

A Case Study on the Compatibility Analysis of Measurement Systems in Automobile Body Assembly

Lee, Myung Duk*

Research and Innovation Center, Ford Motor Co., MI 48202, USA

Lim, Ik Sung

Dept. of Industrial & Management Eng., Namseoul University, S. Korea

Sung, Chun Ja

Dept. of GIS Eng., Namseoul University, S. Korea

Abstract. The dimensional measurement equipment, such as Coordinate Measurement Machine (CMM), Optical Coordinate Measurement Machine (OCMM), and Checking Fixture (CF), take multiple dimensional measurements for each part in an automobile industry. Measurements are also recorded under different measurement systems to see if the responses differ significantly over these systems. Each measurement system (CMM, OCMM, and CF) will be considered as different treatments. This set-up provides massive amounts of process data which are multivariate in nature. Therefore, the multivariate statistical analysis is required to analyze data that are dependent on each other. This research provides step by step methodology for the evaluation procedure of the compatibility of measurement systems and clarify a systematic analyzation among the different measurement system's compatibility followed by number of case studies for each methodologies provided.

Key Words: *Compatibility, Checking Fixture, Optical Coordinate Measurement Machine, MANOVA*

1. INTRODUCTION

The general results of the last section will now be used to extend the analysis of variance for some common linear models to the case of multiple responses. Multivariate Analysis of Variance (MANOVA) model for comparing k population mean vectors $x_{ij} = \mu + \tau_i + \varepsilon_{ij}$, $i=1,2,\dots,k$ and $j=1,2,\dots,n$ in which parameter vector μ is a general level parameter common to all observations, τ_i is an effect due to a treatment or condition

* Corresponding author.
E-mail address: mlee4@ford.com

associated with the i^{th} population, and e_{ij} is a normal random variable with zero mean and Σ for all pairs of i and j . The measurements are assumed to be independent observations on the p -dimensional multinormal variates with mean vectors μ_1, \dots, μ_k under the different treatments and a common unknown covariance matrix Σ for all k conditions. Then the design matrix for the one-way analysis-of-variance model is

$$\zeta = \begin{bmatrix} \tau_{11} & \Lambda & \tau_{1p} \\ \Lambda & \Lambda & \Lambda \\ \tau_{k1} & \Lambda & \tau_{kp} \\ \mu_1 & \Lambda & \mu_p \end{bmatrix} \quad (1.1)$$

The hypothesis to be tested is that of equal-treatment-effect vectors:

$$H_0: \begin{bmatrix} \tau_{11} \\ \text{M} \\ \tau_{1p} \end{bmatrix} = \Lambda = \begin{bmatrix} \tau_{k1} \\ \text{M} \\ \tau_{kp} \end{bmatrix} \quad (1.2)$$

and also a vector of observations decomposed as this;

$$x_{ij} = \bar{x} + (\bar{x}_{ij} - \bar{x}) + (x_{ij} - \bar{x}_{ij}) \quad (1.3)$$

The decomposition in (1.3) leads to the multivariate analog of the univariate sum of squares breakup

$$\sum_{i=1}^k \sum_{j=1}^{n_i} (x_{ij} - \bar{x})(x_{ij} - \bar{x})' = \sum_{i=1}^k n_i (\bar{x}_i - \bar{x})(\bar{x}_i - \bar{x})' + \sum_{i=1}^k \sum_{j=1}^{n_i} (x_{ij} - \bar{x}_i)(x_{ij} - \bar{x}_i)' \quad (1.4)$$

and

$$\sum_{i=1}^k n_i - 1 = (k-1) + \left(\sum_{i=1}^k n_i - k \right) \text{ degrees of freedom} \quad (1.5)$$

One test of $H_0: \tau_1 = \tau_2 = \Lambda = \tau_k = 0$ involves generalized variance and rejection of H_0 if the ratio of generalized variances

$$\Lambda^* = \frac{|E|}{|H + E|} = \frac{\left| \sum_{i=1}^k \sum_{j=1}^{n_i} (x_{ij} - \bar{x}_i)(x_{ij} - \bar{x}_i)' \right|}{\left| \sum_{i=1}^k \sum_{j=1}^{n_i} (x_{ij} - \bar{x})(x_{ij} - \bar{x})' \right|} \quad (1.6)$$

is too small. The quantity $|E|/|H + E|$ corresponds to the F-test of H_0 : *no-treatment effects* in the univariate case. Multivariate analysis of variance was originally developed by Wilks through the generalized likelihood-ratio principle. That approach led to the test statistic (1.6) and Λ^* is the reciprocal of the product of all characteristic roots of $HE^{-1} + I$.

