

FUZZY ERROR MATRIX IN CLASSIFICATION PROBLEMS

S. R. KANNAN* AND S. RAMATHILAGAM

ABSTRACT. This paper concerns a new method called Fuzzy Supervised Method for error matrix, the method has developed based on Adoptive Neuro- Fuzzy Inference Systems(ANFIS). For the performance point of view initially the new method tested with trial data and then this paper applies the proposed method with real world problems. So that this paper generated 1000 random error matrices in programming language [R] and then it tests the new proposed method for the error matrices. The results of Fuzzy Supervised Method given in terms of Kappa Index and Congalton Accuracy Indexes, and performance of Fuzzy Supervised Method has evaluated by using Pearson's test.

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1. Introduction

Fuzzy set theory has been applied to an increasing number of real world problems of considerable complex type. It gives solution to a variety of problem such as, prediction and modeling where the physical processes are not understood or highly complex. In this study, this paper presents a method Fuzzy Supervised Method for error matrix to improve the accuracy of resultant data. Error matrix is a very effective way to represent accuracy in classification analysis, because the accuracy of each category are plainly described along with both the errors of inclusion and errors of exclusion present in the classification problem. Using an error matrix to represent accuracy has been recommended by many researchers [2, 11, 12], as it provides a detailed assessment of the agreement between the sample reference data and classification data at specific locations, together with a complete description of the misclassification's registered for each category. Here each agreement, that is the resultant classes are expressed in

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TABLE 1. Error matrix by hard classification

CD(rows) & RD(columns)				
	R_1	...	R_n	
C_1	e_{11}	...	e_{1q}	e_{1+}
.
.
.
C_m	e_{m1}	...	e_{mq}	e_{q+}
	e_{+1}	...	e_{+q}	

terms of producer and user accuracy. Producer accuracy is indicating the probability of reference classes and it obtain the producer accuracy, with related to errors of omission. User accuracy is indicating the probability of classification classes and it obtain the user accuracy, with related to errors of commission [2]. Thus, the producer and user accuracy of each classes are completely dependent on errors of inclusion and errors of exclusion.

So the main problem in classification method the error matrix has been introduced with uncertainty by soft or hard classifiers. In hard classification each element of sample data is associated with only one class in the classification and only one class in the reference data. Consequently, a class assignment is judged exactly right, or exactly wrong. In soft classification, gradual membership in several classes is allowed for each element of sample data and assignments to classes are judged correct, or incorrect in varying membership degrees. But to apply conventional measures of classification accuracy, these soft classification outputs must be hardened and the comparison limited to crisp reference data, causing a general loss of information. In this way the uncertainty arises in reference and classification data [2] by both the classifiers. By poor classification scheme can result in significant bias being introduced into error matrix which many over or underestimation the true accuracy [12]. Two types of sources of biases in accuracy assessment, they are conservative and optimistic biases in the accuracy assessment, many of the sources of these types, which are not possible to avoid. Hence, it is impossible to get the reference or classified data without error by either classifiers. Also it is clear that the soft and hard classifiers with error matrix is obviously having uncertainty and the error matrix provides accuracy with uncertainty.

So the error matrix needs a filter after produce the entries of it or agreements of error matrix in hard or soft classifiers. To solve the problem of uncertainty in error matrix, this paper introduces Fuzzy Supervised Method with the use of ANFIS, with the aim of reducing uncertainty in the error matrix by soft or hard classifiers and to improve the resultant data. Every agreements of error matrix are expressed in terms of fuzzy membership grades [0,1] and then the new fuzzy agreement will be introduced. The main purpose of this study is for the following reasons:

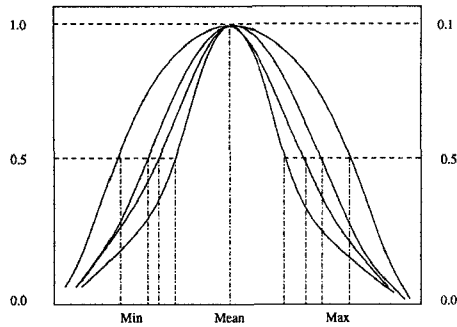


FIGURE 1. New Fuzzy Membership Function

1. to improve the accuracy resultant data;
2. to reduce the uncertainty in error matrix.

