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ON FUZZY FAINTLY PRE-CONTINUOUS FUNCTIONS

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Abstract. The aim of this paper is to introduce a new generalization of fuzzy faintly continuous functions called fuzzy faintly pre-continuous functions and also we have introduced and studied weakly fuzzy pre-continuous functions. Several characterizations of fuzzy faintly pre-continuous functions are given and some interesting properties of the above functions are discussed.

1. Introduction

C. L. Chang [5] introduced and developed the concept of fuzzy topological spaces based on the concept of fuzzy sets introduced by Zadeh in [10]. Since then various important notions in the classical topology such as continuous functions have been extended to fuzzy topological spaces [5].

The concept of faintly continuous functions was introduced and studied by P. E. Long and L. L. Herrington in [7] and it was extended to fuzzy topological spaces by Anjan Muherjee in [1]. In [2], the concept of fuzzy faintly α-continuous functions was also introduced and studied. The purpose of this paper is to introduce and study the concept of fuzzy faintly pre-continuous functions. Section 2 deals with preliminaries. Section 3 deals with the characterizations of fuzzy faintly pre-continuous functions and Section 4 deals with some interesting properties of fuzzy faintly pre-continuous functions.

2. Preliminaries

In this paper by (X, T) (X for short) we mean fuzzy topological space in the sense of [5]. Let λ be a fuzzy set. The fuzzy closure of λ [5] and the fuzzy interior of λ [5] are defined as: fuzzy closure of $\lambda = \mathrm{cl}\,(\lambda) = \wedge\{\mu \,|\, \mu \geq \lambda, \mu \text{ is fuzzy closed}\}$ and fuzzy interior of $\lambda = \mathrm{int}\,(\lambda) = \vee\{\sigma \,|\, \sigma \leq$

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λ, σ is fuzzy open$\}$. A fuzzy point x_p in X is a fuzzy set in X defined by [1]

$$x_p(y) = \begin{cases} p, \quad p \in (0,1], & \text{for } y = x,\ y \in X, \\ 0, & \text{for } y \neq x,\ y \in X. \end{cases}$$

x and p are respectively called the support and the value of the fuzzy point. A fuzzy set λ in X is called fuzzy pre-open if $\lambda \leq \operatorname{int}\operatorname{cl}(\lambda)$ and λ is called regular open if $\lambda = \operatorname{int}\operatorname{cl}(\lambda)$.

Let (X, T) and (Y, S) be any two fuzzy topological spaces. Let $f : (X, T) \rightarrow (Y, S)$ be a function. f is called fuzzy pre-continuous if the inverse image of each fuzzy open set in Y is fuzzy pre-open in X.

A fuzzy topological space X is product related [3] to a fuzzy topological space Y if for any fuzzy set v in X and ξ in Y whenever $\lambda' (= 1 - \lambda) \ngeq v$ and $\mu' (1 - \mu) \ngeq \xi$ imply $\lambda' \times 1 \vee 1 \times \mu' \geq v \times \xi$, where λ is a fuzzy open set in X and μ is a fuzzy open set in Y, there exists a fuzzy open set λ_1 and a fuzzy open set $\mu_1$in Y such that $\lambda_1' \geq v$ or $\mu_1' \geq \xi$ and $\lambda_1' \times 1 \vee 1 \times \mu_1' = \lambda' \times 1 \vee 1 \times \mu'$. If (X, T) and (Y, S) are any two fuzzy topological spaces, we define a product fuzzy topology $T \times S$ on $X \times Y$ to be that fuzzy topology for which $\mathcal{B} = \{\lambda \times \mu / \lambda \in T, \mu \in S\}$ forms a base.

