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Performance Evaluation and Convergence Analysis of a VEDNSS LMS Adaptive Filter Algorithm

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Abstract

This paper investigates noise reduction performance and performs convergence analysis of a Variable Error Data Normalized Step-Size Least Mean Square(VEDNSS LMS) algorithm. Adopting VEDNSS LMS results in higher system complexity, but noise is reduced providing fast convergence speed. Mathematical analysis demonstrates that tap coefficient misadjustment converges. This is confirmed by computer simulation with the proposed algorithm.

Keywords: *Step-size, Filter coefficient, Mean square error, Convergence, Differentiation*

I. Introduction

The Least Mean Square (LMS) algorithm is widely used as an adaptive filtering algorithm, since it is robust and straightforward. When the input signal correlation is large, convergence rate is low and the mean square error (MSE) is large. That is, the closer the white signal is to the input signal, the higher the speed of convergence. Many algorithms were proposed to guarantee fast convergence and stability. Examples include robust variable step-size (RVSS) LMS algorithm and error data normalized step-size (EDNSS) LMS algorithm[1],[2].

Step-size μ in the LMS algorithm controls convergence speed of the filter coefficient and determines excess mse in the convergence state.

Since the convergence rate is in proportional to step-size μ, a large μ is often selected to ensure fast convergence. However, when a large μ is selected, MSE is also large. The basic concept of the above two algorithms is that how each tap coefficient is separated from the optimal coefficient. If the difference between the optimal coefficient and each tap coefficient is large, a large μ is selected, conversely a small μ is selected.

In reference [1],[2] it can be seen that RVSS LMS and EDNSS LMS algorithm outperform the LMS algorithm in convergence speed and mse. In reference [3],[4] it has been reported that adopting variable step-size decreases the convergence rate and misadjustment error. Using this concept and once more adjusting step-size differentiating objective function with respect to step-size, we modified EDNSS LMS as VEDNSS LMS. This paper analyzes the convergence to the optimal filter coefficient and derives the expression for the excess filter coefficient error. The results derived from the analysis are verified numerically through computer simulation.

II. EDNSS LMS Algorithm

The fractional quantity in (1) may be viewed as a time-varying step-size. Clearly, it is controlled

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by normalization of both the error and the input data vectors.

$$\mathbf{w}_{k+1} = \mathbf{w}_k + \frac{\mu}{\alpha\|\mathbf{e}_L(k)\|^2 + (1-\alpha)\|\mathbf{x}_k\|^2}\mathbf{x}_k e_k \quad (1)$$

$$e_k = d_k - \mathbf{w}_k^T \mathbf{x}_k \quad (2)$$

$$\|\mathbf{e}_L(k)\|^2 = \sum_{n=0}^{L-1} |e(k-n)|^2 \quad (3)$$

where subscript k is the iteration number, $\mathbf{w}$ is the vector of adaptive filter weights, d is the desired signal, $\mathbf{x}$ is the adaptive filter input vector, μ is a positive scalar called the step-size and superscript T denotes vector/matrix transpose. α is a positive constant between one and zero, e_k is the output error and (3) is the squared norm of the error vector estimated over its last L values.

III. Proposed VEDNSS LMS Algorithm

We now consider a performance function [5] such as (4). It consists of the squared-error and the additional exponential term to reflect the property that as the error is smaller the step-size decreases, conversely the step-size increases.

In the VEDNSS LMS algorithm, the filter coefficient ($\mathbf{w}_k$) is determined according to (5) and step-size (μ_k) is controlled by (7), (8) and (9). It can be seen that the step-size is controlled by the squared-norm of both the error and the input data vectors in the denominator and by the element in the numerator that is attained by differentiating (4) for step-size.

