(2) $f(xyx) \geq f(x)$ for all $x, y \in S$.

Let $(S, ., \leq)$ be an ordered semigroup and f a fuzzy subset of S. Then the function defined by

$$f^{'} : S \longrightarrow [0,1] | a \longrightarrow f^{'}(a) := 1 - f(x),$$

is a fuzzy subset of S, called the complement of f in S.

As an important generalization of the notion of fuzzy sets in S, Atanassov [2] introduced the concept of an intuitionistic fuzzy set (*IFS* for short) defined on a non-empty set S as objects having the form

$$A := \{\langle x, \langle \mu_A(x), \gamma_A(x) \rangle \rangle | x \in S\},$$

where the functions $\mu_A : S \longrightarrow [0,1]$ and $\gamma_A : S \longrightarrow [0,1]$ denote the *degree* of *membership* (namely $\mu_A(x)$) and the *degree* of *nonmembership* (namely $\gamma_A(x)$) of each element $x \in S$ to the set A respectively, and $0 \leq \mu_A + \gamma_A \leq 1$ for all $x \in S$ [13].

For any two *IFSs* A and B of an ordered semigroup S we define:

(1) $A \subseteq B$ iff $\mu_A(x) \leq \mu_B(x)$ and $\gamma_A(x) \geq \gamma_B(x)$ for all $x \in S$,

(2) $A^c = \{\langle x, \gamma_A(x), \mu_A(x) \rangle | x \in S\}$,

(3) $A \cap B = \{\langle x, \min\{\mu_A(x), \mu_B(x)\}, \max\{\gamma_A(x), \gamma_B(x)\} \rangle | x \in S\}$,

(4) $A \cup B = \{\langle x, \max\{\mu_A(x), \mu_B(x)\}, \min\{\gamma_A(x), \gamma_B(x)\} \rangle | x \in S\}$.

The intuitionistic fuzzy sets are studied by many authors (see for example three journals: 1) *Information Sciences* 2) *Fuzzy Sets and Systems* and 3) *Notes on Intuitionistic Fuzzy Sets*) and have many interesting applications in mathematics (see for example Kim, Dudek and Jun in [21]). Also in [22], [23], Kim and Jun introduced the concept of intuitionistic fuzzy (interior) ideals of semigroups.

Let $(S, ., \leq)$ be an ordered semigroup and $A \subseteq S$, the intuitionistic characteristic function

$$\chi_A := \{\langle x, \mu_{\chi_A}, \gamma_{\chi_A} \rangle | x \in S\},$$

where μ_{χ_A} and γ_{χ_A} are fuzzy subsets defined as follows:

$$\mu_{\chi_A} : S \longrightarrow [0,1] | x \longrightarrow \mu_{\chi_A}(x) := \begin{cases} 1, & \text{if } x \in A, \\ 0, & \text{if } x \notin A, \end{cases}$$

and

$$\gamma_{\chi_A} : S \longrightarrow [0,1] | x \longrightarrow \gamma_{\chi_A}(x) := \begin{cases} 0 & \text{if } x \in A, \\ 1 & \text{if } x \notin A. \end{cases}$$

Definition 10 ([13]). Let $(S, ., \leq)$ be an ordered semigroup, an IFS $A = \langle \mu_A, \gamma_A \rangle$ of S is called an intuitionistic fuzzy subsemigroup of S if:

(1) $\mu_A(xy) \geq \min\{\mu_A(x), \mu_A(y)\}$,

(2) $\gamma_A(xy) \leq \max\{\gamma_A(x), \gamma_A(y)\}$.

Definition 11. Let $(S, ., \leq)$ be an ordered semigroup. An IFS $A = \langle \mu_A, \gamma_A \rangle$ of S is called an intuitionistic fuzzy left ideal of S if:

(1) $x \leq y \Longrightarrow \mu_A(x) \geq \mu_A(y)$ and $\gamma_A(x) \leq \gamma_A(y)$,

(2) $\mu_A(xy) \geq \mu_A(y)$ for all $x, y \in S$,

(3) $\gamma_A(xy) \leq \gamma_A(y)$ for all $x, y \in S$.

Definition 12. Let $(S, ., \leq)$ be an ordered semigroup. An IFS $A = \langle \mu_A, \gamma_A \rangle$ of S is called an intuitionistic fuzzy right ideal of S if:

(1) $x \leq y \Longrightarrow \mu_A(x) \geq \mu_A(y)$ and $\gamma_A(x) \leq \gamma_A(y)$,

(2) $\mu_A(xy) \geq \mu_A(x)$ for all $x, y \in S$,

(3) $\gamma_A(xy) \leq \gamma_A(x)$ for all $x, y \in S$.

An *IFS* $A = \langle \mu_A, \gamma_A \rangle$ of S is called an intuitionistic fuzzy two-sided ideal of S, if it is both an intuitionistic fuzzy left and an intuitionistic fuzzy right ideal of S. Equivalent definition:

Definition 13. Let $(S, ., \leq)$ be an ordered semigroup. An IFS $A = \langle \mu_A, \gamma_A \rangle$ of S is called an intuitionistic fuzzy ideal of S if:

(1) $x \leq y \Longrightarrow \mu_A(x) \geq \mu_A(y)$ and $\gamma_A(x) \leq \gamma_A(y)$,

(2) $\mu_A(xy) \geq \max\{\mu_A(x), \mu_A(y)\}$ for all $x, y \in S$,

(3) $\mu_A(xy) \leq \min\{\gamma_A(x), \gamma_A(y)\}$ for all $x, y \in S$.

4. Relation between intuitionistic fuzzy filters and intuitionistic fuzzy prime ideals of ordered semigroups

It is well known that a fuzzy subset f of an ordered semigroup S is a fuzzy filter of S if and only if the complement f' of f is a fuzzy prime ideal of S [18]. In this paragraph we give a relation between intuitionistic fuzzy filters and intuitionistic fuzzy prime ideals. We show that an *IFS* $A = \langle \mu_A, \gamma_A \rangle$ of S is an intuitionistic fuzzy filter of S if and only if the complement $A^c = \langle \gamma_A, \mu_A \rangle$ of A is an intuitionistic fuzzy prime ideal of S.

