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INTUITIONISTIC FUZZINESS OF STRONG HYPER K-IDEALS AND HYPER K-SUBALGEBRAS

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ABSTRACT. Intuitionistic fuzzifications of strong hyper K-ideals and hyper K-subalgebras in hyper K-algebras are discussed, and related properties are investigated. Relations between intuitionistic fuzzy hyper K-subalgebras, intuitionistic fuzzy weak hyper K-ideals and intuitionistic fuzzy strong hyper K-ideals are provided.

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1. Introduction

The study of BCK-algebras was initiated by K. Iséki in 1966 as a generalization of the concept of set-theoretic difference and propositional calculus. Since then many researches worked in this area. The hyperstructure theory (called also multialgebras) is introduced in 1934 by F. Marty [10] at the 8th congress of Scandinavian Mathematiciens. Around the 40's, several authors worked on hypergroups, especially in France and in the United States, but also in Italy, Russia, Japan and Iran.

Hyperstructures have many applications to several sectors of both pure and applied sciences. Recently in [9] Y. B. Jun et al. introduced and studied hyperBCK-algebra which is a generalization of a BCK-algebra. In [1] and [9] R. A. Borzooei et al. constructed the hyper K-algebras, and studied (weak) implicative hyper K-ideals in hyper K-algebras.

In [7] and [8] Y. B. Jun et al. studied the fuzzy (implicative) hyper K-ideals in hyper K-algebras. Y. B. Jun et al. [6] introduced the notion of fuzzy (weak) implicative hyper K-ideals, and investigated related properties. They gave relations among fuzzy weak implicative hyper K-ideals, fuzzy implicative

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hyper K-ideals, and fuzzy hyper K-ideals. In [4], R. A. Borzooei and Y. B. Jun studied intuitionistic fuzzy hyper BCK-ideals of hyper BCK-algebras.

In [2], R. A. Borzooei and Y. B. Jun discussed intuitionistic fuzzifications of (weak) implicative hyper K-ideals in hyper K-algebras. They gave relations among intuitionistic fuzzy hyper K-ideals, intuitionistic fuzzy weak hyper K-ideals, intuitionistic fuzzy implicative hyper K-ideals and intuitionistic fuzzy weak implicative hyper K-ideals. They provided conditions for an intuitionistic fuzzy hyper K-ideal to be an intuitionistic fuzzy implicative hyper K-ideal, and also discussed conditions for an intuitionistic fuzzy weak hyper K-ideal to be an intuitionistic fuzzy weak implicative hyper K-ideal.

In this paper we consider the intuitionistic fuzzifications of strong hyper K-ideals and hyper K-subalgebras in hyper K-algebras. We give relations between intuitionistic fuzzy hyper K-subalgebras, intuitionistic fuzzy strong hyper K-ideals and intuitionistic fuzzy weak hyper K-ideals.

2. Preliminaries

We include some elementary aspects of hyper K-algebras that are necessary for this paper, and for more details we refer to [3] and [11]. Let H be a non-empty set endowed with a hyper operation "$\circ$", that is, $\circ$ is a function from $H \times H$ to $\mathcal{P}^*(H) = \mathcal{P}(H) \setminus \{\emptyset\}$. For two subsets A and B of H, denote by $A \circ B$ the set $\bigcup_{a \in A, b \in B} a \circ b$.

By a *hyper I-algebra* we mean a non-empty set H endowed with a hyper operation "$\circ$" and a constant 0 satisfying the following axioms:

(H1) $(x \circ z) \circ (y \circ z) \prec x \circ y$,
(H2) $(x \circ y) \circ z = (x \circ z) \circ y$,
(H3) $x \prec x$,
(H4) $x \prec y$ and $y \prec x$ imply $x = y$

for all $x, y, z \in H$, where $x \prec y$ is defined by $0 \in x \circ y$ and for every $A, B \subseteq H$, $A \prec B$ is defined by $\exists a \in A$ and $\exists b \in B$ such that $a \prec b$. If a hyper I-algebra $(H, \circ, 0)$ satisfies an additional condition:

(H5) $0 \prec x$ for all $x \in H$,

then $(H, \circ, 0)$ is called a *hyper K-algebra* (see [3]).

