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FEM Analysis of Factors Influencing the Beat Frequency of Japanese Temple Bells

Takuji Nakanishi*, Hajime Ogushi* Atsushi Yarai*
*Department of Electrical and Electrcnics Engineering, Faculty of Engineering, Osaka Sangyo University
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Abstract

The beat frequency is very important for the sound of Buddhist temple bells, but its concrete origins have not yet been fully clarified. In our research, we considered the beat of the bell at Hojobo Temple (Kanagawa Prefecture). Although its beat frequency has already been measured as 1.6 Hz, no satisfactory explanation has been offered for the factor that determines this value. In our previous research, we investigated the "Doza," the point where the bell is struck, and the "Obi," the vertical stripes around the bell, both of which are circumferential asymmetrical factors that can be visually recognized. Our investigations were carried out by using the Finite Element Method. These factors, however, could not sufficiently explain the beat frequency. Here, we first investigate the "Nyu," the many small projections on the bell, and the deviation between the centers of the inner and outer diameters of the bell. These two factors, however, were also found tobe insufficient explanations of the beat frequency. Through subsequent investigation, however, we finally clarified that the beat frequency's origin lies in the local dimension variation in the "Komazume," which is the bottom part of the bell as well as its thickest part.

Keywords: *Beat frequency, Buddist temple bell, Finite elemn method, Komazume*

1. Introduction

The sound of Buddhist temple bells has long been loved in Japan for its power to give people a serene state of mind. The beat is very important to each bell's distinct sound. It is generally understood that the sound of a bell's beat is generated by formal or material asymmetrical factors on the bell's circumference; however, its concrete origins in various Buddhist temple bells have not yet been fully clarified [1].

In our research, we considered the beat of the bell at Hojobo Temple (Isehara City, Kanagawa Prefecture), which has a typical sound of the bells found in old Japanese temples [1, 2, 3].

Corresponding author: Takuji Nakanishi (naka@eic.osaka-sandai.ac.jp)
Department of Electrical and Electrcnics Engineering, Faculty of Engineering, Osaka Sangyo University, 3-1-1 Nakagaito, Daito Osaka, 574-8530, Japan

Figure 1 shows the outline, the main dimensions, and the names of the principal parts of the Hojobo temple bell [4]. Figure 2 shows the actual sound wave of the bell at Hojobo Temple [5]. A regular beat can be observed in the figure, and its beat frequency is calculated at about 1.6 Hz. Until now, however, our research has not been able to adequately explain the origin of thisbeat frequency [1, 6, 7, 8, 9]. In our previous research, we used the finite element method (FEM) to analyze the visually recognizable asymmetrical factors of the bell around its circumference as the origins of the beat. These factors include the "Doza," where the bell is struck, and the "Obi," the vertical stripes around the bell. The beat frequency obtained by these two factors, however, was 0.4 Hz at most, which is far from the actual value.

In this paper, we begin our search for the origins of the beat by first investigating the "Nyu," the many small projections on the bell; our second investigation looks into the deviation between the centers of the inner and outer diameters of the bell [8]. Neither of these factors, however, can sufficiently explain the beat frequency. As our third investigation into the origins of the beat, we examine the local dimensional variations in the "Komazume," which is the bottom part of the bell [9]. This third investigation clarifies that the origin of the examined bell's beat frequency, or by extension that of other bells, is due to local dimensional variations in the "Komazume." More concretely, the beat frequency can be controlled by slightly increasing or decreasing the thickness of the Komazume. Such local dimensional variations in the Komazume, in terms of thickness, can be easily observed on the actual Komazume of the Hojobo temple bell.

II. Analysis Model

The FEM analysis model of the bell at the Hojobo Temple was structured as follows. First, its two-dimensional FEM cross-sectional shape was made. The full model was configured by rotating its cross-sectional shape by 360 degrees.

Figure 3 shows the analysis model. Each element was composed of 20 contact points. The elements of the Komazume and Doza parts were madesmaller than the other parts. As is shown, although the real Doza shape is circular, the FEM simulated shape of this miniaturized bell was configured in a quadrilateral shape on condition that the mass of each Doza is the same. The number of circumferential divisions was 60, and the two small upper areas were selected as fixed areas, as shown in the figure. The analysis software is Marc K7.3. The Hojobo temple bell is

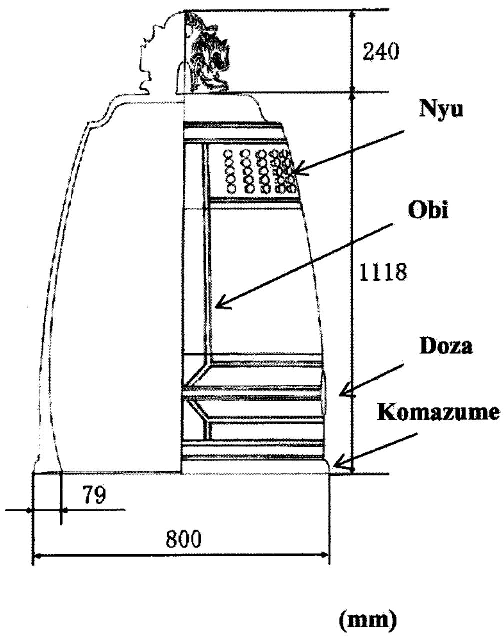

Fig. 1. Hojobo Temple's bell.