The ANFIS is on the Sugeno's [9] fuzzy model, and then it has modified and applied successfully for many real world problems by Roger [6, 7, 8]. He has introduced and used ANFIS for quick and straightforward of input selection for neuro fuzzy modeling [6]. ANFIS employing fuzzy if-then rules can model the qualitative aspects of human knowledge and reasoning processes without employing precise quantitative analysis [7], the model or data is expressed in terms of membership grades and then the rules are used to evaluate a crisp output. Fuzzy if-then rules is a expressions of the form IF M THEN N, where M and N are labels of fuzzy sets [7] characterized by appropriate functions. Due to their concise form, fuzzy if-then rules are often employed to capture the imprecise modes of reasoning that plays an essential role in the human ability to make decisions in an environment of uncertainty and imprecision. This paper has defined a new fuzzy membership function and an effective rules to get new fuzzy agreement for Fuzzy Supervised Method.

The remainder of the paper is divided into two sections. Section 2. describes the error matrix by hard or soft classifiers, defines membership function for Fuzzy Supervised Method, evaluates the new fuzzy agreement, and comparing the results between Fuzzy Supervised Method for error matrix and error matrix. Conclusion of this paper is given in Section 3.

2. Fuzzy supervised method for error matrix

Fuzzy set based Measure is an excellent computational design that gives a mathematical tool for dealing with the uncertainty. There are several developments on Fuzzy set, based on Neural Networks [11], self - organizing map [4] and etc. This section focuses on four-parts in order to reduce the uncertainty of error matrix in hard or soft classifiers.

1. Forming Error Matrix with Hard or Soft classifiers,

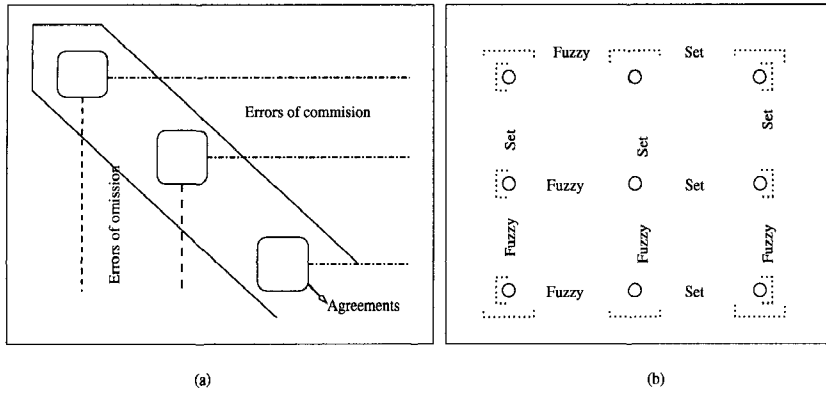


FIGURE 2. Fuzzy set for membership function

2. Designing membership function for each agreement of error matrix along with error inclusions and error exclusions,
3. Forming rules for Fuzzy Supervised Method for evaluating new fuzzy agreements and
4. Comparing the Accuracies of Error Matrix and Fuzzy Supervised Method in Error Matrix.

2.1. Forming error matrix with hard or soft classifiers

Let R_n be the set of reference data assigned to class n , and C_m the set of classification data assigned to class m , with $1 \leq n \leq q$ and $1 \leq m \leq q$ and q as the number of classes. The data R_n and C_m are considered as crisp sets in error matrix. The characteristic function for crisp sets are defined as follows:

$$\mu_{R_n}(x) : X \rightarrow \{0, 1\}, \tag{1}$$

where X is an universal set and for all x in X .

$$\mu_{C_m}(x) : X \rightarrow \{0, 1\}, \tag{2}$$

where X is an universal set and for all x in X .

$$\mu_{R_n} = \left\{ \begin{array}{l} 1 \text{ iff } x \in R_n \\ 0 \text{ otherwise} \end{array} \right\} \tag{3}$$

$$\mu_{C_m} = \left\{ \begin{array}{l} 1 \text{ iff } x \in C_m \\ 0 \text{ otherwise} \end{array} \right\} \tag{4}$$

$$M(m, n) = \sum_{x \in X} \mu_{R_n \cap C_m}(x) \tag{5}$$

$$\mu_{R_n \cap C_m}(x) = \left\{ \begin{array}{l} 1 \text{ iff } x \in R_n \wedge x \in C_m \\ 0 \text{ otherwise} \end{array} \right\} \tag{6}$$