A fuzzy point $x_p \in \lambda$, where λ is a fuzzy subset in X if and only if $p \leq \lambda(x)$. A fuzzy point x_p is quasi-coincident with λ, denoted by $x_p q \lambda$, if and only if $p > \lambda'(x)$ or $p + \lambda(x) > 1$ where λ' denotes the complement of λ defined by $\lambda' = 1 - \lambda$. A fuzzy subset λ in a fuzzy topological space X is said to be q-neighbourhood for a fuzzy point x_p if and only if there exist a fuzzy open subset η such that $x_p q \eta \leq \lambda$. A fuzzy point x_p is said to be a fuzzy θ-cluster point of a fuzzy subset λ if and only if for every open q-neighbourhood η of x_p, $\operatorname{cl}(\eta)$ is quasi-coincident with λ. The set of all fuzzy θ-cluster points of λ is called the fuzzy θ-closure of λ and is denoted by $\operatorname{cl}_\theta(\lambda)$. The complement of a fuzzy θ-closed subset is a fuzzy θ-open which is equivalent to the condition: a fuzzy subset μ is called fuzzy θ-open if and only if $\operatorname{int}_\theta(\mu) = \mu$, where the fuzzy set $\vee \{x_p \in X :$ for some open q-neighborhood η of $x_p, \operatorname{cl}(\eta) \subseteq \mu\ \}$ is the fuzzy θ-interior of μ and is denoted by $\operatorname{int}_\theta(\mu)$ and $fp\operatorname{int}(\mu)$ is the largest fuzzy pre-open set contained in μ.

3. Characterizations of fuzzy faintly pre-continuous functions

The concepts of fuzzy faintly continuous functions and that of fuzzy faintly α-continuous functions are introduced in [1] and [2] respectively.

Definition 3.1. Let (X, T) and (Y, S) be any two fuzzy topological spaces. A function $f : (X, T) \rightarrow (Y, S)$ is said to be *fuzzy faintly continuous* if for each fuzzy point x_p of X and each fuzzy θ-open set μ containing $f(x_p)$, there exists a fuzzy open subset λ containing x_p such that $f(\lambda) \leq \mu$.

Definition 3.2. Let (X, T) and (Y, S) be any two fuzzy topological spaces. A function $f : (X, T) \rightarrow (Y, S)$ is said to be *fuzzy faintly α-continuous* if for each fuzzy point x_p of X and each fuzzy θ-open set μ containing $f(x_p)$, there exists a fuzzy α-open set λ containing x_p such that $f(\lambda) \leq \mu$.

Based on the above two concepts we now define

Definition 3.3. Let (X,T) and (Y,S) be any two fuzzy topological spaces. A function $f:(X,T) \to (Y,S)$ is called *fuzzy faintly pre-continuous* if for each fuzzy point x_p of X and each fuzzy θ-open set μ containing $f(x_p)$, there exists a fuzzy pre-open set γ containing x_p such that $f(\gamma) \leq \mu$.

Definition 3.4. Let (X,T) and (Y,S) be any two fuzzy topological spaces. A function $f:(X,T) \to (Y,S)$ is called *some what fuzzy faintly pre-continuous* [9] if for every fuzzy θ- open set λ in (Y,S) such that $f^{-1}(\lambda) \neq 0$, there exists a fuzzy pre-open set μ in X such that $\mu \neq 0$ and $\mu \leq f^{-1}(\lambda)$.

Proposition 3.5. *Let (X,T) and (Y,S)be any two fuzzy topological spaces. Let $f:(X,T) \to (Y,S)$ be a function. Then the following are equivalent.*

(1) *f is fuzzy faintly pre-continuous.*
(2) *$f^{-1}(\mu)$ is a fuzzy pre-open subset in X for each fuzzy θ-open set μ in Y.*
(3) *$f^{-1}(\mu)$ is a fuzzy pre-closed subset in X for each fuzzy θ-closed set μ in Y.*
(4) *$fp\operatorname{cl}\left(f^{-1}(\gamma)\right) \leq f^{-1}(\operatorname{cl}_\theta(\gamma))$ for any fuzzy subset γ in Y.*
(5) *$f^{-1}(\operatorname{int}_\theta(\gamma)) \leq fp\operatorname{int}\left(f^{-1}(\gamma)\right)$ for any fuzzy subset γ in Y.*
(6) *$f:(X,T) \to (Y,S_\theta)$ is fuzzy pre-continuous where S_θ is the family of all fuzzy θ-open sets.*
(7) *$f:(X,T_p) \to (Y,S)$ is fuzzy faintly continuous where T_p is the family of all fuzzy pre-open sets.*
(8) *$f:(X,T_p) \to (Y,S_\theta)$ is fuzzy continuous.*

Proof. (1) $\Rightarrow$ (2). Let μ be any fuzzy θ-open subset in Y and x_p be any fuzzy point in X such that $x_p \in f^{-1}(\mu)$. Since f is fuzzy faintly pre-continuous, there exists a fuzzy pre-open subset γ in X with $x_p \in \gamma$ such that $f(\gamma) \leq \mu$. Then $x_p \in \gamma \leq f^{-1}(f(\gamma)) \leq f^{-1}(\mu)$, which shows that $f^{-1}(\mu)$ is a fuzzy pre-neighborhood of each of its points and hence $f^{-1}(\mu)$ is a fuzzy pre-open subset in X.