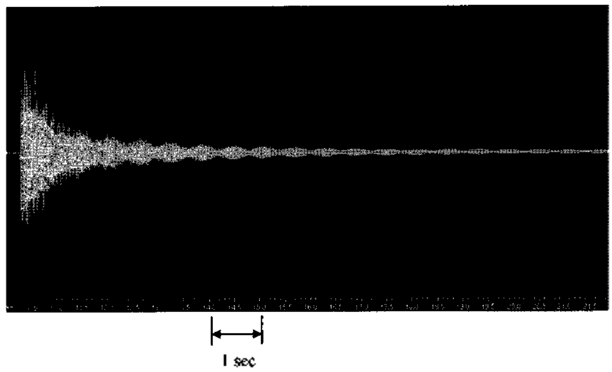

Fig. 2. Actual sound wave of the bell at Hojobo Temple.

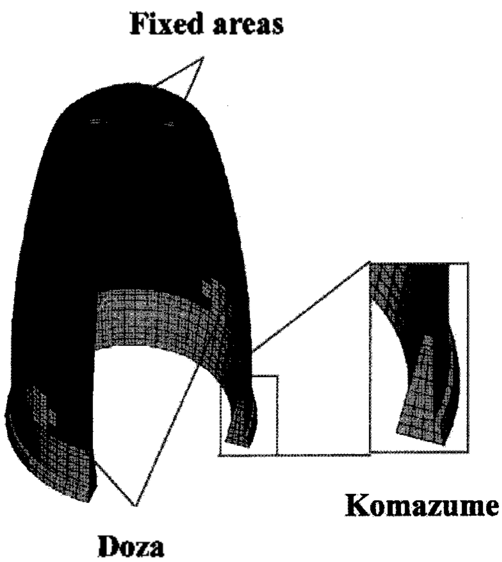

Fig. 3. Analysis model of the bell at Hojobo Temple.

made of bronze, and Table 1 shows the three material constants for bronze that are needed for the analysis [10].

The FEM is an analysis method to solve the characteristic equation to obtain the eigenvalues of, in the case of the present subject, the vibration frequencies as well as the vibration modes. Concretely, the discrete equation for the contact point displacement vector of each finite element can be represented as follows [11].

$$M\ddot{a}+C\dot{a}+Ka+Ka+f=0 \qquad (1)$$

Here, M is the whole mass matrix, K is the whole stiffness matrix, $C\dot{a}$ is a damping term, and f is a load vector. Eliminating both $C\dot{a}$ and f from this equation, and substituting the periodical solution $a=\bar{a}\exp(j\omega t)$ into it, the following characteristic equation can be obtained.

$$|-\omega^2M+K|=0 \qquad (2)$$

The solutions for the vibration frequencies as well as the vibration modes can be obtained by solving Equation (2).

Table 2 compares the measured and FEM-analyzed vibration frequencies of each mode for the Hojobo temple bell. As shown in the table, the differences between them are around 3-5%. The reason for this can be considered the differences in the material constants and in the details of the partial shapes. Such differences, however, are not important in understanding the bell's acoustics.

Table 1. Bronze material constants of bell.

Young's Modulus	8.08×10^{10} N/m^2
Poisson's ratio	0.358
Mass density	8.60×10^{3} kg/m^3

Table 2. Vibration frequencies.

Mode	FEM analyzed	Measured
4-0	143.0	137.8
6-0	319.0	301.8
8-0	395.4	378.6

(Hz)

III. Fundamental Principles for Generation of the Bell's Beat

It has already been clarified that the fundamental principles involved in the generation of a bell's beat, or the beat of similar instruments, are grounded in formal or material circumferential asymmetrical factors [1]. Figure 4 exemplifies these principles for the generation of the 4-0 mode beat when the asymmetrical factor is the Doza. As shown in the figure, if the bell has this circumferential asymmetrical factor, two vibration modes occur. In one mode, the location of the maximum vibration amplitude includes the Doza, while in the other one it does not include the Doza. There is a slight difference in vibration frequency between these two vibration modes, in which the former frequency is slightly higher than the latter frequency. The beat occurs due to the difference between these two modes' frequencies. Here, we name the fundamental vibration mode that involves the Doza the 4-0H mode, while the one that doesn't involve the Doza is named the 4-0L mode. The angle betweeneach loop location for these two