1.1 Case Study for Three Measurement Systems ($\mu_{CF} = \mu_{OCMM} = \mu_{CMM}$)

In our situation, three measurement systems ($k=3$) were administered to the same unit. The responses ($p=4$) were the four measurement points Front Right Hole (FRH), Middle Right Hole (MRH), Middle Left Slot (MLS), and Rear Left Hole (RLH) in automobile underbody assembly. Within each measurement system, observations were recorded on those dimensions for $N_j = 20$ times. From this data, the hypothesis of a

common mean vector will be tested for the four master control points of the three measurement systems population. The MANOVA table takes the following format (Table 1). Using equation(1.6)

$$\Lambda^* = \frac{|E|}{|H + E|} = 0.51239 \tag{1.7}$$

Since $n_i = 60$ measurements, $p = 4$ responses, and $k=3$ treatments, Distribution of Wilks' lambda indicates that an exact test (assuming normality and equal group covariance matrices) of $H_0 : \tau_1 = \tau_2 = \tau_3 = 0$ (no treatment effects) versus $H_1 : \text{at least one } \tau_i \neq 0$ is available. To carry out the test, the test statistic will be compared

$$\left(\frac{\sum n_i - p - 2}{p} \right) \left(\frac{1 - \sqrt{\Lambda^*}}{\sqrt{\Lambda^*}} \right) \sim F_{2p, 2(\sum n_i - p - 2); p \geq 1, k = 3} \tag{1.8}$$

$$= 5.360$$

with a percentage point of an F -distribution having $(8, 108)$ *df*. Since the test statistic exceeds that value $5.360 > F_{8, 108(.05)} = 2.086$, the hypothesis of a common mean vector must be rejected for the three measurement systems. The question of which devices and master control points have contributed to this rejection can be explained by the treatment of simultaneous confidence interval.

Table 1.1. The MANOVA Table

Sources of variation	Matrix of sum of squares and cross-products	Degree of freedom
Treatment	$\begin{bmatrix} 0.2376 & 0.6101 & -0.2454 & -0.1971 \\ 0.6101 & 2.2078 & -0.6864 & -0.5471 \\ -0.2454 & -0.6864 & 0.2583 & 0.2071 \\ -0.1971 & -0.5471 & 0.2071 & 0.1661 \end{bmatrix}$	3
Error	$\begin{bmatrix} 4.541 & 0.1909 & 0.08771 & -1.599 \\ 0.191 & 2.9815 & 0.26353 & 0.496 \\ 0.088 & 0.2635 & 3.21841 & 1.091 \\ -1.599 & 0.4964 & 1.09103 & 4.816 \end{bmatrix}$	56

2. MULTIVARIATE COMPARISON BY THE MANOVA

As in the univariate analysis of variance, rejection of some hypothesis on the parameters of the multivariate linear model does not indicate which treatments or treatment combinations are different and which could be considered as coming from common populations. In the multivariate model, it also needs to be determined which responses or response-treatment combinations may have led to the rejection.

For pairwise comparisons, the Bonferroni approach can be used to construct simultaneous confidence intervals for the components of the differences $\tau_i - \tau_j$. These

intervals are shorter than those obtained for all contrasts, and they require critical values only for the univariate t-statistic.

