Modified Structural Modeling Method and Its Application: Behavior Analysis of Passengers for East Japan Railway Company

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Selected paper from APIEMS 2006

Abstract. In order to cope with the ill-defined problem of human behavior being immanent uncertainty, several methodologies have been studied in game theoretic, social psychological and political science frameworks. As methods to arrange system elements systematically and draw out the consenting structural model concretively, ISM, FSM and DEMATEL based on graph theory etc. have been proposed. In this paper, we propose a modified structural modeling method to recognize the nature of problem. We introduce the statistical method to adjust the establishment levels in group decision situation. From this, it will become possible to obtain effectively and smoothly the structural model of group members in comparison with the traditional methods. Further we propose a procedure for achieving the consenting structural model of group members based on the structural modeling method. By applying the method to recognize the nature of ill-defined problems, it will be possible to solve the given problem effectively and rationally. In order to inspect the effectiveness of the method, we conduct a practical problem as an empirical study: "Behavior analysis of passengers for the Joban line of East Japan Railway Company after new railway service of Tsukuba Express opened".

Keywords: Ill-defined problem, Uncertainty, Structural modeling, Empirical study, Group decision-making, Principal components analysis

1. INTRODUCTION

In most cases, decisions are made by a group or a committee which complicates the decision process since a consensus is required. Particularly, with increasing of the diversified value sense of people, the decision situation has been become extremely complex. Decision makers may have access to different information/knowledge on which to base their decision, and/or they may place a different ordering/weighting on the alternatives since they own differences of values, beliefs, attitudes, and understandings for the given problem set. Therefore, how to deal with the complexity which arises commonly in multi-participant decision situation rationally and efficiently is an important factor since it affects the quality of group decision making.

In order to cope with the problem with respect to human behavior such as human judgment, insight and

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intuition, several methodologies have been studied in game theoretic, social psychological and political science frameworks (Marakas 1999). For instance, such the multivariable analysis of statistical methodology as principal components analysis and cluster analysis are exploited to solve considerable practical problems in several areas (Terano 1985). And also some qualitative techniques such as Brainstorming (Hogg and Tibdale 2001), NGT(Nominal Group Techniques) (Delbecq *et al.*, 1975), Delphi method (Linstone and Turoff 1975), LENS(Leadership Effectiveness and New Strategy)(Clark 1980) have been employed for creating an alternative space from which meaningful and distinct alternatives are likely to be identified.

Among the methodologies used in arranging system elements in a hierarchy, ISM (Interpretive Structural Modeling) (Warfield et al., 1975), DEMATEL (Decision Making Trial and Evaluation Laboratory)(Gabus and Fontela 1975) and FSM (Fuzzy Structural Modeling)(Tazaki and Amagasa 1979) are popular ones. The major advantage of those methodologies is intuitive appeal of the graphical picture to decision makers. ISM, DEMATEL and FSM are based on graph theory to portray system hierarchy with contextual relations among elements such as "purpose and means", "cause and effect". The relations among system elements modeled by ISM through a pair wise comparison, are intuitively and empirically given with binary relation $\{0 \text{ or } 1\}$ to indicate whether or not the element is relative to the other under an assumption that the relations are transitive, that is, if A is relative to B and B relative to C, then A is relative to C, which assumes transitivity inference works in usual while human beings make decision. On the other hand, FSM uses binary fuzzy relation given within the closed interval of [0, 1] to represent the subordination relations among the elements (Klir and Yuan 1995), and relaxes the transitivity constraint in contrast to ISM. Different from ISM and FSM, DE-MATEL structures system elements by ranking the degree to give effect and the degree to get effect between them, which is predefined given on "cause and effect" relations with four grade values in order to incline strong relations to evaluate. Although DEMATEL does not assume that the relations own transitivity property, the decision makers are strongly required having high quality of knowledge background, so-called expert of the area, for achieving the effectiveness of weighting.

In this paper, we aim to propose a modified structural modeling method based on FSM to recognize what the ill-defined problem itself is. For obtaining the structural model of multi-participant decision makers effectively and smoothly, we attempt to adjust the establishment levels in group decision situation statistically. Furthermore, we employ a procedure to synthesize the consenting structural model of group members based on the structural modeling method. From doing this, it will be possible to solve the problem mentioned above effectively and rationally in comparison with the traditional methods.

To inspect the effectiveness of the method, we con-

duct a practical study: "Behavior analysis of passengers for the Joban line of East Japan Railway Company after new opening service of Tsukuba Express railway" to examine whether the new opened railway brings changes to passengers' behavior.

The rest of the paper is organized as follows: the following section proposes the method based on structural modeling method in detail. The practical problem is studied and analyzed in section 3. Finally, a conclusion is discussed in the end after analyzing the results of study.

2. GROUP DECISION MAKING BASED ON STRUCTURAL MODELING METHOD

As described in the former section, the methodologies are elaborated under the assumption/condition for simplicity, and exploited in several areas. According to our practical study in the latter section, we propose a modified structural modeling approach, which is based on FSM. Figure 1 illustrates a flowchart for the approach we propose.

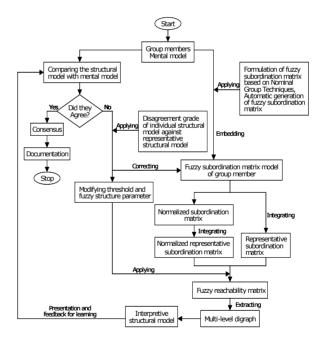


Figure 1. Group decision making based on structural modeling method.

The algorithm shown in figure 1 begins with mental model of individual group member which is determined depending on their intuition to the given problem. Then, each mental model is embedded into a fuzzy subordination matrix on the context on basis of the relaxation of transitivity, reflexivity and symmetry by each group member (Zadeh 1965; Klir and Yuan 1995; Tazaki and Amagasa 1979). Herein, NGT and automatic generation method of subordination matrix are applied to embed entries of the matrices efficiently and effectively. In the mental model, uncertainty arises because group members differ in their understanding of the given problem and their knowledge background while their respective interests converge, and also the scale to set up the elements of the fuzzy subordination matrix is individually different. In other words, it is necessary to adjust the different individual establishment level of group members to the same level on the contextual relation before forwarding the consensus between group members.

For doing this, we formulate the individual fuzzy subordination matrix with the same establishment level. In the proposed approach, we normalize statistically the entries of the matrix embedded by group individual. Then, a representative subordination matrix is formulated by integrating the fuzzy subordination matrices of group members as follows:

Let $S = \{s_1, s_2, \dots, s_i, \dots, s_n\}$ denote a system with *n* elements, and let $A_k = [a_{ij}^k]$ $(i, j = 1, 2, \dots, n, k = 1, 2, \dots, m)$ denote the fuzzy subordination matrices in S, where $a_{ij}^k = f^k(s_i, s_j) (0 \le a_{ij}^k \le 1, i, j = 1, 2, \dots, n, k = 1, 2, \dots, m)$. a_{ij}^k is the grade of which s_i is subordinate to s_j . *m* is the number of group members. Let $NA_k = [h_{ij}^k]$ $(i, j = 1, 2, \dots, n, k = 1, 2, \dots, m)$ denote the normalized fuzzy subordination matrices. And then the given data $A_k = [a_{ij}^k]_{n \times n}$ from group members is normalized by $h_{ij}^k = g(a_{ij}^k, \overline{a}_k, \sigma_a^k)$ such that

$$h_{ij}^{k} = (\frac{a_{ij}^{k} - \overline{a}_{k}}{\sigma_{a}^{k}} \times 10 + 50) \times 10^{-2}$$

(*i*, *j* = 1, 2, ..., *n*, *k* = 1, 2, ..., *m*)

and also shown in $NA_k = N^k (\overline{a}_k, (\sigma_a^k)^2)$ (k = 1, 2, ..., m), where

$$\overline{a}_k = \frac{1}{n^2} \sum_{i=1}^n \sum_{j=1}^n a_{ij}^k \qquad (k = 1, 2, \dots, m) ,$$

and

$$\sigma_a^k = \frac{1}{n} \sqrt{\sum_{i=1}^n \sum_{j=1}^n a_{ij}^{k^2} - \overline{a}_k^2} \quad (k = 1, 2, \dots, m)$$

Now, we use the normalized subordination matrices to compute the representative subordination matrix which holds the data factor from group members. Let $NAR = [d_{ij}]$ $(i, j = 1, 2, \dots, n)$ be a representative subordination matrix, which is computed by

$$d_{ij} = \frac{1}{m} \sum_{k=1}^{m} h_{ij}^k$$
 (*i*, *j* = 1, 2, ..., *n*).

