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Reliability Equivalence of Two Non-identical Components

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Abstract. The aim of this work is to generalize reliability equivalence techniques to apply them to a system consists of two independent and non-identical components connected in series(parallel) system, that have constant failure rates. We shall improve the system by using one component only. We start by establishing two different types of reliability equivalence factors, the survival reliability equivalence (SRE) and mean reliability equivalence (MRE) factors. Our second studies, introducing some applications for our studies in airports and our life. Also, we introduced some numerical results.

Key Words : *Reliability equivalence, series system, redundant methods, α-fractiles, survival reliability equivalence and mean reliability equivalence factor.*

Acronyms

RF	reliability function
MTTF	mean time to failure
RM	reduction method
HRM	hot redundant method
CRM	cold redundant method
ISRM	imperfect switch redundant method
REF	reliability equivalence factor
SREF	survival reliability equivalence factor
MREF	mean reliability equivalence factor

1. INTRODUCTION

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The concept of reliability equivalence factors has been introduced by Råde (1989). Råde (1990, 1991, 1993) has applied such concepts to various reliability systems. Later Sarhan (2000, 2002, 2004, 2005), Mustafa (2002), Sarhan et al. (2004), Sarhan and Mustafa (2006) and Mustafa et al. (2007) applied the same concept on more general and complex systems.

Generally, there are two basic methods to improve a given system, see Sarhan (2000). These methods are: (1) reduction methods, and (2) redundancy method. The redundancy methods includes three possible methods:(2.1) hot duplication methods; (2.2) cold duplication method; and (2.3) imperfect duplication method.

Sarhan and Mustafa (2006) introduced different vectors of the reliability equivalence factor of a series system consists of n independent and non-identical components. They improve the system reliability by using four methods as follows:

(1) **Reduction method**: Reducing the failure rates of some of the system components that belong to the set $B = \{i_1, \cdots, i_m\} \subseteq \{1, 2, \cdots, n\}$, $m \leq n$, each by its own, that is the failure rate of component $i \in B$, will be reduced by a factor ρ_i, $0 < \rho_i < 1$.

(2) **Hot duplication method**: Duplicating each component belongs to the set $A = \{i_1, \cdots, i_m\} \subseteq \{1, 2, \cdots, n\}$, $m \leq n$. Namely, each component belongs to the set A is duplicated by a hot redundant standby component.

(3) **Cold duplication method**: Assuming cold duplication of each component belongs to the set $A = \{i_1, \cdots, i_m\} \subseteq \{1, 2, \ldots, n\}$, $m \leq n$. That is, each component belongs to the set A is duplicated by a cold redundant standby components.

(4) **Imperfect duplication method**: Assuming cold redundant standby component connected by random switch to the each component belongs to the set $A = \{i_1, \cdots, i_m\} \subseteq \{1, 2, \ldots, n\}$, $m \leq n$. This means that, each component in the set A is connected by a cold redundant standby component via an imperfect switch. Note that, different components have different switches.

In the current paper we interest with the system that consist of two components only, because two components series system has important applications such as in metro and airplane and in many other planes. If there are two series roads connected between two (three) towns, can be improve these roads to have the same system as if we add a bridge (new road) connection between these towns. In metro, train station and airport if there are two stations connected by two series roads also we can improve these roads to have the same system as if we add new passage connected between these stations. And other applications in our life. Also, two parallel component system has important applications such as in metro and airplane and in many other planes. If there are two road connected between two towns, can be improve these roads to have the same system as if we add a new road connection between these towns. In metro, train station and airport if there are two passages also we can improve these passages to have the same system as if we add new passage.

The main objective of this paper is derive the reliability equivalence factors of series(parallel)system. The system studied here consists of two independent and non-identical components. This system can be improved according to one of the following four different methods:

(1) **Redaction method**: Reducing the failure rate of each system components that belong to a set $B \subset \{\{1\}, \{1,2\}\}$. That is, reducing the failure rate of component $i \in B$, by the factor ρ_i, $0 < \rho_i < 1$.