Definition 14. Let $(S, ., \leq)$ be an ordered semigroup. An IFS $A = \langle \mu_A, \gamma_A \rangle$ of S is called an intuitionistic fuzzy filter of S if:

(1) $x \leq y \Longrightarrow \mu_A(x) \leq \mu_A(y)$ and $\gamma_A(x) \geq \gamma_A(y)$,

(2) $\mu_A(xy) = \min\{\mu_A(x), \mu_A(y)\}$ for all $x, y \in S$,

(3) $\gamma_A(xy) = \max\{\gamma_A(x), \gamma_A(y)\}$ for all $x, y \in S$.

Lemma 15 ([25]). *Let S be a semigroup and A a non-empty subset of S. Then*

(1) *A is a subsemigroup of S if and only if the intuitionistic characteristic function $\chi_A = \langle \mu_{\chi_A}, \gamma_{\chi_A} \rangle$ of A is an intuitionistic fuzzy subsemigroup of S.*

(2) *A is a left(right, two-sided) ideal of S if and only if $\chi_A = \langle \mu_{\chi_A}, \gamma_{\chi_A} \rangle$ is an intuitionistic fuzzy left(right, two-sided) ideal of S.*

Proposition 16. *Let S be an ordered semigroup, $\emptyset \neq F \subseteq S$. Then F is a filter of S if and only if the intuitionistic characteristic function $\chi_F = \langle \mu_{\chi_F}, \gamma_{\chi_F} \rangle$ of F is an intuitionistic fuzzy filter of S.*

Proof. ($\Longrightarrow$) Let S be an ordered semigroup, F a filter of S, and $\chi_F = \langle \mu_{\chi_F}, \gamma_{\chi_F} \rangle$ the intuitionistic characteristic function of F. Then χ_F is an intuitionistic fuzzy filter of S. Indeed: By Lemma 15, χ_F is an intuitionistic fuzzy subsemigroup of S. Let $x, y \in S$, $x \leq y$. If $x \notin F$ then $\mu_{\chi_F}(x) = 0$ and $\gamma_{\chi_F}(x) = 1$.

Since $\mu_{\chi_F}(y) \geq 0$ and $\gamma_{\chi_F}(y) \leq 1$ for all $y \in S$. Then $\mu_{\chi_F}(x) \leq \mu_{\chi_F}(y)$ and $\gamma_{\chi_F}(x) \geq \gamma_{\chi_F}(y)$. Let $x \in F$, then $\mu_{\chi_F}(x) = 1$ and $\gamma_{\chi_F}(x) = 0$. Since $y \leq x \in F$ and F is a filter of S we have $y \in F$. Then $\mu_{\chi_F}(y) = 1$ and $\gamma_{\chi_F}(y) = 0$. Thus $\mu_{\chi_F}(x) \leq \mu_{\chi_F}(y)$ and $\gamma_{\chi_F}(x) \geq \gamma_{\chi_F}(y)$. Hence condition (1) of definition 14, is satisfied.

Let $x, y \in S$ if $xy \notin F$ then $\mu_{\chi_F}(xy) = 0$ and $\gamma_{\chi_F}(xy) = 1$. Since F is a filter of S and $xy \notin F \Longrightarrow x \notin F$ or $y \notin F$. Hence

$$\begin{aligned} \mu_{\chi_F}(x) &= 0 \text{ and } \gamma_{\chi_F}(x) = 1, \\ \mu_{\chi_F}(y) &= 0 \text{ and } \gamma_{\chi_F}(y) = 1. \end{aligned}$$

Thus

$$\begin{aligned} \min\{\mu_{\chi_F}(x), \mu_{\chi_F}(y)\} &= 0 = \mu_{\chi_F}(xy) \\ \text{and } \max\{\gamma_{\chi_F}(x), \gamma_{\chi_F}(y)\} &= 1 = \gamma_{\chi_F}(xy). \end{aligned}$$

Let $xy \in F$ then $\mu_{\chi_F}(xy) = 1$ and $\gamma_{\chi_F}(xy) = 0$. $xy \in F \Longrightarrow x \in F$ and $y \in F$. Then

$$\begin{aligned} \mu_{\chi_F}(x) &= 1 \text{ and } \gamma_{\chi_F}(x) = 0, \\ \mu_{\chi_F}(y) &= 0 \text{ and } \gamma_{\chi_F}(y) = 1. \end{aligned}$$

Thus

$$\begin{aligned} \min\{\mu_{\chi_F}(x), \mu_{\chi_F}(y)\} &= 1 = \mu_{\chi_F}(xy) \\ \text{and } \max\{\gamma_{\chi_F}(x), \gamma_{\chi_F}(y)\} &= 0 = \gamma_{\chi_F}(xy), \end{aligned}$$

hence conditions (2) and (3) of definition 14, are satisfied.

($\Longleftarrow$) Let $\chi_F = \langle \mu_{\chi_F}, \gamma_{\chi_F} \rangle$ be an intuitionistic fuzzy filter of S. By Lemma 15, F is subsemigroup of S.

Let $x, y \in S$. If $xy \in F$ then $\mu_{\chi_F}(xy) = 1$ and $\gamma_{\chi_F}(xy) = 0$. Since χ_F is an intuitionistic fuzzy filter of S. By conditions (2) and (3) of definition 14, we have

$$\begin{aligned} \min\{\mu_{\chi_F}(x), \mu_{\chi_F}(y)\} &= \mu_{\chi_F}(xy) \\ \text{and } \max\{\gamma_{\chi_F}(x), \gamma_{\chi_F}(y)\} &= \gamma_{\chi_F}(xy). \end{aligned}$$

Hence

$$\begin{aligned} \min\{\mu_{\chi_F}(x), \mu_{\chi_F}(y)\} &= 1 \\ \text{and } \max\{\gamma_{\chi_F}(x), \gamma_{\chi_F}(y)\} &= 0, \end{aligned}$$

and we have

$$\begin{aligned} \mu_{\chi_F}(x) &= 1 \text{ and } \gamma_{\chi_F}(x) = 0, \\ \mu_{\chi_F}(y) &= 1 \text{ and } \gamma_{\chi_F}(y) = 0. \end{aligned}$$

Thus we have $x \in F$ and $y \in F$.