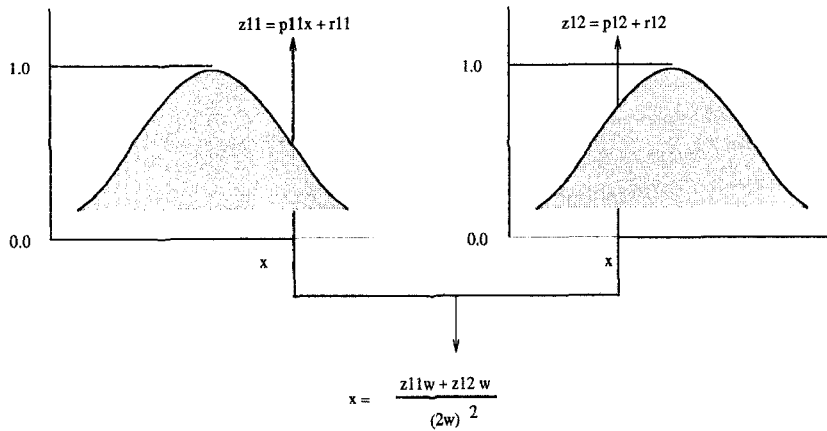


FIGURE 3. Fuzzy Supervised model for getting new fuzzy agreements

Equations 3 and 4 have been used to assign the values of every element of R_n , and the values of every element of C_m , and the equations 5 and 6 have been used to calculate the elements of error matrix in row m and column n . The error matrix is shown in table [1], e_{i+} and e_{+i} are the total assignment to the i th class for classification and reference data, respectively. The disadvantage of hard classification are:

1. A class assignment is judged exactly right or exactly wrong, that means the element has only two possibility with 0 (wrong) or 1 (right), the partial truth has not discussed.
2. Assigning the element of error matrix is not a straight forward way. The error matrix in soft classification also follows the same procedure of hard classification, the element of error matrix is not selected by straight forward way.

2.2. Defining new membership function for fuzzy supervised method

The each fuzzy set is divided into two categories before we start to define a new fuzzy membership for Fuzzy Supervised Method, that is the elements of each sets divided into two parts which are the elements lower and equal to mean, and elements which are equal to mean or above mean. The membership function assigns the highest membership value to the mean value of each fuzzy set, the membership grades for remaining each element of fuzzy set gets according to the strength of it by comparing with mean elements of the fuzzy set. So this paper defines the following fuzzy membership equations to assign the membership grades according to the instructions, which are stated above, also this paper develops programs in [R] programming language to implement the defined fuzzy

membership equations and assign membership grades in perfect way. The new membership equations for Fuzzy Supervised Method are defined as:

$$\mu_R(x) = 1/(1 + [((x_i - m)/(m - n))^2]^b) \tag{7}$$

$$\mu_R(x) = 1/(1 + [((x_j - m)/(o - n))^2]^b) \tag{8}$$

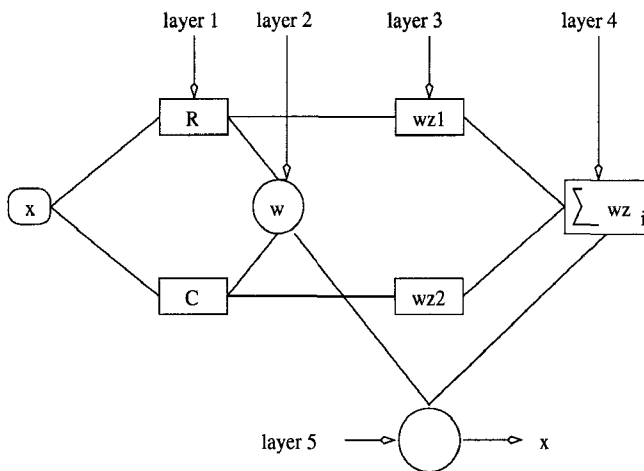


FIGURE 4. Fuzzy Supervised Network for Error Matrix

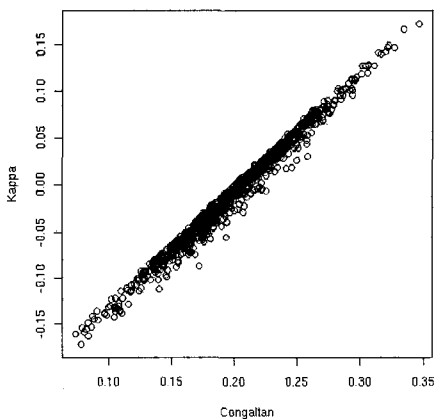


FIGURE 5. Congalton and Kappa accuracies of Fuzzy Supervised Method for 1000 error matrices