Next, the fuzzy reachability matrix is computed on the basis of *NAR*, and multi-level digraph is drawn as an interpretive structural model. In order to compare the structural model with mental model, a feedback for learning will be performed to group members. If an agreement among group members is obtained, the process goes ahead to documentation step. Otherwise, a threshold and fuzzy structure parameter will be modified and the algorithm is iterated until a consenting model is derived.

Here, let p be the threshold, specified by α -cut, which is defined by the modified z-value in standard normal distribution as figure 2 shows. In other words, the percentage of subordination relations among elements which exist in the structural model to be evaluated can be controlled by the value of p. For example, suppose p = 0.53 means that the subordination relations among elements of system stand about 30% in all, also that we need to care about the relations with over 0.53 of degree among system elements. In this way, we can manipulate and interpret the interpretive structural model meaning-fully in comparison with the traditional methods depending on human intuition.

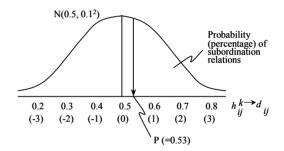


Figure 2. Adjustment of establishment levels of group members.

2.1 Algorithm of Modified Structural Modeling

A decision group consists of several members (decision makers) with either equal or different knowledge background for a given problem.

Let GM_k ($k = 1, 2, \dots, m$) denote group members, and A_k ($k = 1, 2, \dots, m$) be fuzzy subordination matrices of data given by GM_k .

- **Definition**: A row (column) is said to be regular if it contains only a single $a_{ij}^k \ge p$ in A_{ij} .
- Rule 1: If there exist no regular row and /or column in the single hierarchy matrix, the lowest order row (column) must be split into the regular rows (columns).
- **Rule 2:** When all of the regular rows obtained by splitting are eliminated from A and the rows must be recombined on the graph.
- **Proposition 1** (Tazaki and Amagasa 1979): Let s_{i_j} (j = 1, 2, ..., l) be the regular rows corresponding to s_i in a single hierarchy matrix,

then the following operation is carried out for such regular rows.

$$a_{.i}^* = a_{.i} \wedge (\bigwedge_{j=1}^l \overline{a}_{.i_j}) \quad (i = 1, 2, \dots, n),$$

where $\overline{a}_{i_j} = (1 - \overline{a}_{i_j})/(1 + \lambda \overline{a}_{i_j})$ with a given number λ .

The algorithm covers the steps on the basis of the rules and the proposition as follows:

- **Step 1:** Give a fuzzy subordination matrix $A_k = [a_{ij}^k]$ which satisfies the fuzzy irreflexive law and the fuzzy asymmetric law. Express the representative subordination matrix $N = [d_{ij}]$ $(i, j = 1, 2, \dots, n)$ by integrating $A_k(k = 1, 2, \dots, m)$. Further, compute the fuzzy semi-reachability matrix N'_k of A_k and N' satisfying the fuzzy semi-transitive law.
- **Step 2:** Identify the sets of top level, intermediate level, bottom level as well as isolated level on the basis of the semi-reachability matrix N'_k and N'. Further determine the subordination relation sets between the top level set and the bottom level set, and call it the block set with respect to each of N'_k and N', and normalized subordination matrix $N_k = [h_{ii}^k]$.
- Step 3: Eliminate all of the rows concerning elements belonging to the top level set, and all of the columns concerning elements belonging to the bottom level set. And then, eliminate the rows and columns concerning elements belonging to the isolated set in N'_k and N'. The fuzzy subordination matrices consisting of remaining rows and columns are reconstructed as N'_k and N'.
- **Step 4:** From N_k and N obtained in *Step 3*, construct the single hierarchy matrix corresponding to each block set in each of N_k and N.
- **Step 5:** Set up the threshold *p* depending on the given probability which means the percentage of subordination relations in system model to be constructed, and identify the structural model corresponding to each single hierarchy matrix N'_k and N' on the basis of the rules 1, 2 and proposition 1. In the step, assume that the regular rows corresponding to s_i are s_{i_j} ($i = 1, 2, \dots, n, j = 1, 2, \dots, l < n$). After applying the proposition 1 for such regular rows, all of the rows s_{i_j} ($j = 1, 2, \dots, l < n$) can be eliminated by replacing a_i with a_j^* . The same

operation in *step 5* can be also applied to eliminate the columns.

We can uniquely get the structural model by making use of the modified structural modeling method described above.

3. BEHAVIOR ANALYSIS OF PASSEN-GERS FOR EAST JAPAN RAILWAY COMPANY

In this section, we apply the modified structural modeling method proposed in section 2 to analyze the factors of passengers' behavior for the Joban Line of East Japan Railway Company. The situation which we treat is the Joban Line user's trend accompanied by opening railroad: The Tsukuba express (abbreviated as TX) was opened on August 24, 2005 as a means of transportation by the railroad which travels through a northern metropolitan area in Japan.

We asked an Internet research company to conduct a survey on internet homepage from September 28, 2005 for three days after TX's opening about one month. Candidates are about 1000 residents along TX line who will use Joban Line or use TX as a certain use. The questionnaire contains 24 items considered when the respondents choose which line is better.

- s₁: Fare. (basic fare to the destination)
- s₂: Speed to the destination.
- s₃: Frequency of train service.
- s₄: The ease of access to a nearby station.
- s_5 : The congestion degree of people in a train.
- s₆: Whether a seat be taken or not?
- s₇: The number of times of a change.
- s_8 : The ease of carrying out of a change.
- s₉: Whether work is possible in the train or not?
- s_{10} : Equipment of a station. (width of an escalator, an elevator and a passage, etc.)
- s₁₁: Whether the shopping be made on the way or not? (there is a shopping mall contiguous to a station building or a station etc.)
- s₁₂: The image of the railroad company.
- s₁₃: The establishment of a parking lot for bicycles near the station.
- s₁₄: The establishment of a parking lot for cars near the station.
- s_{15} : The ease of using the bus-on-a-regular-route.
- s₁₆: Safety of a platform.
- s₁₇: Atmosphere around a station.
- s₁₈: Width of passenger cars.
- s₁₉: Reliability of time.
- s₂₀: Merit of connection with other lines.
- s₂₁: Whether the shopping can be made around a station yard or not?
- s₂₂: Extra charge. (there is a setup of a special express, specification, etc.)
- s_{23} : The arrangement of the corporate staff of a rail-

		Initial Eigenv	ralue	Squar	re-sum of Facto	or Loadings	Square-sum of Factor Loadings after Varimax rotation		
Factors	Total	Contribution Rate	Cumultative Contribution Rate	Total	Contribution Rate	Cumultative Contribution Rate	Total	Contribution Rate	Cumultative Contribution Rate
1	8.230	34.290	34.290	8.230	34.290	34.290	4.649	19.372	19.372
2	2.553	10.636	44.927	2.553	10.636	44.927	3.934	16.390	35.762
3	1.291	5.380	50.306	1.291	5.380	50.306	2.472	10.300	46.063
4	1.081	4.505	54.811	1.081	4.505	54.811	2.100	8.749	54.811
5	.930	3.875	58.686						
6	.794	3.309	61.995						
7	.733	3.056	65.050						
8	.707	2.946	67.996						
9	.648	2.699	70.695						
10	.637	2.654	73.349						

Table 1. The cumulative contribution rate.

road, and correspondence.

s₂₄: Information services, **concerning** congestion and delay.

