Linearly Constrained Adaptive Array Processing with Alternate Mainbeam Nulling

Byong Kun Chang[1] · Chang Dae Jeon[2] · Dong Hyuk Song[3]

Abstract

This paper concerns with signal cancellation problem in a linearly constrained adaptive array processor in coherent environment. Alternate mainbeam nulling approach was proposed to prevent the signal cancellation phenomenon. The linearly constrained LMS algorithm with a unit gain constraint and that with a null constraint in the direction of the desired signal is alternately implemented to reduce the signal interaction between the desired signal and the interferences, which is the main cause of the signal cancellation. It is shown that the proposed method performs better than a conventional method.

Key words : Adaptive Array, Linear Constraint, LMS Algorithm, Coherent, Signal Cancellation, Mainbeam Nulling.

Ⅰ. Introduction

This paper proposes a simple method to prevent signal cancellation phenomenon in a linearly constrained adaptive array processor in coherent environment. In the well known linearly constrained adaptive array processor proposed by O. L. Frost[1], it is shown that even though a desired signal is almost successfully estimated with the assumption that the desired signal and impinging interferences are uncorrelated, the desired signal is partially cancelled in the array output due to the signal interaction between the desired signal and the interferences during adaptive processing of input signals[2]. In the Frost array processor, the directional and spectral information for the desired signal is assumed to be known a priori, while that for the interference signals is unknown.

To estimate the desired signal, the array weights are updated iteratively by the linearly constrained LMS algorithm to minimize the array output in the least mean square sense while maintaining a unit gain in the look direction(i.e., the direction of the desired signal).

If the interference signals are partially or totally(i.e., coherent) correlated with the desired signal, the Frost array processor can not estimate the desired signal properly such that the desired signal is partially or totally cancelled in the array output depending on the extent of correlation between the desired signal and the interferences[2]. Thus, the Frost array processor is incompetent in nulling the interference signals correlated with the desired signal.

A variety of methods have been proposed to improve the performance of the linearly constrained adaptive array processor in nulling the interferences in correlated interference environment[2]~[5]. A master-slave type array processor proposed by Duvall[2] employs a subtractive preprocessing to eliminate signal cancellation during adaptive process. In the master processor, the desired signal is eliminated through a subtractive preprocessor and thus the array weights in the master processor are updated by the linearly constrained LMS algorithm using the subtractive interferences only. Thus, the signal interaction between the desired signal and the interferences is avoided. The resulting array weights which are not affected by the desired signal are copied into the slave array processor in which the input signals(i.e., the desired signal plus interferences) are processed to yield the array output signal. Since the phase relationship among the input signals are in the master processor may be the same as that in the slave processor, the interferences may be eliminated with almost no cancellation of the desired signal. The shortcoming of this processor is that it requires an additional processor to the Frost array processor.

A spatial smoothing approach proposed by Shan and Kailath[3] employs subarray preprocessing to decorrelate the input signals. To this end, the average of the input correlation matrix of each subarray(i.e., spatially smoothed correlation matrix) is used as the input correlation matrix in the optimal weight vector solution. It is demonstrated that if the number of subarrays is greater than or equal to that of the input signals, the rank of the

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smoothed correlation matrix for coherent interferences is the same as that of the input correlation matrix as if all the incident input signals were uncorrelated. Thus the coherent interferences are successfully eliminated in the array output. It is shown that the suboptimal weight vector solution may be obtained by applying the linearly constrained LMS algorithm to each subarray recursively for each data snapshot. The disadvantage of the spatial smoothing approach is that the number of the sensor elements should be greater than or equal to twice the number of input signals to eliminate all the coherent interferences. Thus for this array processor to work properly, the number of subarrays needs to increase as the number of interferences increases, which results in more subarray preprocessing.

Su and Widrow[4] proposed a parallel processing method which employs a subarray structure similar to the spatial smoothing approach. It is shown that processing time is much reduced due to parallel processing of input signals while proper number of subarrays are required to eliminate the incident coherent interferences successfully.

In this paper, an alternate mainbeam nulling approach is proposed with the linearly constrained LMS algorithm to reduce the signal cancellation in coherent interference environment. This approach may be called algorithmic approach since the linearly constrained LMS algorithm is modified such that the interaction between the desired signal and the interferences is reduced while the conventional methods may be called structural approach since additional hardware needs to be structured.

In the proposed method, a null constraint is added to the unit gain constraint in the look direction in the linearly constrained LMS algorithm, which is implemented alternately for each constraint. It may be said that the signal interaction is reduced as half as that in the linearly constrained LMS algorithm. It is to be noted that in the master-slave type array processor, mainbeam nulling is continuously implemented in the master processor during adaptive process. It is shown that the performance of the proposed method is more robust to the increasing number of elements than that of conventional methods.

Ⅱ. Optimum Solution for the Linearly Constrained Array Processor

Consider a narrowband linear array with N equispaced sensor elements as shown in Fig. 1, where each sensor is followed by a complex weight. The input signal for each sensor is multiplied by the corresponding weight and is combined to produce the array output. It is assumed that the directional and spectral information for the desired signal is known a priori while that for the interference signals is unknown.

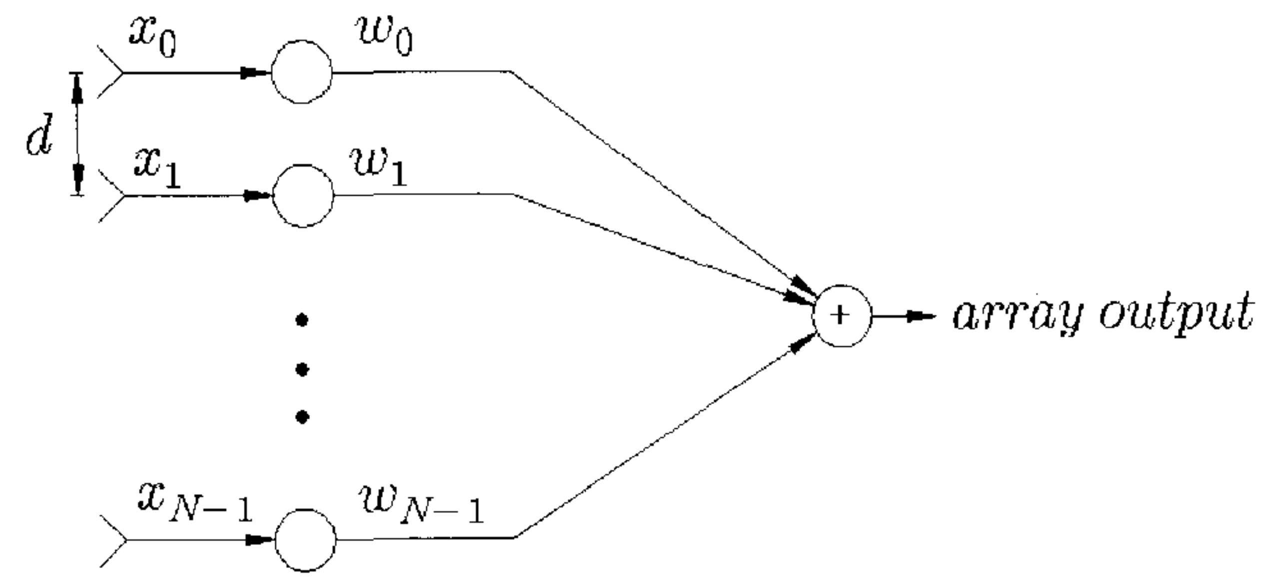

Fig. 1. Narrowband linear array.