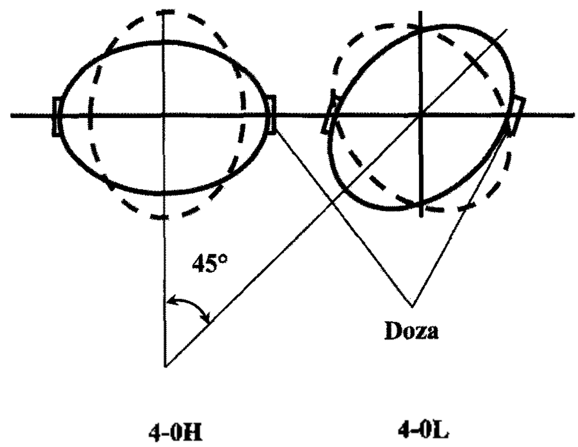

Fig. 4. Vibration aspects of the bell for 4-0L and 4-0H modes.

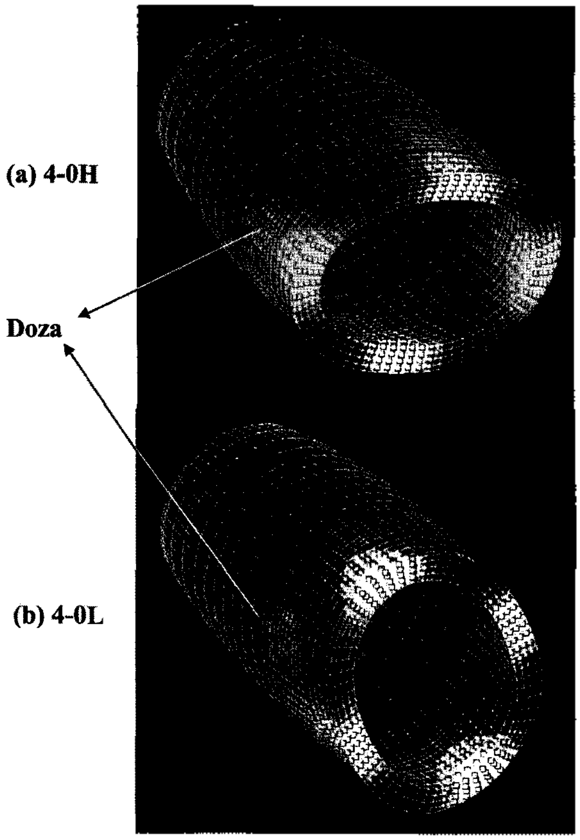

Fig. 5. Vibration aspects of the bell for 4-0L and 4-0H modes obtained by FEM analysis.

modes from the center of the bell is 45° as shown in the figure. The results shown in Figure 5, obtained by FEM analysis whose vibration mode is 4−0, proves the above principle. As clearly shown in the figure, the 4−0H vibration mode (Fig.5 (a)) is the vibration mode whose maximum vibration amplitude includes the Doza, while the 4−0L vibration mode (Fig.5 (b)) is the one that does not include the Doza. The angle of each loop location for these two modes from the center of the bell is definitely 45°.

IV. Influence of Actual Circumferential Asymmetrical Factors on Beat Frequency

In addition to the Doza and Obi, we investigated, as possible origins of the Hojobo temple bell's 1.6−Hz beat frequency, two other circumferential asymmetrical factors of the bell that could be visually recognized. These were the "Nyu," the many small projections on the bell, and the deviation between the centers of the inner and outer diameters of the bell. The finite element method (FEM) model of the Hojobo temple bell was constructed in the same way as that described in our previous research [1].

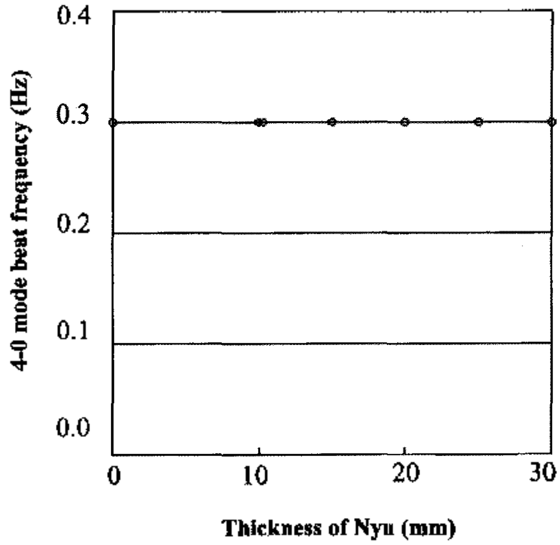

Fig. 6. Relationship between the thickness of the Nyu and 4-0 mode beat frequency.