The degree of the consideration to each item was made to choose from following five levels.

1. Very large.	2. Large.	3. About medium.
4. Small.	5. Completely	nothing.

First, we show the result of the principal components analysis as a conventional statistical method usually applied for an opinion poll questionnaire.

From the principal components analysis, we extract four factors as in at 54.81% of cumulative contribution rate (see Table1).

Although the cumulative contribution rate is no more than 60%, it seems that four or five factors will probably be appropriate (see Figure 3).

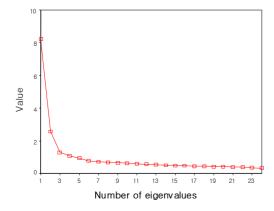


Figure 3. Plot of eigenvalues.

The relation of the four factors and each question item number which were actually extracted is shown in Table 2. Their factor loadings are values after varimax rotation has been performed.

 Table 2. Factor extraction by the principal components analysis.

	5								
Var.	Factors								
vui.	Factor 1	Factor 2	Factor 3	Factor 4					
s2	.769	.074	.025	029					
s20	.736	.136	.062	.126					
s3	.736	.206	.039	.061					
s4	.729	018	.069	.203					
s7	.705	.083	.048	.166					
s8	.699	.238	.048	.181					
s1	.538	.241	.100	.125					
s5	.513	.467	.138	.025					
s6	.432	.287	.263	.050					
s23	.100	.717	.215	.223					
s16	.204	.712	.149	.214					
s18	.217	.685	.217	.145					
s12	.031	.620	.300	.248					
s24	.373	.600	.154	.127					
s10	.214	.578	.215	.295					
s19	.502	.533	.071	010					
s14	.055	.146	.777	.059					
s15	.113	.135	.660	.191					
s13	.185	.065	.617	.162					
s9	055	.319	.581	052					
s22	.066	.341	.492	.056					
s11	.207	.219	.130	.824					
s21	.173	.299	.141	.768					
s17	.230	.440	.191	.580					

Factor Extracting Method: The Principal Component Anlysis with Kaiser's method.

The 1st factor, factor1, is related with attainment nature, such as speed to the destination, merit of connection with other lines, an operation number, and a thing about a change. The 2nd factor, factor2, is a factor about the safety, relief, and comfortable nature to arrangement of a railroad company staff, a platform, passenger car, a railroad company, etc. The 3rd factor, factor3, is the attainment nature to the nearby station and the 4th factor, factor4, expresses the convenience of whether shopping is made not at the railroad itself but at the shopping mall attached to it, etc.

Next, we apply the modified structural modeling method to find the relations of set of items $\{s_1, s_2, \dots, s_{24}\}$. The following is illustrated in order to show how the proposed algorithm works.

We consider $S = \{s_1, s_2, \dots, s_{24}\}\$ as a set of system objects, where each s_k is a question in the survey questionnaire described above. Let $K = \{1, 2, 3\}\$ be the set of decision makers. Let $A_k = [a_{ij}^k]$ $(i, j = 1, \dots, 24, k = 1, 2,$ 3) be the fuzzy subordination matrix for a given object as shown below. In particular, we denote by A_k^n the decision maker *k*'s subordination matrix at n^{th} trial. In an experimentation, we reached an agreement in the third trial.

A_1^1	=										
0.0	0.2	0.3	0.2	0.4	0.5	2.0	0.2	0.7	0.5	0.4	0.5
0.3	0.3	0.4	0.7	0.6	0.7	0.7	0.6	0.6	0.6	0.6	0.5
0.7	0.0	0.7	0.5	0.4	0.5	0.4	0.6	0.3	0.3	0.2	0.7
0.3	0.4	0.5	0.4	0.3	0.4	0.7	0.6	0.2	0.4	0.3	0.4
0.6	0.5	0.0	0.2	0.7	0.7	0.2	0.7	0.3	0.2	0.6	0.6
0.3	0.2	0.5	0.7	0.6	0.3	0.8	0.7	0.6	0.3	0.7	0.5
0.5	0.7	0.3	0.0	0.7	0.7	0.2	0.2	0.2	0.2	0.2	0.7
0.3	0.3	0.6	0.2	0.2	0.2	0.7	0.5	0.2	0.1	0.1	0.3
0.7	0.7	0.3	0.3	0.0	0.8	0.2	0.2	0.7	0.5	0.5	0.8
0.5	0.5	0.5	0.7	0.7	0.5	0.7	0.5	0.4	0.4	0.7	0.7
0.6	0.3	0.3	0.3	0.4	0.0	0.2	0.1	0.8	0.1	0.1	0.7
0.1	0.1	0.2	0.5	0.2	0.1	0.5	0.2	0.1	0.4	0.2	0.2
0.2	0.8	0.2	0.2	0.3	0.1	0.0	0.3	0.6	0.3	0.6	0.6
0.1	0.1	0.3	0.7	0.3	0.3	0.7	0.6	0.7	0.6	0.7	0.8
0.6	0.8	0.3	0.2	0.6	0.1	0.3	0.0	0.2	0.2	0.2	0.7
0.1	0.1	0.5	0.4	0.1	0.2	0.6	0.7	0.6	0.1	0.4	0.4
0.5	0.6	0.4	0.2	0.3	0.4	0.3	0.1	0.0	0.1	0.1	0.7
0.2 0.7	0.2 0.2	0.1 0.3	0.1 0.2	0.1 0.6	0.3 0.1	0.2 0.2	0.1 0.7	0.1 0.1	0.4 0.0	0.2 0.5	0.1 0.7
0.7	0.2	0.5	0.2	0.6	0.1	0.2	0.7	0.1	0.0	0.3	0.7
0.2	0.1	0.1	0.7	0.3	0.1	0.0	0.7	0.0	0.1	0.4	0.2
0.0	0.2	0.8	0.4	0.7	0.4	0.2	0.2	0.1	0.1	0.0	0.0
0.4	0.3	0.3	0.0	0.3	0.1	0.0	0.1	0.0	0.3	0.4	0.0
0.2	0.2	0.2	0.4	0.4	0.2	0.4	0.2	0.2	0.2	0.4	0.4
0.6	0.6	0.2	0.7	0.7	0.1	0.1	0.1	0.1	0.1	0.5	0.3
0.0	0.1	0.4	0.1	0.7	0.1	0.6	0.1	0.2	0.1	0.1	0.1
0.6	0.7	0.4	0.8	0.6	0.1	0.1	0.1	0.1	0.6	0.5	0.4
0.1	0.0	0.4	0.1	0.4	0.1	0.5	0.1	0.4	0.1	0.1	0.1
0.7	0.7	0.6	0.8	0.8	0.6	0.3	0.2	0.1	0.1	0.6	0.4
0.7	0.7	0.0	0.1	0.5	0.1	0.3	0.1	0.6	0.1	0.1	0.1
0.6	0.3	0.3	0.2	0.6	0.1	0.3	0.7	0.1	0.4	0.2	0.7
0.1	0.1	0.1	0.0	0.2	0.2	0.3	0.6	0.1	0.1	0.5	0.5
0.6	0.4	0.4	0.2	0.3	0.1	0.1	0.2	0.1	0.3	0.4	0.7
0.2	0.2	0.2	0.1	0.0	0.1	0.1	0.1	0.2	0.1	0.2	0.1
0.6	0.3	0.5	0.1	0.8	0.2	0.2	0.6	0.7	0.1	0.1	0.6
0.1	0.1	0.1	0.5	0.1	0.0	0.4	0.6	0.5	0.1	0.1	0.1
0.6	0.6	0.6	0.4	0.7	0.6	0.5	0.7	0.5	0.2	0.4	0.8
0.2	0.2	0.3	0.6	0.1	0.1	0.0	0.8	0.7	0.2	0.1	0.5
0.7	0.8	0.5	0.7	0.7	0.3	0.4	0.7	0.1	0.2	0.1	0.4
0.6	0.1	0.1	0.2	0.1	0.3	0.5	0.0	0.1	0.1	0.2	0.3

