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Case Study on the Compatibility of Measurement Systems with Part-to-part Variations in Automobile Industry

Lee, Myung Duk*
Research and Innovation Center, Ford Motor Co., MI 48202, USA

Lim, Ik Sung
Dept. of Industrial & Management Eng,. Namseoul University, S. Korea

Sung, Chun Ja
Dept. of GIS Eng., Namseoul University, S. Korea

Abstract. Analysis of measurement systems is important to determine if the measurement process is adequate to measure the part-to-part variability in the process. Control chart techniques provide an effective, and easy-to-use method for performing this analysis. However, application with the real data for the evaluation procedure for multiple measurement systems have not been demonstrated. This research will provide a methodology for the evaluation of part-to-part variation and variation of different measurement systems step by step followed by number of case studies for each methodologies provided.

Key Words: *Part-to-part, Piece-to-piece, Unit-to-unit, Control chart, Checking Fixture, Optical Coordinate Measurement Machine*

1. INTRODUCTION

1.1 Multivariate Two-Way Fixed-Effects Model with Interaction

The linear model for the general observation is

$$x_{ijk} = \mu + \tau_i + \beta_j + \eta_{ij} + \varepsilon_{ijk} \qquad (1.1)$$
$$i = 1,2,\dots,a$$
$$j = 1,2,\dots,b$$
$$k = 1,2,\dots,n$$

where $\sum_{i=1}^{a}\tau_i = \sum_{j=1}^{b}\beta_j = \sum_{i=1}^{a}\eta_{ij} = \sum_{j=1}^{b}\eta_{ij} = 0$. The vectors are all of order p x 1 and ϵ_{ijk} is assumed to be an $N_p(\mathbf{0}, \Sigma)$ random vector. Thus the two-way analysis of variance on p

* Corresponding author.
E-mail address: mlee4@ford.com

responses with n independent observation vectors in each combination. The restriction to have equal, or at least proportional, cell numbers is essential: otherwise, as in the univariate analysis, the various **H** and **E** matrices will not sum to the total sums of squares and product matrix. The hypotheses tests will be developed in analogy with the univariate two-way balanced layout. The null hypotheses of equal A and B effects are

$$\mathbf{H}_{0a}: \begin{bmatrix} \tau_{11} \\ \vdots \\ \tau_{1p} \end{bmatrix} = \cdots = \begin{bmatrix} \tau_{a1} \\ \vdots \\ \tau_{ap} \end{bmatrix} \tag{1.2}$$

$$\mathbf{H}_{0b}: \begin{bmatrix} \beta_{11} \\ \vdots \\ \beta_{1p} \end{bmatrix} = \cdots = \begin{bmatrix} \beta_{b1} \\ \vdots \\ \beta_{bp} \end{bmatrix} \tag{1.3}$$

The ranks of the respective hypotheses matrices are a-1 and b-1. The observation vectors X_{ijk} can be decomposed as

$$x_{ijk} = \bar{x} + (\bar{x}_{i.} - \bar{x}) + (\bar{x}_{.j} - \bar{x}) + (\bar{x}_{ij} - \bar{x}_{i.} - \bar{x}_{.j} + \bar{x}) + (x_{ijk} - \bar{x}_{ij}) \tag{1.4}$$

where $\bar{x}$ is the overall average of the observation vectors, $\bar{x}_{i.}$ is the average of the observation vectors at the i^{th} level of factor A, $\bar{x}_{.j}$ is the average of the observation vectors at the j^{th} level of factor B, and $\bar{x}_{ij}$ is the average of the observation vectors at the i^{th} level of factor A and the j^{th} level of factor B.

Generalizations of the univariate case give the breakups of the sum of squares and cross-products and degrees of freedom:

$$\begin{aligned}\sum_{i=1}^{a}\sum_{j=1}^{b}\sum_{k=1}^{n}(x_{ijk} - \bar{x})(x_{ijk} - \bar{x})' &= \sum_{i=1}^{a} bn(\bar{x}_{i.} - \bar{x})(\bar{x}_{i.} - \bar{x})' + \sum_{j=1}^{b} an(\bar{x}_{.j} - \bar{x})(\bar{x}_{.j} - \bar{x})' \\ &+ \sum_{i=1}^{a}\sum_{j=1}^{b} n(\bar{x}_{ij} - \bar{x}_{i.} - \bar{x}_{.j} + \bar{x})(\bar{x}_{ij} - \bar{x}_{j.} - \bar{x}_{.j} + \bar{x})' \\ &+ \sum_{i=1}^{a}\sum_{j=1}^{b}\sum_{k=1}^{n}(x_{ijk} - \bar{x}_{ij})(x_{ijk} - \bar{x}_{ij})' \end{aligned} \tag{1.5}$$

