IEMS Vol. 7, No. 2, pp. 182-188, September 2008.

Economic Design of Automated Spiral Parking System

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Selected paper from APIEM 2006

Abstract. Automated parking systems, which automatically park and retrieve vehicles, have been steadily replacing conventional parking systems. The spiral parking system is a type of automated parking systems that has cylindrical parking tower. We develop an economic design model of spiral parking system based on a recursive optimization and simulation procedure in which the dynamic nature of the parking system can be integrated into the mathematical programming model. The optimal values of design parameters are found that gives the minimum total cost while complying with the desired performance of the system

Keywords: Spiral Parking System (SPS), Design Model, Recursive Optimization/Simulation Procedure

1. INTRODUCTION

Automated Parking Systems (APS) represent methods of automatically parking and retrieving cars. At the entrance of the parking structure of APS, driver parks his car and then the vehicle is automatically moved through the parking system and stored in an empty parking cell. All these operations are controlled by a computer system. The driver gets his vehicle back by using a signaling device available at the outside of the parking system. Compared with conventional parking systems, APS have many advantages such as efficient space utilization of parking, improved vehicle and driver safety, decrease in vehicle damages, and better control of vehicles. However, APS are very expensive to build and inflexible to future change and thus a very careful design of the system is justified. Recently, a Swiss design and engineering company, Skyline Innovations (2006), reported in a press release their invention of a fully automatic and compact parking system called the SmartP which they claim is the product of an innovative and creative design concept. In this paper, we deal with the design problem of a spiral parking system (SPS) which we believe has basically the similar structure as that of the SmartP. Contrary to the rectangular shape of conventional APS, SPS has cylindrical modules with a lift in the center of each module. The schematic diagram of the module of SPS is shown in Figure 1.

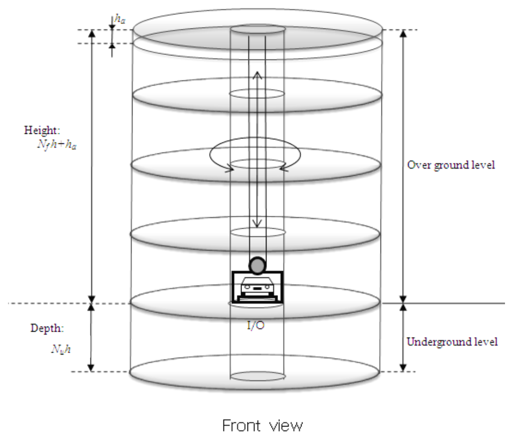

Figure 1. Schematic Diagram of a Spiral Parking System.

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An automated storage and retrieval system (AS/RS) is defined by the Material Handling Institute as a storage system that uses fixed-path storage and retrieval (S/R) machines running on one or more rails between fixed arrays of storage racks. Since APS and AS/RS have many similarities in operations, research articles dealing with the AS/RS could be useful references in solving the design problem of SPS. Karasawa *et al*. (1980) wrote one of the first papers that discuss the optimal design of an AS/RS. He developed a nonlinear mixed integer-programming model for a simple AS/RS in which the S/R machine performs only single command. For dual command mode, Ashayeri *et al*. (1985) developed a mathematical model with the objective of minimizing the total costs over the lifetime assuming that the height of building is already fixed. Bozer and White (1990, 1996) presented an analytical design algorithm to determine the near-optimum number of pickers required in an end-of-aisle order picking operation based on a miniload AS/RS.

For designing AS/RS, two approaches can be adopted, optimization and simulation approaches, each of which has merits as well as shortcomings. Optimization approach has difficulty in dealing with the dynamic nature of system. Simulation approach can evaluate the system throughput in a stochastic environment but does not guarantee that it provides an optimal solution. Several researchers adopted a recursive procedure that combines both optimization and simulation approaches to find the best design parameters. This approach takes advantage of the best features of both optimization and simulation, while minimizing the disadvantages of each method used alone. Nolan and Sovereign (1972) applied a recursive approach for the strategic mobility system problem in reaching decisions for size of transportation forces for the Department of Defense. Carlson *et al*. (1979) applied a recursive optimization/simulation approach in determining the best combination of service, facilities and personnel to maximize the profit in an outpatient health care clinic. Rosenblatt *et al*. (1993) used a similar approach for designing AS/RS and Lee (2003) applied the same approach to the design problem of several well known types of APS.