(2) $\Rightarrow$ (3). Let μ be any fuzzy θ-closed subset in Y. Then $1-\mu$ is fuzzy θ-open in Y. By assumption (2), $f^{-1}(1-\mu)$ is fuzzy pre-open and so $1-f^{-1}(\mu)$ is fuzzy pre-open. Therefore $f^{-1}(\mu)$ is fuzzy pre-closed. This proves (2) $\Rightarrow$ (3).

(3) $\Rightarrow$ (4). Let γ be a fuzzy subset in Y. Then $\gamma \leq \operatorname{cl}_\theta(\gamma)$ (by Remark 2.5 in [2]) and hence $f^{-1}(\gamma) \leq f^{-1}(\operatorname{cl}_\theta(\gamma))$. Now $\operatorname{cl}_\theta(\gamma)$ is fuzzy θ-closed subset in Y and hence by (3), $f^{-1}(\operatorname{cl}_\theta(\gamma))$ is a fuzzy θ-closed subset in X. Thus $fp\operatorname{cl}\left(f^{-1}(\gamma)\right) \leq fp\operatorname{cl}\left(f^{-1}(\operatorname{cl}_\theta(\gamma))\right) = f^{-1}(\operatorname{cl}_\theta(\gamma))$. That is $fp\operatorname{cl}\left(f^{-1}(\gamma)\right) \leq f^{-1}(\operatorname{cl}_\theta(\gamma))$.

(4) $\Rightarrow$ (5). Taking complement of (4), we get

$$\begin{aligned} 1 - fp\,\mathrm{cl}\left(f^{-1}(\gamma)\right) &\geq 1 - f^{-1}\left(\mathrm{cl}_\theta(\gamma)\right) \\ fp\,\mathrm{int}\left(1 - f^{-1}(\gamma)\right) &\geq f^{-1}\left(1 - \mathrm{cl}_\theta(\gamma)\right) \\ f^{-1}\left(fp\,\mathrm{int}\,(1-\gamma)\right) &\geq fp\,\mathrm{int}\left(f^{-1}(1-\gamma)\right) \end{aligned}$$

which implies $f^{-1}(\mathrm{int}_\theta(\mu)) \geq fp\,\mathrm{int}\left(f^{-1}(\mu)\right)$ where $\mu = 1-\gamma$ is a fuzzy subset in Y.

(5) $\Rightarrow$ (1). Let μ be a fuzzy θ-open subset in Y. Then $\mathrm{int}_\theta(\mu) = \mu$. Now by (3)

$$f^{-1}(\mathrm{int}_\theta(\mu)) = f^{-1}(\mu) \geq fp\,\mathrm{int}\left(f^{-1}(\mu)\right).$$

Thus $f^{-1}(\mu) = fp\,\mathrm{int}\left(f^{-1}(\mu)\right)$ [since $fp\,\mathrm{int}\left(f^{-1}(\mu)\right) \leq f^{-1}(\mu)$]. That is $f^{-1}(\mu)$ is fuzzy pre-open.

Let x_p be any fuzzy point in $f^{-1}(\mu) = \sigma$ (say). Then $x_p \in \sigma \in f^{-1}(\mu)$. That is $f(x_p) \in f(\sigma) = ff^{-1}(\mu) \leq \mu$. Thus for any fuzzy point x_p and for each fuzzy θ-open set μ containing $f(x_p)$ there exists a fuzzy pre-open subset σ containing x_p such that $f(\sigma) \leq \mu$. Thus f is fuzzy faintly pre-continuous.

(5) $\Rightarrow$ (6). Let μ be any fuzzy θ-open subset in (Y, S). Then μ is a fuzzy open set in (Y, S_θ) and by (3), $f^{-1}(\mu)$ is fuzzy pre-open. That is inverse image of a fuzzy θ-open set is fuzzy pre-open. This shows that f is fuzzy pre-continuous.