In a hyper I-algebra H, the following hold (see [Proposition 3.4]):

(a1) $(A \circ B) \circ C = (A \circ C) \circ B$. (a2) $x \circ (x \circ y) \prec y$.
(a3) $x \circ y \prec z \Leftrightarrow x \circ z \prec y$. (a4) $A \circ B \prec C \Leftrightarrow A \circ C \prec B$.
(a5) $(x \circ z) \circ (x \circ y) \prec y \circ z$. (a6) $(A \circ C) \circ (B \circ C) \prec A \circ B$.
(a7) $A \circ (A \circ B) \prec B$. (a8) $A \prec A$.
(a9) $A \subseteq B$ implies $A \prec B$.

for all $x, y, z \in H$ and for all nonempty subsets A, B and C of H.

A nonempty subset I of a hyper K-algebra H is called a *weak hyper K-ideal* of H (see [3]) if it satisfies

(I1) $0 \in I$,

(I2) $(\forall x, y \in H)\ (x \circ y \subseteq I,\ y \in I \Rightarrow x \in I)$.

A nonempty subset I of a hyper K-algebra H is called a *hyper K-ideal* of H (see [3]) if it satisfies (I1) and

$$(\forall x, y \in H)\,(x \circ y \prec I,\ y \in I \Rightarrow x \in I). \tag{2.1}$$

A nonempty subset I of a hyper K-algebra H is called a *strong hyper K-ideal* of H (see [5]) if it satisfies (I1) and

$$(\forall x, y \in H)\,((x \circ y) \cap I \neq \emptyset,\ y \in I \Rightarrow x \in I). \tag{2.2}$$

3. Intuitionistic fuzzy strong hyper K-ideals

In what follows let H denote a hyper K-algebra unless otherwise specified. An *intuitionistic fuzzy set* (IFS, for short) in H is an expression α given by

$$\alpha = \{\langle x, \mu_\alpha(x), \gamma_\alpha(x)\rangle \mid x \in H\}$$

where the functions $\mu_\alpha : H \to [0,1]$ and $\gamma_\alpha : H \to [0,1]$ denote the degree of membership (namely $\mu_\alpha(x)$) and the degree of nonmembership (namely $\gamma_\alpha(x)$) of each element $x \in H$ to α, respectively, and

$$0 \le \mu_\alpha(x) + \gamma_\alpha(x) \le 1$$

for all $x \in H$. For the sake of simplicity, we shall use the notation $\alpha = \langle H, \mu_\alpha, \gamma_\alpha\rangle$ instead of $\alpha = \{\langle x, \mu_\alpha(x), \gamma_\alpha(x)\rangle \mid x \in H\}$. Let $\alpha = \langle H, \mu_\alpha, \gamma_\alpha\rangle$ be an IFS in H and let $m, n \in [0,1]$ with $m + n \le 1$. Then the IFS $C_{(m,n)}$ in H is defined by $C_{(m,n)}(x) = (m, n)$, i.e.,

$$\mu_{C_{(m,n)}}(x) = m \ \text{ and } \ \gamma_{C_{(m,n)}}(x) = n$$

for all $x \in H$. The representation "$\alpha(x) \ge (m, n)$" means that $\mu_\alpha(x) \ge m$ and $\gamma_\alpha(x) \le n$. Then the set

$$H_\alpha^{(m,n)} := \left\{x \in H \mid \alpha(x) \ge C_{(m,n)}(x)\right\} = \left\{x \in H \mid \mu_\alpha(x) \ge m,\ \gamma_\alpha(x) \le n\right\}$$

is called an *intuitionistic level set* of α in H.