$\begin{array}{ccccc} 0.7 & 0.2 \\ 0.1 & 0.7 \\ 0.8 & 0.7 \\ 0.1 & 0.1 \\ 0.8 & 0.7 \\ 0.1 & 0.1 \\ 0.7 & 0.4 \\ 0.1 & 0.1 \end{array}$	$\begin{array}{cccccccccccccccccccccccccccccccccccc$	0.6 0.1 0.4 0.1 0.7 0.9 0.1 0.6 0.3 0.2 0.6 0.1 0.3 0.7 0.1 0.1	$\begin{array}{cccccccccccccccccccccccccccccccccccc$	0.1 0.4 0.0 0.1 0.8 0.1 0.1 0.0 0.3 0.1 0.5 0.1 0.2 0.1 0.1 0.1	$\begin{array}{ccccc} 0.5 & 0.5 \\ 0.2 & 0.4 \\ 0.1 & 0.7 \\ 0.3 & 0.7 \\ 0.2 & 0.8 \\ 0.0 & 0.7 \\ 0.1 & 0.8 \\ 0.2 & 0.0 \end{array}$
$A_2^1 =$					
$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	$\begin{array}{cccccccccccccccccccccccccccccccccccc$	$\begin{array}{cccccccccccccccccccccccccccccccccccc$	$\begin{array}{cccccccccccccccccccccccccccccccccccc$	$\begin{array}{cccccccccccccccccccccccccccccccccccc$	$\begin{array}{cccccccccccccccccccccccccccccccccccc$
0.0 0.0	0.0 0.0	0.0 0.0	0.0 0.8	0.3 0.0	0.0 0.0
$\begin{array}{ccccc} 0.0 & 0.8 \\ 0.0 & 0.0 \\ 0.0 & 0.0 \\ 0.0 & 0.7 \\ 0.0 & 0.7 \\ 0.0 & 0.0 \\ 0.0 & 0.0 \\ 0.0 & 0.0 \\ 0.0 & 0.3 \\ 0.0 & 0.0 \\ \end{array}$	$\begin{array}{cccccccccccccccccccccccccccccccccccc$	$\begin{array}{cccccccccccccccccccccccccccccccccccc$	$\begin{array}{ccccc} 0.0 & 0.9 \\ 0.0 & 0.0 \\ 0.0 & 0.0 \\ 0.0 & 0.0 \\ 0.0 & 0.0 \\ 0.0 & 0.0 \\ 0.0 & 0.6 \\ 0.0 & 0.0 \\ 0.0 & 0.7 \\ 0.0 & 0.0 \\ \end{array}$	$\begin{array}{cccccc} 0.0 & 0.0 \\ 0.0 & 0.0 \\ 0.0 & 0.8 \\ 0.0 & 0.0 \\ 0.0 & 0.0 \\ 0.0 & 0.0 \\ 0.0 & 0.4 \\ 0.0 & 0.0 \\ 0.0 & 0.0 \\ 0.7 & 0.0 \\ \end{array}$	$\begin{array}{ccccc} 0.0 & 0.8 \\ 0.0 & 0.0 \\ 0.8 & 0.6 \\ 0.0 & 0.0 \\ 0.0 & 0.0 \\ 0.0 & 0.0 \\ 0.0 & 0.7 \\ 0.0 & 0.0 \\ 0.0 & 0.8 \\ 0.0 & 0.0 \\ \end{array}$
$A_3^1 =$					
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	$\begin{array}{cccc} 0.7 & 0.7 \\ 0.1 & 0.1 \\ 0.6 & 0.9 \\ 0.4 & 0.4 \\ 0.0 & 0.2 \\ 0.0 & 0.0 \\ 0.8 & 0.0 \\ 0.0 & 0.0 \end{array}$	$\begin{array}{cccc} 0.6 & 0.6 \\ 0.1 & 0.1 \\ 0.8 & 0.7 \\ 0.3 & 0.4 \\ 0.0 & 0.1 \\ 0.0 & 0.0 \\ 0.0 & 0.0 \\ 0.0 & 0.0 \end{array}$	$\begin{array}{ccccc} 0.5 & 0.2 \\ 0.2 & 0.2 \\ 0.7 & 0.8 \\ 0.7 & 0.7 \\ 0.1 & 0.0 \\ 0.1 & 0.7 \\ 0.0 & 0.0 \\ 0.0 & 0.0 \end{array}$	$\begin{array}{ccccc} 0.0 & 0.0 \\ 0.1 & 0.0 \\ 0.3 & 0.5 \\ 0.3 & 0.3 \\ 0.0 & 0.0 \\ 0.0 & 0.0 \\ 0.0 & 0.0 \\ 0.0 & 0.0 \\ 0.0 & 0.0 \end{array}$	$\begin{array}{cccc} 0.0 & 0.1 \\ 0.0 & 0.0 \\ 0.3 & 0.3 \\ 0.2 & 0.4 \\ 0.0 & 0.0 \\ 0.0 & 0.0 \\ 0.0 & 0.0 \\ 0.0 & 0.0 \end{array}$