(2) **Hot redundant method**: assuming hot redundant to the system. It means that the system is connected by one different a hot redundant standby component.

(3) **Cold redundant method**: assuming cold redundant to the system. It means that the system is connected by only one different a cold redundant standby components.

(4) **Imperfect redundant method**: assuming cold redundant standby component connected by random switch to the system. That is, the system is connected with only one different component by a cold redundant standby component via an imperfect switch.

In the previous articles the authors improved the system reliability by improving some system components by duplication each component with the same component in the different duplication methods, but in this paper, we improving the system components by add only one component to the original system by using the redundant methods.

The lifetime of the system components is assumed to be exponentially distributed. To derive the reliability equivalence factors of a system, we need the following definition.

Definition 1.1. [Sarhan (2002)] A reliability equivalence factor is a factor by which a characteristic of components of a system design has to be multiplied in order to reach equality of a characteristic of this design a different design.

The RF and MTTF will be used as characteristics of the system performance. In this case the reliability equivalence will be referred as SREF and MREF, respectively.

This paper is organized as follows. Section 2 presents the sf and MTTF to the original system. Section 3 gives the sf and MTTF of the improved designs of the system using different improving methods. The $\alpha-$fractiles of the original and improved systems are calculated in Section 4. The REF are obtained in Section 5. Numerical results and conclusions are given in Section 6.

2. THE ORIGINAL SYSTEM

We consider a system show in Figure 2.1, that consists of two components connected in series. The failure rate of a system component is assumed to be a constant.

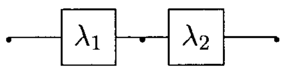

Figure 2.1. 2-components, series system.

Let T_i be the lifetime of the component i, $i = 1, 2$. It is assumed that the failure rate of a system component is constant say λ_i. Therefore T_i has exponentially distributed with parameter λ_i. In what follows, we present the RF and MTTF of this system.

The RF of the system, say $R(t)$, is given by

$$R(t) = e^{-(\lambda_1+\lambda_2)\,t} \tag{2.1}$$

The MTTF, is

$$\text{MTTF} = \int_0^\infty R(t)\,dt = \frac{1}{\lambda_1+\lambda_2}. \tag{2.2}$$

3. THE IMPROVED SYSTEMS

The original system can be improved according to one of the following four different methods:

(1) Reduction method

(2) Hot redundant method

(3) Cold redundant method

(4) Imperfect switch redundant method.

To derive the REFs of the system, we make equivalence between the improved system, which obtained by using reduction method, and that improved system, which obtained according to redundancy methods.

The following subsections gives the RF and MTTF of the improved systems using different methods.

3.1. Reduction Method

Let $R_{\rho_B}(t)$ be the RF of the improved system when reducing the failure rates of the set of components $B \in \{\{1\},\{1,2\}\}$ by the factor ρ_i $0<\rho_i<1$, $i\in B$. One can obtain the function $R_{\rho_B}(t)$ as follows

$$R_{\rho_{\{1\}}}(t) = e^{-(\rho_1\lambda_1+\lambda_2)t} \tag{3.1}$$

$$R_{\rho_{\{1,2\}}}(t) = e^{-(\rho_1\lambda_1+\rho_2\lambda_2)t} \tag{3.2}$$

From Equations (3.1) and (3.2) the MTTF of the improved system say MTTF_{ρ_B}, becomes

$$\text{MTTF}_{\rho_{\{1\}}} = \frac{1}{\rho_1\lambda_1+\lambda_2} \tag{3.3}$$

$$\text{MTTF}_{\rho_{\{1,2\}}} = \frac{1}{\rho_1\lambda_1+\rho_2\lambda_2} \tag{3.4}$$

That is, reducing the failure rates of the set of B components increases the system MTTF by the amount $\triangle_1$:

i) if $B=\{1\}$, then $\triangle_1 = \frac{(1-\rho_1)\lambda_1}{(\rho_1\lambda_1+\lambda_2)(\lambda_1+\lambda_2)}$,

ii) if $B=\{1,2\}$, then $\triangle_1 = \frac{(1-\rho_1)\lambda_1+(1-\rho_2)\lambda_2}{(\rho_1\lambda_1+\rho_2\lambda_2)(\lambda_1+\lambda_2)}$.