Definition 3.1. An IFS $\alpha = \langle H, \mu_\alpha, \gamma_\alpha\rangle$ in H is called an *intuitionistic fuzzy strong hyper K-ideal* of H if it satisfies

$$\begin{aligned} &\inf_{a \in x \circ x} \mu_\alpha(a) \ge \mu_\alpha(x) \ge \min\left\{\mu_\alpha(y),\ \sup_{b \in x \circ y} \mu_\alpha(b)\right\}, \\ &\sup_{c \in x \circ x} \gamma_\alpha(c) \le \gamma_\alpha(x) \le \max\left\{\gamma_\alpha(y),\ \inf_{d \in x \circ y} \gamma_\alpha(d)\right\} \end{aligned} \tag{3.1}$$

for all $x, y \in H$.

Example 3.2. Let $H = \{0, a, b\}$ be a hyper K-algebra with the following Cayley table:

$\circ$	0	a	b
0	$\{0\}$	$\{0\}$	$\{0\}$
a	$\{a\}$	$\{0\}$	$\{a\}$
b	$\{b\}$	$\{b\}$	$\{0, b\}$

Define an IFS $\alpha = \langle H, \mu_\alpha, \gamma_\alpha \rangle$ in H by

$$\alpha = \left\langle H, \left(\frac{0}{0.6}, \frac{a}{0.3}, \frac{b}{0.6} \right), \left(\frac{0}{0.08}, \frac{a}{0.5}, \frac{b}{0.08} \right) \right\rangle.$$

Then $\alpha = \langle H, \mu_\alpha, \gamma_\alpha \rangle$ is an intuitionistic fuzzy strong hyper K-ideal of H.

Proposition 3.3. *Every intuitionistic fuzzy strong hyper K-ideal $\alpha = \langle H, \mu_\alpha, \gamma_\alpha \rangle$ of H satisfies the following assertions.*

(i) $(\forall x \in H) \left(\mu_\alpha(0) \geq \mu_\alpha(x) \ \&\ \gamma_\alpha(0) \leq \gamma_\alpha(x) \right)$.

(ii) $(\forall x, y \in H) \left(x \prec y \Rightarrow \mu_\alpha(x) \geq \mu_\alpha(y) \ \&\ \gamma_\alpha(x) \leq \gamma_\alpha(y) \right)$.

(iii) *For all $x, y \in H$ and $a \in x \circ y$, we have*

$$\mu_\alpha(x) \geq \min\{\mu_\alpha(a), \mu_\alpha(y)\} \ \&\ \gamma_\alpha(x) \leq \max\{\gamma_\alpha(a), \gamma_\alpha(y)\}.$$

Proof. (i) Since $0 \in x \circ x$ for all $x \in H$, we get

$$\mu_\alpha(0) \geq \inf_{a \in x \circ x} \mu_\alpha(a) \geq \mu_\alpha(x),$$

$$\gamma_\alpha(0) \leq \sup_{a \in x \circ x} \gamma_\alpha(a) \leq \gamma_\alpha(x)$$

for all $x \in H$. Thus (i) is valid.

(ii) Let $x, y \in H$ be such that $x \prec y$. Then $0 \in x \circ y$, and so

$$\sup_{u \in x \circ y} \mu_\alpha(u) \geq \mu_\alpha(0) \ \&\ \inf_{v \in x \circ y} \gamma_\alpha(v) \leq \gamma_\alpha(0).$$

It follows from (3.1) and (i) that

$$\mu_\alpha(x) \geq \min\left\{ \mu_\alpha(y), \sup_{u \in x \circ y} \mu_\alpha(u) \right\} \geq \min\{\mu_\alpha(y), \mu_\alpha(0)\} = \mu_\alpha(y),$$

$$\gamma_\alpha(x) \leq \max\left\{ \gamma_\alpha(y), \inf_{v \in x \circ y} \gamma_\alpha(v) \right\} \leq \max\{\gamma_\alpha(y), \gamma_\alpha(0)\} = \gamma_\alpha(y).$$