In this study, we develop an economic design model of spiral parking system and find the optimal solution using a recursive optimization and simulation procedure. The objective is to minimize the initial installation and operation costs while satisfying operational and physical constraints. The operational constraints include the desired level of system performance such as service level, average waiting time of vehicles, and utilization ratio. The physical boundary of width, height, and length of system is also considered as physical constraints.

The remainder of this paper is organized as follows. In section 2, we describe the operation and design characteristics of SPS. Section 3 presents the development of a design model including the assumptions and notations. A recursive optimization and simulation procedure is given in section 4. Section 5 shows the validity of the model and solution procedure through solving an example problem with hypothetical data. Finally, concluding remarks appear in section 6.

2. OPERATION AND DESIGN CHARACTERISTICS OF SPS

It is assumed that SPS consists of N_m number of identical parking modules. Figure 1 shows the schematic diagram of a module of SPS. Each module consists of N_f number of floors, N_u number of underground floors, and N_s number of slots in each floor. Differing from the popular parking systems such as elevator type, rotary type and shuttle type, SPS is cylindrically shaped and has floors both in over and underground. There is a lift through the center circle so that the customer vehicle is able to move up or down to the designated floor and slot.

When vehicle arrives at a module and finds that either the system is busy or other vehicles are waiting for service (the storage queue of the module is not empty), then it waits for its turn in the queue. The vehicles are served on first-come-first-served basis. For storage service the vehicle is driven onto the empty pallet at the input/output (I/O) area and then the center-positioned high-speed lift carries the loaded pallet to the floor allocated by the computer system while rotating horizontally to the pre-assigned empty slot. The loaded pallet is stored in the slot by the sliding device of the lift. It is assumed that all the slots in SPS are of a same size and slots are allocated to incoming vehicles based on the closed open location rule. For the subsequent storage task the lift retrieves an empty pallet from the nearest empty slot and returns with the pallet to the I/O area. If both storage and retrieval requests exist in the queues, dual command is performed. That is, after performing a storage task for incoming vehicle, the lift directly travels to the slot associated with the retrieval request to remove the loaded pallet from the slot and bring it back to the I/O area.

We seek the optimum values of the design parameters that comply with the desired level on the following system performance measures.

1) Service level of the system: The ratio of the order requests that are processed to the total number of order requests during a given time period.
2) Average waiting time per order: For an arriving vehicle, waiting time is defined as the time elapsed between the time vehicle arrives at the system and the time it starts being driven into a module. For a departing vehicle, it begins when the retrieval request is generated and ends when the requested vehicle arrives at the input/output (I/O) point.
3) Utilization ratio of the system: From the viewpoint of the system owner, he may have a target level in terms of the system utilization that makes the parking facility a profitable business.

3. DEVELOPMENT OF OPTIMIZATION AND SIMULATION MODELS

3.1 Notations

We develop a non-linear integer optimization model with the following notations.

Decision variables:

N_s : The number of slots per floor
N_f : The number of over ground floors per module
N_u : The number of underground floors in a module
N_m : The number of modules in a parking system
r : Radius of the module (m)
k : Radius of the central circle (m)

Operational and Physical Parameters:

w, l, h : Width, length and height of a slot allowing for the necessary margins in all dimensions (m)
h_a : An allowance for the necessary extra height (m)
V_v : Lifting velocity (m/sec)
V_θ : Turning velocity (rad/sec)
S_L, S_U : Lower and upper bound of the number of slots of a module
F_L, F_U : Lower and upper bound of the number of floors above the ground
U_L, U_U : Lower and upper bound of the number of underground floors
k_L, k_U : Lower and upper bound of the inner radius of a module
α, β : Adjustment parameter of the underground depth

Investment cost parameters:

C_l : Cost of land (\$/m^2)
C_w : Cost per square meter of wall. (\$/m^2)
C_c : Cost per square meter of ceiling. (\$/m^2)
C_d : Cost of excavation for underground level. (\$/m^3)
C_f : Cost per square meter of floor, including the first floor for the module. (\$/m^2)
C_s : Cost of a slot. (\$/unit)
C_o : Annual cost of an operator per module. (\$/module)
C_{ef} : Fixed cost of a lift (\$/each)
C_{ev} : Variable cost of a lift (\$/m)
C_{af} : Fixed cost of a sliding device (\$/each)
C_{av} : Variable cost of a sliding device (\$/m)

