Optimum Configuration of Multiple Antennas for the Combinded System with Tx. Diversity and Beamforming

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ABSTRACT

The transmit diversity as well as beamforming can increase the performance of wireless communication systems. It is well known that the requirement for the spacing between the neighboring antennas in transmit diversity and beamforming is contradictive to each other. Therefore it is necessary to find the optimum configuration of multiple antennas for getting the maximum performance under the condition that the total number of antennas at transmitter site and the total power of transmitter are fixed. In this paper, the procedure for finding the optimum configuration of multiple antennas was derived through searching the maximum capacity and BER in the combined system with the transmit diversity (Tx diversity) and beamforming.

Key Words : Antenna group, Array gain, Beamforming, Diversity gain, Transmit diversity(Tx diversity)

Ⅰ. Introduction

Antenna spacing is usually required to be large enough, about ten or more times wavelength, to obtain low correlation or independent fading channels in Transmit diversity (Tx diversity) schemes[1], while the beamforming scheme needs the antenna spacing small enough, for example, half wavelength, in order to make sure the signals from all antennas correlated or coherent to achieve spatial directivity[2].

For the contradictive requirements of the antenna spacing, beamforming and diversity scheme were independently exploited at the first research stage, but recently the effort for combining beamforming with Transmit diversity (Tx diversity) at the downlink has been researched[3][4].

When the total number of antennas at the base station is fixed, antennas can be divided as the finite number of groups for Tx diversity, where the spacing between each group is equal to or greater than the required spacing for low correlation between groups. But the spacing between antennas in each group is half wavelength for beamforming. If the number of groups for Tx diversity is increased, the Tx diversity gain can be enhanced, but the beamforming gain will be decreased because the number of antennas in each group gets smaller in the condition of the fixed number of total antennas at the base station. In the reverse case, the opposite result will be given. Therefore it is expected that there will be some optimum configuration between the number of group for Tx diversity and the number of antennas per group. As a performance criteria for finding the optimum configuration of multiple antennas, the theoretical analysis result about capacity and BER expressed with the number of group for Tx diversity and the number of antennas per each group were exploited. In the theoretical analysis result about capacity and BER, it is possible to find the optimum point through the differentiation with respect to one variable, because two variables of the number of group for Tx diversity and the number of antennas per each group are related when the number of total antennas is fixed. Varying the number of groups, the optimum configuration of antenna array for the combined

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system with beamforming and Tx diversity was found for getting some desired performance in this paper.

Ⅱ. The Combined system with Transmit Diversity and Beamforming

It has been published in [3] [4] that the combined system with Tx diversity and beamforming shows better performance than both diversity only and beamforming only system.

The general model of the combined system with Tx diversity and beamforming is illustrated in Fig. 1. In this combined system, the transmit antennas are divided into several groups for Tx diversity. And the antennas in each group will make beamforming. Through encoder in Fig.1, the bit sequence S is mapped into a vector of symbols

$$s' = [s_1, s_2, \ldots, s_2]^T \tag{1}$$

Here, symbol s_k is weighted by the kth weight vector w_k and transmitted using corresponding k th antenna group, in which there are n antennas for beamforming.

The weight vector of k th beamformer w_k can be given as

$$w_k = [w_{k1}, \ldots, w_{kn}]^T \tag{2}$$

where w_{kj} means the weight vector which gives the weighting effect to the signal at j th antenna in k th group.

The transmit signals can be expressed as

$$S_k = s_k w_k, \qquad k = 1, 2, \ldots.m \tag{3}$$

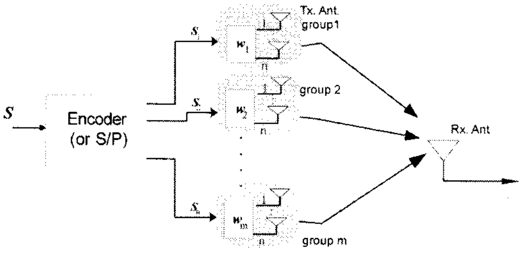

Fig. 1. General model of the combination of Tx diversity and beamforming.

where

$$\sum_{k=1}^{m} |w_k|^2 = 1 \tag{4}$$

So the received signal can be rewritten as

$$y = \sum_{k=1}^{m} h_k w_k s_k + n \tag{5}$$

where

$$h_k = [h_{k1}, \ldots, h_{kn}] \tag{6}$$

Here, h_{kj} is the fading effect corresponding to ith antenna in the kth transmit antenna group. n is the channel noise and is modeled as a zero mean, complex Gaussian random variable with variance σ^2 per dimension.