3.2. Hot Redundant Method

Let $R^H(t)$ be the RF of the improved system assuming hot redundant of the system by the only one component. Figure 3.1, shows the hot redundant of the system.

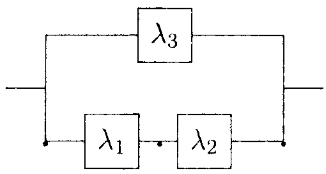

Figure 3.1. Hot redundant of the system.

The function $R^H(t)$ is given by

$$R^H(t) = e^{-\lambda_3 t} + e^{-(\lambda_1+\lambda_2)t} - e^{-(\lambda_1+\lambda_2+\lambda_3)t} \tag{3.5}$$

Let MTTF^H be the MTTF of the improved system assuming hot redundant of the system. Using Equation (3.5), one can deduce MTTF^H as

$$\text{MTTF}^H = \frac{1}{\lambda_1+\lambda_2} + \frac{\lambda_1+\lambda_2}{\lambda_3(\lambda_1+\lambda_2+\lambda_3)} \tag{3.6}$$

That is, hot redundant of the system increases the system MTTF by the amount $\frac{\lambda_1+\lambda_2}{\lambda_3(\lambda_1+\lambda_2+\lambda_3)}$.

3.3. Cold Redundant Method

Let $R^C(t)$ be the RF of the improved system that is obtained by assuming cold redundant of the system.

We mean by the system is cold redundant that a component is connected with the system in such a way that it is activated immediately upon failure of the system. Figure 3.2, shows the cold redundant of the system.

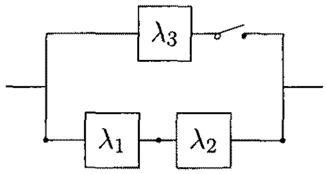

Figure 3.2. Cold redundant of the system.

The function $R^C(t)$ is given by using the joint probability approach, see Billinton and Allan (1982), as

$$R^C(t) = \frac{1}{\lambda_1 + \lambda_2 - \lambda_3} \left[(\lambda_1 + \lambda_2)\, e^{-\lambda_3 t} - \lambda_3\, e^{-(\lambda_1+\lambda_2)\, t}\right] \tag{3.7}$$

From Equation (3.7) the MTTF of the improved system assuming cold redundant of the system is given by

$$\mathrm{MTTF}^C = \frac{1}{\lambda_1 + \lambda_2} + \frac{1}{\lambda_3} \tag{3.8}$$

That is, cold redundant of the system increases the system MTTF by the amount $\frac{1}{\lambda_3}$.

3.4. Imperfect Switch Redundant Method

Let us consider now that the system reliability can be improved assuming imperfect switch redundant of the system. In such method, it is assumed that the system connected by a cold redundant standby component via a random switch having a constant failure rate, say β. Figure 3.3, shows the system after modification using imperfect redundant method.

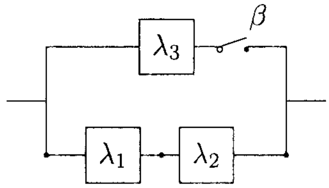

Figure 3.3. Imperfect switch redundant of the system.