Let $x, y \in S$, $x \leq y$. If $x \in F$ then $\mu_{\chi_F}(x) = 1$ and $\gamma_{\chi_F}(x) = 0$. Since $x \leq y$ and $\chi_F = \langle \mu_{\chi_F}, \gamma_{\chi_F} \rangle$ is an intuitionistic fuzzy filter of S, we have $\mu_{\chi_F}(x) \leq$

$\mu_{\chi_F}(y)$ and $\gamma_{\chi_F}(x) \geq \gamma_{\chi_F}(y)$. Thus $\mu_{\chi_F}(y) = 1$ and $\gamma_{\chi_F}(y) = 0$. Therefore $y \in F$. □

Definition 17. Let $(S, ., \leq)$ be an ordered semigroup and $A = \langle \mu_A, \gamma_A \rangle$ an IFS of S. Then A is called an intuitionistic fuzzy prime if:

(1) $\mu_A(xy) \leq \max\{\mu_A(x), \mu_A(y)\}$ for all $x, y \in S$,

(2) $\gamma_A(xy) \geq \min\{\gamma_A(x), \gamma_A(y)\}$ for all $x, y \in S$.

If $A = \langle \mu_A, \gamma_A \rangle$ is an intuitionistic fuzzy ideal of S, then A is called an intuitionistic fuzzy prime ideal of S.

Proposition 18. *Let $(S, ., \leq)$ be an ordered semigroup and $A = \langle \mu_A, \gamma_A \rangle$ an IFS of S. Then $A = \langle \mu_A, \gamma_A \rangle$ is an intuitionistic fuzzy filter of S if and only if the complement $A^c = \langle \gamma_A, \mu_A \rangle$ of A is an intuitionistic fuzzy prime ideal of S.*

Proof. ($\Longrightarrow$) Let $A = \langle \mu_A, \gamma_A \rangle$ be an intuitionistic fuzzy filter of an ordered semigroup S. Then $A^c = \langle \gamma_A, \mu_A \rangle$ is an intuitionistic fuzzy ideal of S. In fact: Let $x, y \in S$, such that $x \leq y$. Then

$$\gamma_A(x) = \mu_A^c(x) = 1 - \mu_A(x) \geq 1 - \mu_A(y) = \mu_A^c(y) = \gamma_A(y),$$

and

$$\mu_A(x) = \gamma_A^c(x) = 1 - \gamma_A(x) \leq 1 - \gamma_A(y) = \gamma_A^c(y) = \mu_A(y).$$

Let $x, y \in S$. Since A is an intuitionistic fuzzy filter of S. Then by conditions (2) and (3) of definition 14, we have

$$\gamma_A(xy) = \min\{\gamma_A(x), \gamma_A(y)\}$$

and

$$\mu_A(xy) = \max\{\mu_A(x), \mu_A(y)\}$$

by conditions (1) and (2) of definition 17, it follows that $A^c = \langle \gamma_A, \mu_A \rangle$ is an intuitionistic fuzzy prime ideal of S.

($\Longleftarrow$) Let $A^c = \langle \gamma_A, \mu_A \rangle$ be an intuitionistic fuzzy prime ideal of S. Then $A = \langle \mu_A, \gamma_A \rangle$ is an intuitionistic fuzzy filter of S. Indeed: Let $x, y \in S$, $x \leq y$. Since $A^c = \langle \gamma_A, \mu_A \rangle$ is an intuitionistic fuzzy ideal of S, we have

$$\mu_A(x) = \gamma_A^c(x) = 1 - \gamma_A(x) \leq 1 - \gamma_A(y) = \gamma_A^c(y) = \mu_A(y),$$

and

$$\gamma_A(x) = \mu_A^c(x) = 1 - \mu_A(x) \geq 1 - \mu_A(y) = \mu_A^c(y) = \gamma_A(y).$$

Let $x, y \in S$. Since $A^c = \langle \gamma_A, \mu_A \rangle$ is an intuitionistic fuzzy ideal of S. By condition (2) and (3) of definition 13, we have

$$\begin{aligned} \gamma_A(xy) &\geq \max\{\gamma_A(x), \gamma_A(y)\} && (*) \\ \text{and } \mu_A(xy) &\leq \min\{\mu_A(x), \mu_A(y)\} && (**). \end{aligned}$$

Since $A^c = \langle \gamma_A, \mu_A \rangle$ is an intuitionistic fuzzy prime ideal of S, by conditions (1) and (2) of definition 17, we have

$$\begin{aligned} \gamma_A(xy) &\leq \max\{\gamma_A(x), \gamma_A(y)\} && (***) \\ \text{and } \mu_A(xy) &\geq \min\{\mu_A(x), \mu_A(y)\} && (****). \end{aligned}$$

By $(*)$, $(**)$, $(***)$ and $(****)$ we have

$$\mu_A(xy) = \min\{\mu_A(x), \mu_A(y)\}$$
$$\text{and } \gamma_A(xy) = \max\{\gamma_A(x), \gamma_A(y)\}.$$

Thus $A = \langle \mu_A, \gamma_A \rangle$ is an intuitionistic fuzzy filter of S. □

5. Relation between intuitionistic fuzzy bi-filters and intuitionistic fuzzy prime bi-ideal subsets of ordered semigroups

In [24], Shabir and Khan have shown that a fuzzy subset f of an ordered semigroup S is a fuzzy bi-filter of S if and only if the complement f' of f is a prime fuzzy bi-ideal subset of S. In this paragraph, we show that an *IFS* $A = \langle \mu_A, \gamma_A \rangle$ of S is a fuzzy bi-filter of S if and only if the complement A^c of A is an intuitionistic fuzzy prime bi-ideal subset of S.