x is a fuzzy set, x_i denotes the set of elements which are below and equal to mean of x , x_j denotes the set of elements of set x which are above and equal to mean of x , m is mean of x , o is a maximum of x and n is a minimum of x .

In the proposed membership function for Fuzzy Supervised Method, the shape or model of membership functions depends on the parameters (please refer the figure 1) of fuzzy set, and changing the parameters will change the shape of membership function.

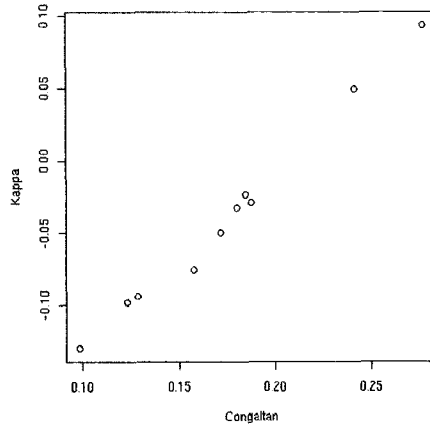


FIGURE 6. Congalton and Kappa accuracies of Fuzzy Supervised Method for 10 Error Matrices

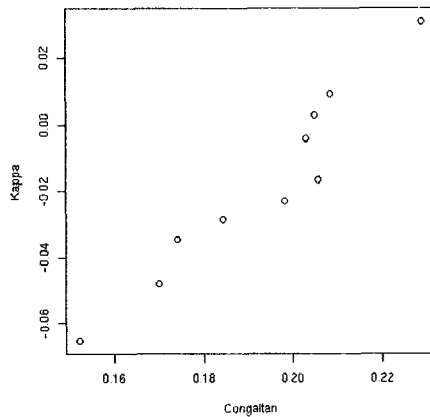


FIGURE 7. Congalton and Kappa accuracies of 10 Error Matrices

Usually before to use the fuzzy set to analyze the data, the suitable model of fuzzy membership function or shape of fuzzy membership function is predicted according to the structure of data by researchers. But in our new fuzzy membership function the shape of membership function is decided by the equations

7 and 8 which we have defined above. So we no need to analyze the suitable model of membership function for input parameters, and we assure that the new membership equations 7 and 8 can be used any type of problems or data by using fuzzy set.

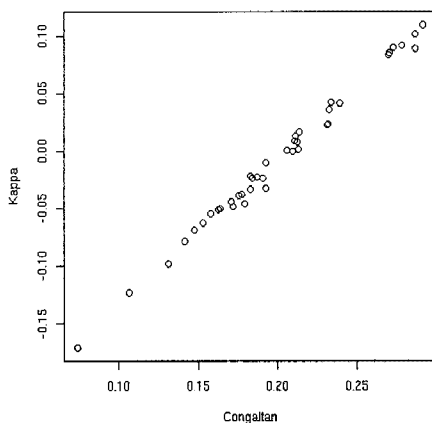


FIGURE 8. Congalton and Kappa accuracies of Fuzzy Supervised Method for 40 Error Matrices

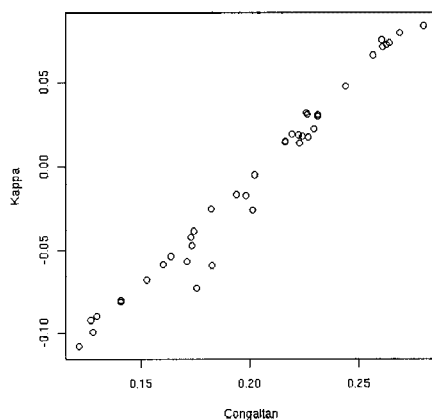


FIGURE 9. Congalton and Kappa accuracies of 40 Error Matrices

2.3. Computing fuzzy agreement from error matrix

The method Fuzzy Supervised Method has been developed with the use of Adaptive Neuro-Fuzzy Inference System [6, 7, 8]. The purpose of this method to