The implications (6) $\Rightarrow$ (7), (7) $\Rightarrow$ (8) and (8) $\Rightarrow$ (1) are obvious. $\square$

From the definitions it is clear that every fuzzy faintly continuous is fuzzy faintly pre-continuous; but however the converse is not true. The following example serves this purpose.

Example 3.6. Let $X = Y = I = [0,1]$. Let μ_1, μ_2, μ_3 be fuzzy sets on I defined as follows:

$$\begin{aligned} \mu_1(x) &= \begin{cases} 0, & 0 \leq x \leq \frac{1}{2}, \\ 2x-1, & \frac{1}{2} \leq x \leq 1, \end{cases} \\ \mu_2(x) &= \begin{cases} 1, & 0 \leq x \leq \frac{1}{4}, \\ -4x+2, & \frac{1}{4} \leq x \leq \frac{1}{2}, \\ 0 & \frac{1}{2} \leq x \leq 1, \end{cases} \\ \mu_3(x) &= \begin{cases} x, & 0 \leq x \leq \frac{1}{4}, \\ 1, & \frac{1}{4} \leq x \leq 1. \end{cases} \end{aligned}$$

Clearly $T_1 = \{0, \mu_2, 1\}, T_2 = \{0, \mu_1, \mu_2, \mu_1 \vee \mu_2, 1\}$ are two fuzzy topologies on I. Let $f : (I, T_1) \to (I, T_2)$ be defined as follows $f(x) = x$ for each $x \in I$. Then $f^{-1}(1) = 1$ and $f^{-1}(\mu_3) = \mu_3$. This example is taken from [2] and it is shown in that μ_3 and 1 are the only fuzzy θ- open sets of (I, T_2). Also it is easy to verify that μ_3 and 1 are fuzzy pre-open sets in (I, T_1). Therefore f is *fuzzy faintly pre-continuous* but it is not *fuzzy faintly continuous* [2], since $f^{-1}(\mu_3) = \mu_3 \notin T_1$.

4. Properties of fuzzy faintly pre-continuous functions

In [5], a fuzzy topological space (X, T) is defined to be fuzzy compact if and only if every fuzzy open cover of X has a finite subcover. Generalizing this, we now define:

Definition 4.1. A fuzzy topological space (X, T) is said to be *fuzzy pre-compact* [4] (*θ-compact* [2]) if every fuzzy pre-open(fuzzy θ-open) cover of X has a finite subcover.

Proposition 4.2. *Let (X, T) and (Y, S) be any two fuzzy topological spaces and suppose that $f : (X, T) \to (Y, S)$ is fuzzy faintly pre-continuous surjective mapping and X is fuzzy pre-compact. Then Y is fuzzy θ-compact.*

Proof. Let $\{\lambda_i | i \in \Delta\}$ be a fuzzy θ-open cover of Y. Since $f : (X, T) \to (Y, S)$ is fuzzy faintly pre- continuous, $\left\{f^{-1}(\lambda_i) \,|\, i \in \Delta\right\}$ is a family of fuzzy pre-open sets in X such that $1_X \le \bigvee_{i\in\Delta} f^{-1}(\lambda_i)$. But by fuzzy pre-compactness of X, there exists a finite subfamily Δ_0 of Δ such that $1_X \le \left\{\bigvee_{i\in\Delta_0} f^{-1}(\lambda_i)\right\}$. Then this implies that $1_Y \le \left\{\bigvee_{i\in\Delta_0} \lambda_i'\right\}$. This shows that Y is fuzzy θ-compact. □

Definition 4.3 ([6])**.** Two fuzzy sets λ and μ of a fuzzy topological space (X, T) are said to be *fuzzy weakly separated* if and only if $\mathrm{cl}(\lambda)q\mu$ and $\mathrm{cl}(\mu)q\lambda$.

Note 4.4 ([6]). The condition of weak separation of two fuzzy sets λ and μ of a fuzzy topological space (X, T) can be restated as follows:

There are fuzzy open sets σ and δ such that $\lambda \le \sigma, \mu \le \delta, \lambda q\delta$ and $\mu q\sigma$.