The problem of eliminating incident coherent interferences to estimate the desired signal may be solved by minimizing the array output power while maintaining a unit gain in the look direction as formulated by

$$\min_{w} \quad w^H R w \qquad \text{subject to} \quad w^H s = 1 \tag{1}$$

where w is the weight vector, $[w_0 \ \ w_1 \ \ \ldots \ \ w_{N-1}]^T$, R is the input correlation matrix, $E[xx^H]$, x is the input signal vector, $[x_0 \ \ x_1 \ \ \ldots \ \ x_{N-1}]^T$, s is the steering vector for the desired signal, $[1 \ e^{-j\Omega\tau_o} \ e^{-j2\Omega\tau_o} \ \ldots \ e^{-j(N-1)\Omega\tau_o}]^T$, Ω is the radian frequency of the desired signal, $\tau_o = d\sin\theta_o/v$, θ_o is the incident angle from the array normal, d is interelement spacing, v is the propagation velocity, and H denotes complex conjugate transpose.

The optimum solution for (1) may be obtained by the Lagrange multiplier method[1] as given by

$$w_o = \frac{R^{-1}s}{s^H R^{-1} s} \tag{2}$$

A suboptimal solution for the optimum weight vector (2) can be obtained iteratively by implementing the linearly constrained LMS algorithm based on the steepest descent method which is given by [1].

$$w_{k+1} = \left[I - \frac{s s^H}{N} \right] [\, w_k - \mu y_k^* x_k] + \frac{s}{N} \tag{3}$$

where w_k, x_k, and y_k are the weight vector, the input signal vector, and the array output at the kth iteration, respectively, μ is convergence parameter, * denotes complex conjugate, and k is iteration index. The operation of the linearly constrained LMS algorithm as represented by (3) may be explained in the weight vector space. The weight vector updated by the unconstrained LMS algorithm is orthogonally projected onto the hyper-subspace which is parallel to the constrained hyper-surface and the resulting vector is added by the steering vector for the desired signal normalized by the number of elements to yield the weight vector for the next iteration.

The problem of the linearly constrained LMS algorithm in (3) is that the desired signal is partially cancelled in the array output even though the desired signal and the interferences are uncorrelated. If the interferences are coherent, the desired signal may be totally cancelled in the array output. Thus, the desired signal may not be estimated properly whether the interferences are correlated with the desired signal or not. To solve this shortcoming of the linearly constrained LMS algorithm, we propose an alternate mainbeam nulling approach for the linearly constrained adaptive array processor.

III. Alternate Mainbeam Nulling Approach

The basic idea of the alternate mainbeam nulling approach is that we reduce the signal interaction between the desired signal and the interferences, which is the main cause of signal cancellation, by removing the desired signal alternately during adaptive process in the linearly constrained LMS algorithm. To this end, a null constraint is employed in the look direction in addition to the unit gain constraint. The problem of minimizing the array output power with a zero gain in the look direction can be formulated as

$$\min_{w} w^H R w \quad \text{subject to} \quad w^H s = 0 \tag{4}$$

The optimum solution for (4) may be solved by applying the Lagrange multiplier method to the following objective function

$$O(w) = w^H R w + \lambda w^H s \tag{5}$$

where λ is the Lagrange multiplier. The gradient of $O(w)$ with respect to w^* is expressed as [6]

$$\nabla_w^*(w) = Rw + \lambda s \tag{6}$$

Setting the gradient and applying the null constraint in (4), we have the optimal weight vector which is given by

$$w_o = 0 \tag{7}$$

The suboptimal solution to (7) may be obtained iteratively by using the steepest descent method as given by

$$w_{k+1} = w_k - \mu(Rw_k + \lambda s) \tag{8}$$

Applying the null constraint in (4) to (8), we can derive the following adaptive algorithm

$$w_{k+1} = \left[I - \frac{ss^H}{N}\right][w_k - \mu y_k^* x_k] \tag{9}$$

The weight vector updated by (9) minimizes the array output power with no effect of the desired signal since it lies on the subspace orthogonal to the look direction.

The alternate mainbeam nulling approach for the linearly constrained adaptive array processor is implemented such that the array weights are updated alternately by the adaptive algorithm with the unit gain constraint in (3) and by that with the null constraint (9) as rewritten in the following with convergence parameters subscripted as μ_u and μ_n to denote that they are different in general for a suboptimal solution.

$$w_{k+1} = \left[I - \frac{ss^H}{N}\right][w_k - \mu_u y_k^* x_k] + \frac{s}{N} \tag{10}$$

$$w_{k+1} = \left[I - \frac{ss^H}{N}\right][w_k - \mu_n y_k^* x_k] \tag{11}$$

where μ_u and μ_n are the convergence parameters for the unit gain and null constraints respectively. It is shown that the performance of the alternate mainbeam nulling approach depends on the values of μ_u and μ_n.

The operation of the alternate mainbeam nulling approach is as follows. The weight update equations (10) and (11) are implemented alternately such that in a steady-state a suboptimal weight vector is obtained by (10). The weights updated by (11) is on the subspace which is parallel to the unit gain constraint surface, and thus orthogonal to the look direction. Thus, the corresponding array output by the weights by (11) is free from the desired signal. As a result, if the array weights by (11) and the corresponding array output are applied in (10), the effect of signal interaction in the multiplication term in the bracket (i.e., the unconstrained LMS algorithm) is that much reduced in the resulting weights. The corresponding output is represented by

$$y_k = w_k^H x_k \tag{12}$$

in which the signal cancellation is reduced.

The final array output signal is obtained by linearly interpolating the output signal samples in (12) generated by the weights from (10) since the array output by the weights updated by (11) mainly consists of reduced interferences.

The weight update operation in the weight vector space is shown in Fig. 2 for a 2-sensor linear array. It is assumed that a sinusoid is incident at the array normal as a desired signal and two coherent interferences are incident at two different non-look directions. A set of level curves are shown with the constraint surface and the subspace parallel to it. Also, the trajectory of the weight vector is shown between the two surfaces.