Definition 19. Let $(S, ., \leq)$ be an ordered semigroup. An IFS $A = \langle \mu_A, \gamma_A \rangle$ of S is called an intuitionistic fuzzy bi-ideal subset of S if:

(1) $x \leq y \Longrightarrow \mu_A(x) \geq \mu_A(y)$ and $\gamma_A(x) \leq \gamma_A(y)$,
(2) $\mu_A(xyx) \geq \mu_A(x)$ for all $x, y \in S$,
(3) $\gamma_A(xyx) \leq \gamma_A(x)$ for all $x, y \in S$.

In definition 17 of intuitionistic fuzzy prime subsets of S, if $A = \langle \mu_A, \gamma_A \rangle$ is an intuitionistic fuzzy bi-ideal subset of S, then A is called an intuitionistic fuzzy prime bi-ideal subset of S.

Proposition 20. *Let $(S, ., \leq)$ be an ordered semigroup, $\emptyset \neq B \subseteq S$. Then B is a bi-ideal subset of S if and only if the intuitonistic characteristic function $\chi_B = \langle \mu_{\chi_B}, \gamma_{\chi_B} \rangle$ of B is an intuitionistic fuzzy bi-ideal subset of S.*

Proof. $(\Longrightarrow)$ Let B be a bi-ideal subset of an ordered semigroup S and $\chi_B = \langle \mu_{\chi_B}, \gamma_{\chi_B} \rangle$ an intuitionistic characteristic function of B. Then χ_B is an intuitionistic fuzzy bi-ideal subset of S. In fact: Let $x, y \in S$, $x \leq y$. If $y \notin B$, then $\mu_{\chi_B}(y) = 0$, $\gamma_{\chi_B}(y) = 1$. Since $\mu_{\chi_B}(x) \geq 0$, $\gamma_{\chi_B}(x) \leq 1$ for all $x \in S$. We have $\mu_{\chi_B}(x) \geq \mu_{\chi_B}(y)$, and $\gamma_{\chi_B}(x) \leq \gamma_{\chi_B}(y)$. Let $y \in B$, then $\mu_{\chi_B}(y) = 1, \gamma_{\chi_B}(y) = 0$. Since B is a bi-ideal subset of S and $x \leq y \in B$ we have $x \in B$. Then $\mu_{\chi_B}(x) = 1, \gamma_{\chi_B}(x) = 0$. Again, we have $\mu_{\chi_B}(x) \geq \mu_{\chi_B}(y)$, and $\gamma_{\chi_B}(x) \leq \gamma_{\chi_B}(y)$. Let $x, y \in S$. If $x \notin B$, then $\mu_{\chi_B}(x) = 0, \gamma_{\chi_B}(x) = 1$. Since $\mu_{\chi_B}(xyx) \geq 0$, and $\gamma_{\chi_B}(xyx) \leq 1$ for all $x, y \in S$, we have $\mu_{\chi_B}(xyx) \geq \mu_{\chi_B}(x)$ and $\gamma_{\chi_B}(xyx) \leq \gamma_{\chi_B}(x)$. Let $x \in B$, then $\mu_{\chi_B}(x) = 1$ and $\gamma_{\chi_B}(x) = 0$. Since $x \in B$ and B is a bi-ideal subset of S, we have $xyx \in B$. Then $\mu_{\chi_B}(xyx) = 1$ and $\gamma_{\chi_B}(xyx) = 0$. Hence $\mu_{\chi_B}(xyx) \geq \mu_{\chi_B}(x)$ and $\gamma_{\chi_B}(xyx) \leq \gamma_{\chi_B}(x)$. Thus $\chi_B = \langle \mu_{\chi_B}, \gamma_{\chi_B} \rangle$ is an intuitionistic fuzzy bi-ideal subset of S.

$(\Longleftarrow)$ Let $\chi_B = \langle \mu_{\chi_B}, \gamma_{\chi_B} \rangle$ be an intuitionistic fuzzy bi-ideal subset of S. Then B is a bi-ideal subset of S. In fact: Let $x, y \in S$. If $x \in B$, then $\mu_{\chi_B}(x) = 1$ and $\gamma_{\chi_B}(x) = 0$. Since χ_B is an intuitionistic fuzzy bi-ideal subset of S, we have $\mu_{\chi_B}(xyx) \geq \mu_{\chi_B}(x)$ and $\gamma_{\chi_B}(xyx) \leq \gamma_{\chi_B}(x)$. Then $\mu_{\chi_B}(xyx) = 1$ and $\gamma_{\chi_B}(xyx) = 0$ and hence $xyx \in B$.

Let $x, y \in S$, $x \leq y$. If $y \in B$, then $\mu_{\chi_B}(y) = 1$ and $\gamma_{\chi_B}(y) = 0$. Since $x \leq y$ and $\chi_B = \langle \mu_{\chi_B}, \gamma_{\chi_B} \rangle$ is an intuitionistic fuzzy bi-ideal subset of S, we have $\mu_{\chi_B}(x) \geq \mu_{\chi_B}(y)$ and $\gamma_{\chi_B}(x) \leq \gamma_{\chi_B}(y)$. Thus $\mu_{\chi_B}(x) = 1$ and $\gamma_{\chi_B}(x) = 0$ and we have $x \in B$. □

Definition 21. Let $(S, ., \leq)$ be an ordered semigroup. An IFS $A = \langle \mu_A, \gamma_A \rangle$ of S is called an intuitionistic fuzzy bi-filter of S if:

(1) $x \leq y \Longrightarrow \mu_A(x) \geq \mu_A(y)$ and $\gamma_A(x) \leq \gamma_A(y)$,
(2) $\mu_A(xy) \geq \min\{\mu_A(x), \mu_A(y)\}$ for all $x, y \in S$,
(3) $\gamma_A(xy) \leq \max\{\gamma_A(x), \gamma_A(y)\}$ for all $x, y \in S$,
(4) $\mu_A(xyx) \leq \mu_A(x)$ for all $x, y \in S$,
(5) $\gamma_A(xyx) \geq \gamma_A(x)$ for all $x, y \in S$.