$$J_k = e_k^2 + \frac{2}{\gamma\|\mathbf{x}_k\|^2}(e^{-(1/2)\gamma e_k^2\|\mathbf{x}_k\|^2} - 1) \quad (4)$$

$$\mathbf{w}_{k+1} = \mathbf{w}_k + \frac{\mu_k}{\alpha\|\mathbf{e}_L(k)\|^2 + (1-\alpha)\|\mathbf{x}_k\|^2}\mathbf{x}_k e_k \quad (5)$$

$$e_k = d_k - \mathbf{w}_k^T \mathbf{x}_k \quad (6)$$

$$\mu_k = \mu_{k-1} - \frac{\rho}{2}\frac{\partial}{\partial \mu_{k-1}} J_k \quad (7)$$

$$= \mu_{k-1} + \rho \frac{(1 - e^{-(1/2)\gamma e_k^2\|\mathbf{x}_k\|^2})}{\alpha\|\mathbf{e}_L(k)\|^2 + (1-\alpha)\|\mathbf{x}_{k-1}\|^2} e_k e_{k-1} \mathbf{x}_k^T \mathbf{x}_{k-1} \quad (8)$$

$$\mu_k = \begin{cases} \mu_{\max} & if\ \mu_k > \mu_{\max} \\ \mu_{\min} & if\ \mu_k < \mu_{\min} \\ \mu_k & otherwise \end{cases} \quad (9)$$

where the parameter ρ is a small positive constant that controls the adaptive behaviour of the step-size μ_k and γ is a damping parameter that determines algorithm sensitivity.

IV. Convergence Analysis of VEDNSS LMS

First, we can derive the following equation from (5).

$$\|\tilde{\mathbf{w}}_{k+1}\|^2 - \|\tilde{\mathbf{w}}_k\|^2 = \frac{-2\mu_k e_k^2}{A} + \frac{\mu_k^2 e_k^2}{A^2}\mathbf{x}_k^T\mathbf{x}_k \quad (10)$$

where

$\tilde{\mathbf{w}}_k = \mathbf{w}_k - \mathbf{w}^o$, $\mathbf{A} = \alpha\|\mathbf{e}_L(k)\|^2 + (1-\alpha)\|\mathbf{x}_k\|^2$,

$\mathbf{w}^o$: optimal coefficient

In order for (10) to converge, it must be smaller than zero. So (11) can be derived.

$$\mu_k < \frac{2\{\alpha\|\mathbf{e}_L(k)\|^2 + (1-\alpha)\|\mathbf{x}_k\|^2\}}{\|\mathbf{x}_k\|^2} \quad (11)$$

Second, the expectation of the filter coefficient error ($E[\tilde{\mathbf{w}}_k]$) must converge toward zero.

$$E[\tilde{\mathbf{w}}_{k+1}] = E[\tilde{\mathbf{w}}_k][\mathbf{I} - \frac{\mu_k}{A}\mathbf{R}_k] \quad (12)$$

where $\mathbf{R}_k = E[\mathbf{x}_k\mathbf{x}_k^T] = \mathbf{M}\Lambda\mathbf{M}^{-1}$ and positive definite.
($\mathbf{M}$: modal matrix of $\mathbf{R}_k$, Λ : eigenvalue matrix of $\mathbf{R}_k$)

Here, according to Schwarz-inequalty (13) can be attained.

$$\left\|\mathbf{M}[\mathbf{I} - \frac{\mu_k}{A}\Lambda]\mathbf{M}^{-1}\right\| \le \|\mathbf{M}\|\|\mathbf{M}^{-1}\|\left\|\mathbf{I} - \frac{\mu_k}{A}\Lambda\right\| \quad (13)$$

$\|M\|\|M^{-1}\|$: conditional number (k(M)) (14)

Solving (13) for one norm case, (15) can be derived.

$$\frac{A}{\lambda_{\min}}\left(1-\frac{1}{k(\mathbf{M})}\right)<\mu_k<\frac{A}{\lambda_{\max}}\left(1+\frac{1}{k(\mathbf{M})}\right) \tag{15}$$

Where $\lambda_{\min}$ is the minimum value of Λ and $\lambda_{\max}$ is the maximum value of Λ.

Finally, in order to converge step-size(μ_k) must be in the common range of (11),(15).

V. Simulation

The simulation results presented in this section will show the potential performance of the VEDNSS LMS. Simulation results are presented for the noisy sinusoid represented as (16). Here $n(t)$ is Gaussian noise with variance 0.3, signal frequency is 50 Hz, sampling frequency is 10 kHz, and filter length is 50.

$$x(t)=\sin(2\pi ft)+n(t) \tag{16}$$

Figure 1(a) is a noisy signal. Figure 1(b),(c),(d) are the results where noise is removed using RVSS LMS, EDNSS LMS, and VEDNSS LMS, respectively. It can be seen that if a noisy signal is reconstructed using the VEDNSS LMS algorithm, noise is reduced compared to the case where any other method is used.