$$abn-1=(a-1)+(b-1)+(a-1)(b-1)+ab(n-1) \tag{1.6}$$

The hypotheses of interaction between the A and B treatments can be written as

$$\begin{aligned} \mathrm{H}_{0ab}: \mathbf{C}(\eta_1 - \eta_a) &= \mathbf{0} \\ \mathbf{C}(\eta_{a-1} - \eta_a) &= \mathbf{0} \end{aligned} \tag{1.7}$$

where **C** is the (c-1) x c matrix. The pattern of the matrix

$$\eta_i = \begin{bmatrix} \eta_{i11} & \Lambda & \eta_{i1p} \\ M & O & M \\ \eta_{ib1} & \Lambda & \eta_{ibp} \end{bmatrix} \quad i = 1,\ldots,a \tag{1.8}$$

contains the interaction parameters for the i^{th} row of the layout. For $\mathbf{H}_{0ab}$ to be true the interaction parameters for each response must satisfy $(a\text{-}1)(b\text{-}1)$ homogeneous linear equations, so that the rank of the interaction hypothesis matrix is $(a\text{-}1)(b\text{-}1)$. For the row (A factor) and column (B factor) hypotheses to be testable it is necessary that the interaction null hypothesis be true. For that reason the test for interaction is carried out before the tests for main factor effects. If interaction effects exists, the factor effects do not have a clear interpretation. The matrices of error and hypotheses sums of squares and products are computed in Table 1.1.

Table 1.1. MANOVA Table for Comparing Factors and Their Interaction

Source of variation	Matrix of sum squares and cross-products	Degrees of freedom
Factor a $\mathbf{H}_a$	$SSP_{fac\ a} = \sum_{i=1}^{a} bn(\bar{x}_{i.} - \bar{x})(\bar{x}_{.j} - \bar{x})'$	a-1
Factor a $\mathbf{H}_b$	$SSP_{fac\ b} = \sum_{j=1}^{b} an(\bar{x}_{.j} - \bar{x})(\bar{x}_{.j} - \bar{x})'$	b-1
Interaction $\mathbf{H}_{ab}$	$SSP_{\text{int}\ ab} = \sum_{i=1}^{a}\sum_{j=1}^{b} n(\bar{x}_{ij} - \bar{x}_{i.} - \bar{x}_{.j} + \bar{x})(\bar{x}_{ij} - \bar{x}_{i.} - \bar{x}_{.j} + \bar{x})'$	(a-1)(b-1)
Error **E**	$SSP_e = \sum_{i=1}^{a}\sum_{j=1}^{b}\sum_{k=1}^{n} (x_{ijk} - \bar{x}\ )(x_{ijk} - \bar{x}\ )'$	ab(n-1)
Total **T**	$SSP_{total} = \sum_{i=1}^{a}\sum_{j=1}^{b}\sum_{k=1}^{n} (x_{ijk} - \bar{x}\ )(x_{ijk} - \bar{x}\ )'$	abn-1

To tests the three hypothesis, it is first necessary to invert **E** and then to form the products; $\mathbf{H}_a\mathbf{E}^{-1}$, $\mathbf{H}_b\mathbf{E}^{-1}$, and $\mathbf{H}_{ab}\mathbf{E}^{-1}$. From these, the greatest characteristic roots C_{ag}, C_{bg}, C_{abg} are extracted. Their statistics C_{ig} /1+C_{ig} are referred to as Heck charts or Pillai tables. The parameters of the relevant greatest-characteristic-root distributions are summarized as follows, Table 1.2:

Table 1.2. Greatest-Root Distribution Parameters

			Parameter	
Source	**Statistic**	s	m	n
Rows(a)	$\frac{C_{ag}}{1+C_{ac}}$	min(a-1,p)	$\frac{\lvert a-1-p\rvert-1}{2}$	$\frac{ab(n-1)-p-1}{2}$
Column(b)	$\frac{C_{bg}}{1+C_{bg}}$	min(b-1,p)	$\frac{\lvert b-1-p\rvert-1}{2}$	$\frac{ab(n-1)-p-1}{2}$
interaction(ab)	$\frac{C_{abg}}{1+C_{abg}}$	min[(a-1)(b-1),p]	$\frac{\lvert(a-1)(b-1)-p\rvert-1}{2}$	$\frac{ab(n-1)-p-1}{2}$

2. CASE STUDY (MULTIVARIATE TWO-WAY MODEL)

By constructing a two-way MANOVA, the comparability of measurement systems and part-to-part variation will be analyzed. In this case, the first way of classification consisted of a=3 measurement systems (factor A) and the second involved b=8 (factor B) parts. The responses p=4 (master control points) was checked n=5 times. The matrices of the appropriate sum of squares and cross-products were calculated, leading to the MANOVA Table 2.1.