Let $R^I(t)$ denote the RF of the system shown in Figure 3.3. This function can be obtained by using the joint probability approach, as

$$R^I(t) = \frac{1}{\lambda_1 + \lambda_2 - \lambda_3 - \beta}\left[(\lambda_1 + \lambda_2)\, e^{-(\lambda_3+\beta)t} - (\lambda_3 + \beta)\, e^{-(\lambda_1+\lambda_2)\, t}\right] \tag{3.9}$$

From Equation (3.9) the MTTF of the improved system, say MTTF^I, is given by

$$\text{MTTF}^I = \frac{1}{\lambda_1 + \lambda_2} + \frac{1}{\lambda_3 + \beta} \tag{3.10}$$

That is, improving the system according to the ISRM increases the system MTTF by the following amount $\frac{1}{\lambda_3+\beta}$.
From the improved methods, we can conclude that:

1. $\text{MTTF}^C - \text{MTTF}^H = \frac{1}{\lambda_1+\lambda_2+\lambda_3} > 0$ then $\text{MTTF}^C > \text{MTTF}^H$, for all λ_i, $i = 1, 2, 3$,

2. $\text{MTTF}^C - \text{MTTF}^I = \frac{\beta}{\lambda_3(\lambda_3+\beta)} > 0$ then $\text{MTTF}^C > \text{MTTF}^I$ for all β, λ_i, $i = 1, 2, 3$,

3. $\text{MTTF}^I - \text{MTTF}^H = \frac{\lambda_3^2-\beta(\lambda_1+\lambda_2)}{\lambda_3(\lambda_3+\beta)(\lambda_1+\lambda_2+\lambda_3)}$ then

   (a) $\text{MTTF}^I \geq \text{MTTF}^H$ if $\lambda_3^2 \geq \beta(\lambda_1 + \lambda_2)$,

   (b) $\text{MTTF}^I < \text{MTTF}^H$ if $\lambda_3^2 < \beta(\lambda_1 + \lambda_2)$.

4. THE α-FRACTILES

The α-fractiles is one of the important measures that measures the performance of reliability of a system. This section presents the $\alpha-$fractiles of the original and improved systems. Let $L(\alpha)$ be the α-fractiles of the original system and $L^D(\alpha)$ be the $\alpha-$fractiles of the improved system assuming HRM (D=H), CRM (D=C) and ISRM (D=I).

The α-fractiles $L(\alpha)$, and $L^D(\alpha)$, are defined as the solution of the following equations, respectively,

$$R\left(\frac{L(\alpha)}{\lambda_1+\lambda_2}\right)=\alpha,\ \ R^D\left(\frac{L(\alpha)}{\lambda_1+\lambda_2}\right)=\alpha. \tag{4.1}$$

It follows from Equation (2.1) and the first Equation of (4.1) that

$$L(\alpha)=-\ln(\alpha). \tag{4.2}$$

From the second Equation of (4.1), when $D=H$, and Equation (3.5), one can verify that $L=L^H(\alpha)$ satisfies the following equation

$$e^{-L}+e^{-\left(\frac{\lambda_3}{\lambda_1+\lambda_2}\right)L}-e^{-\left(\frac{\lambda_1+\lambda_2+\lambda_3}{\lambda_1+\lambda_2}\right)L}-\alpha=0 \tag{4.3}$$

Similarly, from the second Equation of (4.1), when $D=C$, and Equation (3.7), $L=L^C(\alpha)$ can be obtained by solving the following equation

$$(\lambda_1+\lambda_2)\,e^{-\left(\frac{\lambda_3}{\lambda_1+\lambda_2}\right)L}-\lambda_3e^{-L}-(\lambda_1+\lambda_2-\lambda_3)\,\alpha=0 \tag{4.4}$$

Finally, from Equation (3.9)and the second Equation of (4.1), when $D=I$, $L=L^I(\alpha)$ satisfies the following equation

$$(\lambda_1+\lambda_2)\,e^{-\left(\frac{\lambda_3+\beta}{\lambda_1+\lambda_2}\right)L}-(\lambda_3+\beta)\,e^{-L}-(\lambda_1+\lambda_2-\lambda_3-\beta)\,\alpha=0 \tag{4.5}$$

Equations (4.3)-(4.5) have no closed form solutions and can be solved using some numerical program such as Mathematica Program.