(iii) For any $x, y \in H$ and $a \in x \circ y$, we obtain

$$\mu_\alpha(x) \geq \min\left\{ \mu_\alpha(y), \sup_{b \in x \circ y} \mu_\alpha(b) \right\} \geq \min\{\mu_\alpha(y), \mu_\alpha(a)\},$$

$$\gamma_\alpha(x) \leq \max\left\{ \gamma_\alpha(y), \inf_{d \in x \circ y} \gamma_\alpha(d) \right\} \leq \max\{\gamma_\alpha(y), \gamma_\alpha(a)\}.$$

Hence (iii) is valid. □

Corollary 3.4. *For every intuitionistic fuzzy strong hyper K-ideal* $\alpha = \langle H, \mu_\alpha, \gamma_\alpha \rangle$ *of* H, *we have*

$$\mu_\alpha(x) \geq \min\left\{\mu_\alpha(y), \inf_{a \in x \circ y} \mu_\alpha(a)\right\},$$

$$\gamma_\alpha(x) \leq \max\left\{\gamma_\alpha(y), \sup_{b \in x \circ y} \gamma_\alpha(b)\right\}$$

for all $x, y \in H$.

Proof. Since $\mu_\alpha(a) \geq \inf_{u \in x \circ y} \mu_\alpha(u)$ and $\gamma_\alpha(a) \leq \sup_{v \in x \circ y} \gamma_\alpha(v)$ for all $a \in x \circ y$, the result follows from Proposition 3.3. □

Theorem 3.5. *If* $\alpha = \langle H, \mu_\alpha, \gamma_\alpha \rangle$ *is an intuitionistic fuzzy strong hyper K-ideal of* H, *then the nonempty intuitionistic level set* $H_\alpha^{(m,n)}$ *is a strong hyper K-ideal of* H *for all* $m, n \in [0, 1]$ *with* $m + n \leq 1$.

Proof. Let $m, n \in [0, 1]$ be such that $m + n \leq 1$ and $H_\alpha^{(m,n)} \neq \emptyset$. Then there exists $x \in H_\alpha^{(m,n)}$ and so

$$\alpha(x) \geq C_{(m,n)}(x), \quad \text{i.e.,} \quad \mu_\alpha(x) \geq m \quad \text{and} \quad \gamma_\alpha(x) \leq n.$$

Using Proposition 3.3(i), we get

$$\mu_\alpha(0) \geq \mu_\alpha(x) \geq m \quad \text{and} \quad \gamma_\alpha(0) \leq \gamma_\alpha(x) \leq n,$$

which imply that $0 \in H_\alpha^{(m,n)}$. Let $x, y \in H$ be such that $y \in H_\alpha^{(m,n)}$ and $(x \circ y) \cap H_\alpha^{(m,n)} \neq \emptyset$. Then there exists $b \in (x \circ y) \cap H_\alpha^{(m,n)}$, and hence $b \in x \circ y$ and $\alpha(b) \geq C_{(m,n)}(b)$, that is, $\mu_\alpha(b) \geq m$ and $\gamma_\alpha(b) \leq n$. It follows from (3.1) that

$$\mu_\alpha(x) \geq \min\left\{\mu_\alpha(y), \sup_{u \in x \circ y} \mu_\alpha(u)\right\} \geq \min\{\mu_\alpha(y), \mu_\alpha(b)\} \geq m,$$

$$\gamma_\alpha(x) \leq \max\left\{\gamma_\alpha(y), \inf_{v \in x \circ y} \gamma_\alpha(v)\right\} \leq \max\{\gamma_\alpha(y), \gamma_\alpha(b)\} \leq n$$

so that $x \in H_\alpha^{(m,n)}$. Therefore $H_\alpha^{(m,n)}$ is a strong hyper K-ideal of H. □

Definition 3.6. An IFS $\alpha = \langle H, \mu_\alpha, \gamma_\alpha \rangle$ in H is called an *intuitionistic fuzzy hyper K-subalgebra* of H if it satisfies:

$$\begin{aligned} &\inf_{a \in x \circ y} \mu_\alpha(a) \geq \min\{\mu_\alpha(x), \mu_\alpha(y)\}, \\ &\sup_{b \in x \circ y} \gamma_\alpha(b) \leq \max\{\gamma_\alpha(x), \gamma_\alpha(y)\} \end{aligned} \tag{3.2}$$

for all $x, y \in H$.