The inverse of the error matrix is

$$\mathbf{E}^{-1}=\begin{bmatrix} 0.031882 & -0.00838 & 0.001332 & 0.007165 \\ -0.00838 & 0.023159 & 0.007337 & 0.001928 \\ 0.001332 & 0.007337 & 0.020938 & -0.00422 \\ 0.007165 & 0.001928 & -0.00422 & 0.018779 \end{bmatrix}$$

Table 2.1. The Hypothesis Matrices for the Measurement Devices, Parts, and Interaction Effects:

Source of variation	SSP	d.f.
Factor A: Measurement Systems (CF, CMM & OCMM)	$\mathbf{H}_a=\begin{bmatrix} 1.338 & -0.6040 & -2.046 & 0.1990 \\ -0.604 & 5.5745 & 6.223 & 6.8175 \\ -2.046 & 6.2229 & 8.424 & 6.6002 \\ 0.199 & 6.8175 & 6.600 & 9.0285 \end{bmatrix}$	2
Factor B: Part to Part	$\mathbf{H}_b=\begin{bmatrix} 18.81 & 13.35 & -19.27 & -22.07 \\ 13.35 & 14.70 & -16.89 & -21.46 \\ -19.27 & -16.89 & 44.50 & 46.31 \\ -22.07 & -21.46 & 46.31 & 58.19 \end{bmatrix}$	7

Interaction	$\mathbf{H}_{ab}=\begin{bmatrix} 8.451 & 4.16 & -5.46 & -8.203 \\ 4.165 & 25.10 & -13.58 & -7.405 \\ -5.461 & -13.58 & 24.23 & 14.979 \\ -8.203 & -7.40 & 14.98 & 23.646 \end{bmatrix}$	14
Error	$\mathbf{E}=\begin{bmatrix} 42.66 & 21.94 & -14.80 & -21.85 \\ 21.94 & 61.43 & -27.10 & -20.76 \\ -14.80 & -27.10 & 62.73 & 22.51 \\ -21.85 & -20.76 & 22.51 & 68.77 \end{bmatrix}$	96

Note: CF: Checking Fixture, CMM: Coordinate Measurement Machine, OCMM: Optical Coordinate Measurement Machine

The matrix product needed for the interaction test is

$$\mathbf{H}_{ab}\mathbf{E}^{-1}=\begin{bmatrix} 0.168537 & -0.03033 & -0.22659 & -0.06246 \\ -0.14864 & 0.432489 & -0.2337 & -0.00359 \\ 0.079249 & -0.06211 & 0.681716 & 0.113847 \\ -0.01017 & 0.052834 & 0.692492 & 0.307871 \end{bmatrix}$$

The greatest root is C_{ab} = 0.47044 (Table 2.2) and the test statistic in the form required by the percentage - point chart is θ_{ab}= .319. The distribution parameters are s=4 m=4.5 n=45.5, and if test at the 0.01 level is chosen, the critical value for θ_{ab} can be found from the first chart to be $\chi_{0.01}$ = *.350*. Since the observed statistic does not exceed that number, the hypothesis $H_0 : \eta_{ij} = 0$ is not rejected. This means that there is no interaction between measurement systems and part-to-part variation. The additive model for measurement system and part effects can be assumed to hold for all combinations of treatments. Since there is no interaction, the main effects from factor *A* and *B* can be tested.

First, consider the hypotheses for the piece-to-piece part variations (factor *B*): $H_0 : \beta_1=\beta_2=\ldots=\beta_b=0$ and H_1: *at least one* $\beta_i\neq 0$. The procedure of hypotheses test for the piece-to-piece variations will be the same as interaction variations test:

$$\mathbf{H}_{b}\mathbf{E}^{-1}=\begin{bmatrix} 0.304064 & -0.03234 & -0.69494 & -0.17272 \\ 0.12622 & 0.063305 & -0.63102 & -0.20782 \\ -0.08181 & 0.186041 & 1.65183 & 0.511457 \\ -0.047526 & 0.139844 & 1.875571 & 0.698056 \end{bmatrix}$$

where greatest characteristic root C_b = 1.13520 (Table 2.3) and θ_g = 0.53166. For parameters s=4, m=1.0, n=45.5, the $\alpha=0.01$ critical value is $\chi_{0.01}$= .2625. Since $\chi_{0.01}= .2625<\theta_g= .53166$, the hypothesis $\beta_{ij}=0$ is rejected. It can be concluded that there is a major unit-to-unit variation.