Operating cost parameters:

I : Annual maintenance cost, as a percentage of equipment costs. (%)
H : Economic life horizon of the parking system.
d : Discounting rate.
Z : Uniform series, present worth discounting factor,

$$Z = \sum_{i=1}^{H} (1+d)^{-i}$$

Notations for simulation model:

TSL : Target service level (%)
AWT : Allowed waiting time per vehicle (sec)
O : Allowable occupancy ratio of the number of parked vehicles in the systems to the capacity (%)
$S(N_s, N_f, N_u, N_m)$: Service level of the parking system (%)
$UR(N_s, N_f, N_u, N_m)$: Utilization ratio of the parking system (%)
$W(N_s, N_f, N_u, N_m)$: Average waiting time per vehicle (sec)
T : Assuming that infinitely numbers of vehicles are waiting to be parked, the law stipulates that it should not take more than a prescribed length of time (T) for parking the vehicles to full capacity and then evacuating them completely. (sec)
t_f : Drive-in/drive-out time at the I/O point (sec)
t_e : Forking time of the sliding device (sec)
b : Floor area ratio (%)

3.2 Objective Function

The total cost of installing parking tower system is composed of the construction, equipment, and operation cost. Each cost can be represented as follows:

Construction Cost

The construction cost consists of five cost elements, the cost of land, floor, wall, ceiling and excavation. The area of the smallest rectangular shaped area required to cover a cylindrical building is $(2r)^2$ and the area of each floor of the building is $\pi(r^2 - k^2)$ due to the ring shaped floor. The area of the wall of a module is $2\pi r \cdot (h \cdot (N_f + N_u) + h_a)$ and the ceiling is πr^2. The unit excavation cost per cubic meter is approximated by the function $\pi r^2 \alpha \beta^{Nuh}$. Therefore, the total construction cost can be expressed as

$$\begin{pmatrix} (2r)^2 \times C_l + \pi(r^2 - k^2) \times C_f \\ +2\pi r \times \left\{ h(N_f + N_u) + h_a \right\} \times C_w \\ +r^2 \pi \times C_c + \pi r^2 \times \alpha \beta^{N_u h} \times C_d \end{pmatrix} \times N_m. \quad (1)$$

Equipment Cost

The number of slots in a module is $(N_f + N_u) \cdot N_s - 1$ since the space of one slot on the first floor has to be left empty for the drive in/out way. The cost of a lift and a sliding device is proportional to the height of the lift, $h \cdot (N_f + N_u)$, and the radius of the central circle, k, respectively.

Thus the total cost of slots, lift, and turn table with sliding devices can be expressed as follow:

$$\begin{pmatrix} \{(N_f + N_u) \times N_s - 1\} \times C_s \\ +(C_{ef} + C_{af}) \times N_m + k \times C_{av} \times N_m \\ +h(N_f + N_u) \times C_{ev} \times N_m \end{pmatrix} \times N_m. \quad (2)$$

Operating Cost

Each module requires one operator for its operation and the discounted annual labor cost of operators is $C_o \cdot Z \cdot N_m$. In addition, maintenance is required for lifts as well as for turn tables with sliding devices and slots. Therefore, the discounted value of operating costs becomes:

$$C_o \times Z \times N_m + [Equiment\ \ Cost] \times I \times Z \tag{3}$$

3.3 Optimization Model

Given the above notations and objective function, A non-linear mixed integer optimization model is developed for the designing problem of the spiral parking system.

Min TC = (Construction Cost) + (Equipment Cost) + (Operating Cost)

subject to

$$S_L \leq (N_f + N_u)\, N_s \leq S_U \tag{4}$$

$$25(\pi N_s) \leq b \tag{5}$$

$$F_L \leq N_f \leq F_U \tag{6}$$

$$U_L \leq N_u \leq U_U \tag{7}$$

$$k_L \leq k \leq k_U \tag{8}$$

$$S(N_s, N_f, N_u, N_m) \geq TSL \tag{9}$$

$$UR(N_s, N_f, N_u, N_m) \geq O \tag{10}$$

$$W(N_s, N_f, N_u, N_m) \leq AWT \tag{11}$$

$$2[\{N_f(\min(hN_f/V_v, \pi/V_\theta) / \max(hN_f/V_v, \pi/V_\theta))^2/3 + 1\} \max(hN_f/V_v, \pi/V_\theta) + \{N_u(\min(hN_u/V_v, \pi/V_\theta) / \max(hN_u/V_v, \pi/V_\theta))^2/3 + 1\} \max(hN_u/V_v, \pi/V_\theta)]\, N_s + (2t_f + 2t_e)(N_f + N_u)\, N_s \leq T \tag{12}$$