Proposition 22. *Let S be an ordered semigroup, $\emptyset \neq F \subseteq S$. Then F is a bi-filter of S if and only if the intuitionistic characteristic function $\chi_F = \langle \mu_{\chi_F}, \gamma_{\chi_F} \rangle$ of F is an intuitionistic fuzzy bi-filter of S.*

Proof. ($\Longrightarrow$) Let S be an ordered semigroup, F a bi-filter of S, and $\chi_F = \langle \mu_{\chi_F}, \gamma_{\chi_F} \rangle$ the intuitionistic characteristic function of F. Then χ_F is an intuitionistic fuzzy bi-filter of S. Indeed: By Lemma 15, χ_F is an intuitionistic fuzzy subsemigroup of S. Let $x, y \in S$, $x \leq y$. If $x \notin F$ then $\mu_{\chi_F}(x) = 0$ and $\gamma_{\chi_F}(x) = 1$. Since $\mu_{\chi_F}(y) \geq 0$ and $\gamma_{\chi_F}(y) \leq 1$ for all x, $y \in S$. Then $\mu_{\chi_F}(x) \leq \mu_{\chi_F}(y)$ and $\gamma_{\chi_F}(x) \geq \gamma_{\chi_F}(y)$. Let $x \in F$, then $\mu_{\chi_F}(x) = 1$ and $\gamma_{\chi_F}(x) = 0$. Since $y \leq x \in F$ and F is a bi-filter of S we have $y \in F$. Then $\mu_{\chi_F}(y) = 1$ and $\gamma_{\chi_F}(y) = 0$. Thus $\mu_{\chi_F}(x) \leq \mu_{\chi_F}(y)$ and $\gamma_{\chi_F}(x) \geq \gamma_{\chi_F}(y)$. Hence condition (1) of definition 21, is satisfied.

Let $x, y \in S$ if $xyx \notin F$ then $\mu_{\chi_F}(xyx) = 0$ and $\gamma_{\chi_F}(xyx) = 1$. Since F is a bi-filter of S and $xyx \notin F \Longrightarrow x \notin F$. Hence $\mu_{\chi_F}(x) = 0$ and $\gamma_{\chi_F}(x) = 1$. Then $\mu_{\chi_F}(xyx) \leq \mu_{\chi_F}(x)$ and $\gamma_{\chi_F}(xyx) \geq \gamma_{\chi_F}(x)$.

Let $xyx \in F$ then $\mu_{\chi_F}(xyx) = 1$ and $\gamma_{\chi_F}(xyx) = 0$. $xyx \in F \Longrightarrow x \in F$. Then

$$\mu_{\chi_F}(x) = 1 \text{ and } \gamma_{\chi_F}(x) = 0.$$

Thus $\mu_{\chi_F}(xyx) \leq \mu_{\chi_F}(x)$ and $\gamma_{\chi_F}(xyx) \geq \gamma_{\chi_F}(x)$.

($\Longleftarrow$) Let $\chi_F = \langle \mu_{\chi_F}, \gamma_{\chi_F} \rangle$ be an intuitionistic fuzzy bi-filter of S. By Lemma 15, F is a subsemigroup of S.

Let $x, y \in S$. If $xyx \in F$ then $\mu_{\chi_F}(xyx) = 1$ and $\gamma_{\chi_F}(xyx) = 0$. Since χ_F is an intuitionistic fuzzy bi-filter of S, we have $\mu_{\chi_F}(xyx) \leq \mu_{\chi_F}(x)$ and $\gamma_{\chi_F}(xyx) \geq \gamma_{\chi_F}(x)$. Thus $\mu_{\chi_F}(x) = 1$ and $\gamma_{\chi_F}(x) = 0$, and so $x \in F$.

Let $x, y \in S$, $x \leq y$. If $x \in F$ then $\mu_{\chi_F}(x) = 1$ and $\gamma_{\chi_F}(x) = 0$. Since $x \leq y$ and $\chi_F = \langle \mu_{\chi_F}, \gamma_{\chi_F} \rangle$ is an intuitionistic fuzzy bi-filter of S, we have $\mu_{\chi_F}(x) \leq \mu_{\chi_F}(y)$ and $\gamma_{\chi_F}(x) \geq \gamma_{\chi_F}(y)$. Thus $\mu_{\chi_F}(y) = 1$ and $\gamma_{\chi_F}(y) = 0$. Therefore $y \in F$. □

In definition 17, of intuitionistic fuzzy prime subsets of an ordered semigroup S if we consider the *IFS* $A = \langle \mu_A, \gamma_A \rangle$ as an intuitionistic fuzzy bi-ideal subset of S. Then A is called an intuitionistic fuzzy prime bi-ideal subset of S.

Proposition 23. *Let $(S, ., \leq)$ be an ordered semigroup and $A = \langle \mu_A, \gamma_A \rangle$ an IFS of S. Then $A = \langle \mu_A, \gamma_A \rangle$ is an intuitionistic fuzzy bi-filter of S if and only if the complement $A^c = \langle \gamma_A, \mu_A \rangle$ of A is an intuitionistic fuzzy prime bi-ideal subset of S.*

Proof. ($\Longrightarrow$) Let $A = \langle \mu_A, \gamma_A \rangle$ be an intuitionistic fuzzy filter of an ordered semigroup S. Then $A^c = \langle \gamma_A, \mu_A \rangle$ is an intuitionistic fuzzy bi-ideal subset of S. In fact: Let $x, y \in S$, such that $x \leq y$. Then

$$\gamma_A(x) = \mu_A^c(x) = 1 - \mu_A(x) \geq 1 - \mu_A(y) = \mu_A^c(y) = \gamma_A(y),$$

and

$$\mu_A(x) = \gamma_A^c(x) = 1 - \gamma_A(x) \leq 1 - \gamma_A(y) = \gamma_A^c(y) = \mu_A(y).$$

Let $x, y \in S$. Since A is an intuitionistic fuzzy bi-filter of S. Then by conditions (4) and (5) of definition 21, we have

$$\gamma_A(xyx) \leq \gamma_A(x), \text{ and } \mu_A(xyx) \geq \mu_A(x)$$

by conditions (2) and (3) of definition 17, it follows that $A^c = \langle \gamma_A, \mu_A \rangle$ is an intuitionistic fuzzy bi-ideal subset of S. Since $A = \langle \mu_A, \gamma_A \rangle$ is an intuitionistic fuzzy bi-filter of S, by conditions (2) and (3) of definition 21, we have

$$\begin{aligned} \gamma_A(xy) &\leq \max\{\gamma_A(x), \gamma_A(y)\}, \\ \text{and } \mu_A(xy) &\geq \min\{\mu_A(x), \mu_A(y)\}. \end{aligned}$$

By conditions (1) and (2) of definition 19, it follows that $A^c = \langle \gamma_A, \mu_A \rangle$ is an intuitionistic fuzzy prime bi-ideal subset of S.