Definition 4.5 ([6])**.** A fuzzy set λ in a fuzzy topological space (X, T) is said to be *fuzzy disconnected* if and only if there are two non-zero fuzzy sets δ_1 and δ_2 in X such that δ_1 and δ_2 are weakly separated in X and $\lambda = \delta_1 \vee \delta_2$.

A fuzzy set λ in a fuzzy topological space (X, T) is said to be *fuzzy connected* if and only if λ is not fuzzy disconnected. In other words, a fuzzy set λ in a fuzzy topological space (X, T) is said to be fuzzy connected if λ can not be expressed as the union of two fuzzy weakly separated fuzzy sets.

Based on the above definition, we now define weakly θ-separated sets (weakly pre-separated sets) as follows:

Definition 4.6. Two fuzzy sets λ and μ of a fuzzy topological space (X, T) are said to be *weakly θ-separated* (*weakly pre-separated*) if there are fuzzy θ-open sets (fuzzy pre-open sets) σ and δ such that $\lambda \le \sigma$, $\mu \le \delta$, $\lambda q\delta$ and $\mu q\sigma$.

Definition 4.7 ([2])**.** A fuzzy set λ in a fuzzy topological space (X, T) is said to be *θ-connected* if λ can not be expressed as the union of two fuzzy weakly θ-separated sets.

Based on this we now define the following:

Definition 4.8. A fuzzy set λ in a a fuzzy topological space (X, T) is said to be *fuzzy pre-connected* if and only if λ can not be expressed as the union of two fuzzy weakly pre-separated sets.

Definition 4.9. Two fuzzy sets λ_1, λ_2 in a fuzzy topological space in X is said to be *fuzzy pre-separated* [4] if $\mathrm{cl}(\lambda_1) + \lambda_2 \leq 1$ and $\lambda_1 + \mathrm{cl}(\lambda_2) \leq 1$.

Definition 4.10. A fuzzy set λ in a fuzzy topological space (X, T) is said to be *fuzzy pre-connected* if λ can not be expressed as the union of two fuzzy pre-separated sets [4].

Proposition 4.11. *Let (X, T) and (Y, S) be any two fuzzy topological spaces. If $f : (X, T) \to (Y, S)$ is fuzzy faintly pre-continuous mapping and X is fuzzy connected, then Y is fuzzy connected.*

Proof. Suppose $f : (X, T) \to (Y, S)$ is fuzzy faintly pre-continuous mapping and X is fuzzy connected. We want to show that Y is fuzzy connected. Suppose that Y is not fuzzy connected. Then there exists non-empty fuzzy open sets λ_1 and λ_2 such that $\lambda_1 \wedge \lambda_2 = 0$ and $\lambda_1 \vee \lambda_2 = 1_Y$. Then λ_1 and λ_2 are fuzzy clopen and so fuzzy θ-open in Y [8]. Since f is fuzzy faintly pre-continuous, $f^{-1}(\lambda_1)$ and $f^{-1}(\lambda_2)$ are fuzzy pre-open sets in X. Now put $\gamma_i = \mathrm{cl}\,\mathrm{int}\,\mathrm{cl}\, f^{-1}(\lambda_i)\,, i = 1, 2$. Then we have $0 \neq f^{-1}(\lambda_1) < \gamma_1$ and $0 \neq f^{-1}(\lambda_2) < \gamma_2$ and $\gamma_1 \wedge \gamma_2 = 0$ and $\gamma_1 \vee \gamma_2 = 1_X$. This shows that X is not fuzzy connected, which is a contradiction. This proves Y is fuzzy connected. □

Proposition 4.12. *Let (X, T) and (Y, S) be any two fuzzy topological spaces and suppose that $f : (X, T) \to (Y, S)$ is fuzzy faintly pre-continuous surjective mapping. If λ is a fuzzy pre-connected set in X, then $f(\lambda)$ is a fuzzy θ-connected set in Y.*

Proof. Suppose $f(\lambda)$ is not fuzzy θ-connected in Y. Then there exists fuzzy θ-separated sets γ and σ in Y such that $f(\lambda) = \gamma \vee \sigma$. Since f is fuzzy faintly pre-continuous, $f^{-1}(\gamma)$ and $f^{-1}(\sigma)$ are fuzzy pre-open in X and

$$\lambda = f^{-1}[f(\lambda)] = f^{-1}[\gamma \vee \sigma] = f^{-1}(\gamma) \vee f^{-1}(\sigma).$$