Lemma 3.7. [3] *Let S be a nonempty subset of H. Then S is a hyper K-subalgebra of H if and only if $x \circ y \subseteq S$ for all $x, y \in S$.*

Theorem 3.8. *An IFS $\alpha = \langle H, \mu_\alpha, \gamma_\alpha \rangle$ in H is an intuitionistic fuzzy hyper K-subalgebra of H if and only if the nonempty intuitionistic level set $H_\alpha^{(m,n)}$ is a hyper K-subalgebra of H for all $m, n \in [0,1]$ with $m + n \leq 1$.*

Proof. Suppose that $\alpha = \langle H, \mu_\alpha, \gamma_\alpha \rangle$ is an intuitionistic fuzzy hyper K-subalgebra of H and let $m, n \in [0,1]$ be such that $m + n \leq 1$ and $H_\alpha^{(m,n)} \neq \emptyset$. Let $x, y \in H_\alpha^{(m,n)}$. For any $a \in x \circ y$, we have

$$\mu_\alpha(a) \geq \inf_{u \in x \circ y} \mu_\alpha(u) \geq \min\{\mu_\alpha(x), \mu_\alpha(y)\} \geq m,$$

$$\gamma_\alpha(a) \leq \sup_{v \in x \circ y} \gamma_\alpha(v) \leq \max\{\gamma_\alpha(x), \gamma_\alpha(y)\} \leq n.$$

Thus $a \in H_\alpha^{(m,n)}$, which shows that $x \circ y \subseteq H_\alpha^{(m,n)}$. Hence $H_\alpha^{(m,n)}$ is a hyper K-subalgebra of H by Lemma 3.7. Conversely suppose that $H_\alpha^{(m,n)} (\neq \emptyset)$ is a hyper K-subalgebra of H for all $m, n \in [0,1]$ with $m + n \leq 1$. Let $k := \min\{\mu_\alpha(x), \mu_\alpha(y)\}$ and $l := \max\{\gamma_\alpha(x), \gamma_\alpha(y)\}$. Then $x, y \in H_\alpha^{(k,l)}$, and so $x \circ y \subseteq H_\alpha^{(k,l)}$. It follows that $\mu_\alpha(z) \geq k$ and $\gamma_\alpha(z) \leq l$ for all $z \in x \circ y$ so that

$$\inf_{a \in x \circ y} \mu_\alpha(a) \geq k = \min\{\mu_\alpha(x), \mu_\alpha(y)\},$$

$$\sup_{b \in x \circ y} \gamma_\alpha(b) \leq l = \max\{\gamma_\alpha(x), \gamma_\alpha(y)\}.$$

Hence $\alpha = \langle H, \mu_\alpha, \gamma_\alpha \rangle$ is an intuitionistic fuzzy hyper K-subalgebra of H. □

Lemma 3.9. [5] *Every strong hyper K-ideal is a hyper K-subalgebra.*

Theorem 3.10. *Every intuitionistic fuzzy strong hyper K-ideal is an intuitionistic fuzzy hyper K-subalgebra.*

Proof. Let $\alpha = \langle H, \mu_\alpha, \gamma_\alpha \rangle$ be an intuitionistic fuzzy strong hyper K-ideal of H. Then the nonempty intuitionistic level set $H_\alpha^{(m,n)}$ is a strong hyper K-ideal of H for all $m, n \in [0,1]$ with $m + n \leq 1$. By Lemma 3.9, $H_\alpha^{(m,n)}$ is a hyper K-subalgebra of H. It follows from Theorem 3.8 that $\alpha = \langle H, \mu_\alpha, \gamma_\alpha \rangle$ is an intuitionistic fuzzy hyper K-subalgebra of H. □

The converse of Theorem 3.10 is not true as seen in the following example.