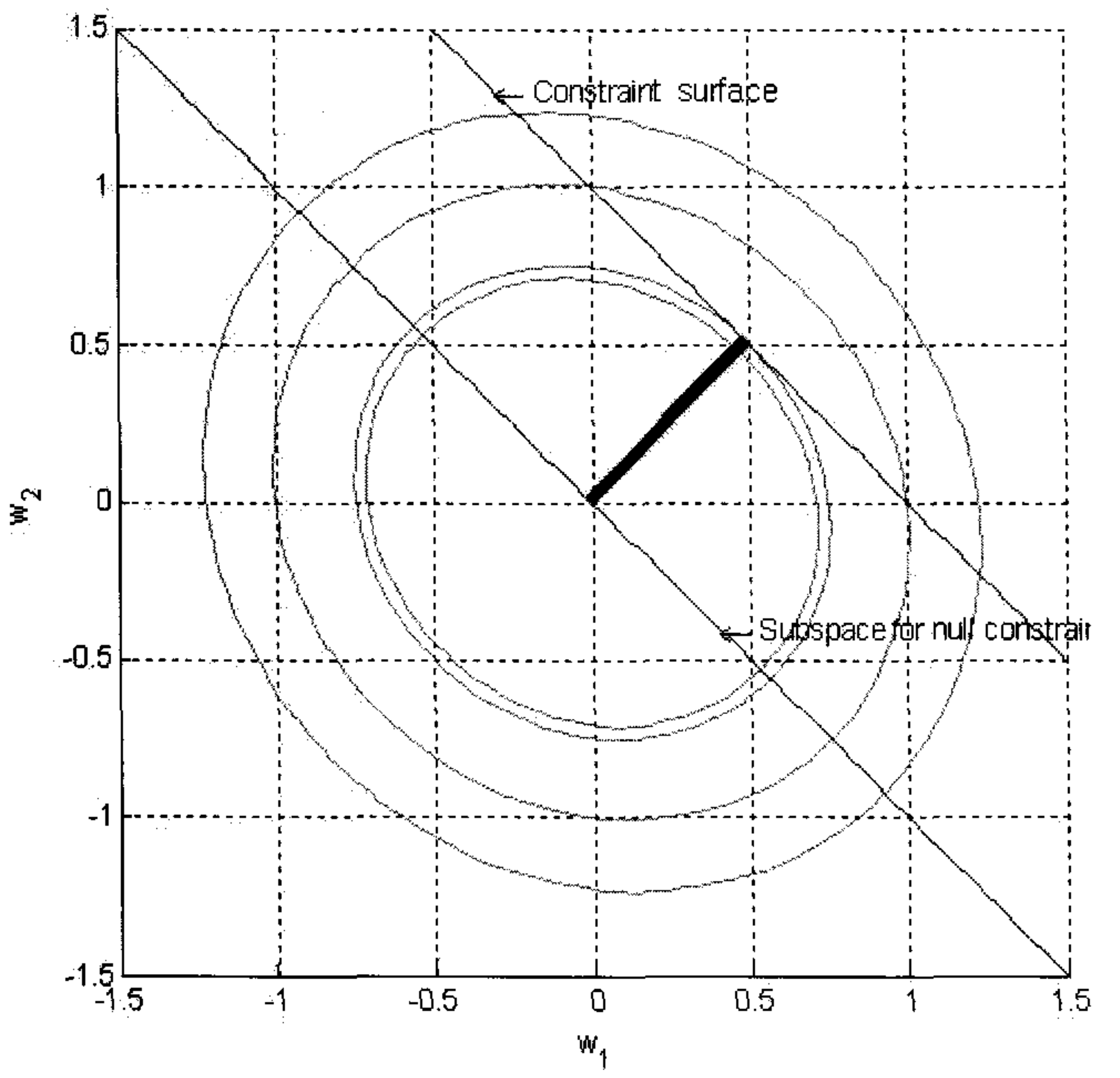

Fig. 2. Weight update in weight vector space.

Ⅳ. Simulation Results

The performance of the alternate mainbeam nulling approach is examined in terms of the convergence parameters μ_u and μ_n. and is compared to a conventional method, the spatial smoothing approach[3]. A 7-sensor linear array is employed with inter-element spacing a half wavelength. A desired sinusoid with magnitude of 0.1 is assumed to be incident at the array normal. Two cases for the coherent interference signals are simulated, one case for a sinusoid at 45.9° from the array normal, the other case for 6 sinusoids at −41°, −38.6°, −36.4°, −34.2°, 27.9°, 43.4°. It is assumed that in the spatial smoothing approach, the number of sensors is 11, which results in 5 subarrays with 7 sensors per each subarray.

To find the effect of the convergence parameters in (10) and (11) on the performance of the proposed method, the variations of μ_u and μ_n for the suboptimal solution are examined for a coherent interference with respect to its magnitude, which is shown in Fig. 3.

In Fig. 3, the magnitude of the interference is increased by 0.1 from the initial magnitude of 0.1 up to 3.5. For each case, the optimal values for μ_u and μ_n are numerically found in the range of 0.0001 to 0.01. It is shown in the figure that μ_u decreases and μ_n increases as the magnitude increases. Thus it is demonstrated that if the incident interference is large in magnitude, the nulling performance is improved with large μ_n and small μ_u.

To examine the variations of μ_u and μ_n for the suboptimal solution in terms of the number of interferences,

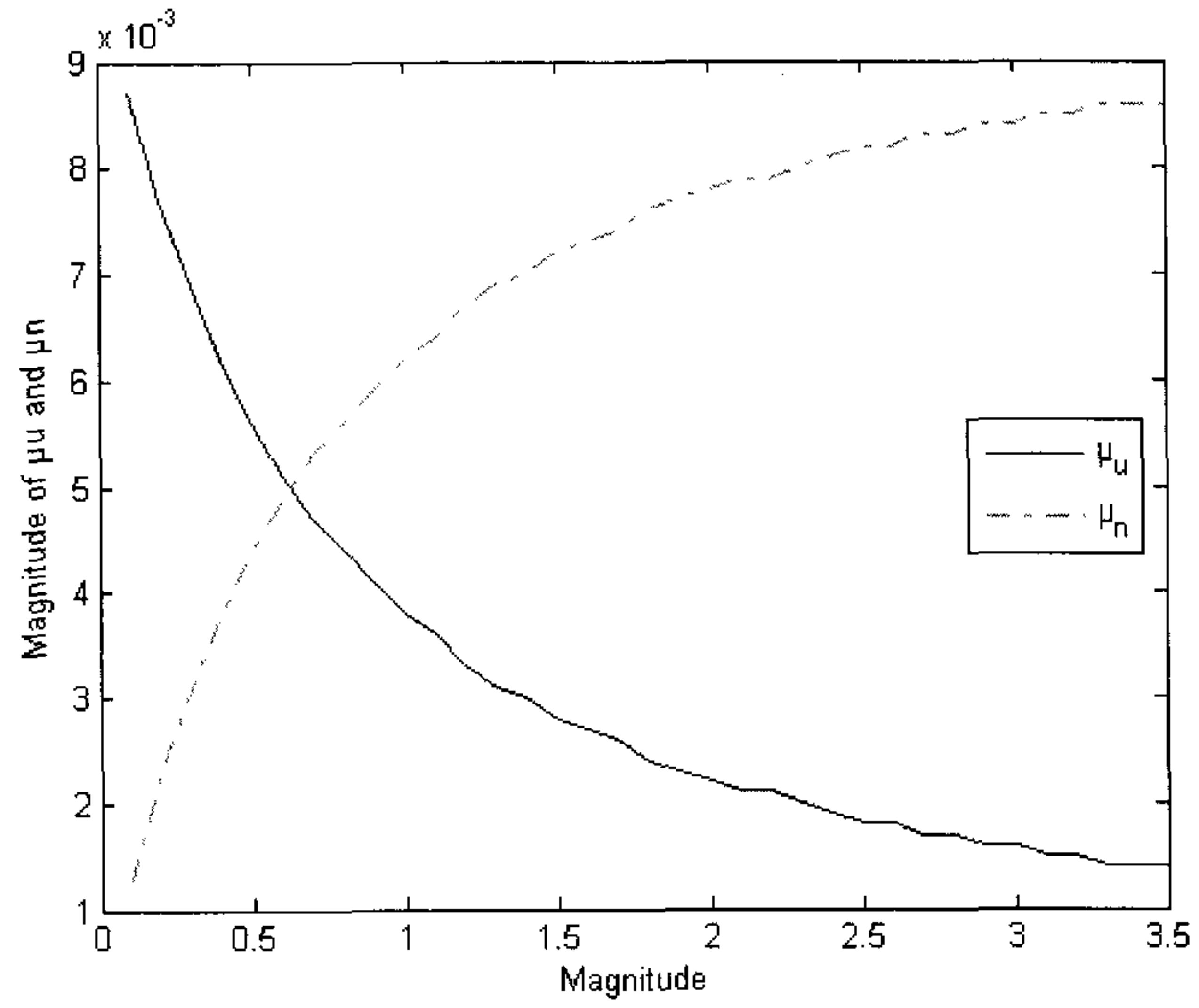

Fig. 3. Variations of convergence parameters in terms of the magnitude of interference.