5. RELIABILITY EQUIVALENCE FACTORS

Now we ready to derive the possible REFs of the system. The following subsections give SREF and MREF of the underlying system

5.1. The SREF

In this subsection we derive three possible SREF, hot SREF and cold SREF, and imperfect SREF. These factors can be derived by solving the following system of two equations

$$R_{\rho_B}(t)=\alpha,\ \ R^D(t)=\alpha \tag{5.1}$$

with respect to t and ρ_B, for a given α.

Using possible set B and a redundancy method, one can derive the SREF, say ρ_B^D.

1. Using (5.1), and $B=\{1\}$

i) when $D = H$, together with Equations (3.1) and (3.5), one can verify that the factor $\rho = \rho^H_{\{1\}}(\alpha)$ satisfies the following equation

$$\alpha^{\frac{\lambda_1+\lambda_2}{\rho\,\lambda_1+\lambda_2}} + \alpha^{\frac{\lambda_3}{\rho\,\lambda_1+\lambda_2}} - \alpha^{\frac{\lambda_1+\lambda_2+\lambda_3}{\rho\,\lambda_1+\lambda_2}} - \alpha = 0 \tag{5.2}$$

ii) Similarly, when $D = C$, together with (3.1) and (3.7), one can deduce the following equation

$$(\lambda_1 + \lambda_2)\,\alpha^{\frac{\lambda_3}{\rho\,\lambda_1+\lambda_2}} - \lambda_3\,\alpha^{\frac{\lambda_1+\lambda_2}{\rho\,\lambda_1+\lambda_2}} - (\lambda_1 + \lambda_2 - \lambda_3)\alpha = 0 \tag{5.3}$$

The factor $\rho = \rho^C_{\{1\}}(\alpha)$ can be obtained by solving the above equation with respect to ρ.

iii) Finally, when $D = I$, together with Equations (3.1) and (3.9), to verify the factor $\rho = \rho^I_{\{1\}}(\alpha)$ satisfies the following equation

$$(\lambda_1 + \lambda_2)\,\alpha^{\frac{\lambda_3+\beta}{\rho\,\lambda_1+\lambda_2}} - (\lambda_3 + \beta)\,\alpha^{\frac{\lambda_1+\lambda_2}{\rho\,\lambda_1+\lambda_2}} - (\lambda_1 + \lambda_2 - \lambda_3 - \beta)\alpha = 0 \tag{5.4}$$

2. Also, by using Equation (5.1), $B = \{1, 2\}$

i) when $D = H$, together with Equation (3.2) and (3.5), one can verify that the factor $\Lambda_\rho = \rho^H_1\lambda_1 + \rho^H_2\lambda_2$ satisfies the following equation

$$\alpha^{\frac{\lambda_1+\lambda_2}{\Lambda_\rho}} + \alpha^{\frac{\lambda_3}{\Lambda_\rho}} - \alpha^{\frac{\lambda_1+\lambda_2+\lambda_3}{\Lambda_\rho}} - \alpha = 0 \tag{5.5}$$

ii) Similarly, when $D = C$, together with (3.2) and (3.7), one can verify that the factor $\Lambda_\rho = \rho^C_1\lambda_1 + \rho^C_2\lambda_2$ satisfies the following equation

$$(\lambda_1 + \lambda_2)\,\alpha^{\frac{\lambda_3}{\Lambda_\rho}} - \lambda_3\,\alpha^{\frac{\lambda_1+\lambda_2}{\Lambda_\rho}} - (\lambda_1 + \lambda_2 - \lambda_3)\alpha = 0 \tag{5.6}$$

iii) Finally, when $D = I$, together with Equations (3.2) and (3.9), to verify the factor $\Lambda_\rho = \rho^I_1\lambda_1 + \rho^I_2\lambda_2$ satisfies the following equation

$$(\lambda_1 + \lambda_2)\,\alpha^{\frac{\lambda_3+\beta}{\Lambda_\rho}} - (\lambda_3 + \beta)\,\alpha^{\frac{\lambda_1+\lambda_2}{\Lambda_\rho}} - (\lambda_1 + \lambda_2 - \lambda_3 - \beta)\alpha = 0 \tag{5.7}$$

Equations (5.2)-(5.7) have no closed form solutions and can be solved using some numerical program such as Mathematica Program.