Let τ_{ih} be the h^{th} component of τ_i , since τ_i is estimated by $\bar{\tau} = \bar{x}_i - \bar{x}$,

$$t_{ij} = \bar{x}_{ih} - \bar{x}_{jh} \quad (2.1)$$

Because (2.1) gives the difference between two independent sample means, the two-sample t-based confidence interval is valid with an appropriately modified α . Notice that

$$\text{Var}(t_{ih} - t_{jh}) = \text{Var}(\bar{X}_{ih} - \bar{X}_{jh}) = (1/n_i + 1/n_j)\sigma_{hh} \quad (2.2)$$

where σ_{hh} is the h^{th} diagonal element of Σ . $\text{Var}(\bar{X}_{ih} - \bar{X}_{jh})$ is estimated by dividing the corresponding element of E by its degrees of freedom. That is,

$$\text{Var}(\bar{X}_{ih} - \bar{X}_{jh}) = \left(\frac{1}{n_i} + \frac{1}{n_j} \right) \frac{E_{hh}}{n-k} \quad (2.3)$$

where E_{hh} is the h^{th} diagonal element of E and $n = n_1 + \dots + n_k$. There are p variables and $k(k-1)/2$ pairwise differences, so each two sample t-interval will employ the critical value $t_{n-k}(\alpha/pk(k-1))$, is the number of simultaneous confidence statements.

2.1 Case Study (Confidence Intervals for Treatment Effects)

This case study shows that the average values for measuring devices differ. Also, the degree of difference depends on the type of machines compared. Here, again is the matrix of sum of squares and cross-products for E :

$$E = \begin{bmatrix} 4.541 & 0.1909 & 0.08771 & -1.599 \\ 0.191 & 2.9815 & 0.26353 & 0.496 \\ 0.088 & 0.2635 & 3.21841 & 1.091 \\ -1.599 & 0.4964 & 1.09103 & 4.816 \end{bmatrix}$$

$$\bar{x}_{1=cf} = \begin{bmatrix} 0.92 \\ 0.95 \\ -0.74 \\ -0.83 \end{bmatrix}, \bar{x}_{2=cm} = \begin{bmatrix} 0.76 \\ 0.57 \\ -0.58 \\ -0.70 \end{bmatrix}, \bar{x}_{3=ocmm} = \begin{bmatrix} 0.83 \\ 0.52 \\ -0.63 \\ -0.75 \end{bmatrix}, \bar{x} = \frac{n_1\bar{x}_1 + n_2\bar{x}_2 + n_3\bar{x}_3}{n_1 + n_2 + n_3} = \begin{bmatrix} 0.84 \\ 0.68 \\ -0.65 \\ -0.76 \end{bmatrix} \quad (2.4)$$

consequently,

$$t_1 = (\bar{x}_1 - \bar{x}) = \begin{bmatrix} 0.08 \\ 0.27 \\ -0.09 \\ -0.07 \end{bmatrix}, t_2 = (\bar{x}_2 - \bar{x}) = \begin{bmatrix} -0.08 \\ -0.11 \\ 0.07 \\ 0.06 \end{bmatrix}, t_3 = (\bar{x}_3 - \bar{x}) = \begin{bmatrix} 0.16 \\ 0.16 \\ 0.02 \\ 0.01 \end{bmatrix}$$

As an example, $t_{11} - t_{21} = 0.08 - (-0.08) = 0.16$. Since $n=60$, $p=4$ and $k=3$ for 95% simultaneous confidence statements we require $t_{57}(.05/4(3)2) = 2.87$. The 95% simultaneous confidence interval is

$$\begin{aligned} \tau_{11} - \tau_{21} \text{ belongs to } & \hat{t}_{11} - \hat{t}_{21} \pm t_{57}(.00208)\sqrt{(1/n_1 + 1/n_2)e_{11}/(n-k)} \\ & = 0.16 \pm 2.87(0.08926) \\ & = 0.16 \pm 0.26 \end{aligned} \quad (2.5)$$

These intervals were obtained and summarized in this form:

<i>cf-cmm</i>	<i>cmm-ocmm</i>	<i>cf-ocmm</i>
$-0.10 < \tau_{11} - \tau_{21} < 0.42$	$-0.33 < \tau_{21} - \tau_{31} < 0.19$	$-0.35 < \tau_{11} - \tau_{31} < 0.35$
$0.17 < \tau_{12} - \tau_{22} < 0.59$	$-0.16 < \tau_{22} - \tau_{32} < 0.26$	$0.22 < \tau_{12} - \tau_{32} < 0.64$
$-0.38 < \tau_{13} - \tau_{23} < 0.06$	$-0.17 < \tau_{23} - \tau_{33} < 0.27$	$-0.33 < \tau_{13} - \tau_{33} < 0.11$
$-0.39 < \tau_{14} - \tau_{24} < 0.13$	$-0.21 < \tau_{24} - \tau_{34} < 0.31$	$-0.34 < \tau_{14} - \tau_{34} < 0.18$