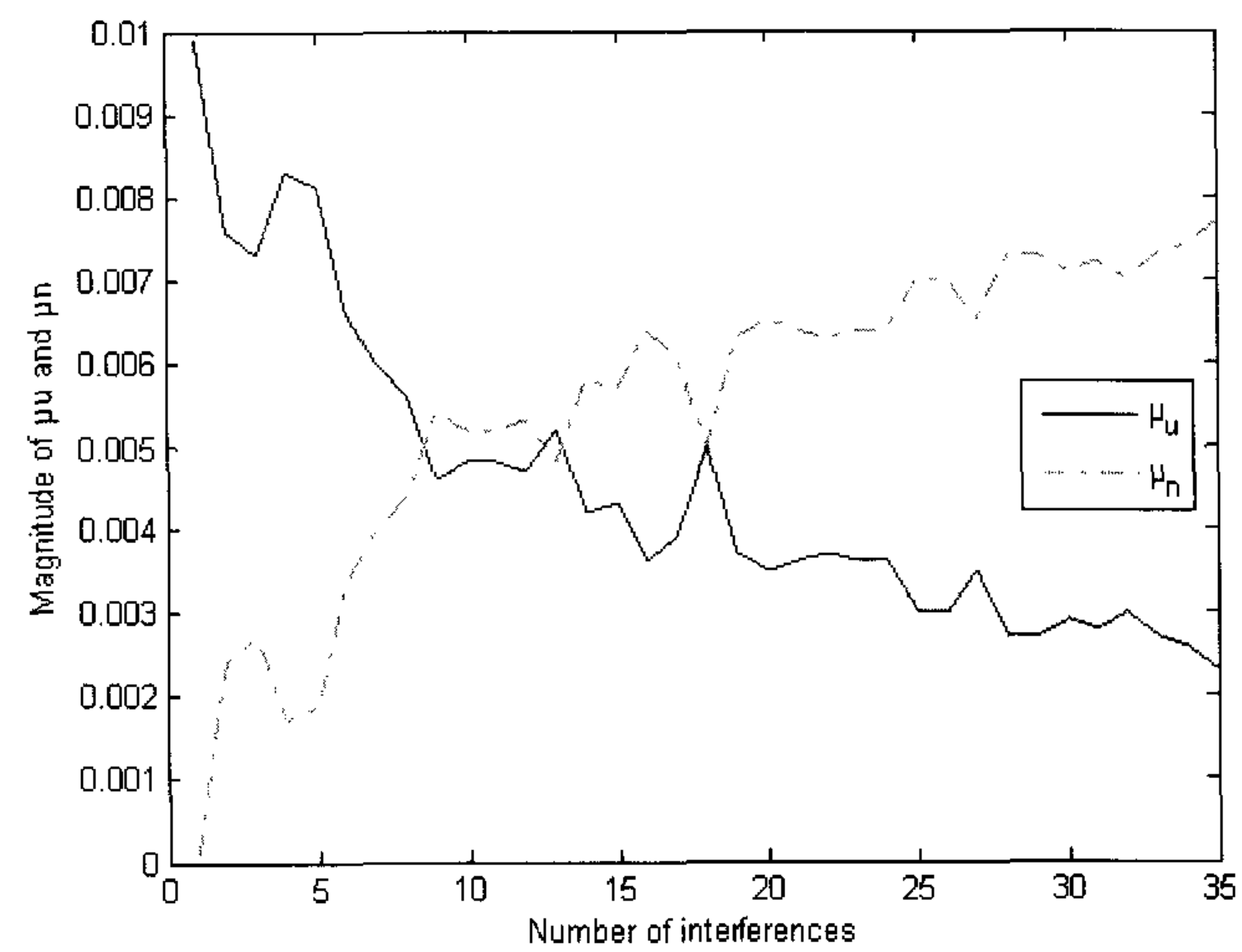

Fig. 4. Variations of convergence parameters in terms of the number of interferences.

the number of interferences from one through 35 are simulated in the range of μ_u and μ_n from 0.0001 to 0.01, which is shown in Fig. 4. It is assumed that the interferences are randomly distributed over the visual range of the array.

It is shown that the μ_u decreases and μ_n increases in general as the number of interferences increases. The variation is shown to be not monotonic, the reason of which seems to be that the distribution of the magnitude at each sensor is affected by the randomly distributed incident angles of the interferences.

The nulling performance of the alternate mainbeam nulling approach with respect to values for μ_u and μ_n is examined for one coherent interference at 45.9° from the array normal. From Fig, 3, the optimal values for μ_u

and μ_n for the magnitude of 0.1 are shown to be 0.0092 and 0.00149, respectively. The performance with these optimal values and that with μ_u and μ_n 0.0015 are compared in Figs. 5~8, where output signals, error power, and beam patterns are shown. The optimal values yield a better performance in terms of magnitude as shown in Figs. 5 and 6. The beam patterns are compared in Figs. 7 and 8, in which the sidelobe for the optimal values gets lower at the direction of the interference compared to that for the value of 0.0015. In the figure, the incident angle of the interference is denoted by an arrow.

To compare the nulling performance of the proposed method with the spatial smoothing approach in the presence of multiple interferences, the case for 6 coherent interferences is simulated. The optimal values for μ_u and μ_n for 6 interferences are shown to be 0.0066 and 0.0034 respectively from Fig. 4. The output signals, error power, and beam patterns are compared in Figs. 9~12, respectively.