Since $f^{-1}(\gamma)$ and $f^{-1}(\sigma)$ are fuzzy pre-separated in X, λ is not fuzzy pre-connected in X which is a contradiction to the assumption. This proves $f(\lambda)$ is a fuzzy θ-connected set in Y. □

Lemma 4.13 ([3])**.** *Let (X, T) and (Y, S) be any two fuzzy topological spaces and let $g : X \to X \times Y$ be the graph mapping [4] of $f : (X, T) \to (Y, S)$. If λ is a fuzzy set in X and μ is a fuzzy set in Y, then $g^{-1}(\lambda \times \mu) = \lambda \wedge f^{-1}(\mu)$.*

Proposition 4.14. *Let (X, T) and (Y, S) be any two fuzzy topological spaces. Let $f : (X, T) \to (Y, S)$ be a mapping. Assume X is product related to Y and let $g : X \to X \times Y$ be its graph mapping. If g is fuzzy faintly pre-continuous, then f is also fuzzy faintly pre-continuous.*

Proof. Suppose that g is a fuzzy faintly pre-continuous mapping and λ is a fuzzy θ-open set in Y. Then

$$\begin{aligned} f^{-1}(\lambda) &= 1 \wedge f^{-1}(\lambda) \\ &= g^{-1}(1 \times \lambda) \\ &\leq \operatorname{int}\left(\operatorname{cl} g^{-1}(1 \times \lambda)\right) \\ &\leq \operatorname{int}\left(\operatorname{cl}\left(1 \times f^{-1}(\lambda)\right)\right) \\ &\leq \operatorname{int}\left(\operatorname{cl} f^{-1}(\lambda)\right) \end{aligned}$$

which implies $f^{-1}(\lambda)$ is fuzzy pre-open in X. Hence f is fuzzy faintly pre-continuous. □

Proposition 4.15. *Let (X,T) and (Y,S) be any two fuzzy topological spaces. If $f:(X,T)\to(Y,S)$ is fuzzy faintly pre-continuous and Y is fuzzy regular space* [1]*, then f is fuzzy pre-continuous.*

Proof. Let f be fuzzy faintly pre-continuous and λ be a fuzzy open set in (Y,S). Since (Y,S) is fuzzy regular, every fuzzy open set in (Y,S) is fuzzy θ-open. Hence $f^{-1}(\lambda)$ is fuzzy pre-open in (X,T). This proves f is fuzzy pre-continuous. □

Definition 4.16. A fuzzy topological space X is called *fuzzy θ-T_2* [2] if for every pair of distinct fuzzy points x_p and x_q there exists fuzzy θ-open sets β and γ containing x_p and x_q respectively such that $\beta \wedge \gamma = 0$.

Definition 4.17. A fuzzy topological space X is called *fuzzy Hausdorff* if for every pair of distinct fuzzy points x_p and x_q there exists fuzzy θ-open sets λ and μ such that $x_p \in \lambda$ and $x_q \in \mu$ and $\lambda \wedge \mu = 0$.

Remark 4.18. In a fuzzy topological space (X,T), the concepts of fuzzy open set and that of fuzzy θ-open set need not coincide but however if the fuzzy topological space (X,T) is fuzzy regular, the above concepts are the same. Therefore generally the above two definitions are not the same.

Definition 4.19. Let (X,T) and (Y,S) be any two fuzzy topological spaces. A function $f:(X,T)\to(Y,S)$ is called *fuzzy faintly precontinuous* if for each fuzzy θ-open set μ containing $f(x_p)$, there exists a fuzzy pre open set γ containing x_p such that $f(\gamma) \leq \mu$.