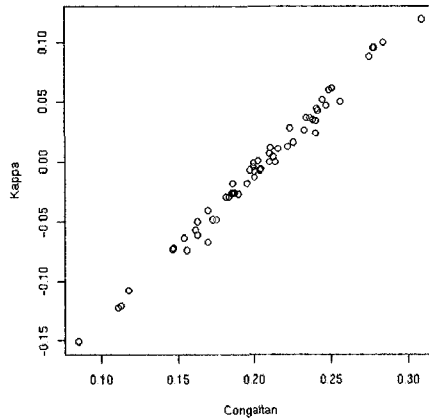


FIGURE 10. Congalton and Kappa accuracies of Fuzzy Supervised Method for 60 Error Matrices

reduce the uncertainty in error matrix and to improve the accuracy of resultant data.

Two membership functions are defined for each agreement (refer the figure 2(a) and (b)) to reduce the uncertainty of error matrix. Each fuzzy set of membership function consists the parameters or classes of both error inclusion and error exclusion of error matrix.

Let x be the agreements between matches and mismatches, that is x is the element of the classes of reference and classified data. Here the agreement x has been represented by two membership functions, please refer the figures 2(b). Consider R_n is the reference data(Matches) and C_m is the classified data (Mismatches), where n varies from 1 to n and m varies from 1 to m . The agreement x in the classes of n and m as indicated in the matches and mismatches. The fuzzy membership functions for the R_n and C_m are:

$$\mu_{R_n} = \left\{ \begin{array}{ll} 1/(1 + [((x_i - m)/(m - n))^2]^b) & \text{if } x < \text{mean}(x) \\ 1/(1 + [((x_j - m)/(o - n))^2]^b) & \text{if } x \geq \text{mean}(x) \end{array} \right\} \quad (9)$$

$$\mu_{C_m} = \left\{ \begin{array}{ll} 1/(1 + [((x_i - m)/(m - n))^2]^b) & \text{if } x < \text{mean}(x) \\ 1/(1 + [((x_j - m)/(o - n))^2]^b) & \text{if } x \geq \text{mean}(x) \end{array} \right\} \quad (10)$$

The membership grades of each agreement between matches and mismatches are computed with the use of equations 9 and 10.

$$x \in \{R_n, C_m\}.$$

The following rules are introduced for x to express both the character of R and C:

- if $x \in R_n$, then $z = px$
- if $x \in C_n$, then $z = qx$,

where p and q are the fuzzy parameters. Figures. 3 and 4 illustrate graphically the fuzzy reasoning mechanism to drive the fuzzy agreement from given agreement x with two membership grades. The computation method of each of layers given below:

Layer 1 :

Every node i in this layer is a square node, and each node generates a membership grades of a linguistic label, with the node function

$$r \in \{R, C\} \tag{11}$$

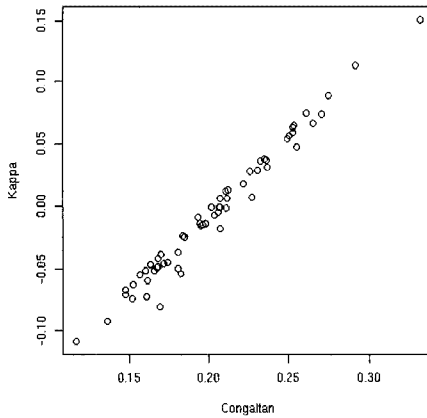


FIGURE 11. Congalton and Kappa accuracies of 60 Error Matrices

Layer 2:

Every node in this layer is a circle node labeled [w], and the node in this layer calculates the ring strength of a rule via multiplication(For information, here the i = 1 is only for first input selection, and i = 2 is for second input and etc.):

$$n_i^2 = \mu_{R_i}(x) * \mu_{C_i}(x) \tag{12}$$

where * denotes product.

Layer 3:

Every node in this layer is a square node with the node function

$$n_i^3 = wz_{ij}, j = 1, 2, \tag{13}$$

$$wz_{i1} = w(p_{i1}x + r_{i1}), \tag{14}$$

$$wz_{i2} = w(p_{i2}x + r_{i2}). \tag{15}$$

Layer 4:

The single node in this layer is a square with the node function:

$$n_i^4 = \sum wz_{ij} = w(p_{ij}x + r_{ij}) \tag{16}$$

summation of rules given in layer 3.

