Simulation of a Non-Directional Wave Spectrum Analysis with Welch's Method

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Abstract—Simulation and signal conditioning on the time domain surface elevation records are conducted to verify the proposed Welch's method in non-directional ocean wave spectrum analysis. These spectrum data are further conditioned to provide wave characteristic that better describe the sea states. Comparison of significant wave height and zero crossing period between the proposed method and a reference toolkit are presented.

Index Terms—Heave Motion, Significant Wave Height, Spectrum Analysis, Zero Crossing Period

I. INTRODUCTION

Ocean wave state can often be described by its wave characteristics from observed power spectra of the sea. Similar research on wave characteristics simulation had been conducted by many other institutes [1][2][3][4]. However most of these researches focus on the complex scientific computations of oceanography and failed to provide the fundamental computation method on determining the wave characteristic from its surface elevation. These wave characteristics are the fundamental parameters for various probabilistic wave modeling algorithms.

Problem occurs when other researchers try to advance and reproduce the work but ended up consuming a huge amount of time due to loss of original computation parameters. It is hoped that this paper will assist researchers from similar field by providing the required fundamental information on signal conditioning procedures needed for determining the significant wave height and zero crossing period.

Theoretical background on the characteristic of significant wave height and zero crossing period is discuss through computation of spectral moments in section II. Signal conditioning following closely with Welch's method in estimation of power spectrum density through modified periodogram is shown with its mathematical representation in section III. Section IV covers the overall spectrum analysis result where the proposed spectrum analysis method is compared with WAFO (Wave Analysis for Fatigue and Oceanography) toolkit [1] for validation. These mathematical representations cover the basic computational requirement for further implementation of an off shore standalone wave monitoring system.

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II. NON DIRECTIONAL WAVE ANALYSIS

If a sensor were to measure waves at a fixed location on the ocean, the wave surface record would be rather irregular and random. Although individual waves can be identified, there is significant variability in height and period from wave to wave. Consequently, definitions of wave characteristics must be statistical or probabilistic, indicating the severity of wave conditions. Wave height is defined in [5] as the vertical difference between wave trough and wave crest. The wave field is a combination of waves with different height and period. The significant wave height, H_{m0} is a useful way to describe the sea state. Significant wave height is calculated from the energy spectrum of the measured waves which corresponds to one third of the highest waves in a measured wave record. H_{m0} is computed using the following equation

$$H_{m0} = 4.0\sqrt{m_0} \tag{1}$$

In addition to significant wave height, another parameter that better describes the wave state is wave period. The zero crossing periods, T_z can be determined from the heave motion and is defined as the average time interval between similar direction crossings of mean water level for wave records. The zero crossing periods can be calculated from the moments of wave frequency spectra with the following equation

$$T_z = L\sqrt{\frac{m_0}{m_2}} \tag{2}$$

T_z is normalized with the length of the accumulated spectrum segment, L. Spectral moments m_0 and m_2 is computed from the wave periodogram. Derivation of H_{m0} and T_z can be found in [6].

III. SIGNAL CONDITIONING

Signal conditioning is needed to represent raw heave motion gathered from the ocean surface to spectral moment. Heave data is sampled with a sampling frequency of Fs. A total number of N heave samples are needed to represent the wave profile. These heave samples are split into K number of segments with 50%

overlapping. Each segment has a length or window size of L. Segmenting the whole record involves the transformation of sequences which are shorter than the whole record which is an advantage when computations are to be performed on a machine with limited core storage. Derivation of Welch's method is shown in [7]. The equation for K is as follow

$$K = \frac{N}{L/2} - 1 \tag{3}$$

Data windowing is required to provide a close form for the input signal so that once converted to the frequency series, no leakage will extend beyond the adjacent frequency bins. Different window types are available to choose for with certain signal types and applications. A "Hann Window" is chosen for its good frequency resolution and it is applied to each K segment of the heave data [8]. Equation of Hann window function is written as

$$w(k) = \frac{1}{2}[1 - \cos\frac{2\pi k}{L}] \qquad k = 0, 1, 2 \ldots (L-1) \tag{4}$$