In this combined system with Tx diversity and beamforming, diversity gain as well as array gain can be acquired for the performance enhancement.

Ⅲ. Optimum Configuration of Multiple antennas for the combined systems

When the total number of antennas in the base station is fixed as M, the more the number of beamforming group , the larger the Tx diversity gain can be obtained, but the less the beamforming gain reversely. Now it is necessary to find the optimum configuration of multiple antennas for getting not only diversity gain but also beamforming gain. In the following section, we will analyze the performance of the combined scheme via BER and capacity to find the optimum configuration of multiple antennas.

3.1 Optimum configuration according to BER Analysis

The SNR of the channels in combined systems which illustrate in Fig. 1 is given by

$$SNR(h_1, \ldots, h_m) = \frac{E\left|\sum_{k=1}^{m}\sum_{j=1}^{n} h_{kj} w_{kj} s_k\right|^2}{E|n|^2} \tag{7}$$

Assuming the signal which was transmitted in each antenna has same power, the SNR of the channel is given by

$$SNR(h_1,\ldots,h_m) = SNR\left|\sum_{k=1}^{m} h_k w_k\right|^2 \tag{8}$$

where SNR is the signal to noise power ratio,

$$SNR = \frac{E\{|S_k|^2\}}{E\{|n|^2\}} = \frac{E_s}{\sigma^2} \tag{9}$$

To maximize the SNR, setting

$$w_{kj} = \frac{h_{kj}^*}{\sqrt{\sum_{k=1}^{m} |h_{kj}|^2}} \tag{10}$$

It is assumed that the channel fadings for antennas in each group satisfy

$$|h_{k1}|^2 = |h_{k2}|^2 =, \ldots, = |h_{km}|^2 \tag{11}$$

Therefore, the maximum value of SNR can be rewritten as:

$$\begin{aligned} SNR(h_1,\ldots,h_m) &= SNR\sum_{k=1}^{m}\sum_{j=1}^{n} |h_{kj}|^2 \\ &= nSNR\sum_{k=1}^{m} |h_{kj}|^2 \end{aligned} \tag{12}$$

Following the exactly same steps as in [5], the optimal receiver in the combined system, referred to as the maximum likelihood (ML) receiver with the maximal ratio combining (MRC) scheme, achieves error probability satisfying

$$\begin{aligned} p_e &\le \exp\left\{-\frac{SNR(h_1,\ldots,h_m)}{m}\right\} \\ &= \exp\left\{-\frac{nSNR\sum_{k=1}^{m} |h_{kj}|^2}{m}\right\} \end{aligned} \tag{13}$$

Let

$$z_i = |h_i|, \quad i = 1,2,\ldots,m \tag{14}$$

then

$$p_e \le \exp\left\{-\frac{nSNR\sum_{k=1}^{m} z_i^2}{m}\right\} \tag{15}$$

z_i is statistically independent, Rayleigh distributed random variable. Thus, their joint density is simply given by the product of their individual densities

$$f(z_1,\ldots,z_m) = \prod_{i=1}^{m} 2z_i \exp\{-z_i^2\} \tag{16}$$

Therefore, we readily obtain that the averaged bit error rate (BER) of the combined system, by averaging (15) with respect to (16), yields

$$\begin{aligned} \bar{pe} &\le \int_0^{\infty}\cdots\int_0^{\infty} \exp\left\{-\frac{nSNR\sum_{k=1}^{m} z_i^2}{m}\right\} \\ &\quad \times \prod_{i=1}^{m} 2z_i \exp\{-z_i^2\} dz_1 \ldots dz_m \\ &= \frac{1}{(1+\frac{nSNR}{m})^m} \end{aligned} \tag{17}$$

Throughout the paper, we provide upper bounds on the error probability. However, these bounds are tight for the mid to high SNR range[6], which is the interesting SNR range when the reliable communication is required.