4.1. Influence of "Nyu" on Beat Frequency

Figure 6 shows the relationship between the thickness of the Nyu and the 4−0 mode beat frequency obtained by FEM analysis. Here, the FEM analysis model of the Nyu was structured so that the cross−sectional area corresponds to the real one of the Hojobo temple bell, whose model includes the Doza but not the Obi. As shown in the figure, the 4−0 mode beat frequency of 0.3 Hz clearly corresponds with that of the Doza [1] and does not depend on the thickness of the Nyu at all. This result shows that the "Nyu" had absolutely no influence on beat frequency.

4.2. Influence of Deviation between the Centers of the Inner and Outer Diameters of the Bell on Beat Frequency

We next analyzed the influence of the deviation between the centers of the inner and outer diameters of the bell on beat frequency. Figure 7 shows how this deviation is defined as the value r. Since the average thickness of Hojobo's bell is 12 mm, we analyzed the influence of the deviation up to a maximum 6 mm. Two cases were investigated: with two Dozas and with one Doza attached to the bell.

Figure 8 shows the relationship between the deviation length r and the 4–0 mode beat frequency. Although figure 8clarifies that the deviation is indeed one of the origins of the beat frequency, the beat frequency reaches 0.6 Hz at most, which is far from its real value of 1.6 Hz. Therefore, the deviation between the centers of the inner and outer diameters cannot be the controlling factor in beat frequency.

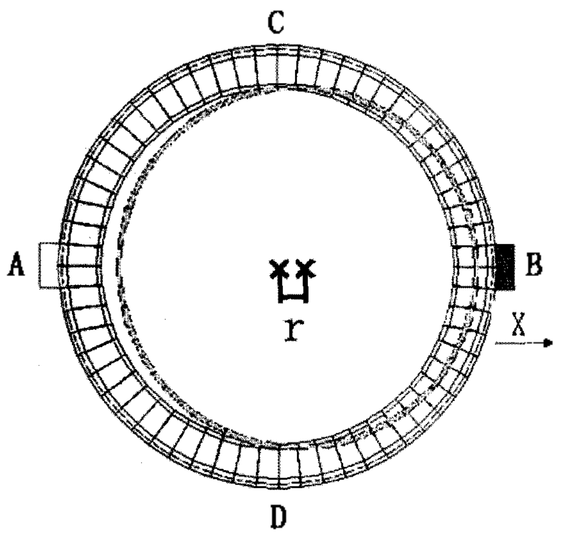

Fig. 7. Definition of the deviation between the centers of the inner and outer diameters of the bell (A~D are Doza locations).

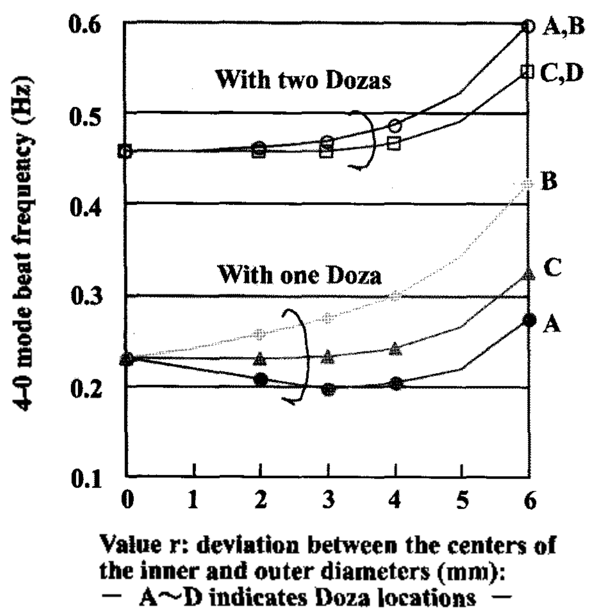

Fig. 8. Relationship between deviation r and 4-0 mode beat frequency.

V. Influence of Dimensional Variation near the Komazume on Beat Frequency: When the Number of Dimension-Varied Locations is one

It was clarified through the above analyses that none of the obtained results could fully explain the real 4–0 mode beat frequency of 1.6 Hz for the Hojobo temple bell. Consequently, it became necessary to investigate other asymmetrical factors on the circumference of the bell as possible explanations of the beat frequency's origins. By newly measuring the Komazume's bottom thickness at different positions on the actual Hojobo bell, it was found that these values scatter from a minimum of 73 mm to a maximum of 83 mm. This section presents the results of our investigations into the influence of the dimensional variations at the Komazume's bottom area on beat frequency.