TABLE 2. Sample Misclassification Matrix

Error Matrix					
	A	B	C	D	E
A	80	4	0	15	7
B	2	17	0	9	2
C	12	5	9	4	8
D	7	8	0	65	0
E	3	2	1	6	38

Layer 5:

The single node in this layer is a circle node labeled $[\Sigma]$ that computes the overall output as the summation of all incoming signals,

$$n_i^5 = x = \sum w_{z_{ij}} / (2w)^2. \quad (17)$$

Layer 5 is using to compute the fuzzy agreement for error matrix. After obtained the fuzzy agreements of error matrix via Fuzzy Supervised Method. To show the performance of the Fuzzy Supervised Method this paper gives the following example. Error matrix from USGS - NPS Vegetation Mapping Program [13] has shown in the table 2, and the Fuzzy Supervised Method for the same error matrix has shown in the table 3. The columns of the matrices define the classes in the reference data, and the rows define the classes in the data being evaluated for accuracy. The classes on the diagonal of the error matrix depict the number of the sample units correctly classified in agreement with the reference data category. The accuracies of Fuzzy Supervised Method for error matrix and the accuracy of error matrix given in the table 3. To verify the efficient of the method, this paper has tested it with 1000 error matrices. So this paper developed a program in software [R], to build up 1000 random error matrices. Also there is an another program developed in programming language [R] to apply the Fuzzy Supervised Method to 1000 error matrices, which are generated randomly by program in programming language [R]. The accuracies of Congalton and Kappa Index generated by Fuzzy Supervised Method for Error Matrices compared with accuracies for the same matrices by Error matrix, shown in the figure.5, and it is very clear that from the figure whenever the accuracy of Congalton increases, the kappa also increases.

2.4. Comparison of accuracies of error matrix and fuzzy supervised method in error matrix

The accuracies of the both the methods are compared with the use of Pearson's test [5]. The correlation coefficient and critical value [5] have been computed in the following way for 10 & Fuzzy Supervised Method in 10 Error

TABLE 3. Fuzzy Supervised Method for Sample Misclassification Matrix

Fuzzy Supervised Method for Error Matrix					
	A	B	C	D	E
A	76.226415	2.468085	3.877358	10.22270	5.000000
B	2.474359	14.548077	2.083333	8.50000	1.177180
C	6.000000	2.500000	9.000000	1.5000	4.272727
D	8.141021	6.742089	2.000000	62.94304	3.000000
E	4.133838	1.853535	4.318182	3.00000	38.000000

TABLE 4. Accuracies of Fuzzy Supervised Method for Error Matrix & Error Matrix

Accuracy Comparison		
	Fuzzy Supervised Method	Error Matrix
Congalton	70.8 %	68.8 %
Kappa	60.7 %	58.3 %

TABLE 5. Accuracies for 10 error matrices

Congolton and Kappa Indexes	
Congolton	Kappa
0.1956818	-0.013695489
0.2368019	0.031970310
0.1978120	-0.007777403
0.1898993	-0.014282091
0.2230795	0.003888935
0.1813564	-0.031905400
0.1223392	-0.113278834
0.1707520	-0.060901326
0.2272145	0.030103108
0.1590911	-0.059804434

Matrices. The correlation coefficient for the Congolton Index and Kappa Index for 10 error matrices: One of the indexes is labeled x and one y. The table 6 has given the square and cross product of the each variable. Index of Covariation:

$$(N * \sum xy) - (\sum x * \sum y) = 0.1366401 \tag{18}$$

Variation of x:

$$(N * \sum x^2) - (\sum x)^2 = 0.3277168 \tag{19}$$

TABLE 6. Square and Cross product of each Congoltan and Kappa Indexes

<i>x</i>	<i>y</i>	<i>x</i> ²	<i>y</i> ²	<i>xy</i>
0.195681	0.0382913	-0.01369548	1.875664e-04	-0.002679957
0.236801	0.0560751	0.03197031	1.022101e-03	0.007570630
0.197812	0.0391295	-0.00777740	6.048800e-05	-0.001538463
0.189899	0.0360617	-0.01428209	2.039781e-04	-0.002712159
0.223079	0.0497644	0.00388893	1.512382e-05	0.000867541
0.181356	0.0328901	-0.03190540	1.017955e-03	-0.005786248
0.122339	0.0149668	-0.11327883	1.283209e-02	-0.013858441
0.170752	0.0291562	-0.06090132	3.708972e-03	-0.010399023
0.227214	0.0516264	0.03010310	9.061971e-04	.006839862