$$N_s \leq 360° / 2\arctan(w/2k), \tag{13}$$

where N_s, N_f, N_u, N_m: are integers

Constraint (4) represents the user requirements of the total number of slots in a module. Constraint (5) is about the floor area ratio, a traffic regulation for the construction of parking tower. It represents the ratio of the total over-ground floor area of all modules to the size of the land of the SPS, $((\pi r^2 N_s N_m)/4r^2 N_m)$. Constraint (6) and (7) are the lower and upper bound of the number of floors above and below the ground, respectively. Constraint (8) represents the lower and upper bound of the radius of the center circle. Constraints (9) and (10) provide the lower bounds on the service level and utilization ratio of the system, respectively. Constraint (11) ensures that the average waiting time per vehicle must not be larger than a given value of AWT. Constraint (12) represents a traffic regulation for safety measures against emergency situations for the parking system. Utilizing the expected travel time of single command in AS/RS, an approximate time is found that requires the system to park the vehicles to its full capacity and then evacuate them completely. Constraint (13) limits the number of slots in a floor. In order to calculate the maximum number of slots in a floor, the relationship $\tan\theta = w/2k$ is utilized in Figure 2. We adapt a total enumeration approach to solve the optimization model.

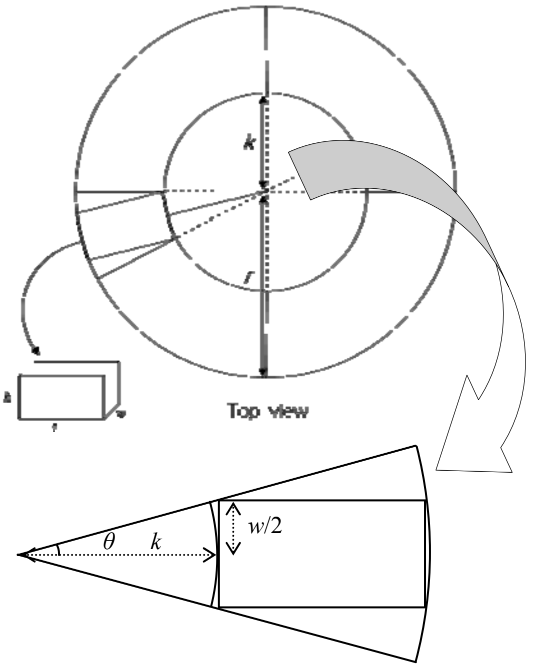

Figure 2. A slice of a floor.

3.4 Simulation Model

A simulation model is developed to test the design parameters obtained from the optimization model and also to estimate the performance measures mentioned previously.

The SPS under our simulation study is composed of a series of events related to many stochastically different vehicles. Our simulation model reflects the real world situation and it was written in Visual C#. When a vehicle arrives at the parking system when the storage queue is not empty, it has to wait tp get its storage service at the queue. The vehicles are stored on the first-come-first-served basis. For the storage of the vehicles, the system seeks for a module which is idle and has relatively smaller number of parked vehicles. When the vehicle gets its turn, it is driven to the slot which is used for the input and output point and the sliding device pick the vehicle and move it to the closest empty slot. After taking the vehicle down off the lift, the lift is sent back to the I/O point with the closest empty pallet or parked car which is first ordered to retrieve among the vehicles on the retrieval queue.

The input data to the simulation program are the values of (N_m, N_f, N_u, N_s) from the optimization model in addition to a given set of design data. As a result of the simulation, we can obtain the values of the three performance measures. The performance measures, for each batch of simulation runs, serve to define the appropriate constraints to be incorporated into the subsequent optimization model. When these values fall within the prescribed acceptable bounds, the entire optimization/simulation process is terminated.