Figure 9 shows the model used for the dimensional variations of the Komazume's bottom area. Based on

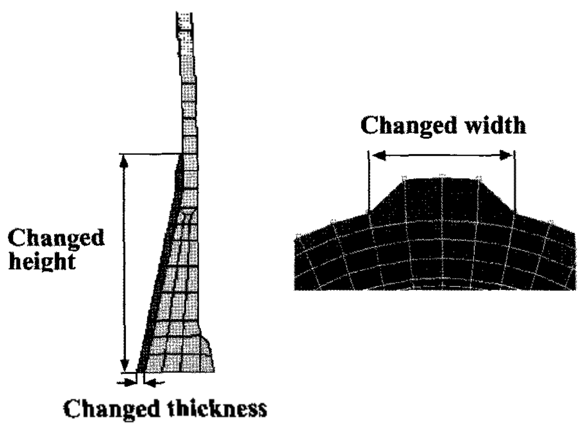

Fig. 9. Definitions for the dimensional variations of "height," "thickness" and "width" at the Komazume.

the locally changed thickness of the Komazume, an irregular region forms over a certain portion of the bell, beginning from the bottom. Three factors were defined to quantify the dimensional changes in the Komazume: "height" of the irregular region from the bottom of the bell, "thickness" of the irregular region, and "width" of the irregular region as a portion of the bell's circumference.

Here, two cases were analyzed: 1) one location and 2) two locations of dimensional change on the circumference (i.e. irregular regions). The three variable factors ("height," "thickness" and "width") for these dimensionally changed regions were not changed simultaneously but separately. For instance, when the "height" was changed, the changed values for both "thickness" and "width" of the studied part were fixed to center values. These center values of change for "height," "thickness," and "width" were set to 198 mm, plus or minus 4 mm, and 231.4 mm, respectively.

When the height or the width of the dimensionally changed region (s) was varied, the model was analyzed under two thickness conditions. Here, these conditions are termed "thicker change," where the thickness value was fixed to +4 mm from the Komazume's center value, and "thinner change," where the thickness value was fixed to −4 mm from the Komazume's center value. (Note: as can be seen in Fig. 9, the absolute thickness of the irregular region naturally decreases from bottom to top; however, in our model the amount of changed thickness was kept to a constant 4 mm.) On the contrary, when the thickness of the dimensionally changedregion was varied, the height and the width were set to their center values of change at 198 mm and 231.4 mm, respectively.

Figure 10 shows the relationship between the changed thickness of the Komazume's bottom part and the 4−0 mode vibration frequency. In this model, although two Dozas were attached at the same heights from the bottom of the bell as those of the real bell, their circumferential locations were varied for a few typical cases. None of the results, however, showed dependence on the circumferential locations.

As shown in the figure, the "Thicker changed; H mode" and "Thinner changed; L mode" frequencies changed much more than the "Thicker changed; L mode" and "Thinner changed; H mode" frequencies, respectively. (Recall that H mode refers to vibration involving the Doza, while L mode refers to vibration without the Doza.) The reasons for these results can be easily understood by referring to the fundamental principles discussed in section 2.

From the results of Figure 10, the relationship between the changed thickness of the Komazume's bottom part and the 4−0 mode beat frequency could be obtained as shown in Figure 11. The target 4−0 mode beat frequency of 1.6 Hz appeared when the changed thicknesses of the Komazume's bottom part were around 3 mm and 3.6 mm for the thinner and thicker changes, respectively.

On the contrary, we actually measured the Komazume's bottom part thickness values on the whole circumference of the bell at Hojobo Temple by using a vernier caliper. As a result, it was clarified that the values scattered from a minimum 73 mm to a maximum

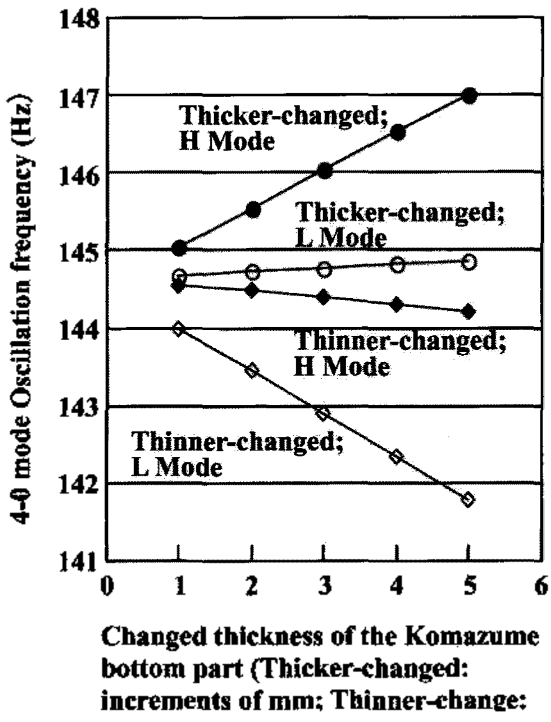

Fig. 10. Relationship between changed thickness of Komazume's bottom part and 4-0 mode"@vibration frequency.