($\Longleftarrow$) Let $A^c = \langle \gamma_A, \mu_A \rangle$ is an intuitionistic fuzzy prime bi-ideal subset of S. Then $A = \langle \mu_A, \gamma_A \rangle$ is an intuitionistic fuzzy bi-filter of S. Indeed: Let $x, y \in S$, $x \leq y$. Since $A^c = \langle \gamma_A, \mu_A \rangle$ is an intuitionistic fuzzy bi-ideal subset of S, we have

$$\mu_A(x) = \gamma_A^c(x) = 1 - \gamma_A(x) \leq 1 - \gamma_A(y) = \gamma_A^c(y) = \mu_A(y),$$

and

$$\gamma_A(x) = \mu_A^c(x) = 1 - \mu_A(x) \geq 1 - \mu_A(y) = \mu_A^c(y) = \gamma_A(y).$$

Let $x, y \in S$. Since $A^c = \langle \gamma_A, \mu_A \rangle$ is an intuitionistic fuzzy bi-ideal subset of S. By condition (2) and (3) of definition, we have $\mu_A(xyx) \leq \mu_A(x)$ and $\gamma_A(xyx) \geq \gamma_A(x)$. Since $A^c = \langle \gamma_A, \mu_A \rangle$ is an intuitionistic fuzzy prime bi-ideal subset of S, we have

$$\begin{aligned} \mu_A(xy) &\geq \min\{\mu_A(x), \mu_A(y)\}, \\ \text{and } \gamma_A(xy) &\leq \max\{\gamma_A(x), \gamma_A(y)\}. \end{aligned}$$

Thus $A = \langle \mu_A, \gamma_A \rangle$ is an intuitionistic fuzzy bi-filter of S. $\square$

6. Relation between intuitionistic fuzzy left (resp. right) filters and intuitionistic fuzzy prime left(resp. right) ideal of ordered semigroups

It is well known that a fuzzy subset f of an ordered semigroup S is a left filter of S if and only if the complement f' of f is a fuzzy prime left ideal of S [24]. In this paragraph, we show that an *IFS*, $A = \langle \mu_A, \gamma_A \rangle$ of an ordered semigroup S is a left(resp. right) filter of S if and only if the complement $A^c = \langle \gamma_A, \mu_A \rangle$ of A is an intuitionistic fuzzy prime left (resp. right) ideal of S.

Definition 24. Let $(S, ., \leq)$ be an ordered semigroup. An IFS $A = \langle \mu_A, \gamma_A \rangle$ of S is called an intuitionistic fuzzy left filter of S if:
(1) $x \leq y \Longrightarrow \mu_A(x) \geq \mu_A(y)$ and $\gamma_A(x) \leq \gamma_A(y)$,
(2) $\mu_A(xy) \geq \min\{\mu_A(x), \mu_A(y)\}$ for all $x, y \in S$,
(3) $\gamma_A(xy) \leq \max\{\gamma_A(x), \gamma_A(y)\}$ for all $x, y \in S$,
(4) $\mu_A(xy) \geq \mu_A(y)$ for all $x, y \in S$,
(5) $\gamma_A(xy) \leq \gamma_A(y)$ for all $x, y \in S$.

Definition 25. Let $(S, ., \leq)$ be an ordered semigroup. An IFS $A = \langle \mu_A, \gamma_A \rangle$ of S is called an intuitionistic fuzzy right filter of S if:
(1) $x \leq y \Longrightarrow \mu_A(x) \geq \mu_A(y)$ and $\gamma_A(x) \leq \gamma_A(y)$,
(2) $\mu_A(xy) \geq \min\{\mu_A(x), \mu_A(y)\}$ for all $x, y \in S$,
(3) $\gamma_A(xy) \leq \max\{\gamma_A(x), \gamma_A(y)\}$ for all $x, y \in S$,
(4) $\mu_A(xy) \geq \mu_A(x)$ for all $x, y \in S$,
(5) $\gamma_A(xy) \leq \gamma_A(x)$ for all $x, y \in S$.

Proposition 26. *Let S be an ordered semigroup, $\emptyset \neq F \subseteq S$. Then F is a left filter of S if and only if the intuitionistic characteristic function $\chi_F = \langle \mu_{\chi_F}, \gamma_{\chi_F} \rangle$ of F is an intuitionistic fuzzy left filter of S.*

Proof. ($\Longrightarrow$) Let F be a left filter of S, and $\chi_F = \langle \mu_{\chi_F}, \gamma_{\chi_F} \rangle$ the intuitionistic characteristic function of F. Then χ_F is an intuitionistic fuzzy left filter of S. Indeed: By Lemma 15, χ_F is an intuitionistic fuzzy subsemigroup of S. Let $x, y \in S$, $x \leq y$. If $x \notin F$ then $\mu_{\chi_F}(x) = 0$ and $\gamma_{\chi_F}(x) = 1$. Since $\mu_{\chi_F}(y) \geq 0$ and $\gamma_{\chi_F}(y) \leq 1$ for all $y \in S$. Then $\mu_{\chi_F}(x) \leq \mu_{\chi_F}(y)$ and $\gamma_{\chi_F}(x) \geq \gamma_{\chi_F}(y)$. Let $x \in F$, then $\mu_{\chi_F}(x) = 1$ and $\gamma_{\chi_F}(x) = 0$. Since $y \leq x \in F$ and F is a left filter of S we have $y \in F$. Then $\mu_{\chi_F}(y) = 1$ and $\gamma_{\chi_F}(y) = 0$. Thus $\mu_{\chi_F}(x) \leq \mu_{\chi_F}(y)$ and $\gamma_{\chi_F}(x) \geq \gamma_{\chi_F}(y)$. Hence condition (1) of definition 24, is satisfied.