Proposition 4.20. *Let (X,T) and (Y,S) be any two fuzzy topological spaces. If $f:(X,T)\to(Y,S)$ is fuzzy faintly precontinuous injective mapping and Y is fuzzy θ-T_2, then X is fuzzy Hausdorff.*

Proof. Since f is injective, $f(x_p) \neq f(x_q)$ for distinct fuzzy points x_p and x_q in X. Since Y is fuzzy θ-T_2, there exists fuzzy θ-open sets λ and μ such that $f(x_p) \in \lambda$, $f(x_q) \in \mu$ and $\lambda \wedge \mu = 0$. Since f is fuzzy faintly precontinuous, we have $x_p \in f^{-1}(\lambda)$, $x_q \in f^{-1}(\mu)$ and $f^{-1}(\lambda) \leq \operatorname{int}\operatorname{cl} f^{-1}(\lambda)$, $f^{-1}(\mu) \leq \operatorname{int}\operatorname{cl} f^{-1}(\mu)$. Also $\operatorname{int}\operatorname{cl} f^{-1}(\lambda) \wedge \operatorname{int}\operatorname{cl} f^{-1}(\mu) = 0$. This shows that X is fuzzy Hausdorff. □

Definition 4.21. Let (X, T) and (Y, S) be any two fuzzy topological spaces. A function $f : (X, T) \to (Y, S)$ is called *fuzzy almost pre-continuous* if the inverse image of fuzzy regular open set in Y is fuzzy pre-open in X.

Definition 4.22. A fuzzy topological space (X, T) is said to be *fuzzy extremally disconnected* if the closure of each fuzzy open set in (X, T) is fuzzy open.

Proposition 4.23. *Let (X, T) and (Y, S) be any two fuzzy topological spaces. If a function $f : (X, T) \to (Y, S)$ is fuzzy faintly pre-continuous and Y is fuzzy extremally disconnected space, then f is fuzzy almost pre-continuous.*

Proof. Let λ be any fuzzy regular open set. We want to show that $f^{-1}(\lambda)$ is fuzzy pre-open. Now since (Y, S) is fuzzy extremally disconnected, λ is a fuzzy θ-open set (by Theorem 2.6 of [1]). Also since f is fuzzy faintly pre-continuous it follows that $f^{-1}(\lambda)$ is fuzzy pre-open. Hence the proposition. □

Proposition 4.24. *Let X_1, X_2, Y_1 and Y_2 be fuzzy topological spaces such that Y_1 is product related to Y_2 and X_1 is product related to X_2. Let $f_1 : X_1 \to Y_1$, $f_2 : X_2 \to Y_2$ be fuzzy faintly pre-continuous functions. Then the product $f_1 \times f_2 : X_1 \times X_2 \to Y_1 \times Y_2$ is fuzzy faintly pre-continuous.*

Proof. Let $\lambda = \vee(\lambda_i \times \lambda_j)$ where λ_i's and λ_j's are of fuzzy θ-open sets of Y_1 andY_2 respectively be a fuzzy θ-open set of $Y_1 \times Y_2$. Then

$$\begin{aligned}
f^{-1}(\lambda) &= \vee\left(f_1^{-1}(\lambda_i) \times f_2^{-1}(\lambda_j)\right) \\
&\leq \vee\left(\operatorname{int}\operatorname{cl} f_1^{-1}(\lambda_i) \times \operatorname{int}\operatorname{cl} f_2^{-1}(\lambda_j)\right) \\
&\leq \vee\left(\operatorname{int}\operatorname{cl}\left(f_1^{-1}(\lambda_i) \times f_2^{-1}(\lambda_j)\right)\right) \\
&\leq \operatorname{int}\operatorname{cl}\left(\vee\left(f_1^{-1}(\lambda_i) \times f_2^{-1}(\lambda_j)\right)\right) \\
&\leq \operatorname{int}\operatorname{cl}(f_1 \times f_2)(\vee(\lambda_i \times \lambda_j)) \\
&\leq \operatorname{int}\operatorname{cl}\Big((f_1 \times f_2)^{-1}(\lambda)\Big).
\end{aligned}$$

This proves that $f_1 \times f_2$ is fuzzy faintly pre-continuous. □

The concept of weakly continuous function is introduced in [7]. Based on this concept we now define the following.

Definition 4.25. Let (X, T) and (Y, S) be any two fuzzy topological spaces. A function $f : (X, T) \to (Y, S)$ is said to be *weakly fuzzy pre-continuous* if for each fuzzy open set λ in (Y, S) there exists a fuzzy pre-open set μ in (X, T) such that $f(\mu) \leq \operatorname{cl}(\lambda)$.