83 mm. Therefore, it is easily known that the changed thickness values of 3~3.6 mm obtained by FEM analysis in each direction are within those actually measured in the real Hojobo temple bell.

Here, to avoid any misunderstanding related to the method of "modeling" the size variations around the Komazume part, we explain in more detail why the measured data themselves were not directly used for the FEM model. As is well known, the method used to construct these bells is casting. Consequently, this process surely produces unevenness (scattering) in the values for the bell's various dimensions, including the thickness of the Komazume's bottom part. Moreover, this resulting unevenness never shows any regularity. Therefore, the possible measured values in this case wereonly the minimum and maximum values obtained by using vernier calipers as the measuring tool (even if any other tools were used, the various sizes on the bell would always be too random for use in constructing an FEM model).

Figure 12 shows a similar relationship between the changed width of the Komazume's bottom part and the 4−0 mode beat frequency. Similarly, as in the case of changing the thickness of the Komazume's bottom part, it was clarified that the target 4−0 mode beat frequency of 1.6 Hz appeared when the changed width of the Komazume's bottom part was roughly within 100~400 mm.

As in both of the cases above, it was clarified that the target 4−0 mode beat frequency of 1.6 Hz appeared when the changed height of the Komazume's bottom part was roughly over 100 mm as shown in Figure 13.

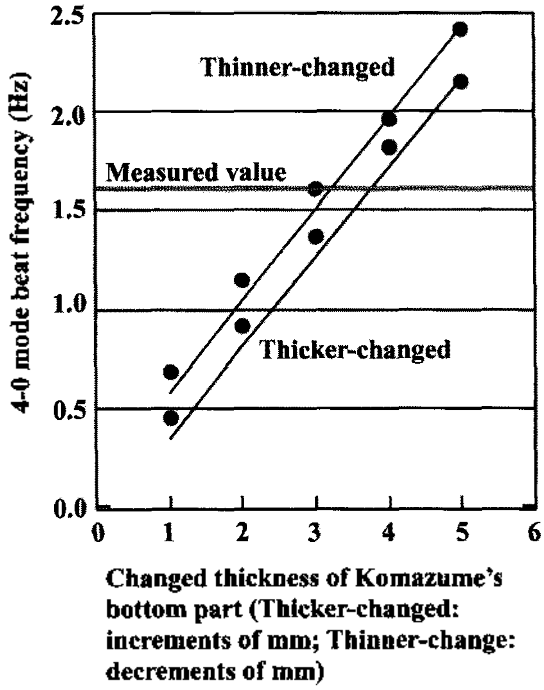

Fig. 11. Relationship between changed thickness of Komazume's bottom part and 4-0 mode beat frequency.

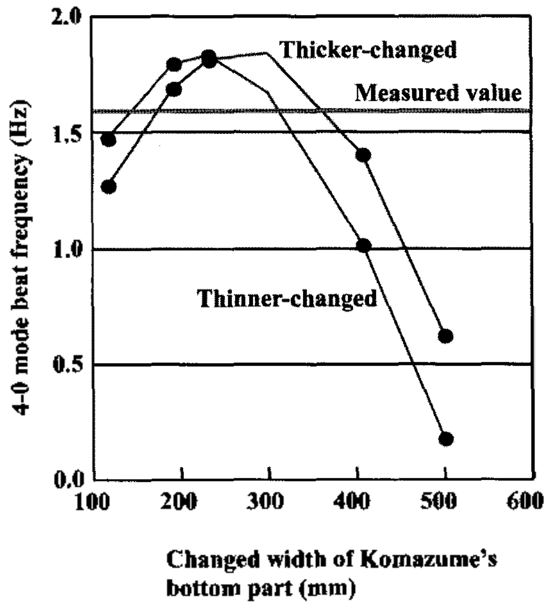

Fig. 12. Relationship between changed width of Komazume's bottom part and 4-0 mode beat frequency.

VI. Influence of Dimensional Variation near the Komazume on Beat Frequency: When the Number of Dimension-Varied Locations is Two

This section shows the results for investigations similar to those shown above for the case when the number of dimension−changed locations is two. Figure 14 shows four (A~D) bottom views of the bell we investigated, each representing a different configuration of the two dimension−change locations relative to the locations of the two Dozas.

Figure 15shows the relationship between the thicker-changed thickness at each location of the Komazume's bottom part and the 4-0 mode beat frequency. The symbols A, B, C, D in thefigure correspond to the configurations shown in Figure 14. Here, the height and width at the dimension-changed locations were fixed to 198 mm and 231.4 mm, respectively. Figure 16 shows a similar relationship between the thinner-changed thickness for each location of the Komazume's bottom part and the 4-0 mode beat frequency.