Figure 2(a) is the FFT result of a signal where the Gaussian noise with variance 0.3 is added and Figure 2 (b),(c),(d) is the FFT result for the case where the noise is removed through the RVSS LMS, EDNSS LMS, and VEDNSS LMS adaptive filter, respectively. We can see that other frequency components except the 50Hz frequency are markedly reduced through the VEDNSS LMS adaptive filtering.

Figure 3 is the variation of the step-size using the VEDNSS LMS algorithm. It can be seen that the step-size increases considerably in the initial iteration state since the error is large. However, as the iteration number increases the error and the input vector inner product are much small compared to the denominator of (8), so the step-size is nearly constant. Here maximum step size is set to 0.26 and minimum is set to 0.001.

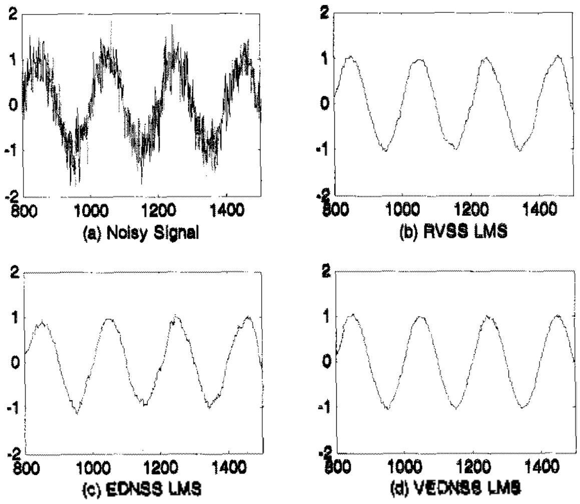

Fig. 1. Noise filtering results using three adaptive algorithms.

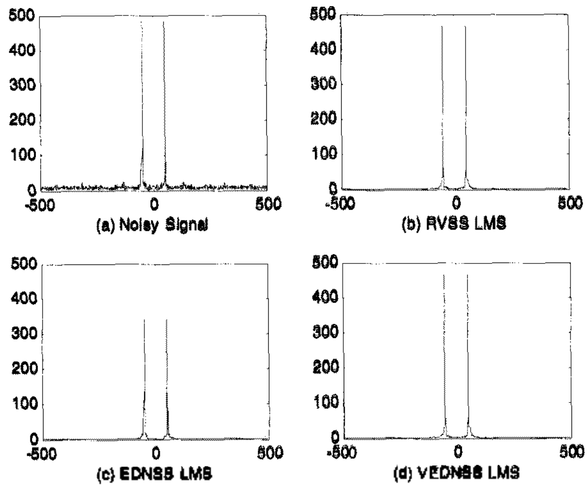

Fig. 2. FFT results of noisy signal and filtered signal.

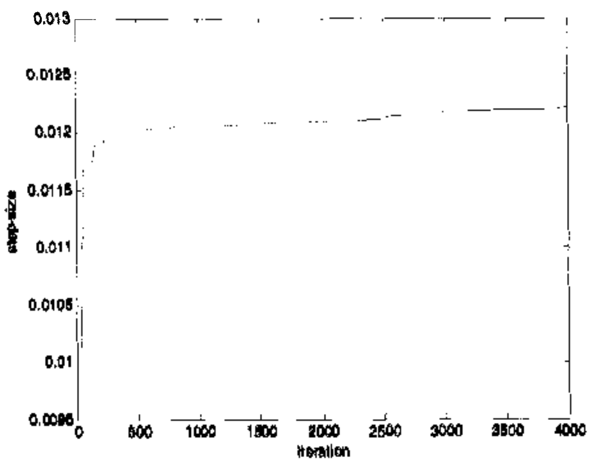

Fig. 3. Step-size variation for VEDNSS filter.

In Table 1, it can be seen that mse of the VEDNSS LMS after 3500 samples (required to count mse in a steady state) is smaller than that of the existing algorithms.

Figure 4 shows the mse variation when the noise variance is changed (0.05, 0.1, 0.15, 0.2, 0.25, 0.3). It can be seen that mse of VEDNSS LMS is smaller than that of other methods.