4. SOLUTION PROCEDURE

Since it turns out to be impossible to solve analytically the proposed design model, we adopt the heuristic based on a recursive optimization and simulation procedure. Through running a batch of simulations, regression functions are defined between changes in the design parameters and the resultant performance measures. These regression functions are entered as additional constraints into the optimization model. Solving the model, we obtain an adjusted series of the design parameters. Generally, we can find a cost-wise optimal solution through the optimization model and the respective variable design parameters are determined. These are fed into the simulation model where the performance measures are generated. If these measures satisfy the prescribed requirements, the procedure is terminated. The above procedure is reiterated until a satisfactory result is obtained. Followings are the steps of the recursive optimization and simulation solution approach.

Step 1 : Solve the optimization model with no constraints related with the performance measures.
Step 2 : Run the simulation model with the optimal design parameters and check whether the simulation results satisfy the target levels of three performance measures. If satisfied, the procedure is terminated. Otherwise, go to step 3.
Step 3 : Determine two points around each of design parameters obtained from step 1. And then with each combination of the design parameters, run the simulation model and obtain values of the respective performance measures.
Step 4 : Apply multiple linear regression analysis to the simulation results to find regression equations.
Step 5 : Add the regression equations to the optimization model as constraints.
Step 6 : Solve the newly defined optimization model with updated constraints. Through simulation examine whether the target performance levels are met or not. If satisfied, the procedure is terminated. Otherwise, go to step 3.

Figure 3 illustrates the overall procedure of the recursive optimization/simulation procedure. The assumption of linear relationship between the design parameters and performance measures is made to facilitate computations in both the simulation and optimization phases. It is our experience that the above recursive procedure converges to a final acceptable solution within a relatively small number of iterations.

5. NUMERICAL EXAMPLE

To show the validity of the model and solution procedure, we solved an example problem with the data obtained from those who have many years of experiences in construction business including the building tower parking systems. The data reflect more or less the real-world situation. They were obtained from compa-

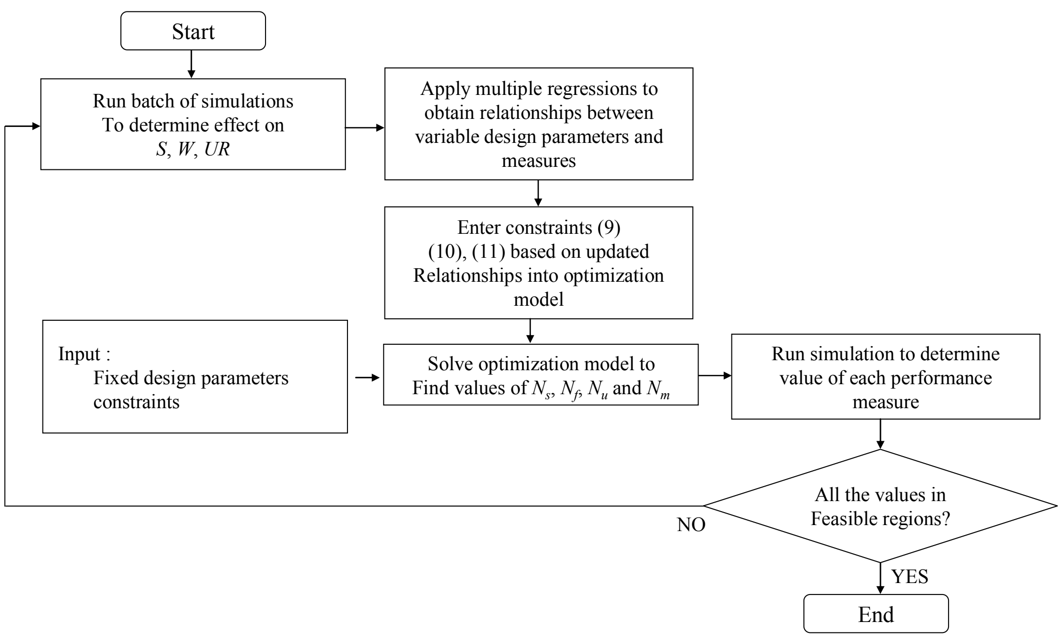

Figure 3. The recursive optimization / simulation procedure.

nies specialized in constructing parking tower system. The numerical values of the pertinent parameters are:

w = 2.1, l = 5.4, h = 1.65, h_a = 2, V_v = 1, V_θ = π/10 S_L = 50, S_U = 100, F_L = 1, F_U =10, U_L = 1, U_U = 5, k_L = 2.7, k_U = 6, α = 1.5, β = 1.2, C_l = 112.5, C_w = 70, C_c = 100, C_d = 11, C_f = 87, C_s = 100, C_o = 10000, C_{ef} = 10, C_{ev} = 5, C_{af} = 10, C_{av} = 5, I = 0.3, H = 10, d = 0.12, TSL = 80, AWT = 300, O = 50, T = 9000, t_f = 10, t_e = 10, b = 1000.