Both Figures 15 and 16show that the target 4-0 mode beat frequency of 1.6 Hz appeared when the changed thicknesses were around 1.5~2 mm for the thinner- and thicker-changed locations of the Komazume's bottom part. These 1.5~2 mm values are almost half those for the case when the number of dimension-changed locations is one, as shown in Figure 11. These results can thus better explain the

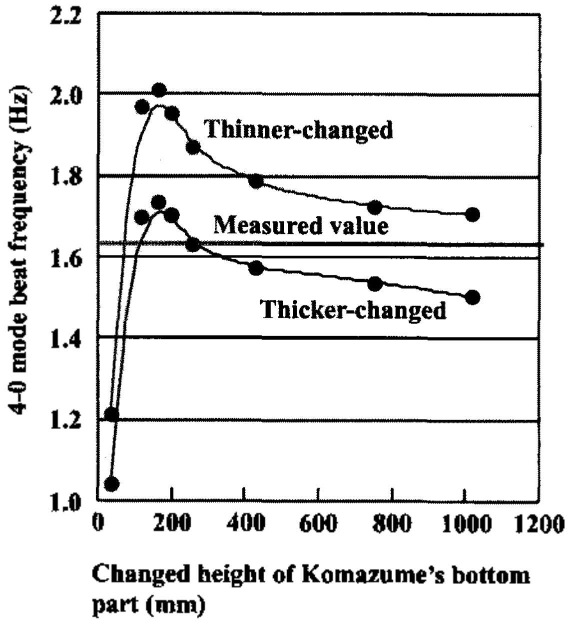

Fig. 13. Relationship between changed height of Komazume's bottom part and 4-0 mode beat frequency.

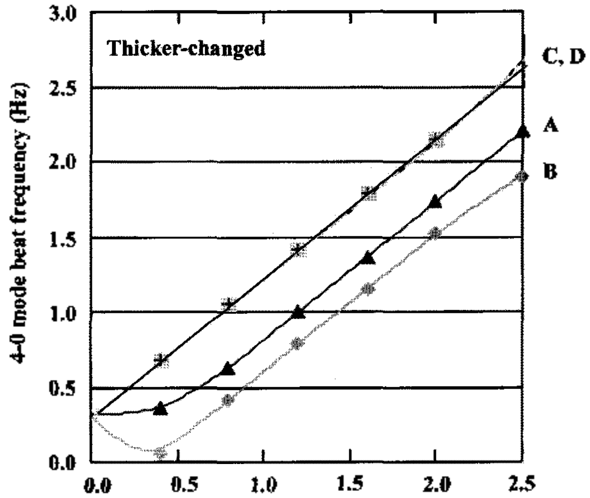

Fig. 15. Relationship between the thicker-changed thickness at each location of the Komazume's bottom part and 4-0 mode beat frequency.

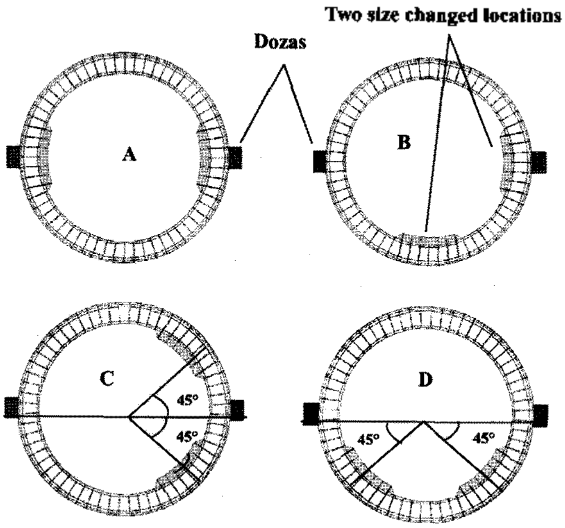

Fig. 14. Four cases (A~D) investigated when the number of dimension-scattered locations is two (bottom views of bell).

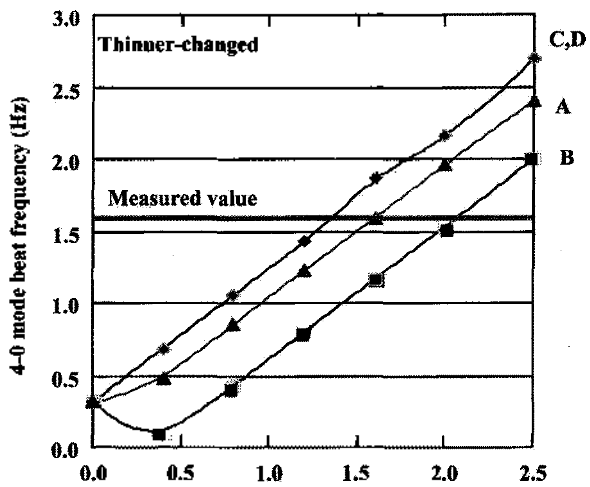

Fig. 16. Relationship between thinner-changed thickness at each location of the Komazume's bottom part and 4-0 mode beat frequency.

origins of the actually measured beat frequency than those for the case when the number of dimension—changed locations is one, since the thickness of the real Komazume's bottom part randomly scatters from a minimum 73 mm to a maximum 83 mm, as described above. In other words, the model with two points of variation more closely approximates the Komazume structure of the actual bell.