5.2. The MREF

The MREF is that reliability equivalence factor which can be obtained when the system mean time to failure is used as a performance measure.

Let $\rho = \xi^D_B$, $D = H$, C and I, denoted the MREF of the system improved by reducing the set B component by the factor ρ to get a system with the same mean time to failure as that system improved according to a redundance method D. Therefore, ξ_B^D can be obtained by solving the following equation with respect to ρ.

$$\mathrm{MTTF}_{\rho_B} = \mathrm{MTTF}^D \tag{5.8}$$

Based on (5.8) and using the results previously obtained for the MTTF of the systems improved according to reduction, hot redundant, cold redundant and imperfect redundant method, one can derive the following equations to get $\xi_B^D = \sum_{i\in B}\lambda_i\xi_i$ as follows

$$\xi_B^D = \frac{1}{\mathrm{MTTF}^D} \tag{5.9}$$

Equation (5.9) can solved numerically by using Mathematica System to get ξ_B^D for given MTTF^D and B.

6. NUMERICAL RESULTS AND CONCLUSION

To explain how one can utilize the previously obtained theoretical results we introduce a numerical example. In such example, we calculate the two different reliability equivalence factors of a two-component series system under the following assumptions:

1. for $\lambda_3^2 \geq \beta(\lambda_1 + \lambda_2)$,
   (a) the failure rates of the first, second and third components are $\lambda_1 = 0.02$, $\lambda_2 = 0.01$ and $\lambda_3 = 0.06$, respectively,
   (b) in imperfect switch redundant method $\beta = 0.04$.
2. for $\lambda_3^2 < \beta(\lambda_1 + \lambda_2)$,
   (a) the failure rates of the first, second and third components are $\lambda_1 = 0.04$, $\lambda_2 = 0.05$ and $\lambda_3 = 0.07$, respectively,
   (b) in imperfect switch redundant method $\beta = 0.09$.

For this example we have found out that:
The MTTF of the original and improved systems are presented in Table 6.1.

Table 6.1. The MTTF of the original and improved systems.

	$\lambda_3^2 \geq \beta(\lambda_1+\lambda_2)$	$\lambda_3^2 < \beta(\lambda_1+\lambda_2)$
MTTF	33.333	11.11
MTTF^H	38.889	19.145
MTTF^I	43.333	17.361
MTTF^C	50.000	25.397

From the above table one can conclude that,

(i) for $\lambda_3^2 \geq \beta(\lambda_1+\lambda_2)$ the mean time to failure of the original and improved system satisfies the following relation

$$\text{MTTF} < \text{MTTF}^H < \text{MTTF}^I < \text{MTTF}^C.$$

(ii) for $\lambda_3^2 < \beta(\lambda_1+\lambda_2)$ the mean time to failures satisfies the following relation

$$\text{MTTF} < \text{MTTF}^I < \text{MTTF}^H < \text{MTTF}^C.$$

The α-fractiles $L(\alpha)$, $L^D(\alpha)$, and the REFs $\rho_B^D(\alpha)$, ξ_B^D, for $D = H, I, C$ and $B \in \{\{1\},\{1,2\}\}$ are calculated using Mathematica Program system according to the previous theoretical formulae.In such calculations the level α is chosen to be 0.1, 0.3, ..., 0.9.

Table 6.2 represent the α-fractiles of the original system and improved systems that are obtained by improving two components according to the previously mentioned methods.