0.4	0.1	0.8	0.6	0.0	0.0	0.7	0.7	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	0.0	0.8	0.7	0.0	0.0	0.0	0.0
0.1	0.1	0.7	0.4	0.8	0.0	0.7	0.6	0.0	0.1	0.1	0.1
0.5	0.0	0.0	0.1	0.0	0.1	0.7	0.7	0.1	0.0	0.1	0.1
0.2	0.1	0.3	0.6	0.0	0.1	0.0	0.1	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.1	0.0	0.0	0.3	0.8	0.0	0.0	0.1	0.1
0.6	0.0	0.8	0.5	0.0	0.1	0.8	0.0	0.0	0.0	0.0	0.0
0.1	0.0	0.0	0.1	0.0	0.0	0.2	0.7	0.1	0.0	0.0	0.2
0.2	0.0	0.3	0.2	0.3	0.5	0.4	0.5	0.0	0.5	0.6	0.7
0.4	0.4	0.4	0.3	0.3	0.3	0.1	0.1	0.3	0.5	0.3	0.3
0.3	0.0	0.4	0.2	0.4	0.4	0.4	0.7	0.0	0.0	0.7	0.3
0.4	0.2	0.1	0.4	0.5	0.2	0.2	0.2	0.5	0.3	0.5	0.5
0.3	0.0	0.3	0.2	0.3	0.0	0.4	0.6	0.0	0.2	0.0	0.1
0.2	0.2	0.2	0.5	0.7	0.7	0.3	0.2	0.8	0.3	0.4	0.4
0.3	0.0	0.5	0.1	0.5	0.3	0.5	0.5	0.0	0.4	0.7	0.0
0.6	0.6	0.5	0.5	0.8	0.4	0.4	0.3	0.4	0.0	0.4	0.4
0.5	0.0	0.5	0.1	0.5	0.1	0.5	0.5	0.0	0.2	0.5	0.2
0.0	0.2	0.2	0.6	0.4	0.7	0.3	0.2	0.3	0.2	0.4	0.3
0.3	0.0	0.4	0.1	0.7	0.6	0.5	0.5	0.2	0.4	0.7	0.2
0.7	0.0	0.3	0.4	0.4	0.5	0.2	0.2	0.2	0.2	0.3	0.3
0.5	0.0	0.3	0.1	0.4	0.6	0.5	0.6	0.1	0.6	0.7	0.1
0.6	0.6	0.0	0.3	0.2	0.4	0.2	0.2	0.3	0.3	0.5	0.4
0.4	0.0	0.3	0.1	0.4	0.7	0.5	0.7	0.0	0.1	0.2	0.1
0.0	0.1	0.0	0.0	0.6	0.1	0.4	0.2	0.0	0.1	0.2	0.4
0.4	0.0	0.2	0.1	0.6	0.7	0.8	0.6	0.0	0.0	0.0	0.1
0.2	0.2	0.1	0.2	0.0	0.2	0.2	0.2	0.2	0.0	0.2	0.4
0.4	0.0	0.2	0.1	0.8	0.6	0.7	0.7	0.0	0.5	0.1	0.2
0.1	0.0	0.1	0.5	0.7	0.0	0.4	0.2	0.5	0.2	0.4	0.5
0.7	0.0	0.6	0.4	0.7	0.0	0.4	0.7	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.4	0.0	0.0	0.0	0.2
0.7	0.1	0.1	0.7	0.1	0.0	0.0	0.1	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.2	0.0	0.0	0.1
0.3	0.0	0.3	0.1	0.7	0.6	0.6	0.6	0.0	0.1	0.1	0.1
0.2	0.1	0.1	0.7	0.7	0.2	0.8	0.4	0.0	0.1	0.2	0.5
0.4	0.0	0.4	0.1	0.5	0.8	0.6	0.6	0.0	0.4	0.4	0.7
0.3	0.6	0.2	0.8	0.6	0.4	0.2	0.4	0.6	0.0	0.4	0.2
0.5	0.0	0.5	0.1	0.7	0.6	0.4	0.7	0.0	0.2	0.3	0.0
0.2	0.1	0.0	0.6	0.0	0.2	0.8	0.3	0.5	0.0	0.0	0.4
0.7	0.0	0.5	0.2	0.4	0.8	0.7	0.5	0.0	0.0	0.1	0.1
0.1	0.0	0.0	0.2	0.1	0.0	0.1	0.5	0.0	0.0	0.0	0.0

The following figure4, figure5, and figure6 illustrate the graphic structures of each decision makers' fuzzy reachablility matrices at the 1st trial derived from A₁, ¹ A₂, ¹ A₃⁻¹ respectively with the α -cut value p = 0.8.

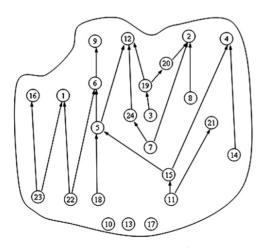


Figure 4. Graphic structure of the 1^{st} decision maker's mental model in the 1^{st} trial (p = 0.80).

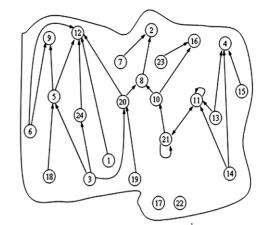


Figure 5. Graphic structure of the 2^{nd} decision maker's mental model in the 1^{st} trial (p = 0.80).

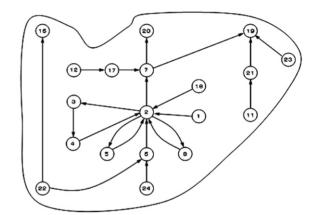


Figure 6. Graphic structure of the 3^{rd} decision maker's mental model in the 1^{st} trial (p = 0.80).

We see that the mental models of decision makers are fairy different with each other in the 1^{st} trail. Then we had a discussion to promote a better understanding on the problem and adjusted each mental models.

According to the flow chart, illustrated in figure 1, we went on the 2^{nd} trail. Even after the 2^{nd} trail, decision makers could not have a consented model. So we went on the 3^{rd} trail, and finally had a consented model described below.

In the third trial, the following subordination matrices were obtained.

A_1^3	=										
0.0	0.2	0.3	0.2	0.4	0.5	0.2	0.2	0.7	0.5	0.4	0.8
0.3	0.3	0.4	0.7	0.6	0.7	0.7	0.6	0.6	0.6	0.6	0.5
0.6	0.0	0.7	0.5	0.4	0.5	0.4	0.6	0.3	0.3	0.2	0.7
0.3	0.4	0.5	0.4	0.3	0.4	0.7	0.6	0.2	0.4	0.3	0.4
0.6	0.5	0.0	0.2	0.8	0.7	0.2	0.7	0.3	0.2	0.6	0.6
0.3	0.2	0.5	0.7	0.6	0.3	0.7	0.8	0.6	0.3	0.7	0.5
0.5	0.7	0.3	0.0	0.7	0.7	0.2	0.2	0.2	0.2	0.2	0.7
0.3	0.3	0.6	0.2	0.2	0.2	0.7	0.5	0.2	0.1	0.1	0.3
0.7	0.7	0.3	0.3	0.0	0.8	0.2	0.2	0.7	0.5	0.5	0.8
0.5	0.5	0.5	0.7	0.7	0.5	0.7	0.5	0.4	0.4	0.7	0.7

$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	$\begin{array}{cccccccccccccccccccccccccccccccccccc$	$\begin{array}{ccccccc} 0.4 & 0.0 \\ 0.2 & 0.1 \\ 0.3 & 0.1 \\ 0.3 & 0.3 \\ 0.6 & 0.1 \\ 0.1 & 0.2 \\ 0.3 & 0.4 \\ 0.1 & 0.3 \\ 0.6 & 0.1 \\ 0.5 & 0.1 \\ 0.7 & 0.4 \\ 0.7 & 0.1 \\ 0.7 & 0.4 \\ 0.7 & 0.1 \\ 0.7 & 0.1 \\ 0.7 & 0.1 \\ 0.7 & 0.1 \\ 0.7 & 0.1 \\ 0.7 & 0.1 \\ 0.7 & 0.1 \\ 0.7 & 0.1 \\ 0.7 & 0.1 \\ 0.7 & 0.1 \\ 0.7 & 0.1 \\ 0.7 & 0.1 \\ 0.7 & 0.1 \\ 0.7 & 0.1 \\ 0.7 & 0.1 \\ 0.7 & 0.1 \\ 0.7 & 0.1 \\ 0.0 & 0.1 \\ 0.7 & 0.3 \\ 0.1 & 0.3 \\ 0.1 & 0.3 \\ 0.6 & 0.1 \\ 0.3 & 0.7 \\ 0.1 & 0.1 \\ 0.1 &$	$\begin{array}{cccccccccccccccccccccccccccccccccccc$	$\begin{array}{cccccccccccccccccccccccccccccccccccc$	$\begin{array}{cccccccccccccccccccccccccccccccccccc$
$\begin{array}{c} {A_2}^3 = \\ \hline 0.0 & 0.0 \\ 0.0 & 0.0$	$\begin{array}{cccccccccccccccccccccccccccccccccccc$	$\begin{array}{cccccccccccccccccccccccccccccccccccc$	$\begin{array}{cccccccccccccccccccccccccccccccccccc$	$\begin{array}{cccccccccccccccccccccccccccccccccccc$	$\begin{array}{cccccccccccccccccccccccccccccccccccc$