The window weight or window squared and summed, W_{ss} use to normalize the periodogram estimator which will be discussed later is shown as

$$W_{ss} = L\sum_{k=0}^{L-1} w(k)^2 \tag{5}$$

Time-series record collected from the heave sensor needs to be transformed into a summation of simple sine waves. These sine waves are the components of the sea state, each with a distinct height and frequency. In other words, the spectral analysis method determines the distribution of wave energy and average statistics for each wave frequency by converting the time series of the wave record into a wave spectrum. This is essentially a transformation from the time-domain to the frequency-domain, and is accomplished most conveniently using a mathematical tool known as the Fast Fourier Transform (FFT). It is applied to each segment of the time-series heave data. The first segment of input heave data, $x(n)$ with length L is multiplied by the window function $w(n)$ mentioned above following the FFT equation.

$$X(k) = \sum_{n=0}^{L-1} w(n)x(n)e^{-2\pi i/L} \qquad k = 0, 1, 2 \ldots (L-1) \tag{6}$$

Although the input signal $x(n)$ is a real number, the output of FFT, $X(k)$ is a complex number array with the same length as the input array and window size, L. Almost half the information is redundant in standard and double-sided output formats. The data is conjugated symmetric about the $(n/2)$ harmonic, also known as the Nyquist component. Therefore, all the information above the Nyquist component can be discarded. Displaying only the positive frequencies gives the single-sided output [8]. The single-sided output is shown in the periodogram estimator equation as.

$$Sxx_i(0) = \frac{1}{KW_{ss}}|X(0)|^2 \qquad i = 1, 2, 3 \tag{7}$$

$$Sxx_i(k) = \frac{1}{KW_{ss}}\left[|X(n)|^2 + |X(L-n)|^2\right] \tag{8}$$

$$Sxx_i(\frac{L}{2}) = \frac{1}{KW_{ss}}\left|X(\frac{L}{2})\right|^2 \tag{9}$$

The computed periodogram from first segment of heave data, Sxx_1 with the size of L, has a $(L/2+1)$ number of frequency bin ranges from the dc component $Sxx_1(0)$ to the Nyquist component $Sxx_1(L/2)$. Each element of the periodogram is normalized with the window weight, Wss and number of segments K. The remaining segments of heave data (Sxx_2 and Sxx_3) is repeated with the same signal conditioning algorithm. These periodogram is accumulated to produce the final periodogram Sxx. Zero-order spectral moment m_0 and its first derivate m_2 are defined by

$$m_k = \sum_{n=0}^{L/2} n^k Sxx(n) \qquad k = 0, 2 \tag{10}$$

n corresponds to the frequency bin available in the analyzed wave spectrum.

IV. RESULT

Mathematical expression on the signal conditioning method mentioned above was simulated and coded with Matlab. A subroutine was developed to output significant wave height and zero-crossing period by inputting a time series elevation record. The input parameter was generated from the WAFO toolkit following a JONSWAP spectrum.