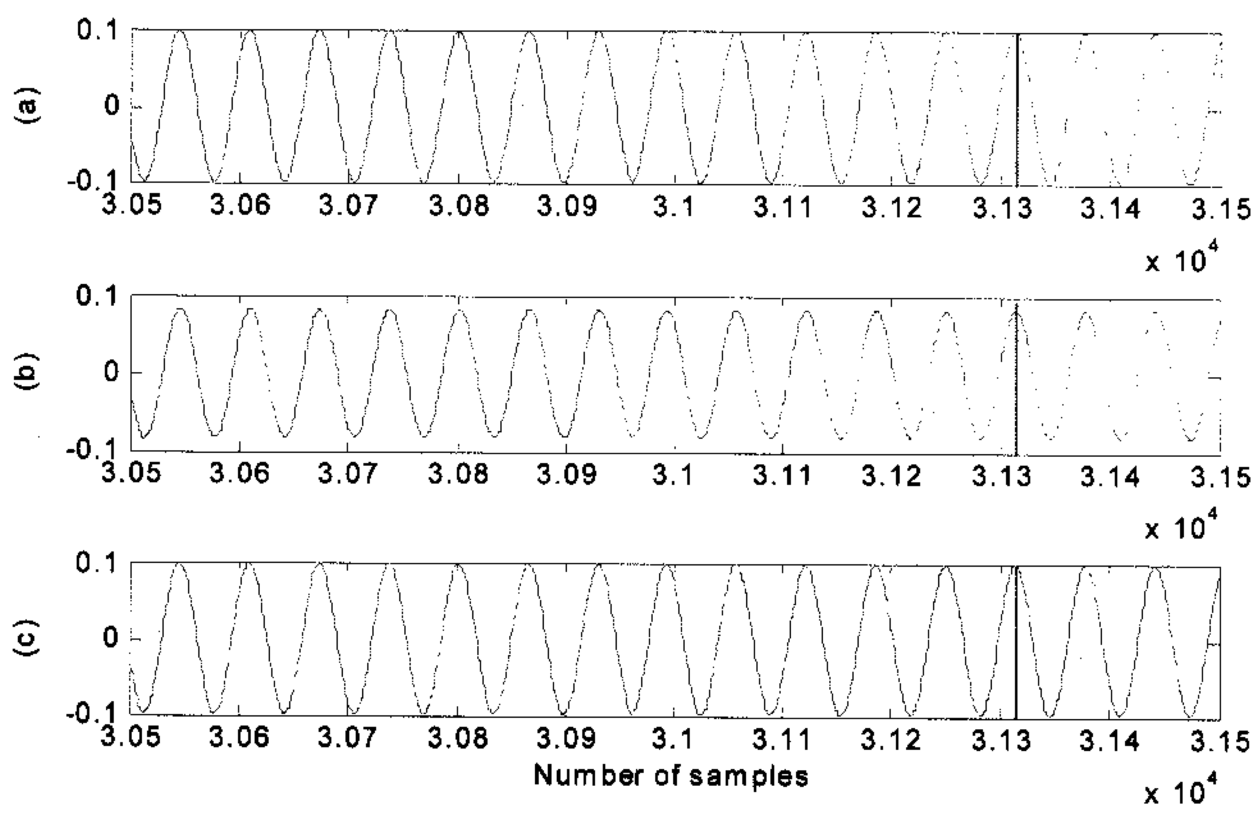

Fig. 5. (a) Desired signal, and array outputs, (b) for 0.0015 (c) for optimal values.

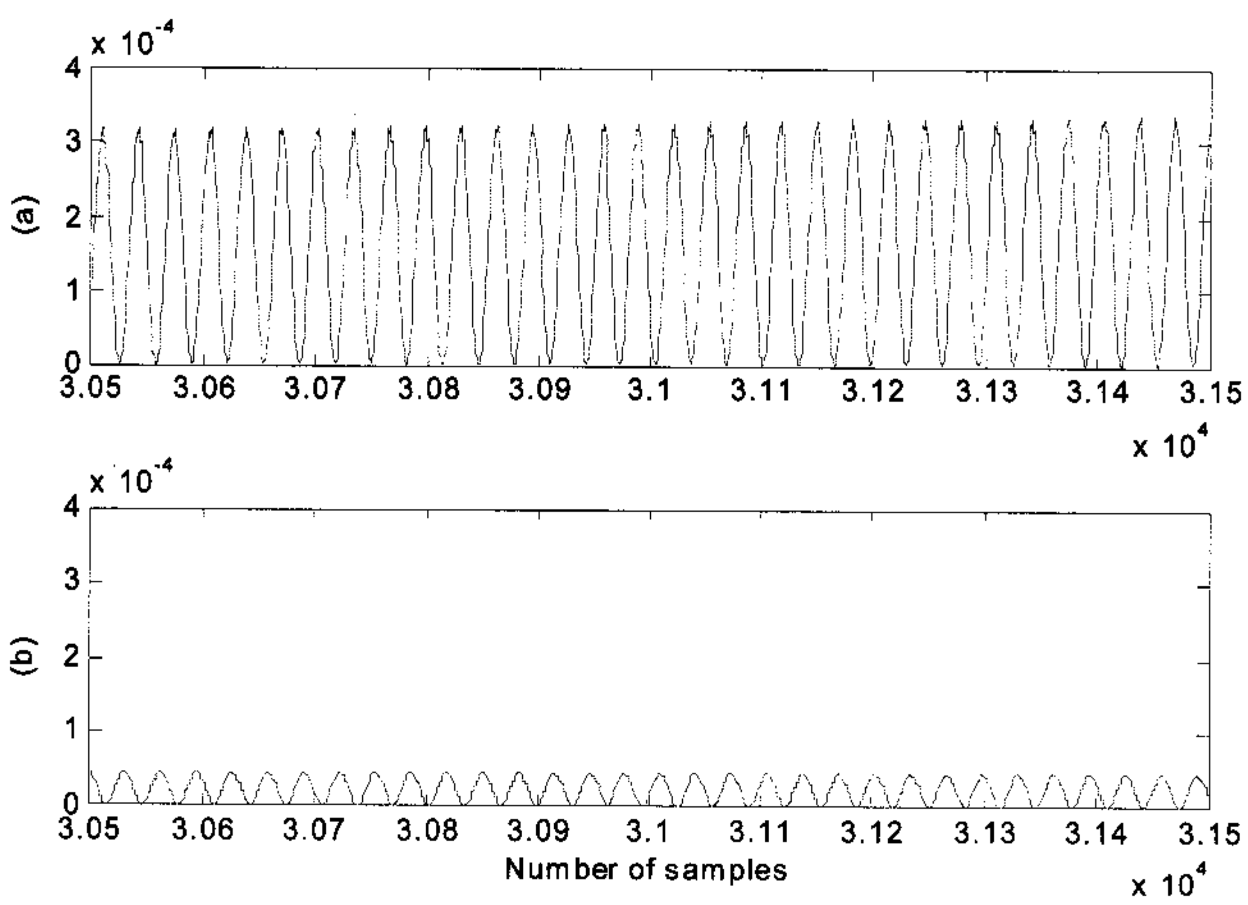

Fig. 6. Error power; (a) for 0.0015, (b) for optimal values.