We also assumed that on the average 30 vehicles arrive per hour following a Poisson distribution and each vehicle stays on the average 2 hours in the system following an exponential distribution. The results of iterative procedure are presented in Tables 1 and 2. The first table shows the linear equations obtained from the regression analysis of the result data coming from the simulation. They were incorporated into the optimization model for the next iteration. The second table shows the design parameters, total cost, and performance measures for the design parameters. Performance measures SL, UR, and AWT indicate service level, utilization ratio, and average waiting time, respectively.

Solving the optimization model without the performance related constraints, we obtained N_s = 10, N_f = 4, N_u = 1, N_m = 1, k = 3.231, and r = 8.695 which means 10 slots per floor, 4 over ground floors, 1 underground floor, and 1 module with 8.695m radius. These design parameters gave the performance level of (SL = 68.43, UR = 91.51, AWT = 186043) and they were unsatisfactory due to the low service level and long average waiting time. We applied the multiple regression analysis to the simulation results and developed linear regression equations between the design parameters and performance measures. We add those regression equations to the optimization model, and solved the model again. With the added constraints, we got N_s = 18, N_f = 1, N_u = 3, N_m = 1, k = 5.954, and r = 11.403. Those design parameters also did not satisfy the target performance level and thus another iteration was made. At the end of third iteration, we finally found the solution, N_s = 17, N_f = 2, N_u = 1, N_m = 2, k = 5.617, and r = 11.067 which means that 17 slots per floor, 2 over ground floors, 1 underground floors, and 2 module with 11.067m radius. The system with the design parameter values turned out to be acceptable since they satisfy the prescribed requirements. The total cost is 562304 dollars and service level, utilization ratio, and average waiting time are 93.79, 51.6, and 96.91, respectively.

6. CONCLUSIONS

This paper dealt with the design problem of an automated spiral parking system. The optimal design solution was found based on a recursive optimization and simulation procedure. This approach takes advantage of the best features of both the optimization and simulation, while minimizing the disadvantages of each method used alone. The objective is to minimize the initial installation and operation costs while satisfying the operational and physical constraints. The operational constraints include the desired level of system performance such as service level, average waiting time of vehicles, and utilization ratio. The physical boundary of width, height, and length of system is also considered as the physical constraints.

Numerical example was solved to show the validity of the model and the proposed algorithm using the real world data. A satisfactory solution was found within a few iterations. And the assumption of a linear relationship between the design parameters and the performance measures could be justified by high correlation coefficients.

Table 1. Regression functions on the outputs from the simulation model.

Iteration	Regression Function		R^2
1	Service Level	: $6.917 + 2.435N_s + 6.341N_f + 4.81N_u + 11.609N_m$	0.527
	Utilization Ratio	: $235.146 - 6.573N_s - 16.572N_f - 9.321N_u - 20.368N_m$	0.832
	Avg. Waiting Time	: $460807.6 - 16055.1N_s - 36000.6N_f - 27144.3N_u - 57358.4N_m$	0.465
2	Service Level	: $73.417 - 0.819N_s + 2.094N_f + 1.036N_u + 12.987N_m$	0.685
	Utilization Ratio	: $149.122 - 1.518N_s - 14.578N_f - 13.161N_u - 13.558N_m$	0.721
	Avg. Waiting Time	: $1067.167 - 13.03N_s - 62.83N_f - 41.756N_u - 248.17N_m$	0.572

Table 2. Result of the optimization model.

Iteration	N_s	N_f	N_u	N_m	k	r	Total Cost($)	Service Level(%)	Utilization Ratio(%)	Avg Waiting Time(s)
1	10	4	1	1	3.231	8.695	261493	68.43	91.51	186043
2	18	1	3	1	5.954	11.403	371280	81.79	44.58	291.5
3	17	2	1	2	5.617	11.067	562304	93.79	51.6	96.91

There are many possibilities for further researches for SPS. It is based on a new and creative concept and recently started being implemented in the real world situation. There could be many variations in design as well as operating aspects. Also, there exist many design problems that could be solved through the recursive optimization and simulation procedure.

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