0.0 0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0 0.4	1 0.3	0.0	0.0	0.0	0.0	0.4	0.0	0.0	0.0	0.7
0.0 0.0		0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0 0.0	0.0	0.7	0.0	0.0	0.0	0.0	0.0	0.0	0.3	0.4
0.0 0.0		0.0	0.0	0.0	0.0	0.0	0.8	0.0	0.0	0.0
0.0 0.0	0.0	0.0	0.8	0.7	0.0	0.0	0.7	0.0	0.0	0.6
0.0 0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0 0.5	5 0.7	0.2	0.3	0.0	0.7	0.4	0.0	0.0	0.0	0.8
0.0 0.0										
0.0 0.0		0.0	0.0	0.0	0.0	0.8	0.3	0.0	0.0	0.0
0.0 0.8	3 0.0	0.0	0.0	0.0	0.0	0.6	0.0	0.0	0.0	0.3
0.0 0.0		0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0 0.0	0.0	0.3	0.0	0.0	0.0	0.0	0.0	0.0	0.8	0.6
			0.3				0.0	0.0		
0.0 0.0		0.0		0.0	0.0	0.0			0.0	0.0
0.8 0.7	7 0.0	0.0	0.3	0.5	0.0	0.0	0.0	0.0	0.0	0.0
0.0 0.0		0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
		0.0								
0.0 0.0	0.0	0.3	0.0	0.0	0.0	0.6	0.0	0.4	0.0	0.7
0.0 0.0		0.8	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
	0.0	0.0	0.0					0.0		
0.0 0.3	3 0.5	0.5	0.3	0.5	0.0	0.7	0.0	0.0	0.0	0.8
0.0 0.0		0.7	0.0	0.0	0.0	0.0	0.7	0.0	0.0	0.0
0.0 0.0	0.0	0.7	0.0	0.0	0.0	0.0	0.7	0.0	0.0	0.0
2										
$A_3^3 =$										
0.0 0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.8
0.0 0.0		0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.2 0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
				0.0						
0.0 0.0		0.0	0.0		0.0	0.0	0.0	0.0	0.0	0.0
0.0 0.8	3 0.0	0.0	0.8	0.8	0.0	0.2	0.8	0.0	0.0	0.8
0.0 0.0		0.0	0.0	0.0	0.2	0.8	0.0	0.0	0.0	0.0
0.0 0.3	3 0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.1
0.0 0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0 0.0	0.0	0.0								
0.0 0.1	0.0	0.0	0.0	0.8	0.0	0.0	0.9	0.0	0.0	0.8
0.0 0.0		0.0	0.0	0.0	0.1	0.0	0.0	0.0	0.0	0.0
	0.0									
0.0 0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.9	0.0	0.0	0.2
0.0 0.0		0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0 0.0										
0.2 0.8	8 0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0 0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0 0.8	3 0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0 0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0 0.0		0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.2
0.0 0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
										0.0
0.0 0.8		0.0	0.0	0.0	0.0	0.8	0.0	0.0	0.0	0.2
0.0 0.0	0.0	0.8	0.0	0.0	0.0	0.0	0.1	0.0	0.0	0.2
		0.0	0.0	0.0			0.1			0.0
0.0 0.0		0.1	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0 0.0	0.0	0.0	0.2	0.0	0.0	0.0	0.8	0.0	0.0	0.0
										0.0
0.0 0.0		0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0 0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.1	0.0	0.1	0.0
0.0 0.2	2 0.0	0.8	0.1	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0 0.0		0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0 0.2	2 0.0	0.8	0.1	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0 0.0		0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0 0.2	2 0.0	0.8	0.1	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0 0.0		0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0 0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.1	0.0	0.3
0.0 0.0		0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0 0.0		0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.2	0.2
0.0 0.0		0.0	0.0	0.0	0.0	0.0	0.8	0.0	0.0	0.0
0.0 0.0		0.0	0.8	0.8	0.0	0.0	0.8	0.0	0.0	0.8
0.0 0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0 0.8		0.0	0.2	0.1	0.0	0.0	0.0	0.0	0.0	0.8
0.0 0.0	0.0	0.0	0.0	0.0	0.0	0.8	0.0	0.0	0.0	0.0
			0.2		0.1			0.0	0.0	
0.0 0.8		0.0		0.0		0.0	0.0			0.0
0.0 0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0 0.0		0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.8	0.0
0.0 0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
		0.0	0.0	0.3	0.0	0.0	0.3	0.0	0.0	0.8
0 X 0										
0.8 0.2	0.0	0.0	0.0	0.1	0.0	0.0	0.0	0.0	0.0	0.0
0.8 0.2			0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.3
0.0 0.0) (11)				0.0	V.V	0.0			
0.0 0.0 0.0 0.0		0.0								
0.0 0.0 0.0 0.0 0.0 0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.1
0.0 0.0 0.0 0.0 0.0 0.0	0.0	0.8	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.1
$\begin{array}{cccc} 0.0 & 0.0 \\ 0.0 & 0.0 \\ 0.0 & 0.0 \\ 0.0 & 0.0 \end{array}$	0.0 0.0	0.8 0.0	0.0 0.0	0.0 0.0	$\begin{array}{c} 0.0 \\ 0.0 \end{array}$	0.0 0.0	0.0 0.0	0.0 0.0	$\begin{array}{c} 0.0 \\ 0.0 \end{array}$	0.1 0.8
0.0 0.0 0.0 0.0 0.0 0.0	0.0 0.0	0.8	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.1

Let NA_k be the adjusted standard deviation score of decision maker k. Let NAR be the average of NA_k (k = 1, 2, 3), that is.,