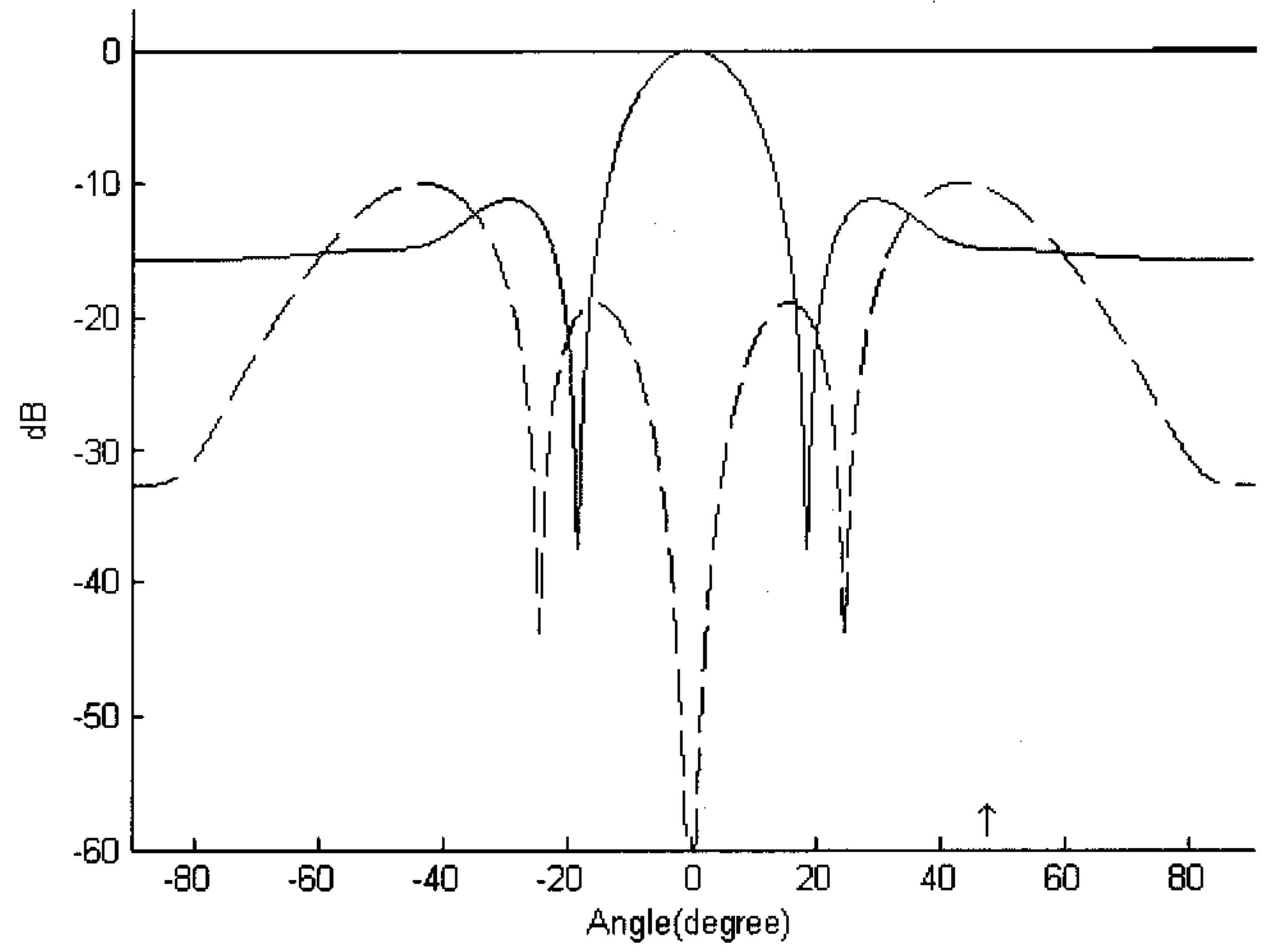

Fig. 7. Beam patterns for 0.0015; unit gain constraint(solid line), null constraint(dashed line).

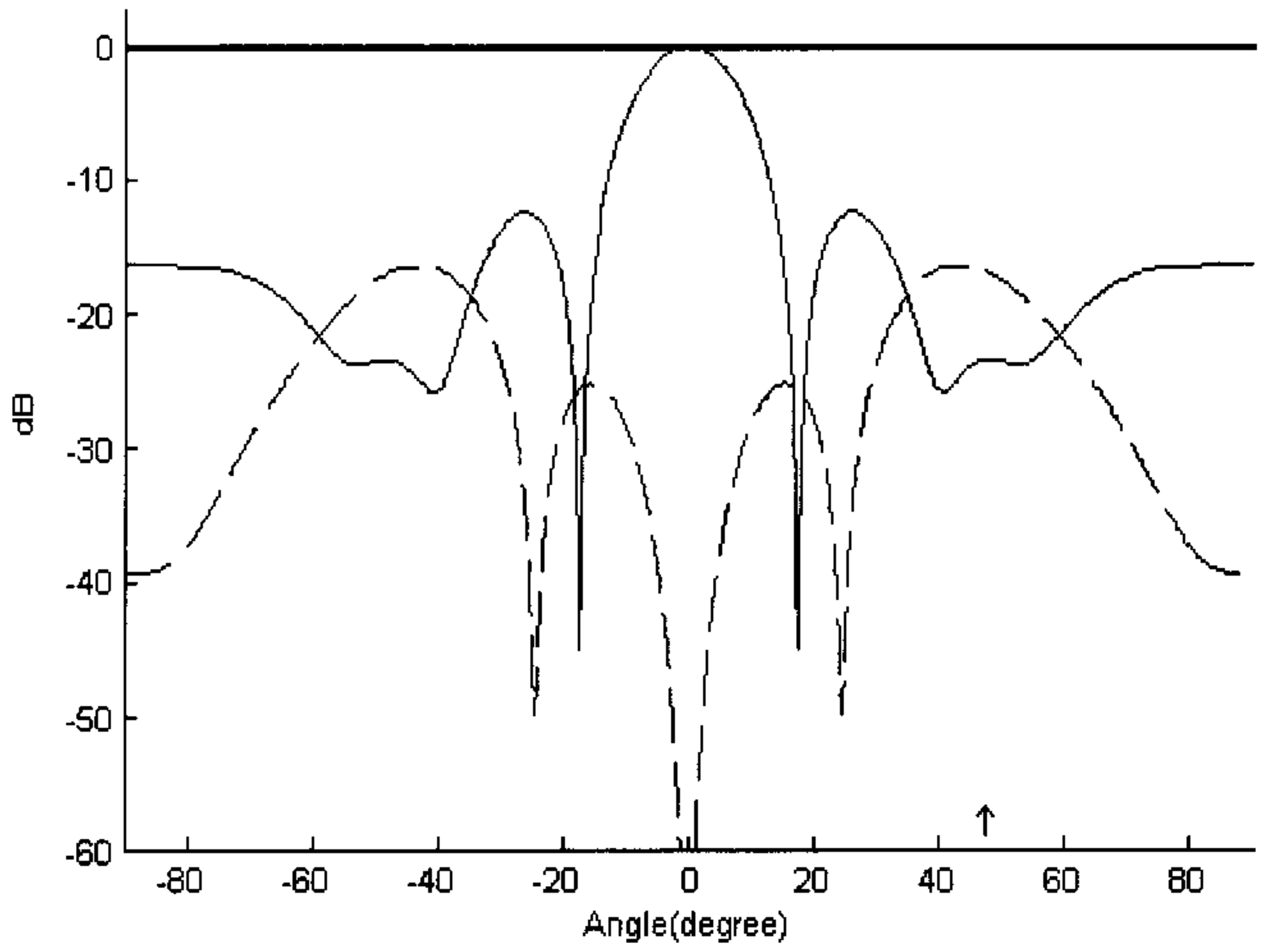

Fig. 8. Beam patterns for optimal values; unit gain constraint(solid line), null constraint(dashed line).