Variation of y:

$$(N * \sum y^2) - (\sum y)^2 = 0.423986 \tag{20}$$

Correlation coefficient(r):

$$\frac{((N * \sum xy) - (\sum x * \sum y)) / ((N * \sum x^2) - (\sum x)^2 * (N * \sum y^2) - (\sum y)^2)}{(\sum y)^2} = 0.9834 \tag{21}$$

Degrees of freedom:

$$df = N - 2 = 8 \tag{22}$$

r-critical for = .05

$$r - critical(\alpha = .05, df = 8) = .632. \tag{23}$$

Here the value(or absolute value) of r is larger than the r-critical value, so this paper has decided to reject the null hypothesis, and it concluded there is a strong linear relationship between two accuracies. In the same way, the correlation coefficient for the Fuzzy Supervised Method for the same 10 error matrices has obtained. The correlation coefficient for the Congoltan Index and Kappa Index for Fuzzy Measure for 10 Error Matrices have given in the table 7.

Fuzzy Supervised Method for Error Matrices

$$\text{Correlation coefficient}(r) = -0.2984111 \tag{24}$$

$$\text{r-critical}(= .05; df = 8) = .632 \tag{25}$$

Here the value(or absolute value) of r is smaller than the r-critical value. So this paper has decided do not reject the null hypothesis, and there is no strong linear relationship between two accuracies. Correlation coefficients and critical values of 10, 20, 40, 60 matrices of two methods have given in the table 8 and Kappa and Congoltan accuracies of 10, 40, 60 of Fuzzy Supervised Method for Error Matrices & Error Matrices have given the figures 6, 7, 8, 9, 10 and 11. In

TABLE 7. Accuracies for Fuzzy Supervised Method for error matrices

Congoltan and Kappa Indexes	
Congoltan	Kappa
x	y
0.1641291	-0.0498711602
0.1955637	-0.0154156562
0.1788437	-0.0313234003
0.1940971	-0.0078827136
0.1947574	-0.0103425398
0.1057447	-0.1224260573
0.2038157	0.0002470564
0.2348116	0.0433234565
0.2090713	0.0041218117
0.1775195	-0.0395759773

TABLE 8. Correlation coefficients for Fuzzy Supervised Method for Error Matrix and Error Matrix & Critical Values

Comparison of Fuzzy Supervised Method & Error Matrix			
No. of matrices	Fuzzy Measure For Error Matrix	Error matix	Critical Value
10	-0.2984111	0.983395	0.632
20	-0.1136743	-1.000000	0.444
40	0.20459	-1.000000	0.324
60	-0.1290964	-1.000000	0.273

the case of Error matrix the accuracies of Kappa and Congoltan have a linear relationship. Since results or accuracies of both methods have linear relationship in error matrix, we can immediately give the accuracy of one method for error matrix when we have the accuracy of error matrix with the use of other one method. Hence there is no use to give the both methods to find the accuracy of error matrix. In the case of Fuzzy Supervised Method, there is no linear relationship between the accuracies of Kappa and Congoltan, also it is clear from the graphs (Figures 5, 6, 8 and 10) whenever Kappa increases, congaltan also increases. And also the proposed method gives always better accuracy than the accuracy of error matrix with soft or hard classifiers.

3. Conclusions

This paper has given a new method called fuzzy supervised method to supervise the error matrix by hard or soft classifiers. Mainly this paper developed new membership functions and rules to evaluate fuzzy agreement for error matrix with soft or hard classifiers in order to reduce the uncertainty of result in error matrix and to improve the accuracy of resultant data. This paper has constructed 1000 random error matrices in programming language [R] and shown explicitly the performance of Fuzzy Supervised Method in Error Matrix, and also this paper has compared the accuracies of both Error matrix and Fuzzy Supervised Method with error matrix by using Pearson's test. Of course we always have better accuracy in Fuzzy Supervised Method than Error Matrix. The results have given explicitly with the use of graphs.

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