Figure 5 is the mse result of RVSS LMS, EDNSS LMS, and VEDNSS LMS according to various frequency components (0.1~0.5 KHz) with interval 0.05 KHz. It can be seen that mse of VEDNSS LMS is smaller than that of other methods.

Figure 6 shows the MSE behaviours of three algorithms. As seen from Figure 6, the proposed VEDNSS LMS algorithm provides similar convergence speed with RVSS LMS algorithm, and faster convergence rate than EDNSS LMS algorithm. Therfore, we can conclude that the proposed VEDNSS LMS has convergence speed as fast as two existing algorithms and smaller mse than that of two methods.

Ⅵ. Conclusion

This paper examines a variable error data normalized step-size LMS adaptive filtering algorithm. The algorithm overcomes the disadvantages of a fixed step-size μ in the existing algorithms. Simulation results show a mean square error improvement under a Gaussian noise environment even after reaching a steady state attaining the convergence rate as fast as the existing two adaptive filtering algorithms.

Future work will concentrate on improving the convergence rate. Experiments will be conducted under a non-stationary noise environment.

Acknowledgements

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Table 1. MSE of three adaptive algorithms.

($\times 10^{-3}$)

	RVSS LMS	EDNSS LMS	VEDNSS LMS
MSE	3.7	3.9	3.5

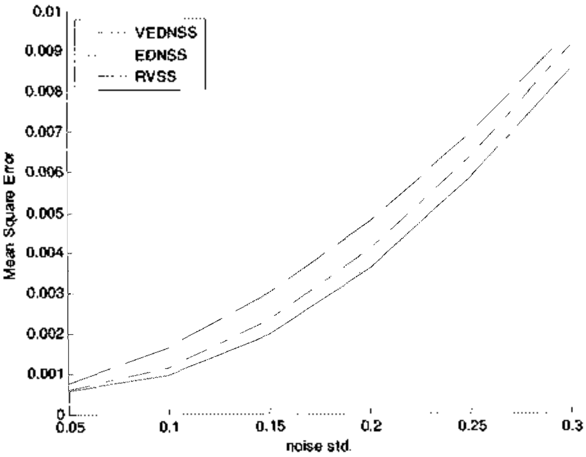

Fig. 4. MSE variation of adaptive algorithms with respect to several noise variances.

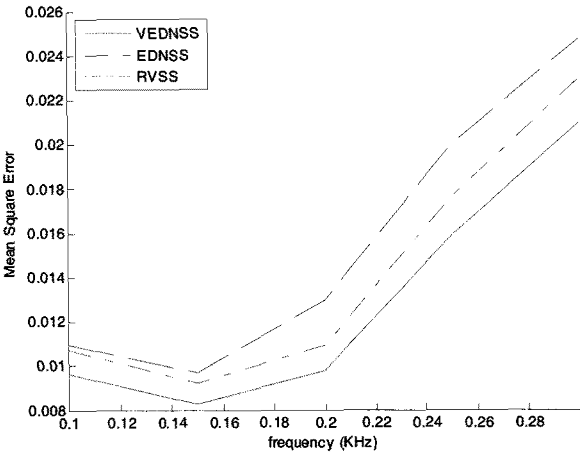

Fig. 5. MSE variation of adaptive algorithms with respect to several frequency components.

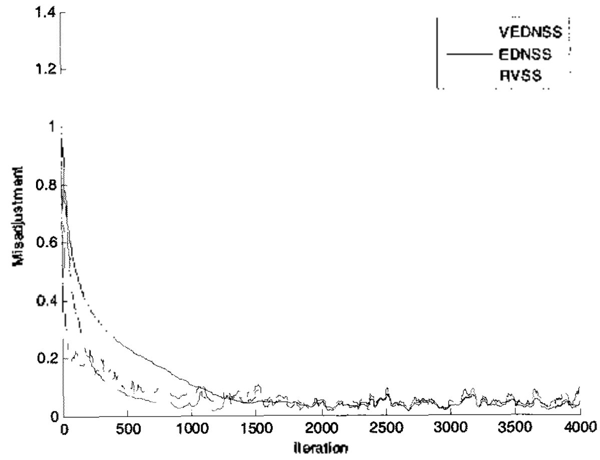

Fig. 6. Weight error of three adaptive algorithms.

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[Profile]

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