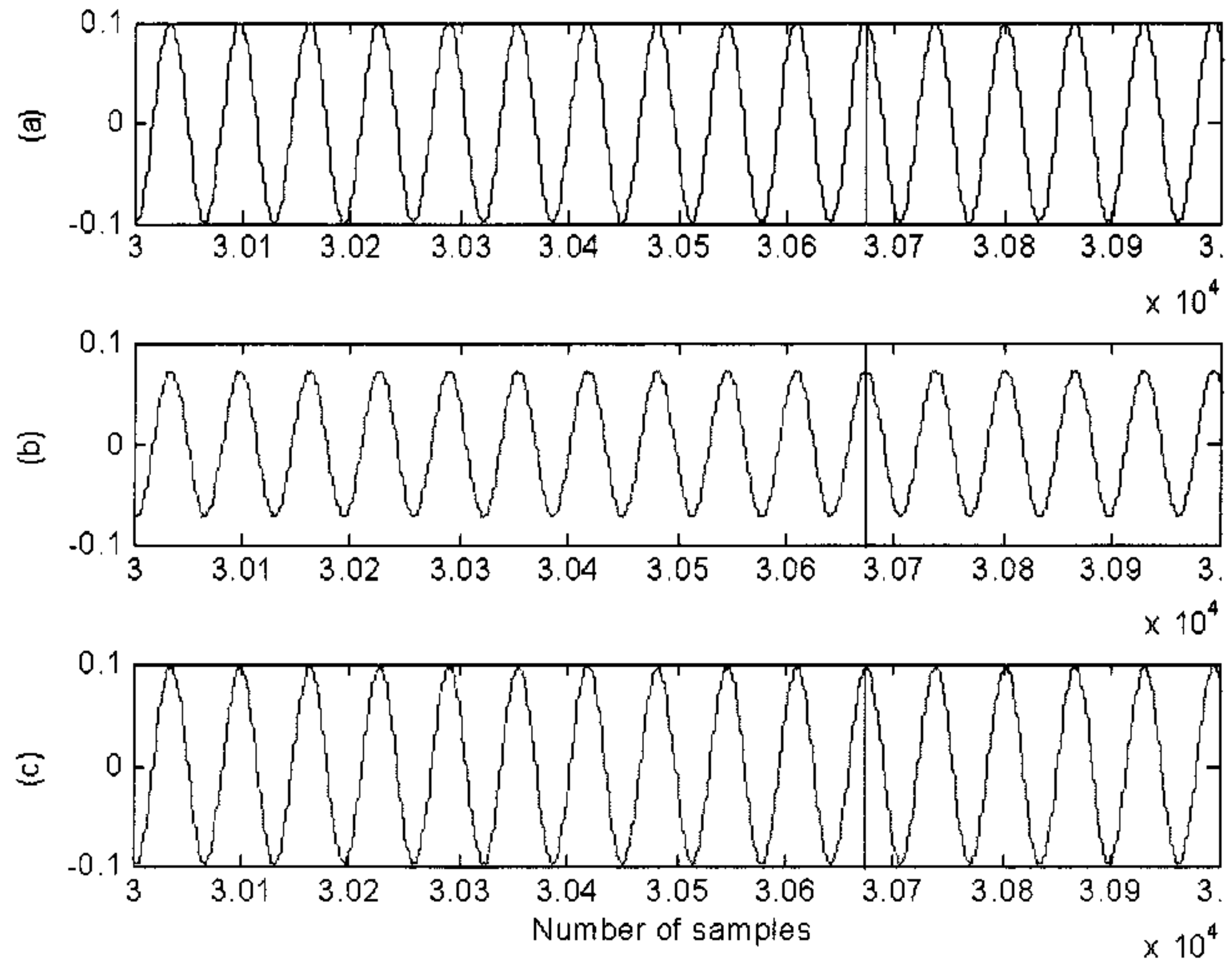

Fig. 9. (a) Desired signal, and array outputs, (b) spatial smoothing approach, (c) proposed method.

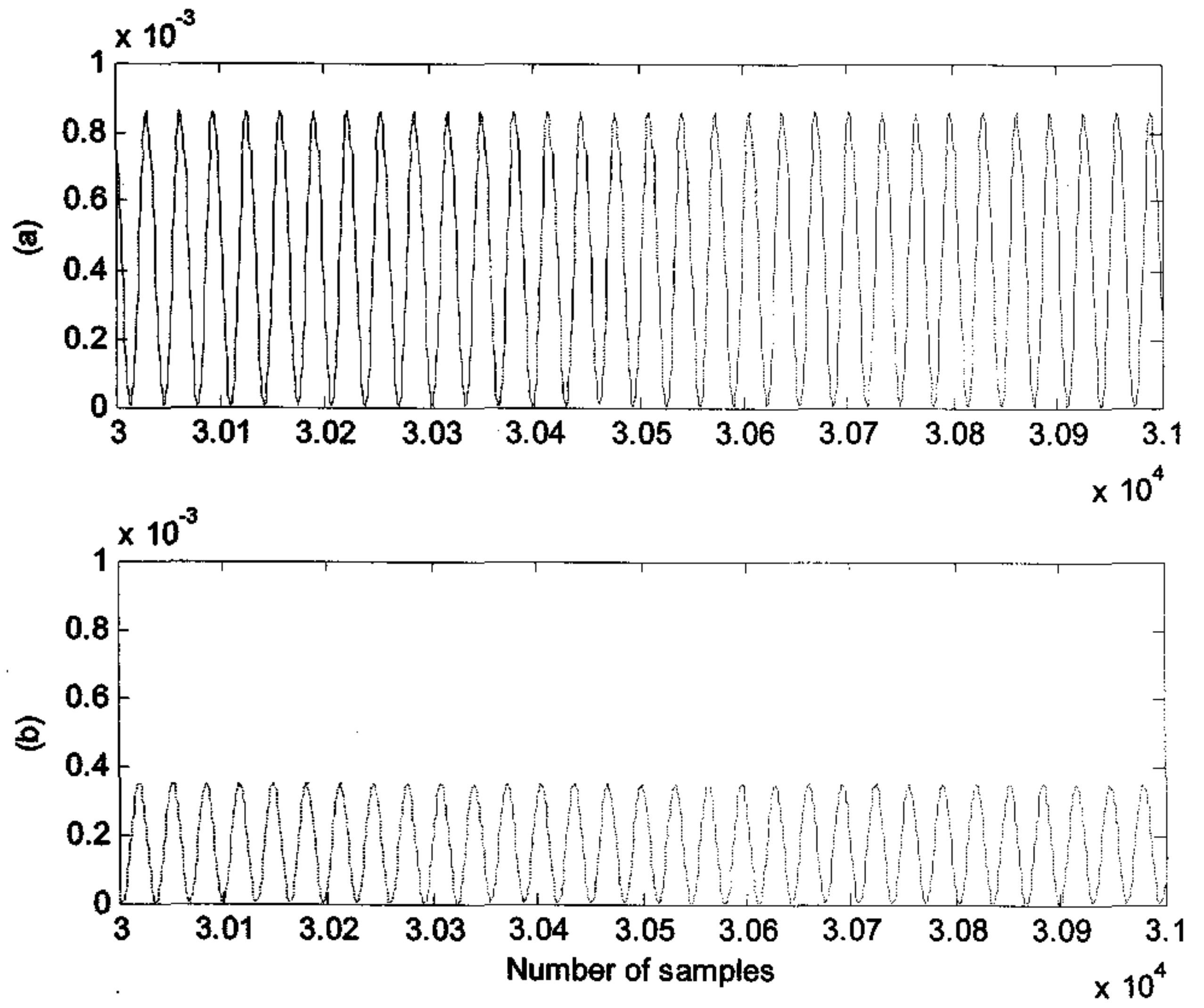

Fig. 10. Error power; (a) spatial smoothing approach, (b) proposed method.