Let $x, y \in S$ if $xy \in F$ then $\mu_{\chi_F}(xy) = 1$ and $\gamma_{\chi_F}(xy) = 0$. Since F is a left filter of S and $xy \in F \Longrightarrow y \in F$. Hence $\mu_{\chi_F}(y) = 1$ and $\gamma_{\chi_F}(y) = 0$. Thus $\mu_{\chi_F}(xy) \geq \mu_{\chi_F}(y)$ and $\gamma_{\chi_F}(xy) \leq \gamma_{\chi_F}(y)$.

($\Longleftarrow$) Let $\chi_F = \langle \mu_{\chi_F}, \gamma_{\chi_F} \rangle$ be an intuitionistic fuzzy left filter of S. By Lemma 15, F is a subsemigroup of S. Let x, $y \in S$. If $xy \in F$ then $\mu_{\chi_F}(xy) = 1$ and $\gamma_{\chi_F}(xy) = 0$. Since χ_F is an intuitionistic fuzzy left filter of S we have $\mu_{\chi_F}(xy) \geq \mu_{\chi_F}(y)$ and $\gamma_{\chi_F}(xy) \leq \gamma_{\chi_F}(y)$. Thus $\mu_{\chi_F}(y) = 1$ and $\gamma_{\chi_F}(y) = 0$ and so $y \in F$.

Let $x, y \in S$, $x \leq y$. If $x \in F$ then $\mu_{\chi_F}(x) = 1$ and $\gamma_{\chi_F}(x) = 0$. Since $x \leq y$ and χ_F is an intuitionistic fuzzy left filter of S, we have $\mu_{\chi_F}(x) \leq \mu_{\chi_F}(y)$ and $\gamma_{\chi_F}(x) \geq \gamma_{\chi_F}(y)$. Thus $\mu_{\chi_F}(y) = 1$ and $\gamma_{\chi_F}(y) = 0$. Therefore $y \in F$. □

In a similar way we can prove the following:

Proposition 27. *Let S be an ordered semigroup, $\emptyset \neq F \subseteq S$. Then F is a right filter of S if and only if the intuitionistic characteristic function $\chi_F = \langle \mu_{\chi_F}, \gamma_{\chi_F} \rangle$ of F is an intuitionistic fuzzy right filter of S.*

In definition 17, of an intuitionistic fuzzy prime subsets of an ordered semigroup S, if $A = \langle \mu_A, \gamma_A \rangle$ is an intuitionistic fuzzy left(resp. right) ideal of S. Then $A = \langle \mu_A, \gamma_A \rangle$ is an intuitionistic fuzzy prime left(resp. right) ideal of S.

Proposition 28. *Let $(S, ., \leq)$ be an ordered semigroup and $A = \langle \mu_A, \gamma_A \rangle$ an IFS of S. Then $A = \langle \mu_A, \gamma_A \rangle$ is an intuitionistic fuzzy left filter of S if and only if the complement $A^c = \langle \gamma_A, \mu_A \rangle$ of A is an intuitionistic fuzzy prime left ideal subset of S.*

Proof. ($\Longrightarrow$) Let $A = \langle \mu_A, \gamma_A \rangle$ be an intuitionistic fuzzy left filter of an ordered semigroup S. Then $A^c = \langle \gamma_A, \mu_A \rangle$ is an intuitionistic fuzzy left ideal of S. In fact: Let $x, y \in S$, such that $x \leq y$. Then

$$\gamma_A(x) = \mu_A^c(x) = 1 - \mu_A(x) \geq 1 - \mu_A(y) = \mu_A^c(y) = \gamma_A(y),$$

and

$$\mu_A(x) = \gamma_A^c(x) = 1 - \gamma_A(x) \leq 1 - \gamma_A(y) = \gamma_A^c(y) = \mu_A(y).$$

Let $x, y \in S$. Since A is an intuitionistic fuzzy left filter of S. Then by conditions (4) and (5) of definition 24, we have

$$\mu_A(xy) \geq \mu_A(y) \text{ and } \gamma_A(xy) \leq \gamma_A(y),$$

by conditions (2) and (3) of definition 11, it follows that $A^c = \langle \gamma_A, \mu_A \rangle$ is an intuitionistic fuzzy left ideal of S. Since A is an intuitionistic fuzzy left filter of S. By conditions (2) and (3) of definition 24, we have

$$\begin{aligned} \mu_A(xy) &\geq \min\{\mu_A(x), \mu_A(y)\} \\ \text{and } \gamma_A(xy) &\geq \max\{\gamma_A(x), \gamma_A(y), \end{aligned}$$

by conditions (1) and (2) of definition 17, it follows that A^c is an intuitionistic fuzzy prime left ideal of S.

($\Longleftarrow$) Let $A^c = \langle \gamma_A, \mu_A \rangle$ is an intuitionistic fuzzy prime left ideal of S. Then $A = \langle \mu_A, \gamma_A \rangle$ is an intuitionistic fuzzy left filter of S. Indeed: Let $x, y \in S$, $x \leq y$. Since $A^c = \langle \gamma_A, \mu_A \rangle$ is an intuitionistic fuzzy left ideal of S, we have

$$\mu_A(x) = \gamma_A^c(x) = 1 - \gamma_A(x) \leq 1 - \gamma_A(y) = \gamma_A^c(y) = \mu_A(y),$$

and

$$\gamma_A(x) = \mu_A^c(x) = 1 - \mu_A(x) \geq 1 - \mu_A(y) = \mu_A^c(y) = \gamma_A(y).$$