The hypothesis of equal master control points (FRH, MLS, and RLH) can be accepted at the 0.05 level for the CF & CMM and CF & OCMM. There is also no difference between CMM and OCMM. By applying simultaneous confidence intervals, it is easily verified that two intervals which are $\tau_{12} - \tau_{22}$ and $\tau_{12} - \tau_{32}$ do not contain zero. Thus, it is reasonable to conclude that there is a difference on measurement point, MRH, between CF& CMM and CF & OCMM.

3. PROFILE ANALYSIS

This section is concerned with questions related to similarity between vectors(profiles). Profile analysis pertains to situations where a battery of P treatments are administered to two or more groups of subjects. It is assumed that the responses for the different groups are independent of one another, but all responses must be expressed in similar units. In our situation, four master control points (responses) have been collected from automotive underbody assembly based on three measurement systems (treatment). Let the model for the i^{th} observation on the h^{th} response under treatment j be

$$\begin{aligned}
 x_{ijh} &= \zeta_{jh} + e_{ijh} & (3.1) \\
 i &= 1, \dots, N_j \text{ (samples)} \\
 j &= 1, \dots, k \text{ (treatment)} \\
 h &= 1, \dots, p \text{ (responses)}
 \end{aligned}$$

The parameter matrix is

$$\zeta = \begin{bmatrix} \zeta_{11} & \Lambda & \zeta_{12} \\ \Lambda & \Lambda & \Lambda \\ \zeta_{k1} & \Lambda & \zeta_{kp} \end{bmatrix} \quad (3.2)$$

The vector of residuals $e'_{ij} = [e_{ij1}, \dots, e_{ijp}]$ of the ij^{th} sampling unit has the multinormal distribution with null mean vector and some unknown nonsingular covariance matrix Σ .

To test parallelism of the treatment mean, the profile hypothesis will be tested.

$$\mathbf{H}_0 = \begin{bmatrix} \zeta_{11} - \zeta_{12} \\ \Lambda \\ \zeta_{1, p-1} - \zeta_{1p} \end{bmatrix} = \begin{bmatrix} \zeta_{21} - \zeta_{22} \\ \Lambda \\ \zeta_{2, p-1} - \zeta_{2p} \end{bmatrix} = \Lambda = \begin{bmatrix} \zeta_{k1} - \zeta_{k2} \\ \Lambda \\ \zeta_{k, p-1} - \zeta_{kp} \end{bmatrix} \quad (3.3)$$

The matrix statement of (3.3) is $H_0 : C_1 \zeta M_1 = 0$, where

$$C_1 = \begin{bmatrix} 1 & -1 & 0 & \Lambda & 0 \\ 0 & 1 & -1 & \Lambda & 0 \\ & & \Lambda & & \\ 0 & 0 & 0 & & \Lambda - 1 \end{bmatrix} \quad (3.4)$$

$$M_1 = \begin{bmatrix} 1 & 0 & \Lambda & 0 \\ -1 & 1 & \Lambda & 0 \\ 0 & -1 & \Lambda & 0 \\ & & \Lambda & \Lambda \\ 0 & 0 & \Lambda & -1 \end{bmatrix} \quad (3.5)$$

are the $(k-1) \times k$ and $p \times (p-1)$ cases of the transformation matrix. The test of H_0 amounts to a one-way multivariate analysis of variance on the $p-1$ difference of the observations of the adjacent responses from each sample unit.

The parameters of the distribution of the greatest characteristic root of HE^{-1} when H_0 is true are $s = \min(k-1, p-1)$, $m = (|k-p|-1)/2$, and $n = (N-k-p)/2$. And by the usual decision rule the parallelism hypothesis would be rejected for large values of the test statistic.