Table 6.2. The α-fractiles

		$\lambda_3^2 \geq \beta(\lambda_1+\lambda_2)$			$\lambda_3^2 < \beta(\lambda_1+\lambda_2)$		
α	$L(\alpha)$	$L^H(\alpha)$	$L^I(\alpha)$	$L^C(\alpha)$	$L^I(\alpha)$	$L^H(\alpha)$	$L^C(\alpha)$
0.1	2.303	2.383	2.659	2.969	3.0765	3.4245	4.4558
0.3	1.231	1.377	1.553	1.812	1.8924	2.0736	2.7837
0.5	0.693	0.909	1.022	1.228	1.2886	1.4005	1.9115
0.7	0.357	0.577	0.644	0.793	0.8357	0.9029	1.2479
0.9	0.105	0.272	0.301	0.380	0.4018	0.4317	0.6039

Based on the results presented in Table 6.2, it seems that

1. for $\lambda_3^2 \geq \beta(\lambda_1+\lambda_2)$, then $L(\alpha) < L^H(\alpha) < L^I(\alpha) < L^C(\alpha)$,
2. for $\lambda_3^2 < \beta(\lambda_1+\lambda_2)$, then $L(\alpha) < L^I(\alpha) < L^H(\alpha) < L^C(\alpha)$,

in all studied cases. This is confirmed by the results obtained for MTTF.

Tables 6.3 and 6.4 show the SREF when the system is improved according to HRM, CRM and ISRM and reducing the failure rates of the components, belong to the set $B \in \{\{1\},\{1,2\}\}$.

Table 6.3. The SREF

	$\lambda_3^2 \geq \beta(\lambda_1+\lambda_2)$					
	$B=\{1\}$			$B=\{1,2\}$		
α	ρ^H	ρ^I	ρ^C	Λ_ρ^H	Λ_ρ^I	Λ_ρ^C
0.1	0.9494	0.7991	0.6638	0.0289	0.0259	0.0233
0.3	0.8117	0.6630	0.4967	0.0262	0.0233	0.0199
0.5	0.6441	0.5176	0.3467	0.0229	0.0204	0.0169
0.7	0.4276	0.3304	0.1743	0.0186	0.0166	0.0135
0.9	0.0807	0.0247	0.0083	0.0116	0.0105	0.0083

Table 6.4. The SREF

	$\lambda_3^2 < \beta(\lambda_1+\lambda_2)$					
	$B=\{1\}$			$B=\{1,2\}$		
α	ρ^I	ρ^H	ρ^C	Λ_ρ^I	Λ_ρ^H	Λ_ρ^C
0.1	0.2629	0.4339	NA	0.0674	0.0605	0.0465
0.3	0.0564	0.1815	NA	0.0573	0.0523	0.0389
0.5	NA	NA	NA	0.0484	0.0445	0.0326
0.7	NA	NA	NA	0.0384	0.0356	0.0257
0.9	NA	NA	NA	0.0236	0.0305	0.0194

According to the results presented in Tables 6.2 to 6.4, it may be observed that:

1. for $\lambda_3^2 \geq \beta(\lambda_1+\lambda_2)$ then

   (a) Hot redundant of the system will increase $L(0.1)$ from $\frac{2.303}{\lambda_1+\lambda_2}$ to $\frac{2.383}{\lambda_1+\lambda_2}$, see Table 6.2. The same effect on $L(0.1)$ can occur by reducing the failure rates of:

   (i) the components 1, $B=\{1\}$, by the factor 0.9494,

   (ii) the components 1 and 2, $B=\{1,2\}$ by the factor ρ_1, ρ_2, such that $0.02\rho_1+0.01\rho_2=0.0289$,where ρ_1 take any value on [0, 1] and $\rho_2=\frac{0.289-0.02\rho_1}{0.01}$ see Table 6.3.

   (b) In the same manner one can read the rest obtained results assuming HRM, CRM and ISRM.