Table 2.2. MANOVA for Measurement Systems and Parts

CRITERION Roy's	TEST STATISTIC 0.47044			
s = 4 m = 4.5 n = 45.5				
EIGEN Analysis for Measurement Systems*Parts				
Eigenvalue	0.4704	0.4353	0.1914	0.1491
Proportion	0.3775	0.3493	0.1536	0.1196
Cumulative	0.3775	0.7268	0.8804	1.0000
Eigenvector	1	2	3	4
FRH	-0.06799	-0.02724	0.06867	0.14764
MRH	0.11560	0.08495	0.04959	-0.01091
MLS	-0.04302	0.07795	0.10475	-0.04514
RLH	-0.00658	0.08062	-0.06407	0.09017

Table 2.3. MANOVA for Part-to-Part

CRITERION Roy's	TEST STATISTIC 1.13520			
s = 4 m = 1.0 n = 45.5				
EIGEN Analysis for Parts				
Eigenvalue	1.1352	0.2988	0.16009	0.05852
Proportion	0.6869	0.1808	0.09687	0.03541
Cumulative	0.6869	0.8677	0.96459	1.00000
Eigenvector	1	2	3	4
FRH	0.00830	-0.1566	-0.0606	0.0601
MRH	-0.02415	-0.0281	0.1087	-0.0998
MLS	-0.07656	-0.0275	0.1015	0.0633
RLH	-0.08138	-0.0471	-0.0739	-0.0669

Note: FRH: Front Right side Hole, MRH: Middle Right side Hole, MLS: Middle Left side Slot, RLH: Rear Left side Hole.

In a similar manner, factor A effects (compatibility of measurement systems) are tested by considering $H_0: \tau_1=\tau_2=...=\tau_a=0$ and H_1: *at least one* $\tau_j \neq 0$. The matrix product will be needed as follows:

$$\mathbf{H}_a\mathbf{E}^{-1} = \begin{bmatrix} 0.046419 & -0.03982 & -0.04175 & 0.020784 \\ -0.00883 & 0.192957 & 0.298417 & 0.108214 \\ -0.05886 & 0.235783 & 0.343257 & 0.085773 \\ 0.022701 & 0.222045 & 0.35802 & 0.156295 \end{bmatrix}$$

and extract its greatest characteristic root C_a=0.48832 (Table 2.4). The associated test statistic is $\theta_g = .328$. For parameters s=2, m=0.5, n=45.5, the α=0.01 critical value is $\chi_{0.01} = .175$. Since $\chi_{0.01} = .175 < \theta_g = .328$, the compatibility of the three measurement systems are rejected. It is reasonable to conclude that both the part-to-part variation and the measurement systems affect the responses. The multiple-comparisons problem of determining which measurement systems are different will be determined by the next section.

Table 2.4. MANOVA for Measurement Systems

CRITERION Roy's	TEST STATISTIC 0.48832			
s = 2 m = 0.5 n = 45.5				
EIGEN Analysis for Measurement Systems				
Eigenvalue	0.4883	0.09883	0.00000	0.00000
Proportion	0.8317	0.16832	0.00000	0.00000
Cumulative	0.8317	1.00000	1.00000	1.00000
Eigenvector	1	2	3	4
FRH	0.0111	-0.1263	0.0590	-0.1109
MRH	-0.1188	0.0269	-0.0900	-0.0149
MLS	-0.0894	0.0676	0.0669	-0.0624
RLH	-0.0611	-0.1059	0.0178	0.0593

3. COMPARISON BETWEEN TWO MEASURING INSTRUMENTS

Multivariate paired comparison concepts will be used to find which measurement systems are different. In our situation, two treatments will be administered to the same units and responses will be compared to assess the effects of the treatments. The paired responses will then be analyzed by computing their differences, thereby eliminating much of the influence of extraneous unit-to-unit variation.

3.1 Paired Comparison for Univariate Case

In the univariate case, let y_{1j} denote the response treatment 1 and let y_{2j} denote the response to treatment 2 for the j^{th} unit trial. That is, (y_{1j}, y_{2j}) are measurements recorded on the j^{th} unit or j^{th} pair of like units. The differences between each pair of observations are defined as $d_j = y_{1j} - y_{2j}$, $j=1,2, ..., n$. Therefore,

$$\mu_d = E(y_{1j}-y_{2j}) = E(y_{1j})-E(y_{2j}) = \mu_{1j}-\mu_{2j} \tag{3.1}$$

The mean and standard deviation of these differences are denoted as $\bar{d}$ and S_d. The test statistic for the hypothesis is