Proposition 4.26. *Let (X, T) and (Y, S) be any two fuzzy topological spaces. If $f : (X, T) \to (Y, S)$ is a weakly fuzzy pre-continuous, then f is some what fuzzy faintly pre-continuous.*

Proof. Let λ be any fuzzy θ-open set in (Y, S). Also since λ is fuzzy θ-open set in Y, there exists a fuzzy open set σ in Y such that, $\sigma \leq \operatorname{cl}\sigma \leq \lambda$. Since f is weakly fuzzy pre-continuous, there exists a fuzzy pre-open set μ in X such that $f(\mu) \leq \mathrm{cl}\,\sigma \leq \lambda$. This means $\mu \leq f^{-1}(\lambda)$ and therefore f is some what fuzzy faintlyy pre-continuous. □

Example 4.27. Let $X = \{0,1\}$, $T = \{0,1,f\}$ where $f : X \to I$ is such that $f(0) = 0$, $f(1) = 1$. Let $Y = \{a,b,c\}$, $S = \{\bar{0},\bar{1},g,h,i\}$ where g,h and i are defined as follows: $g : Y \to I$ is such that $g(a) = 1$, $g(b) = g(c) = 0; h : Y \to I$ is such that $h(a) = 0$, $h(b) = 1$, $h(c) = 0$ and $i : Y \to I$ is such that $i(a) = 1 = i(b)$, $i(c) = 0$. Define $f : (X,T) \to (Y,S)$ as follows: $f(0) = a$; $f(1) = b$. The only fuzzy θ-open in Y is 1 and $f^{-1}(1) = 1$. This shows that f *is fuzzy faintly pre-continuous.*

Now consider the fuzzy open set g in (Y,S). Then $\mathrm{cl}(g) = 1 - h$ and there is no non-zero fuzzy open set μ in X such that $f(\mu) \leq \mathrm{cl}(g)$ (For $f(1) = 1 \nleq 1 - h$). This shows that f *is not weakly fuzzy pre-continuous.*

Definition 4.28. Let (X,T) and (Y,S) be any two fuzzy topological spaces. A function $f : (X,T) \to (Y,S)$ is said to be *fuzzy almost pre-continuous* if for each fuzzy regular open set λ in Y, there exists a fuzzy pre-open set μ in X such that $f(\mu) \leq \lambda$.

Definition 4.29. A fuzzy topological space (X,T) is called *fuzzy almost regular space* if $T_\theta = T_R$ (where T_R is the family of all fuzzy regular open sets in X).

Proposition 4.30. *Let $f : (X,T) \to (Y,S)$ be fuzzy faintly pre-continuous. Assume (Y,T) is fuzzy almost regular space. Then f is fuzzy almost pre-continuous.*

Proof. Suppose $f : (X,T) \to (Y,S)$ is fuzzy faintly pre-continuous. Then by Proposition 3.5 (f), $f : (X,T) \to (Y,S_\theta)$ is fuzzy pre-continuous. Now (Y,S) is fuzzy almost regular space implies that $S_\theta = S_S$ and thus $f : (X,T) \to (Y,S_S)$ is fuzzy pre-continuous means $f : (X,T) \to (Y,S)$ is fuzzy almost pre-continuous. □

The following proposition is easy to establish.

Proposition 4.31. *If $f : (X,S) \to (Y,T)$ be fuzzy faintly pre-continuous and $A \subset X$, then $f/A : A \to Y$ is fuzzy faintly pre-continuous.*

Proposition 4.32. *Suppose $f : (X,S) \to (Y,T)$ be weakly fuzzy pre-continuous and X is product related to Y. Then the graph map $g : X \to X \times Y$ is some what fuzzy faintly pre-continuous.*

Proof. Let λ be any fuzzy θ-open set in $X \times Y$. Then there exists a fuzzy open set μ_1 in X and fuzzy open set λ_1 in Y such that $\mathrm{cl}(\mu_1 \times \lambda_1) = \mathrm{cl}(\mu_1) \times \mathrm{cl}(\lambda_1) \leq \lambda$.

Since f is weakly fuzzy pre-continuous, there exists a fuzzy pre-open set μ_0 in X such that $f(\mu_0) \leq \mathrm{cl}(\mu_2)$. Consequently $g(\mu_0) \leq \mathrm{cl}(\mu_1) \times \mathrm{cl}(\mu_2) \leq \lambda$. In other words $\mu_0 \leq g^{-1}(\lambda)$. This proves that g is some what fuzzy faintly pre-continuous. □

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