The random thickness variation in the real Komazume's bottom part, which ranges within 10 mm, can be regarded by extension as the consequence of having a number of dimension—varied locations.

Therefore, we can conclude that the origin of the Hojobo temple bell's beat frequency of 1.6 Hz is mainly due to the slight variation in the thickness of the Komazume part. Generally, Japanese temple bells have similar fundamental shape factors, which include those of the Doza, Nyu, Obi and Komazume. From our investigation, therefore, the origin of the beat of Japanese temple bells is assumed to be generally the same as that found for the Hojobo temple bell.

Finally, based on the investigations described above, we give the reason why the vibration mode for the measured beat frequency of 1.6 Hz corresponds to the 4—0 vibration mode. Of course, all of the FEM simulated raw data for Figs. 6, 8~13, 15, and 16, as well as Fig. 13 in Ref. [1], were obtained after confirming each vibration mode from the simulated vibration motions, which was the 4—0 mode. On the other hand, as shown in Ref. [1], the vibration center position for each vibration mode differs from those of the others (the higherthe mode number, the higher the vibration center position measured from the bottom of the bell). The Doza's center position on the Hojobo temple bell is located about 240 mm from the bottom, which is very close to that of the 6—0 vibration mode. Furthermore, as described in Ref. [1], the Doza's thickness is 11.5 mm, which can be considered sufficient and much more conspicuous as the bell's asymmetrical factor than the local size variations at the Komazume, which is the thick bottom part of the bell. The FEM simulated 6—0 mode beat frequency, however, is 0.65 Hz at most (that of the 8—0 vibration mode is 0.53 Hz), which is far from 1.6 Hz. Considering these factors and the investigations in the paper, it can be concluded that the measured 1.6 Hz beat frequency is mainly due to the local size variations at the Komazume and, moreover, that the vibration mode is the 4—0 mode.

VII. Conclusions

We investigated the origins of the Japanese temple bell's beat frequency by using FEM analysis, adopting Hojobo Temple's bell as an example. In searching for these origins, we first investigated circumferential asymmetrical factors of the bell that are visually recognizable, such as the Nyu and the deviation between the inner and outer diameters of the bell. Neither of these factors, however, could sufficiently explain the origin of the beat frequency.

Our subsequent research has clarified that the origin of the 1.6—Hz beat frequency of the Hojobo bell, or by extension that of other Japanese temple bells, is mainly due to the local size variations at the Komazume, which is the thick bottom part of the bell. More concretely, the beat frequency is controlled by slightly increasing or decreasing the thickness at local regions of the Komazume. Such local dimensional variations in the Komazume can be easily observed on the actual Komazume of the Hojobo temple bell. From our investigations, the origin of the beat of Japanese temple bells is assumed to be mostly the same as that of Hojobo Temple's bell, considering the fact that Japanese temple bells have similar fundamental shape factors.

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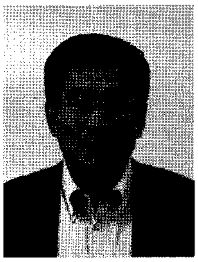
[Profile]

• **Takuji Nakanishi**



1965.4~1969.3: B.S. degree in communication engineering, Osaka University
1969.4~1971.3: M.S. degree in communication engineering, Osaka University
1984: Ph.D. degree in communication engineering, Osaka University
1971.4~1993.3: The Electrical Communication Laboratories, Nippon Telegraph and Telephone Corporation (NTT)
1993.4~present: Professor, Dept. of Electric and Electronics Engineering, Osaka Sangyo University

• **Hajime Ogushi**

1999.4~2003.3: B.S. degree in electric and electronics engineering, Osaka Sangyo University
2003.4~2005.3: M.S. degree in electric and electronics engineering, Osaka Sangyo University
2005.4~present: Matsusada Precision Corporation

• **Atsushi Yarai**

1985.4~1989.3: B.S. degree in electric and electronics engineering, Osaka Sangyo University
1989.4~1991.3: M.S. degree in electric and electronics engineering, Osaka Sangyo University
1996: Ph.D. Degree in applied physics and chemistry, University of Electro-Communications
1991.4~present: associate professor, Dept. of Electric and Electronics Engineering, Osaka Sangyo University