$$t = \frac{\bar{d} - \delta}{s_d / \sqrt{n}} \tag{3.2}$$

where

$$\bar{d} = \frac{\sum_{j=1}^{n} d_j}{n} \tag{3.3}$$

and

$$s_d^2 = \frac{\sum_{j=1}^{n} (d_j - \bar{d})^2}{n-1} \tag{3.4}$$

has a t-distribution with n-1 d.f. Consequently, an α-level test of

$$H_0 : \mu_d = 0 \tag{3.5}$$

versus

$$H_1 : \mu_d \neq 0 \tag{3.6}$$

may be conducted by comparing $|t|$ with t_{n-1} (α/2)- the upper $100(\alpha/2)^{th}$ percentile of a t-distribution with n-1 d.f.

3.2 Paired Comparison for Multivariate Case

In the case of p-variate observations from two multivariate populations, the data can be arranged as in Table 3.1. Where the first subscript indicates the treatment, the second subscript indicates the measured characteristic (responses), and the third subscript indicates the experimental element that has been measured. These may also be arranged as the p-dimensional vectors;

$$y_{11}, y_{12}, \cdots, y_{1j}, \cdots, y_{1n1}$$
$$y_{21}, y_{22}, \cdots, y_{2j}, \cdots, y_{2n1}$$

where $y_{11}=(y_{111}\ y_{121} \ldots y_{1p1})$ and it is assumed that $y_{1j} \sim N_p(\mu_1, \Sigma)$ and $y_{2j} \sim N_p(\mu_2, \Sigma)$ with equal covariance matrices.

Table 3.1. Layout of Data for Multivariate Analysis

Treatment															
1								2							
Sample No			Characteristic					Sample No.			Characteristic				
	1	2	…	k	…	p			1	2	…	k	…	p	
1	y_{111}	y_{121}	…	y_{1k1}	…	y_{1p1}		1	y_{211}	y_{221}	…	y_{2k1}	…	y_{2p1}	
2	y_{112}	y_{122}	…	y_{1k2}	…	y_{1p2}		2	y_{212}	y_{222}	…	y_{2k2}	…	y_{2p2}	
⋮	⋮	⋮		⋮		⋮		⋮	⋮	⋮		⋮		⋮	
j	y_{11j}	y_{12j}	…	y_{1kj}	…	y_{1pj}		j	y_{21j}	y_{22j}	…	y_{2kj}	…	y_{2pj}	
⋮	⋮	⋮		⋮		⋮		⋮	⋮	⋮		⋮		⋮	
n	y_{11n}	y_{12n}	…	y_{1kn}	…	y_{1pn}		n_2	y_{21n}	y_{22n}	…	y_{2kn}	…	y_{2pn}	

The p paired difference random variables become

$$\begin{aligned} d_{1j} &= x_{11j} - x_{21j} \\ d_{2j} &= x_{12j} - x_{22j} \\ \vdots \quad & \quad \vdots \quad \vdots \\ d_{pj} &= x_{1pj} - x_{2pj} \end{aligned} \tag{3.7}$$

Let $\boldsymbol{d}' = [d_{1j}, d_{2j}, ..., d_{pj}]$ and assume, for $j = 1, 2, ..., n$, that

$$\mathbf{E}(d_j) = \boldsymbol{\delta}(\mu_{1j} - \mu_{2j}) = \begin{bmatrix} \delta_1 \\ \delta_2 \\ \vdots \\ \delta_p \end{bmatrix} \tag{3.8}$$

and

$$\mathrm{Cov}(\mathbf{d}_j) = \Sigma_d \tag{3.9}$$

If, in addition, $d_1, d_2, ..., d_n$ are independent $N_p(\delta, \Sigma_d)$ random vectors, inferences about the vector of mean difference δ can be based upon a T^2 statistic. Specifically,

$$T^2 = n(\bar{d} - \delta)' S_d^{-1} (\bar{d} - \delta) \tag{3.10}$$

where

$$\bar{d} = \frac{1}{n} \sum_{j=1}^{n} d_j \tag{3.11}$$

and

$$S_d = \frac{1}{n-1} \sum_{j=1}^{n} (d_j - \bar{d})(d_j - \bar{d})' \tag{3.12}$$

Given the observed differences $\bar{d}_j = [d_{1j},\ d_{2j},\ \mathrm{K},\ d_{pj}]$ and let $\delta=(\mu_{1j}-\mu_{2j})$, an α-level test of $H_0 : \delta = 0$ vs. $H_1 : \delta \neq 0$ for an $N_p(\delta, \Sigma_d)$ population rejects H_0 if the observed $T^2 = n\bar{d}'S_d^{-1}\bar{d} > \frac{(n-1)p}{n-p}F_{p,n-p(\alpha)}$ where $F_{p,n-p(\alpha)}$ is the upper $(100\alpha)^{th}$ percentile of an F-distribution with p and n-p degree of freedom.