2. for $\lambda_3^2 < \beta(\lambda_1+\lambda_2)$:

   (a) Hot redundant of the system will increase $L(0.1)$ from $\frac{2.303}{\lambda_1+\lambda_2}$ to $\frac{3.4245}{\lambda_1+\lambda_2}$, see Table 6.2. The same effect on $L(0.1)$ can occur by reducing the failure rates of:

   (i) the components 1, $B=\{1\}$, by the factor 0.4339,

   (ii) the components 1 and 2, $B=\{1,2\}$, by the factor $\rho 1, \rho_2$, such that, $0.04\rho_1+0.05\rho_2=0.0605$, where $\rho_1 \in [0,1]$, and $\rho_2=\frac{0.0605-0.04\rho_1}{0.05}$, see Table 6.4.

   (b) In the same manner one can read the rest obtained results assuming hot, cold and imperfect switch redundant methods.

The notation NA in Tables 6.4, means that the value of ρ_B is not available and therefore there is possible equivalence between the system improved by reduction method and that system improved by using the redundancy methods.
Table 6.5 show the MREF when the system is improved according to HRM, CRM and ISRM and reducing the failure rates of the components, belong to the set $B \in \{\{1\},\{1,2\}\}$.

Table 6.5. The MREF

B	$\lambda_3^2 \geq \beta(\lambda_1+\lambda_2)$			$\lambda_3^2 < \beta(\lambda_1+\lambda_2)$		
	ξ_B^H	ξ_B^I	ξ_B^C	ξ_B^I	ξ_B^H	ξ_B^C
{1}	0.0157	0.0131	0.0010	0.0076	0.0023	NA
{1,2}	0.0257	0.0231	0.0200	0.0576	0.0523	0.0394

Based on the results presented in Table 6.5, it may be observed that:

1. for $\lambda_3^2 \geq \beta(\lambda_1+\lambda_2)$ then the improved system that can be obtained by improving the system according to hot redundant method, has the same mean time to failure of that system which can be obtained by doing one of the following:
   (i) reducing the failure rate of component 1, $B = \{1\}$, by the factor $\xi_1^H = 0.7857$, where $\xi_{\{1\}}^H = 0.02\xi_1 = 0.0157$
   (ii) reducing the failure rates of components 1 and 2, $B = \{1,2\}$, by the factor $\xi_{\{1,2\}} = 0.0257$, such that $0.02\xi_1 + 0.01\xi_2 = 0.0257$, where $\xi_1 \in [0,1]$ and $\xi_2 = \frac{0.0257-0.02\xi_1}{0.01}$
2. for $\lambda_3^2 > \beta(\lambda_1+\lambda_2)$ then the improved system that can be obtained by improving the system according to hot redundant method, has the same mean time to failure of that system which can be obtained by doing one of the following:
   (i) reducing the failure rate of component 1, $B = \{1\}$, by the factor $\xi_1^H = 0.0557$, where $\xi_{\{1\}}^H = 0.04\xi_1 = 0.0023$
   (ii) reducing the failure rates of components 1 and 2, $B = \{1,2\}$, by the factor $\xi_{\{1,2\}} = 0.0523$, such that $0.04\xi_1 + 0.05\xi_2 = 0.0523$, where $\xi_1 \in [0,1]$ and $\xi_2 = \frac{0.0523-0.05\xi_1}{0.04}$
3. The notation NA in Tables 6.4, means that the value of ρ_B is not available and therefore there is possible equivalence between the system improved by reduction method and that system improved by using the redundancy methods.

In the same manner one can read the rest obtained results assuming HRM, CRM and ISRM.

In the same way, one can derive the REFs for two components parallel system. As pervious, we improve the system by add one component only by using the hot redundant, cold redundant and imperfect redundant methods.

7. FUTURE STUDIES

In the future, we will study the reliability equivalence technique in the different cases:

Case 1: In the our life and in industry any component (system) has not only one type of failure rates but there are many types of failures such as human, industry and operating failures,... etc. We must study the reliability system under these types of failure rates, and his failure rate is mixture of these types.

Case 2: In some system with n components the life time of the system components sometimes are not independent. Also, each component has different distribution.

Case 3: In sometimes the failure rate of the system components are depend on the time.

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