$$NAR = \frac{NA_1 + NA_2 + NA_3}{3}.$$

NAR =

NAR =	
0.43 0.45 0.47 0.45 0.48 0.50 0.45 0.45 0.52 0.50 0.48 0.	79
0.47 0.47 0.48 0.52 0.51 0.52 0.52 0.51 0.51 0.51 0.51 0.51 0.51	50
0.54 0.43 0.52 0.50 0.48 0.50 0.48 0.51 0.47 0.47 0.52 0.4	
0.47 0.48 0.50 0.48 0.47 0.48 0.62 0.51 0.45 0.48 0.47 0.4	
0.51 0.73 0.43 0.45 0.79 0.76 0.45 0.66 0.69 0.45 0.59 0.7	
0.47 0.45 0.61 0.52 0.51 0.47 0.56 0.79 0.61 0.47 0.52 0.5	
0.50 0.66 0.47 0.43 0.52 0.52 0.45 0.45 0.45 0.45 0.45 0.5	
0.47 0.47 0.51 0.45 0.45 0.45 0.52 0.50 0.45 0.44 0.44 0.4	
0.52 0.54 0.47 0.47 0.43 0.79 0.45 0.55 0.79 0.50 0.50 0.7	
0.50 0.50 0.50 0.52 0.52 0.50 0.54 0.50 0.48 0.48 0.52 0.5	
0.51 0.47 0.52 0.47 0.48 0.43 0.50 0.44 0.82 0.44 0.44 0.4	
0.44 0.44 0.45 0.50 0.45 0.44 0.50 0.45 0.44 0.48 0.45 0.4	
0.49 0.79 0.45 0.45 0.47 0.44 0.43 0.47 0.61 0.47 0.51 0.4	
0.44 0.44 0.47 0.52 0.47 0.47 0.52 0.51 0.52 0.51 0.52 0.5	
0.51 0.79 0.47 0.45 0.51 0.44 0.47 0.43 0.45 0.51 0.45 0.0	
0.44 0.44 0.57 0.48 0.44 0.45 0.51 0.52 0.51 0.44 0.48 0.4	
0.50 0.51 0.48 0.45 0.47 0.48 0.47 0.44 0.43 0.44 0.44 0.4	
0.45 0.45 0.44 0.44 0.44 0.47 0.45 0.44 0.44 0.48 0.45 0.4	
0.52 0.63 0.47 0.54 0.51 0.44 0.45 0.79 0.44 0.43 0.50 0.0	
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0.52 0.52 0.51 0.51 0.63 0.44 0.51 0.44 0.79 0.44 0.44 0.4	
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$0.31 \ 0.00 \ 0.48 \ 0.79 \ 0.33 \ 0.44 \ 0.44 \ 0.44 \ 0.44 \ 0.51 \ 0.52 \ 0.3$	
0.52 0.64 0.51 0.79 0.54 0.51 0.47 0.52 0.44 0.44 0.51 0.4	
0.52 0.64 0.51 0.77 0.54 0.51 0.47 0.52 0.44 0.44 0.4	
0.51 0.52 0.51 0.45 0.51 0.44 0.47 0.58 0.44 0.50 0.45 0.4	
0.44 0.44 0.44 0.43 0.45 0.45 0.47 0.51 0.44 0.44 0.50 0.5	
0.51 0.48 0.48 0.55 0.47 0.44 0.44 0.45 0.44 0.47 0.61 0.4	
0.45 0.45 0.45 0.44 0.43 0.44 0.44 0.44 0.70 0.44 0.45 0.4	
0.51 0.47 0.50 0.44 0.79 0.69 0.45 0.51 0.76 0.44 0.44 0.7	
0.44 0.44 0.44 0.50 0.44 0.43 0.48 0.51 0.50 0.44 0.44 0.4	
0.51 0.72 0.61 0.51 0.60 0.53 0.59 0.58 0.50 0.45 0.48 0.7	
0.45 0.45 0.47 0.51 0.44 0.44 0.43 0.79 0.56 0.45 0.44 0.4	
0.52 0.79 0.50 0.52 0.56 0.47 0.50 0.61 0.44 0.45 0.44 0.5	
0.51 0.44 0.44 0.45 0.44 0.47 0.50 0.43 0.44 0.44 0.45 0.4	
0.52 0.45 0.45 0.51 0.51 0.44 0.45 0.48 0.44 0.48 0.79 0.5	
0.44 0.52 0.48 0.44 0.52 0.44 0.47 0.44 0.43 0.44 0.45 0.4	
0.79 0.66 0.53 0.45 0.56 0.64 0.45 0.44 0.57 0.44 0.44 0.4	66
0.44 0.44 0.44 0.44 0.44 0.53 0.52 0.44 0.44 0.43 0.47 0.4	52
0.52 0.52 0.47 0.48 0.47 0.45 0.45 0.59 0.47 0.50 0.45 0.6	
0.44 0.44 0.44 0.79 0.51 0.44 0.44 0.51 0.50 0.44 0.43 0.4	
0.52 0.52 0.59 0.57 0.51 0.59 0.47 0.62 0.45 0.44 0.44 0.4	
0.44 0.44 0.44 0.62 0.44 0.44 0.52 0.52 0.54 0.44 0.45 0.4	43

Let NAR^* be the reachability matrix.

NAR*=	
0.54 0.54 0.54 0.5	4 0.54 0.54 0.54 0.54 0.54 0.51 0.54 0.79
0.52 0.52 0.54 0.5	2 0.54 0.52 0.54 0.54 0.54 0.51 0.52 0.52
0.54 0.62 0.61 0.6	0 0.61 0.61 0.59 0.61 0.61 0.51 0.61 0.62
0.52 0.52 0.61 0.5	2 0.61 0.52 0.62 0.62 0.61 0.51 0.52 0.52
0.54 0.79 0.61 0.6	0.79 0.79 0.59 0.66 0.79 0.51 0.61 0.79
0.52 0.52 0.61 0.5	2 0.61 0.52 0.62 0.79 0.61 0.51 0.52 0.52
0.54 0.66 0.61 0.6	0.61 0.61 0.59 0.61 0.61 0.51 0.61 0.62
0.52 0.52 0.61 0.5	2 0.61 0.52 0.62 0.62 0.61 0.51 0.52 0.52
0.54 0.55 0.55 0.5	5 0.55 0.79 0.55 0.55 0.79 0.51 0.55 0.79
	2 0.55 0.52 0.55 0.55 0.55 0.51 0.52 0.52
	4 0.54 0.54 0.54 0.54 0.82 0.51 0.54 0.63
	2 0.54 0.52 0.54 0.54 0.54 0.51 0.52 0.52
	0 0.61 0.61 0.59 0.61 0.61 0.51 0.61 0.62
	2 0.61 0.52 0.62 0.62 0.61 0.51 0.52 0.52
	01 0.61 0.61 0.59 0.61 0.61 0.51 0.61 0.62
	2 0.61 0.52 0.62 0.62 0.61 0.51 0.52 0.52
	4 0.54 0.54 0.54 0.54 0.54 0.51 0.54 0.60
	2 0.54 0.52 0.54 0.54 0.54 0.51 0.52 0.52
	1 0.61 0.61 0.59 0.79 0.61 0.51 0.61 0.67
	9 0.61 0.52 0.62 0.62 0.61 0.51 0.52 0.52
	5 0.55 0.55 0.55 0.55 0.55 0.51 0.79 0.61
	2 0.63 0.52 0.55 0.55 0.79 0.51 0.52 0.52
	4 0.54 0.54 0.54 0.54 0.54 0.51 0.54 0.54
	2 0.54 0.52 0.54 0.54 0.54 0.51 0.52 0.52
	9 0.61 0.61 0.59 0.61 0.61 0.51 0.61 0.62
	2 0.61 0.52 0.62 0.62 0.61 0.51 0.52 0.52
	9 0.61 0.61 0.59 0.61 0.61 0.51 0.61 0.62
	2 0.61 0.52 0.62 0.62 0.61 0.51 0.52 0.52
	'9 0.61 0.61 0.59 0.61 0.61 0.51 0.61 0.62 '2 0.61 0.52 0.62 0.61 0.51 0.52 0.52
	8 0.58 0.58 0.58 0.58 0.58 0.51 0.52 0.67
	2 0.58 0.52 0.58 0.58 0.58 0.51 0.52 0.52
	5 0.55 0.55 0.55 0.55 0.55 0.51 0.70 0.61
	2 0.63 0.52 0.55 0.55 0.50 0.51 0.70 0.51
	5 0.79 0.79 0.55 0.55 0.79 0.51 0.52 0.52
	2 0.55 0.52 0.55 0.55 0.55 0.51 0.52 0.52
	1 0.61 0.61 0.59 0.61 0.61 0.51 0.61 0.79
	2 0.61 0.52 0.62 0.79 0.61 0.51 0.52 0.52
	1 0.61 0.61 0.59 0.61 0.61 0.51 0.61 0.62
	2 0.61 0.52 0.62 0.62 0.61 0.51 0.52 0.52
	5 0.55 0.55 0.55 0.55 0.55 0.51 0.79 0.61
	2 0.63 0.52 0.55 0.55 0.79 0.51 0.52 0.52
	0 0.61 0.64 0.59 0.61 0.64 0.51 0.61 0.79
	2 0.61 0.53 0.62 0.62 0.61 0.51 0.52 0.52
	9 0.59 0.59 0.59 0.59 0.59 0.51 0.59 0.67
	9 0.59 0.52 0.59 0.59 0.59 0.51 0.52 0.54
	0 0.61 0.61 0.59 0.62 0.61 0.51 0.61 0.79
0.52 0.52 0.61 0.6	2 0.61 0.52 0.62 0.62 0.61 0.51 0.52 0.52

We obtain the level sets from NAR.* Let L_t , L_i , L_b and L_{is} be a top level set, an intermediate level set, a bottom level set and an isolation level set, respectively. Then, we have

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$\mathbf{L}_{t} = \{\mathbf{s}_{2}, \mathbf{s}_{4}, \mathbf{s}_{9}, \mathbf{s}_{12}, \mathbf{s}_{16}\},$
$L_i = \{s_1, s_5, s_6, s_8, s_{11}, s_{20}, s_{21}\},\$
$L_b = \{s_3, s_7, s_{10}, s_{13}, s_{14}, s_{15}, s_{17}, s_{18}, s_{19}, s_{22}, s_{23}, s_{24}\}$ and
$L_{is} = \phi$ (the empty set).