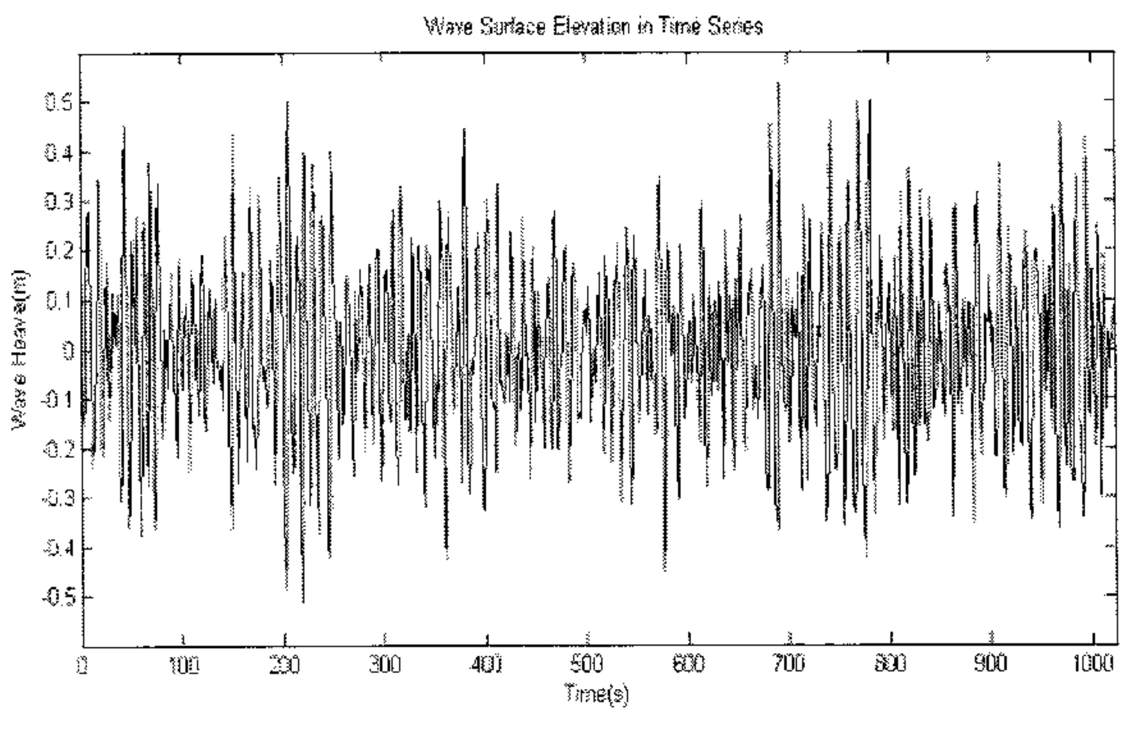

Fig. 3 Simulated surface elevation records from WAFO

The generated heave data which had a significant wave height of 0.72m and zero-crossing period of 7.38s from WAFO is shown in Fig. 3. At the end of this section, these two parameters are used to verify the proposed signal conditioning method.

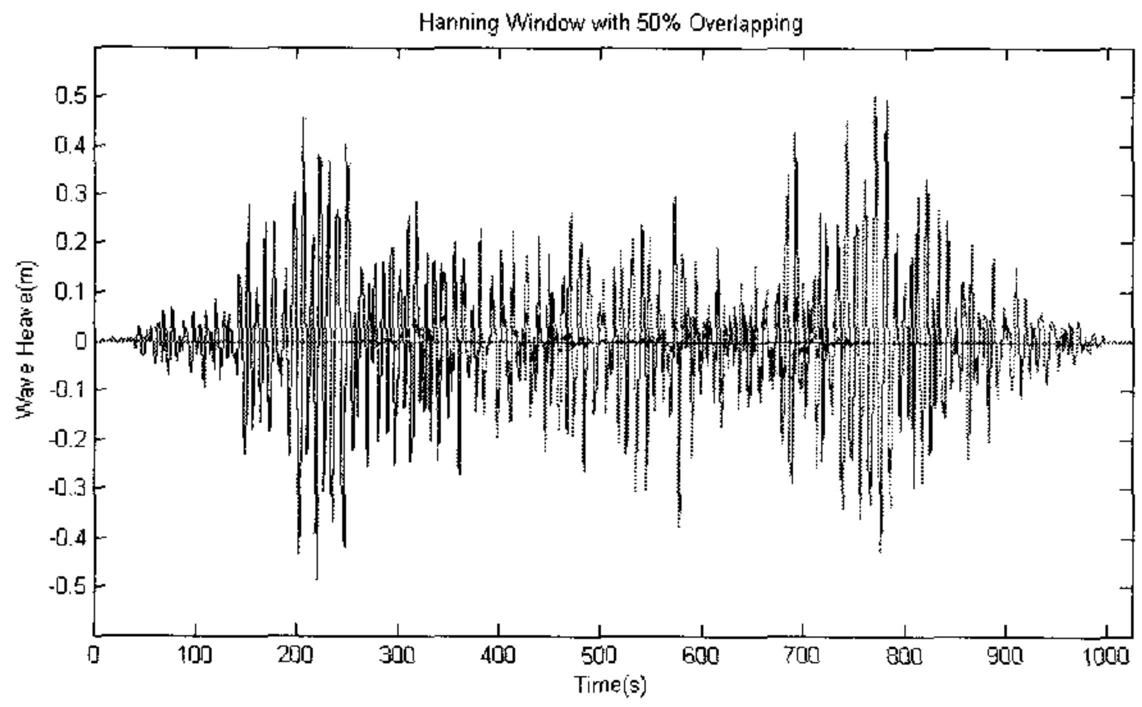

Fig. 4 Segmentation of heave data with Hann window

A total number of 1024 simulated heave data samples were divided into 3 segments following equation (3). Hann window was applied to each segment with 50% overlapping. The heave data resulting from Hann windowing is shown in Fig. 4 with segment 1 in blue, segment 2 in green and segment 3 in red. As shown in Fig. 4, data windowing changed the signal gradually from zero to a maximum and then fall smoothly back to zero providing a closed form of the signal. Each of these time series segments was then transformed into frequency domain through FFT with equation (6). PSD of each spectrum was again computed through periodogram method and accumulated independently following equation (7-9).