3.2 Optimum Configuration according to Capacity Analysis

Reference[7] described the capacity analysis method of multipath channels. According to the transmitted signals in multipath channels, wireless channels can be classified as single channel and separate channel cases. In the combined systems, the serial to parallel transformer(S/P) at the transmitter is used in the single channel case. Or, if we use encoder at the transmitter, it turns to separate channel case.

1) Single channel case: In case of single channel, the capacity of the combined system can be derived as:

$$\begin{aligned} C &= W\log_2\left(1 + \frac{\left(\sum_{i=1}^{m} s_{beam}\right)^2}{2\sigma^2}\right) \\ &= W\log_2(1 + MSNR) \end{aligned} \tag{18}$$

where $s_{beam} = \sum_{j=1}^{n} s_j$, here s_j is the transmit signal in the jth antenna in each group, and W is the bandwidth which is available in the system.

Equation (18) shows that, for a fixed number of transmit antennas with an average SNR, system capacity is constant. The divided number of groups does not affect the system capacity.

2) Separate channel case: The capacity of the combined system under separate channel case can be given as:

$$
\begin{aligned}
C &= mW\log_2\left(1+\frac{\left(\sum_{i=1}^{m} s_i\right)^2}{2\sigma^2}\right) \qquad (19)\\
&= W\log_2\left(1+\frac{n}{m}SNR\right)\\
&= mW\log_2\left(1+\frac{n}{m^2}SNR\right)
\end{aligned}
$$

From equation (19), it is easy to find that, for a fixed number of transmit antennas with an average SNR, system capacity only changes according to the number of divided group. Then, we can propose the optimum configuration using this equation.

Ⅳ. Numerical Examples for Finding Optimum Configuration

In the above theoretical analysis result about capacity and BER, it is possible to find the optimum point through the differentiation with respect to one variable, because two variables of the number of group for Tx diversity and the number of antennas per each group are related as M=nm when the number of total antennas is fixed as M. Actually the total number of antennas at a base station is limited. In this paper, it was assumed that there are 12 antennas at base station.

When the channel condition, like SNR, can be prejudged as some average value, the configuration of grouping transmit antennas at transmitter can be obtained in that condition. Fig. 2 and Fig. 3 show the numerical example when SNR is set as 5.

In Fig. 2, the smallest value of BER can be found when the number of groups achieves an optimal value 4. It is also observed in Fig. 3 that largest capacity can be achieved when the number of groups achieves the same optimal value 4 in separate channel case.

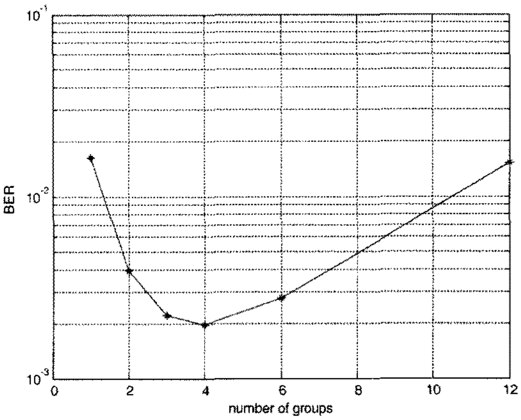

Fig. 2. BER according to the number of groups with average SNR.

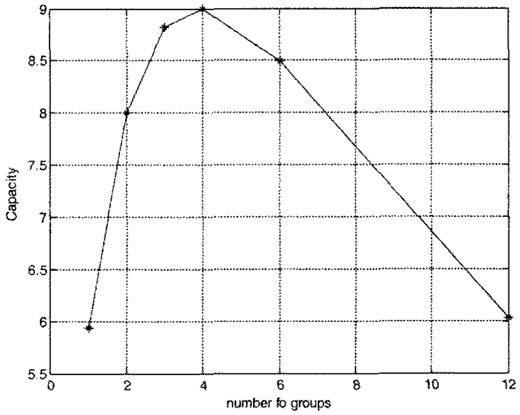

Fig. 3. Capacity according to the number of groups with average SNR in separate channel case.

As shown before, because system capacity is constant in single channel, we will decide the optimum configuration using BER analysis method. However, in separate channel case, we can use either of the two methods and the result is same.