Let $x, y \in S$. Since $A^c = \langle \gamma_A, \mu_A \rangle$ is an intuitionistic fuzzy prime left ideal of S. By conditions (1) and (2) of definition 17, we have

$$\begin{aligned} \gamma_A(xy) &\geq \max\{\gamma_A(x), \gamma_A(y)\} \\ \text{and } \mu_A(xy) &\leq \min\{\mu_A(x), \mu_A(y)\} \end{aligned}$$

By conditions (1) and (2) of definition 10, it follows that A is an intuitionistic fuzzy subsemigroup of S. Since $A^c = \langle \gamma_A, \mu_A \rangle$ is an intuitionistic fuzzy left ideal of S, by condition (2) and (3) of definition 11, we have $\gamma_A(xy) \geq \gamma_A(y)$ and $\mu_A(xy) \leq \mu_A(y)$. Thus $A = \langle \mu_A, \gamma_A \rangle$ is an intuitionistic fuzzy left filter of S. □

Similarly we can prove the following:

Proposition 29. *Let $(S, ., \leq)$ be an ordered semigroup and $A = \langle \mu_A, \gamma_A \rangle$ an IFS of S. Then $A = \langle \mu_A, \gamma_A \rangle$ is an intuitionistic fuzzy right filter of S if and only if the complement $A^c = \langle \gamma_A, \mu_A \rangle$ of A is an intuitionistic fuzzy prime right ideal of S.*

References

1. Y. S. Ahn, K. Hur, D. S. Kim, *The lattice of intuitionistic fuzzy ideals of a ring,* J. Appl. Math. Comput. 19 (2005) 551 – 572.
2. K. T. Atanassov, *Intuitionistic fuzzy sets, Fuzzy Sets and Syst.* 20 (1986) 87 – 96.
3. K. T. Atanassov, *New operations defined over the intuitionistic fuzzy sets,* Fuzzy Sets and Syst. 61 (1994) 137 – 172.
4. K. T. Atanassov, *Intuitionistic fuzzy sets, Theory and Applications Studies in Fuzziness and Soft Computing,* Vol. 35, Physica-Verlag, Heidelberg, 1999.
5. B. Banerjee, D. Kr. Basnet, *Intuitionistic fuzzy subrings and ideals,* J. Fuzzy Math. 11 (1) (2003) 139 – 155.
6. R. Biswas, *On fuzzy sets and intuitionistic fuzzy sets,* NIFS 3 (1997) 3 – 11.
7. Y. Cao, *Chain decompositions of ordered semigroups,* Semigroup Forum Vol. 65 (2002) 83 – 106.
8. B. Davvaz, W. A. Dudek Y. B. Jun, *Intuitionistic fuzzy H_v-submodules,* Inform. Sci. 176 (2006) 285 – 300.
9. S. K. De, R. Biswas, A. R. Roy, *An application of intuitionistic fuzzy sets in medical diagnosis,* Fuzzy Sets Syst. 117 (2001) 209 – 213.
10. W. A. Dudek, B. Davvaz, Y. B. Jun, *On intuitionistic fuzzy subhyper-quasigroups of hyperquasigroups,* Inform. Sci. 170 (2005) 251 – 262.
11. L. Dengfeng, C. Chuntian, *New similarity measures of intuitionistic fuzzy sets and application to pattern recognitions,* Pattern Recognition Letters, 23(2002), 221 – 225.
12. W. L. Gau, D. J. Buehrer, *Vague sets,* IEEE Trans. Systems Man Cybernet, 23(1993), 610 – 614.
13. Y. B. Jun, *Intuitionistic fuzzy bi-ideals of ordered semigroups,* KYUNGPOOK Math. J. 45(2005), 527 – 537.
14. Y. B. Jun, *On (Φ,Ψ)-intuitionistic fuzzy subgroups,* KYUNGPOOK Math. J. 45(2005), 87 – 94.
15. N. Kehayopulu, *On weakly prime ideals of ordered semigroups,* Math. Japonica 35 No. 6 (1990) 1051 – 1056.
16. N. Kehayopulu, *Remarks on ordered semigroups,* Math., Japonica 35 (1990) 1061 – 1063.

17. N. Kehayopulu, Xiang-Yun Xie and M. Tsingelis, *A characterization of prime and semiprime ideals of semigroups in terms of fuzzy subsets,* Soochow J. Math. 27 (2001) $139 - 144$.
18. N. Kehayopulu, M. Tsingelis, *Fuzzy sets in ordered groupoids,* Semigroup Forum, 65(2002), $128 - 132$.
19. N. Kehayopulu, M. Tsingelis, *The embedding of an ordered groupoids into a poe-groupoid in terms of fuzzy sets,* Inform. Sci. 152 (2003) $231 - 236$.
20. N. Kehayopulu and M. Tsingelis, *Fuzzy bi-ideals in ordered semigroups,* Inform. Sci., 171 (2005) $13 - 28$.
21. K. H. Kim, W. A. Dudek and Y. B. Jun, *On intuitionistic fuzzy subquasigroups of quasigroups,* Quasigroups and Related Systems, 7(2000), $15 - 28$.
22. K. H. Kim and Y. B. Jun, *Intuitionistic fuzzy interior ideals of semigroups,* Int. J. Math. Math. Sci., 27(5)(2001), $261 - 267$.
23. K. H. Kim and Y. B. Jun, *Intuitionistic fuzzy ideals of semigroups,* Indian J. Pure Appl. Math., 33(4)(2002), $443 - 449$.
24. M. Shabir and A. Khan, *Fuzzy filters of ordered semigroups,* accepted.
25. M. Shabir and A. Khan, *Intuitionistic fuzzy interior ideals of ordered semigroups,* submitted.
26. E. Szmidt, J. Kacprzyk, *Entropy for intuitionistic fuzzy sets,* Fuzzy Sets Syst. 118 (2001) $467 - 477$.
27. Yongfa Hong, Changjun Jiang, *Characterizing regular semigroups using intuitionistic fuzzy sets,* Notes on *IFS*, $10 : 20 - 31$(2004).
28. L. A. Zadeh, *Fuzzy sets,* Inform. Control, 8(1965), $338 - 353$.

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