Moreover, it is known that we can give a kind of weight to each item by calculating an eigenvector of the absolute value maximum eigenvalue of the matrix. In our case, the weights of s_i 's are given in the table 3.

e												
	s ₂	S ₃	s_4	S ₅	s ₆	\mathbf{S}_7						

 Table 3. An eigenvector of NAR.*

0.192	0.205	0.220	0.201	0.201	0.194	0.207	0.207
S9	s ₁₀	s ₁₁	s ₁₂	s ₁₃	s ₁₄	s ₁₅	s ₁₆
0.190	0.214	0.200	0.189	0.208	0.208	0.208	0.199
S ₁₇	s ₁₈	S ₁₉	s ₂₀	s ₂₁	s ₂₂	s ₂₃	s ₂₄
0.197	0.205	0.212	0.207	0.200	0.212	0.205	0.209

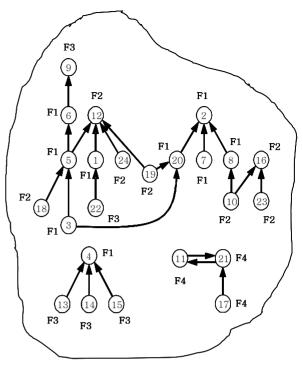
Figure 7 illustrates the graphic structure in the group decision. Read the number *i* in each circle of the figure as s_i . The α -cut p is determined so that the consenting structural model is obtained from our final agreement. In this case p is 0.7.

Now we reconsider the result of the principal components analysis based on the structural model (see figure 7). In figure 7, for indicating each element of the digraph is in which factor extracted from the principal components analysis, we make F1 stand for factor1, F2 for factor2, F3 for factor3 and F4 for factor4, and put them around each circle.

As shown in figure 7, we have three clusters. Almost all the composition items of the first biggest cluster are included in the factor1 or in the factor2 of table2. All the items in the intermediate level set are in the factor1 and the items in the top level set except s2 in this cluster are not in the factor1, which proves that the factor1 is the core factor.

The second cluster is composed by $\{s_4, s_{13}, s_{14}, s_{15}\}$ which is a set of the principal components of factor3, and one more item in the factor1, s_4 . Since the factor3 is the attainment nature to the nearby station, this, of course, causes the attainment nature to the destination, which is explained by the hierarchy system (that is, arrows from items s_{13} , s_{14} , s_{15} to s_4) in this cluster. The items s_9 and s_{22} were in the factor3 simply from table2, but we notice that they should be in the factor2 when we reconsider the meaning and their factor loadings.

The third cluster's composition items are completely the same as that of the factor4 which expresses the convenience of whether shopping is made not at the railroad itself but at the shopping mall attached to it, etc. Our hierarchy system claims that such kind of convenience is essentially based on s_{17} (the atmosphere around a station).



(F1 stands for factor1, F2 for factor2, F3 for factor3 and F4 for factor4. (see Table 2)).

Figure 7. Graphic structure in the group decision (p = 0.70).

If it says about the reconsideration on the number of factors, suitable semantic attachment corresponding to the structural model cannot be found out from the 5 factor model. So we can conclude that the 4 factor model is appropriate one.

Only by the principal components analysis, a traditional statistical method, we just extract factors which are latent in the reply to a questionnaire and we cannot find any relationships between items. Although the principal components analysis with varimax method gives us a simple factor structure, all factors are orthogonal and we should not consider any relationships between factors. Of course, we can use another kind of method such as oblimin method to see the relationships between factors, but we may sacrifice the simple nature of factor structure. In the practical problem, we saw that these problems were solved when we used the modified structural modeling method with the principal components analysis, and the structural model in figure 7 helped us reconsider the interpretation of the factors.

Consider the hierarchy structure in figure 7. We can interpret the class relation of items, with which we may make a proposal leading to an operating strategy in a railroad company.

For instance, look at the hierarchy from s_3 to s_9 , described by vertical arrows. We see that people need to be seated (s_6) if they want to work in the train (s_9), the congestion degree must not be so high (s_5) that they can get a seat (s_6), and these conditions are guaranteed by the fre-

 \mathbf{S}_1

quency of train service (s_3) . Considering the weight, we see that the weight of s_3 is the greatest among those of s_3 , s_5 , s_6 , and s_9 . Then the railroad company intending to satisfy these kinds of passengers' needs should increase the frequency of train service. But it may sometimes cost a lot, so they try to find a method to reduce the congestion degree. The arrow diagram implies that this is reasonably the second best way to choose.

Now we consider the hierarchy structure with s_{12} as the top. Since the item s_{12} refers to the image of a railroad company, this structure explains which items contribute to improve the railroad company's image. The fare (s_1) is important of course. The extra charge (s_{22}) , such as a setup of a special express, may raise the fare, but this does not have direct influence on the image, although the weight of this item is one of the greatest in the hierarchy. And also it seems that the congestion degree (s_5) , the reliability of time (s_{19}) , and the information services on them (s_{24}) are very important factors for the image. Talking of the weight, the reliability of time might be the most important item among them for the image of railroad company.

From the principal components analysis, that is like saying that, those items do not play an important role in their factors (see Table 2), but the structural model in figure 7 implies that we should not ignore them. From this, it can be seen that the structural model complements the principal components analysis with the graphic structure of relations among elements (items) within it.

4. CONCLUSION AND REMARKS

In studying complex problems, in developing plans, in managing organizations, in working with systems and various kinds of human endeavor, it is often desirable and sometimes essential to synthesize hierarchies. For arranging elements in a hierarchy rationally, the problem with respect to human behavior such as human judgment and intuition has to be dealt with. Several methodologies such as ISM, FSM and DEMATEL, etc. have been exploited in several areas.

In this paper, we proposed the modified structural modeling method based on FSM for structuring the consenting model for group decision making. In the method, we introduced " α -cut" concept to control the manipulation of threshold value, which makes possible that the value of threshold is modified meaningfully, not only depending on decision-maker's intuition. And then we proposed the procedure to construct the consenting structural model from group decision-makers. Moreover, we conducted the practical study to analyze whether the passengers' behavior changed after the new opened service of Tsukuba Express railway for East Japan Railway Company.

In the empirical study, we extracted 4 factors by making use of the principal components analysis based on a web-designed questionnaire. However, the traditional statistical method, such as principle components analysis we used, cannot help us see what relations among elements (items) within each factor. By applying the modified structural modeling method we proposed, it is possible to investigate the relations in internal factor through the consenting structural model.

In summary, the main advantage of the proposed method is as follows:

- A consenting structural model is effectively and rationally constructed for multi-participant decision making by introducing α-cut for the threshold;
- (2) The result computed by the principal components analysis can be expressed explicitly through the structural model built by the proposed method.

In this paper, we show that our modified structural modeling method can be applied to see the structure of items in a questionnaire. The method has high potential for applying to many kinds of problem, e.g. structure analysis, decision making, and clarifying ill-defined problem, performed by a team of some members with following limitations;

- the team consists of relatively small number of people, e.g. from 3 up to 10
- each member of the team has common understanding on the problem and has wide views on the elements in various perspectives (but not necessarily an expert)
- one person can be chosen from the team as a coordinator who would lead the discussion to a consensus
- the consisting elements of the problem can be comparable with each other in at least one valuation basis

We are now on researching how to apply our method to an information security evaluation system (Nagata K. *et al.*, 2007).

We used SPSS Release 12.0.1 j for the principal components analysis.

ACKNOWLEDGMENT

The second author is supported by Grant-in-Aid for Young Scientists (B) (19710135) of the Ministry of Education, Culture, Sports, Science and Technology of Japan.

We thank for financial support from East Japan Railway Company.

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