However, in the practical situation, the channel condition changes sometimes. Therefore the optimal number of groups should be changed based on channel condition. Fig. 4 and Fig. 5 illustrate the numerical example with different SNR by analyzing the BER performance and capacity according to the number of groups. As expected, the optimal number of groups can be derived differently at different SNR.

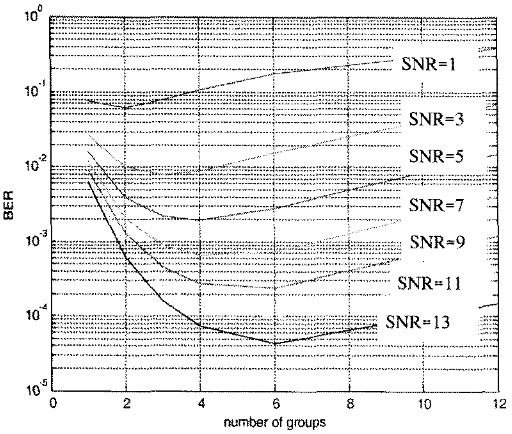

Fig. 4. BER according to the number of groups with different SNR.

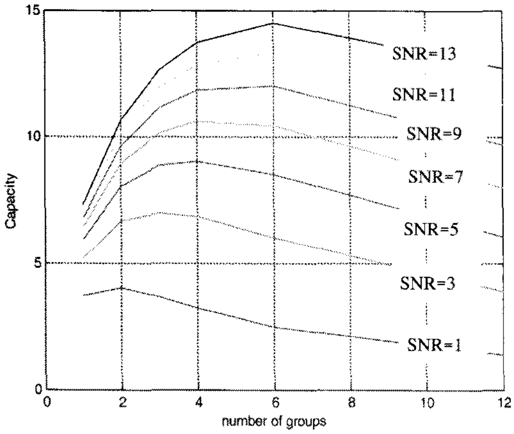

Fig. 5. Capacity according to the number of groups with different SNR in separate channel case.

V. Conclusion

Earlier researches on the combined system with transmit diversity and beamforming have manifested the benefits of diversity gain as well as beamforming gain. However, when the total number of multiple antennas at base station is fixed, there is a contradictive issue in assigning the number of antennas to Tx diversity part and beamforming part each other. In this paper, we derived the optimum configuration of multiple antennas for exploiting Tx diversity gain as well as beamforming gain to get the best performance and highest capacity.

References

〔1〕 W. C. Y. Lee, "Antenna Spacing Requirement for a Mobile Radio Base-Station Diversity," *Bell System Tech. Journal*, Vol.50, pp.1859-1876, July/Aug. 1971.

〔2〕 R. B. Ertel, P. Cardieri, K. Sowerby, T. S. Rappaport, J.Reed, "Overview of spatial channel models for antenna array communication systems", *IEEE Personal Communication Magazine*, Vol.5, pp.10-22, Feb. 1998.

〔3〕 Zhong ding Lei, Chin F. P. S., Ying- Chang Liang, *IEEE 56th, Conf. on Vehicular Technology Proceedings*, Vol.4, pp.24-28, Sept. 2002.

〔4〕 Zhu F., Lim M. S., "Combined beamforming with space- time blockcoding using double antenna array group," *IEE Electronics Letters*, Vol.40, Issue 13, pp.811-813, Jun. 2004.

〔5〕 Ilan Hen, "MIMO architecture for wireles communication," *Intel technology J.*, Vol.10 Issue 02, pp.157-166, May 2006.

〔6〕 A.J. Viterbi and J.K. Omura, "Principles of Digital Communication and Coding," pp.54-64, *McGraw-Hill, Inc.*, NewYork, USA.

〔7〕 Rashid-Farrokhi, F., Liu, K.J.R. and Tassiulas, L., "Transmit," *IEEE Journal on Selected Areasin Communications*, vol.16, Issue8, pp.1437-1450, Oct. 1998.

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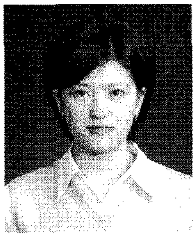

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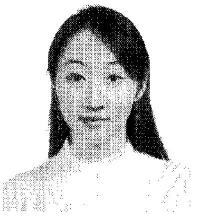

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