3.2.1 Case Study 1: Comparison between CF and OCMM

Data was collected by taking $N_j=30$ measurements from each measurement system on the same part for the $p=4$ master control locating points. The covariance matrix and its inverse matrix for CF and OCMM are as follows:

$$\mathbf{\bar{d}}=\begin{bmatrix}0.19\\0.45\\0.21\\0.73\end{bmatrix}\quad \mathbf{S}=\begin{bmatrix}1.84202 & 1.04621 & -0.60821 & -0.79913\\ 1.4621 & 1.85983 & -1.18121 & -1.24483\\ -0.60821 & -1.18121 & 1.88162 & 1.35972\\ -0.79913 & -1.24483 & 1.35972 & 2.86823\end{bmatrix}\quad \mathbf{S}^{-1}=\begin{bmatrix}0.80551 & -0.45806 & -0.06952 & 0.05858\\ -0.45806 & 1.22468 & 0.50025 & 0.16675\\ -0.06952 & 0.50025 & 1.03453 & -0.29269\\ 0.05858 & 0.16675 & -0.29269 & 0.57609\end{bmatrix}$$

$$T^2 = n(\bar{d}-\delta)'S_d^{-1}(\bar{d}-\delta) \tag{3.13}$$

$$=30\begin{bmatrix}0.19 & 0.45 & 0.21 & 0.73\end{bmatrix}\begin{bmatrix}0.80551 & -0.45806 & -0.06952 & 0.05858\\ -0.45806 & 1.22468 & 0.50025 & 0.16675\\ -0.06952 & 0.50025 & 1.03453 & -0.29269\\ 0.05858 & 0.16675 & -0.29269 & 0.57609\end{bmatrix}\begin{bmatrix}0.19\\0.45\\0.21\\0.73\end{bmatrix}$$
$$=20.29305$$

Upon comparing T^2 value with the critical value 18.47, the hypothesis will be rejected $H_0 : \mu_1 = \mu_2$ and concluded that $\mu_{11} \neq \mu_{21}$ or $\mu_{12} \neq \mu_{22}$ or $\mu_{13} \neq \mu_{23}$ or $\mu_{14} \neq \mu_{24}$, or any one or more (or possible all) of the means are not equal to the values specified. Apparently, the measurement instruments (CF and OCMM) affects the characteristics which are master locating points.

It would not be proper to test the four individual mean differences by univariate t statistics, for it must be protected against the effects of positive correlations among the subsets as well as the tendency for individual differences to be significant merely by chance as more responses are included in the variate vectors.

The conclusions which would have incorrectly assumed independence of the characteristics should be examined. From the value $\bar{d}$ in the formula and the computations already completed for testing

$$H_{01} : \mu_{11} = \mu_{21}$$
$$t_{01} = \frac{\bar{d}}{s_d/\sqrt{n}} = \frac{0.19}{1.84202/\sqrt{30}} = 0.56 \tag{3.14}$$

and, to test $H_{02} : \mu_{12} = \mu_{22}$, as same procedure, we have $t_{02} = 1.33$, and $t_{03} = 0.61$, $t_{04} = 1.39$. Upon comparing the calculated t with $t_{0.05,29} = 1.699$ at the level $\alpha = 0.05$, There is no evidence to reject any hypothesis. Because the value of t_{01} through t_{04} is less than the critical value 1.699. Clearly this numerical example serves to illustrate the need for multivariate procedures where they are applicable. Univariate procedures incorrectly applied may lead to completely erroneous conclusions.

3.2.3 Case Study 2: Comparison between CF and CMM

Again, data was collected by taking $N_j=30$ measurements from each measurement systems on the same part for the $p=4$ master control locating points. The covariance matrix and its inverse matrix for CF and CMM are as follows:

$$\bar{\mathbf{d}}=\begin{bmatrix}0.26\\0.64\\0.47\\0.92\end{bmatrix}\quad \mathbf{S}=\begin{bmatrix}1.51628 & 0.59731 & -0.14710 & -0.58400\\ 0.59731 & 1.31495 & -0.47728 & -0.38503\\ -0.14710 & -0.47728 & 2.10838 & 0.03924\\ -0.58400 & -0.38503 & 0.03924 & 1.03683\end{bmatrix}\quad \mathbf{S}^{-1}=\begin{bmatrix}0.94028 & -0.31009 & -0.01231 & 0.41493\\ -0.31009 & 1.03653 & 0.20924 & 0.20235\\ -0.01231 & 0.20924 & 0.51985 & 0.05109\\ 0.41493 & 0.20235 & 0.05109 & 1.27140\end{bmatrix}$$