Accumulated spectrum intensity from the three segments is shown in Fig. 5. Zero-order spectral moment *m0* and first derivative *m2* were used to compute the significant wave height and zero-crossing period following equation
(1-2). Comparison between the WAFO toolkit and developed Matlab routine is shown in Table 1.

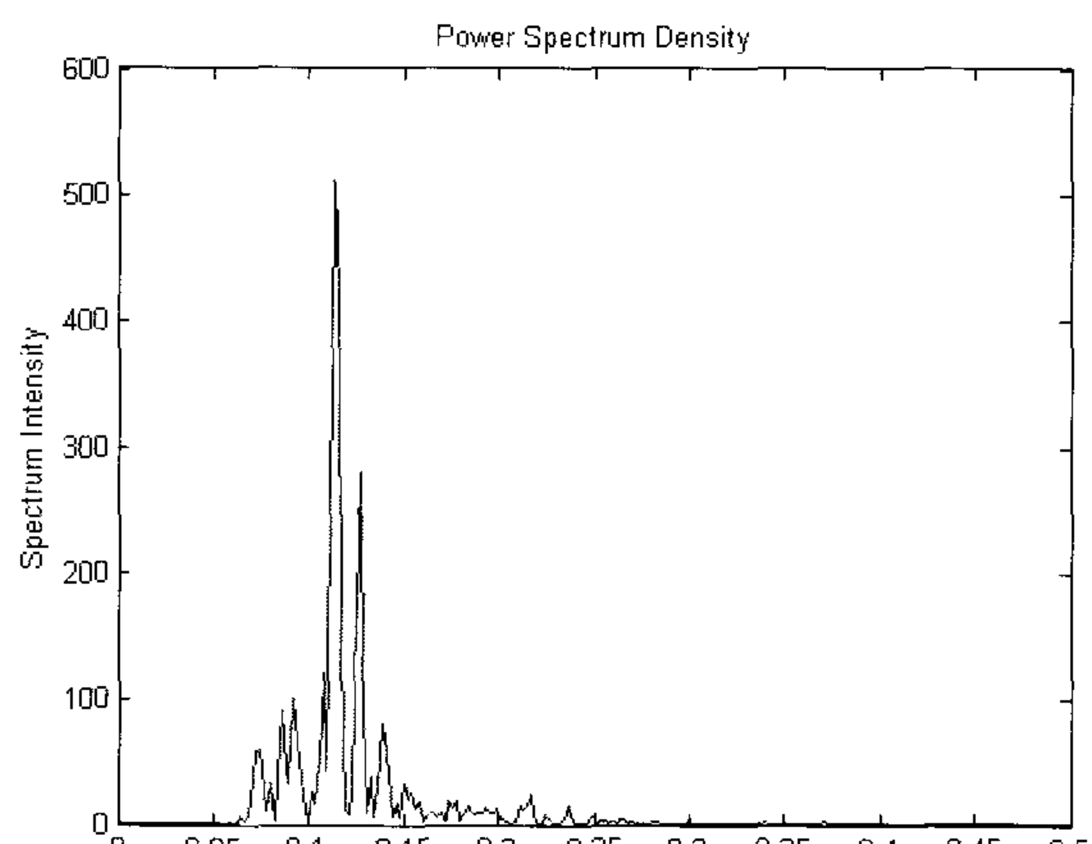

Fig. 5 Spectrum intensity of the periodogram for each frequency ranging until the Nyquist component

Table 1 Wave Measurement Comparison Table

Model	H_{m0} (m)	T_z (s)
WAFO	0.72	7.38
MATLAB	0.73	7.40

Table 1 shows a slight difference in wave measurements between the two models. Further research on increasing the frequency resolution by introducing more segments on the time series heave data will be conducted.

V. CONCLUSION

Development of a non-directional wave spectrum analysis embedded module represents an alternative controller for a wave monitoring buoy system. Future research on directional wave analysis computation will be conducted to better reveal the wave characteristic of a certain sea states. Further investigation is currently being conducted to better determine the most suitable window size and overlapping segment.[9],[10],[11]

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