3.1 Case study (Parallelism for Measurement Systems)

To see the compatibility of three different measurement systems, parallelism can be conducted by profile analysis. The four responses (master control points) have been collected from the underbody assembly. Also, three measurement systems, CF, CMM, and OCMM, will be considered as three treatment. The general observations are arranged as in Table 3.1. Also, the population means $\mu_1' = [\mu_{11}, \mu_{12}, \mu_{13}, \mu_{14}]$ represent the average responses to four treatments for the first group. The mean vectors for each measurement system are plotted in Figure 3.1.

Table 3.1. Observations for the Profile Analysis

	Responses			
	1	2	...	p
Treatment 1	x_{111}	x_{112}	Λ	x_{11p}
	Λ	Λ	Λ	Λ
	$x_{N,11}$	$x_{N,12}$	Λ	$x_{N,1p}$
Treatment 2	x_{121}	x_{122}	Λ	x_{12p}
	Λ	Λ	Λ	Λ
Treatment k	x_{1k1}	...	Λ	x_{1kp}
	Λ	...
	$x_{N_k k1}$...	Λ	$x_{N_k kp}$

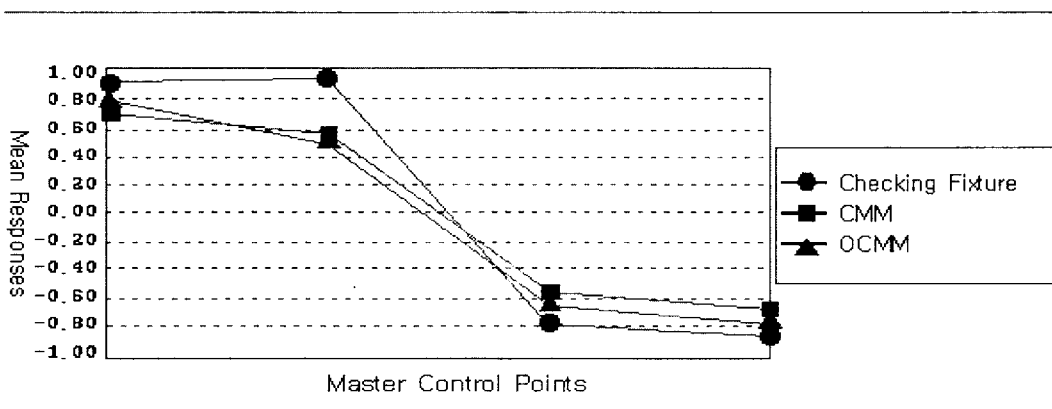


Figure 3.1. Mean Vectors Profiles for Measurement System

The analysis is started with the parallelism test, and for that step the observations are transformed to the difference of measuring points FRH & MRH & MLS, and MLS & RLH. The greatest root of $|\mathbf{H} - \lambda\mathbf{E}| = 0$ will be denoted by C_g , where $g = \min(k-1, p)$. The critical values have been tabulated for the greatest root of $|\mathbf{H} - \lambda(\mathbf{H} + \mathbf{E})| = 0$, and so the charts must be entered with the test statistic $\theta_g = C_g / (1 + C_g)$. The hypothesis and error sums of squares and product matrices are

$$\mathbf{H} = \begin{bmatrix} 1.225 & -2.039 & 0.0911 \\ -2.615 & 5.673 & -2.360 \\ 1.920 & -2.360 & 5.853 \end{bmatrix}$$

$$\mathbf{E} = \begin{bmatrix} 7.141 & -2.615 & 1.920 \\ -0.615 & 5.673 & -0.230 \\ 1.920 & -2.360 & 5.853 \end{bmatrix} \quad \mathbf{E}^{-1} = \begin{bmatrix} 0.17261 & 0.0673 & -0.02949 \\ 0.673 & 0.23804 & 0.07390 \\ -0.02949 & 0.07390 & 0.21032 \end{bmatrix}$$

From the characteristic equation of \mathbf{HE}^{-1} , its largest root will be $C_g = 0.7948$. The statistic for the percentage-point chart is $\theta_g = 0.4428$. The requisite parameters are $s=2$, $m=0$, and $n=26.5$. If the $\alpha = 0.05$ level is chosen, the critical value for θ_g can be found approximately $\chi_{0.05; 2, 0, 26.5} = 0.18$. Since the sample value exceeds that number, it was rejected at the 5 percent level. It is reasonable to conclude that the three measurement systems were not comparable to each other.