$$T^2=n(\bar{d}-\delta)'S_d^{-1}(\bar{d}-\delta) \tag{3.15}$$

$$=30\begin{bmatrix}0.26 & 0.64 & 0.47 & 0.92\end{bmatrix}\begin{bmatrix}0.94028 & -0.31009 & -0.01231 & 0.41493\\ -0.31009 & 1.03653 & 0.20924 & 0.20235\\ -0.01231 & 0.20924 & 0.51985 & 0.05109\\ 0.41493 & 0.20235 & 0.05109 & 1.27140\end{bmatrix}\begin{bmatrix}0.26\\0.64\\0.47\\0.92\end{bmatrix}$$

$$=65.93155$$

The two-sample T^2 statistics had the value 65.39; the associate F is 4.14, at $\alpha=0.01$, with degrees of freedom 4 and 26 and the critical value is 18.47. H_0 is rejected and concluded there is a nonzero mean difference between the measurements of the two measuring systems.

Although the null hypothesis has been rejected, it is still not known which of the four mean differences may have contributed to the significant T^2 or for which it might be reasonable to conclude that their population means are equal. The simultaneous confidence intervals will be used to test the individual differences.

In order to determine which of the measurements may be contributing to the rejection of H_0, the simultaneous confidence intervals will be used to test the individual differences. A 100 (1-α)% simultaneous confidence intervals for the individual mean differences δ_i are given by

$$\delta_i=\bar{d}_i\pm\sqrt{\frac{(n-1)p}{(n-p)}F_{p,n-p(\alpha)}}\sqrt{\frac{S_{d_i}^2}{n}} \tag{3.16}$$

$$=0.26\pm\sqrt{18.47}\sqrt{\frac{1.51628}{30}}$$

$$=0.26\pm0.97$$

and the confidence interval for the mean FRH becomes -0.26±0.97. Observing that the hypothetical value $\delta=0$ is included with this confidence interval, i.e. $-0.71\le\delta\le1.23$, it is reasonable to conclude that the FRH measurement did not contribute to the rejection of H_0. For the MRH and MLS measurement it is given that $\delta_2=0.64\pm0.90$ and $\delta_3=0.47\pm1.14$. As the confidence interval does contain the hypothetical value 0, it is also reasonable to

conclude that the observed mean MRH and MLS did not differ from its hypothetical value sufficiently to have caused the original hypothesis to be rejected.
Finally, for RLH measurement we obtain

$$\delta_4 = 0.92 \pm \sqrt{18.47}\sqrt{1.03683/30}$$
$$= 0.92 \pm 0.80$$
$$= 0.12 \le \mu_{14}\text{-}\mu_{24} \le 1.72$$

Since zero is not included in the interval, it will be concluded at the one percent joint significance level that the means for the RLH measuring point subtest differ between the measuring devices. When the underbody assembly is located and secured in the checking fixture, clamping unit (heavy spring loaded hand clamp) apply too much force on the RLH control point. This causes deformation of sheet metal parts.

From these results, the confidence interval of the FRH, MRH, and MLS subtest mean difference. Thus, it concludes that the two measurement systems (CF and CMM) differ with respect to the mean subtest scores on RLH measurements, but not with respect to the three other measuring points.

3.2.4 Case Study 3: Comparison between OCMM and CMM

By taking the same steps, data was collected by taking $N_j=30$ measurements from each measurement systems on the same part for the p=4 master control locating points. The covariance matrix and its inverse matrix for OCMM and CMM are as follows:

$$\bar{\mathbf{d}} = \begin{bmatrix} 0.07 \\ 0.19 \\ 0.26 \\ 0.19 \end{bmatrix} \quad \mathrm{S} = \begin{bmatrix} 0.73609 & -0.06092 & -0.01655 & 0.08632 \\ -0.06092 & 0.59651 & -0.17269 & -0.11349 \\ -0.01655 & -0.17269 & 1.53145 & 0.05352 \\ 0.08632 & -0.11349 & 0.05352 & 1.30340 \end{bmatrix} \quad \mathrm{S}^{-1} = \begin{bmatrix} 1.37994 & 0.13507 & 0.03297 & -0.08098 \\ 0.13507 & 1.77329 & 0.19662 & 0.13739 \\ 0.03297 & 0.19662 & 0.67595 & -0.01292 \\ -0.08098 & 0.13739 & -0.01282 & 0.78508 \end{bmatrix}$$