Example 3.11. Let $H = \{0, a, b\}$ be a hyper K-algebra with the following Cayley table:

$\circ$	0	a	b
0	$\{0\}$	$\{0\}$	$\{0\}$
a	$\{a\}$	$\{0, a\}$	$\{0, a\}$
b	$\{b\}$	$\{a, b\}$	$\{0, a, b\}$

Define an IFS $\alpha = \langle H, \mu_\alpha, \gamma_\alpha \rangle$ in H by

$$\alpha = \left\langle H, \left(\frac{0}{0.6}, \frac{a}{0.6}, \frac{b}{0.2} \right), \left(\frac{0}{0.08}, \frac{a}{0.08}, \frac{b}{0.7} \right) \right\rangle.$$

Then $\alpha = \langle H, \mu_\alpha, \gamma_\alpha \rangle$ is an intuitionistic fuzzy hyper K-subalgebra of H. But it is not an intuitionistic fuzzy strong hyper K-ideal of H since

$$\mu_\alpha(b) = 0.2 < 0.6 = \min \left\{ \mu_\alpha(a), \sup_{u \in b \circ a} \mu_\alpha(u) \right\}$$

and/or

$$\gamma_\alpha(b) = 0.7 > 0.08 = \max \left\{ \gamma_\alpha(a), \inf_{v \in b \circ a} \gamma_\alpha(v) \right\}.$$

Lemma 3.12. [2] *Every intuitionistic fuzzy hyper K-ideal is an intuitionistic fuzzy weak hyper K-ideal.*

Theorem 3.13. *Every intuitionistic fuzzy strong hyper K-ideal is an intuitionistic fuzzy weak hyper K-ideal.*

Proof. Let $\alpha = \langle H, \mu_\alpha, \gamma_\alpha \rangle$ be an intuitionistic fuzzy strong hyper K-ideal of H. Let $x, y \in H$ be such that $x \prec y$. Then $0 \in x \circ y$, and so $\sup_{u \in x \circ y} \mu_\alpha(u) \geq \mu_\alpha(0)$ and $\inf_{v \in x \circ y} \gamma_\alpha(v) \leq \gamma_\alpha(0)$. It follows from (3.1) and Proposition 3.3(i) that

$$\begin{aligned} \mu_\alpha(x) &\geq \min \left\{ \mu_\alpha(y), \sup_{u \in x \circ y} \mu_\alpha(u) \right\} \\ &\geq \min\{\mu_\alpha(y), \mu_\alpha(0)\} = \mu_\alpha(y), \end{aligned}$$

$$\begin{aligned} \gamma_\alpha(x) &\leq \max \left\{ \gamma_\alpha(y), \inf_{v \in x \circ y} \gamma_\alpha(v) \right\} \\ &\leq \max\{\gamma_\alpha(y), \gamma_\alpha(0)\} = \gamma_\alpha(y). \end{aligned}$$

Obviously, we have

$$\begin{aligned} \mu_\alpha(x) &\geq \min \left\{ \mu_\alpha(y), \sup_{a \in x \circ y} \mu_\alpha(a) \right\} \\ &\geq \min \left\{ \mu_\alpha(y), \inf_{a \in x \circ y} \mu_\alpha(a) \right\}, \end{aligned}$$

$$\begin{aligned}\gamma_\alpha(x) &\leq \max\left\{\gamma_\alpha(y), \inf_{b\in x\circ y}\gamma_\alpha(b)\right\}\\ &\leq \max\left\{\gamma_\alpha(y), \sup_{b\in x\circ y}\gamma_\alpha(b)\right\}.\end{aligned}$$

Hence $\alpha = \langle H, \mu_\alpha, \gamma_\alpha\rangle$ is an intuitionistic fuzzy hyper K-ideal of H, and thus $\alpha = \langle H, \mu_\alpha, \gamma_\alpha\rangle$ is an intuitionistic fuzzy weak hyper K-ideal of H by Lemma 3.12. □

Note that the IFS $\alpha = \langle H, \mu_\alpha, \gamma_\alpha\rangle$ in Example 3.11 is also an intuitionistic fuzzy weak hyper K-ideal of H. Hence the converse of Theorem 3.13 is not true in general.

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