4. SUMMARY

By applying one-way multivariate analysis, the compatibility of three (CF, OCMM and CMM) measurement systems with same unit has been analyzed. The analysis reveals that the measurement systems are not comparable to each other. To find out which system and which master control points have caused this rejection, the Bonferroni approach is used to construct simultaneous confidence intervals. The major differences between the

several multivariate population MANOVA and the two populations case is that in the former all group differences cannot be summarized by a single number such as T^2 . Instead, one must look for the possible existence of significant between-population variation of more than one dimension. By applying these concepts, it is found that the MRH control point has put a significant impact on the rejection of the compatibility study for the three measurement systems. In the two populations case, the centroids always fall along a single dimension so all structures are concentrated because two points (centroids) must fall along a straight line. When there are more than populations, the population centroids (vector, profile) need not fall along a straight line. By using profile analysis, it has been proved that the three measurement systems are not comparable to each other. By using this methodology, one can track down and correct root causes without any difficulty. This result could save a tremendous amount of effort, product cycle time and improve the quality of automotive body assembly.

REFERENCES

- Baron, J. (1992). *Dimensional Analysis and Process Control of Body-In-While processes*, Ph.D. Dissertation, University of Michigan, Ann Arbor, USA.
- Bennich, P., Cvesto, P. C., Soons, H. and Trapet, E.(1992). Calibration of Co-ordinate Measuring Machines, *WELL Technical Guidelines*, draft version, October.
- Burdick, R.K., Borrer C.M., and Montgomery, D.C. (2003). A review of methods for measurement systems capability analysis, *Journal of Quality Technology*, **35**, 342~354.
- Folan, P. and Browne,J. (2005). A review of performance measurement: Towards performance management, *Computers in Industry*, **56**, 663~680.
- Gimmi, K. J. (1993). Measurement Uncertainty, *ASQC Quality Congress Transactions*, Boston, 916-921.
- Greer, D. (1988). On-line Machine Vision Sensor Measurements in a Coordinate System, *SME paper # IO 88-289*.
- Holmes, D. S. and Mergen, A. E. (1993). Improving the Performance of the T^2 Control Chart, *Quality Engineering*, **5**, 619-625.
- Hotelling, H. (1933). Analysis of a Complex of Statistical variables into Principal Components, *Journal of Educational Psychology*, **24**, 498-520.
- Joung, S.H. and Byun, J.H. (2004). Stability Evaluation of In-Line Measurement System with Repeated Measurements, *J. of the Korean Institute of Industrial Engineers*, **30**, 36~43.

- Knowles, G., Vickers, G. and Anthony, J. (2003). Implementing Evaluation of the Measurement Process in an Automotive Manufacturer: a Case Study, *Quality and Reliability Engineering International*, **19**, 397~410.
- Lee, Myung Duk (1996). *Methodology for Measurement Systems Analysis and Dimensional Control Process of Automotive Body Manufacturing*, Ph.D. Dissertation, Wayne State University, Detroit, Michigan.
- Lee, Seunghoon (1993). A Multivariate Calibration Procedure When the Standard Measurements is Also Subject to Error, *J. of Korean Institute of Industrial Engineers*, Vol. 19, No. 2, 35-41.
- Mast, J. and Trip, A. (2005). Gauge R & R studies for destructive measurements, *Journal of Quality Technology*, **37**, 40~49.
- Morris, A. S. (1991). *Measurement and Calibration for Quality Assurance*, Prentice Hall International (UK) Ltd.
- Proud, P. and Ermer, D. S. (1993). A Geometrical Analysis of Measurement system Variations, *ASQC Quality Congress Transactions*, Oct., 929-935.
- Wu, S. K. (1991). *A Methodology for Optimal Door Fit in Automobile Body Manufacturing*, Ph. D. Dissertation, The University of Michigan, Ann Arbor, USA.
- Yang, K. (1996). Improving Automotive Dimensional Quality by Using Principal Component Analysis, *Quality and Reliability Engineering International*, **12**, 401-409.