$$T^2 = n(\bar{d}-\delta)' S_d^{-1}(\bar{d}-\delta)$$

$$= 30\begin{bmatrix} 0.09 & 0.19 & 0.26 & 0.19 \end{bmatrix} \begin{bmatrix} 1.37994 & 0.13507 & 0.03297 & -0.08098 \\ 0.13507 & 1.77329 & 0.19662 & 0.13739 \\ 0.06297 & 0.19662 & 0.67595 & -0.01292 \\ -0.08098 & 0.13739 & -0.01282 & 0.78508 \end{bmatrix} \begin{bmatrix} 0.07 \\ 0.19 \\ 0.26 \\ 0.19 \end{bmatrix}$$
$$= 5.265929$$

Taking α=0.01, it is found that $[p(n-1)/(n-p)]F_{p,n-p}(.01)=18.47$. Since $T^2 = 5.27<18.47$, H_0 is accepted and concluded that there is no mean difference between the measurements of the two measuring devices (OCMM and CMM). This means that the two measurement systems, OCMM and CMM, are very comparable to each other.

4. SUMMARY

By applying two-way multivariate analysis, the compatibility of three (CF, OCMM and CMM) measurement systems and also part-to-part variation has been analyzed. In our case study, it has been found that there is no interaction between measurement systems and part-to-part variation. However, there is a major unit-to-unit variation. This variation may come from assembly process or incoming parts. It needs further investigation. Analysis has also found that measurement systems are not comparable with each other. By decomposing the analysis further with paired comparison analysis, it is found that OCMM and CMM are comparable to each other and CF is not comparable to either OCMM or CMM. This suggests that CF needs recalibration to improve accuracy and repeatability.

REFERENCES

Baron, J. (1992). *Dimensional Analysis and Process Control of Body-In-While processes,* Ph.D. Dissertation, University of Michigan, Ann Arbor, USA.

Bennich, P., Cvesto, P. C., Soons, H. and Trapet, E.(1992). Calibration of Co-ordinate Measuring Machines, *WELL Technical Guidelines,* draft version, October.

Burdick, R.K., Borror C.M., and Montgomery, D.C. (2003). A review of methods for measurement systems capability analysis, *Journal of Quality Technology*, **35**, 342~354.

Folan, P. and Browne, J. (2005). A review of performance measurement: Towards performance management, *Computers in Industry*, **56**, 663~680.

Gimmi, K. J. (1993). Measurement Uncertainty, *ASQC Quality Congress Transactions,* Boston, 916-921.

Greer, D. (1988). On-line Machine Vision Sensor Measurements in a Coordinate System, *SME paper* # IO 88-289.

Holmes, D. S. and Mergen, A. E. (1993). Improving the Performance of the T^2 Control Chart, *Quality Engineering,* **5**, 619-625..

Hotelling, H. (1933). Analysis of a Complex of Statistical variables into Principal Components, *Journal of Educational Psychology,* **24**, 498-520.

Joung, S.H. and Byun, J.H. (2004). Stability Evaluation of In-Line Measurement System with Repeated Measurements, *J. of the Korean Institute of Industrial Engineers*, **30**, 36~43.

Knowles, G., Vickers, G. and Anthony, J. (2003). Implementing Evaluation of the Measurement Process in an Automotive Manufacturer: a Case Study, *Quality and Reliability Engineering International*, **19**, 397~410.

Lee, Myung Duk (1996). *Methodology for Measurement Systems Analysis and Dimensional Control Process of Automotive Body Manufacturing*, Ph.D. Dissertation, Wayne State University, Detroit, Michigan.

Lee, Seunghoon (1993). A Multivariate Calibration Procedure When the Standard Measurements is Also Subject to Error, J. of Korean Institute of Industrial Engineers, Vol. 19, No. 2, 35-41.

Mast, J. and Trip, A. (2005). Gauge R & R studies for destructive measurements, *Journal of Quality Technology*, **37**, 40~49.

Morris, A. S. (1991). *Measurement and Calibration for Quality Assurance,* Prentice Hall International (UK) Ltd.

Proud, P. and Ermer, D. S. (1993). A Geometrical Analysis of Measurement system Variations, *ASQC Quality Congress Transactions,* Oct., 929-935.

Wu, S. K. (1991). *A Methodology for Optimal Door Fit in Automobile Body Manufacturing,* Ph. D. Dissertation, The University of Michigan, Ann Arbor, USA.

Yang, K. (1996). Improving Automotive Dimensional Quality by Using Principal Component Analysis, *Quality and Reliability Engineering International*, **12**, 401-409.