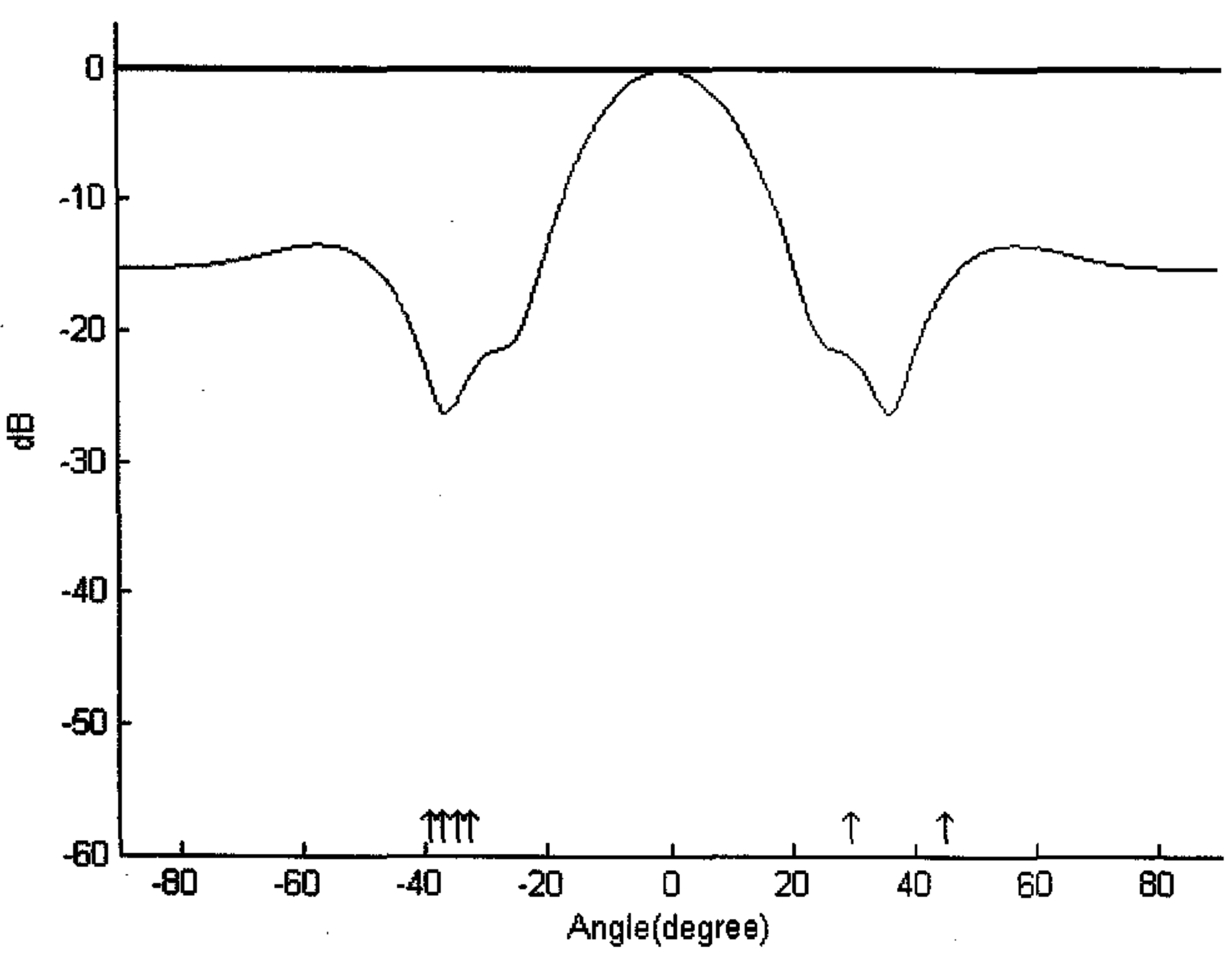

Fig. 11. Beam pattern of the spatial smoothing approach.

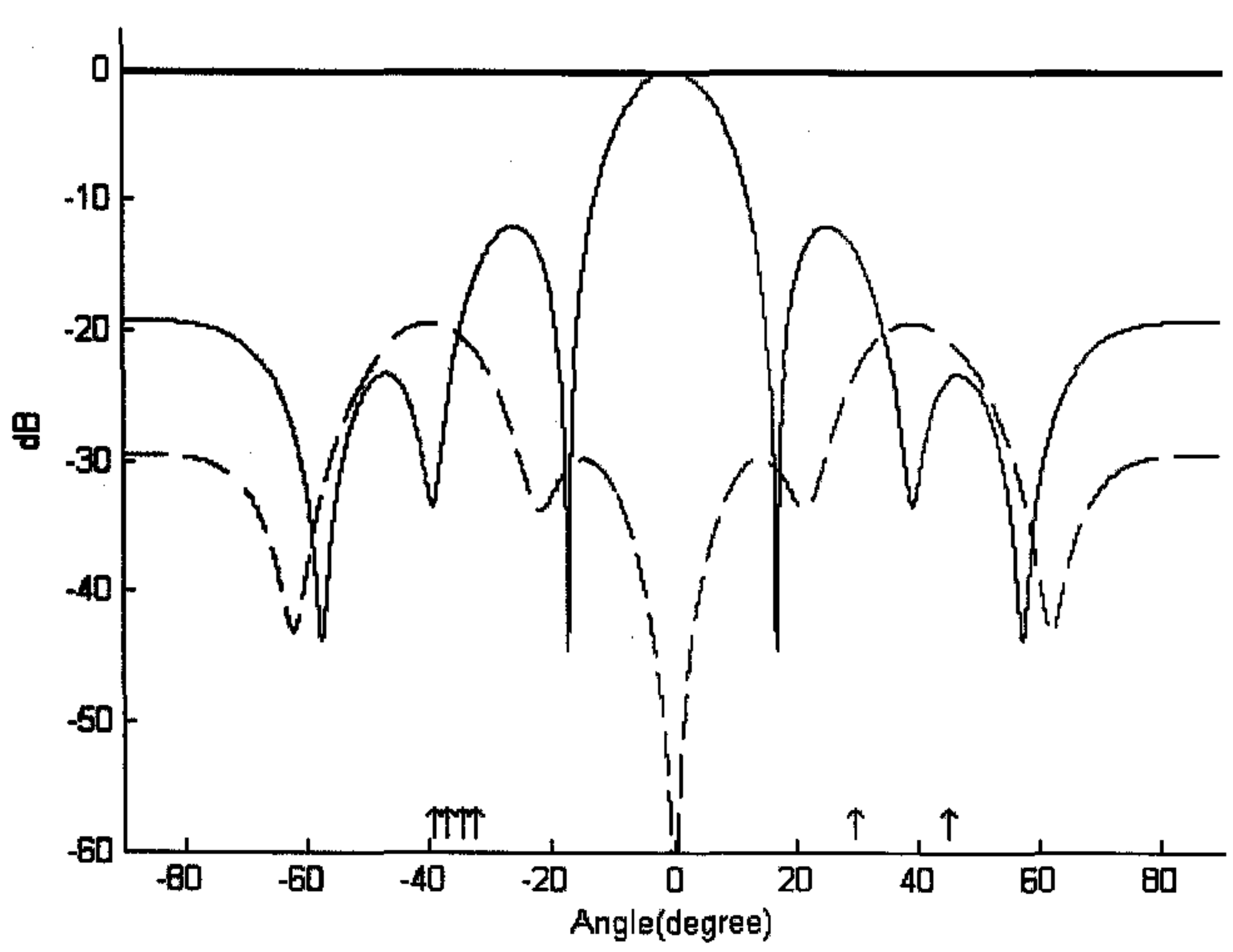

Fig. 12. Beam patterns of the proposed method; unit gain constraint(solid line), null constraint(dashed line).

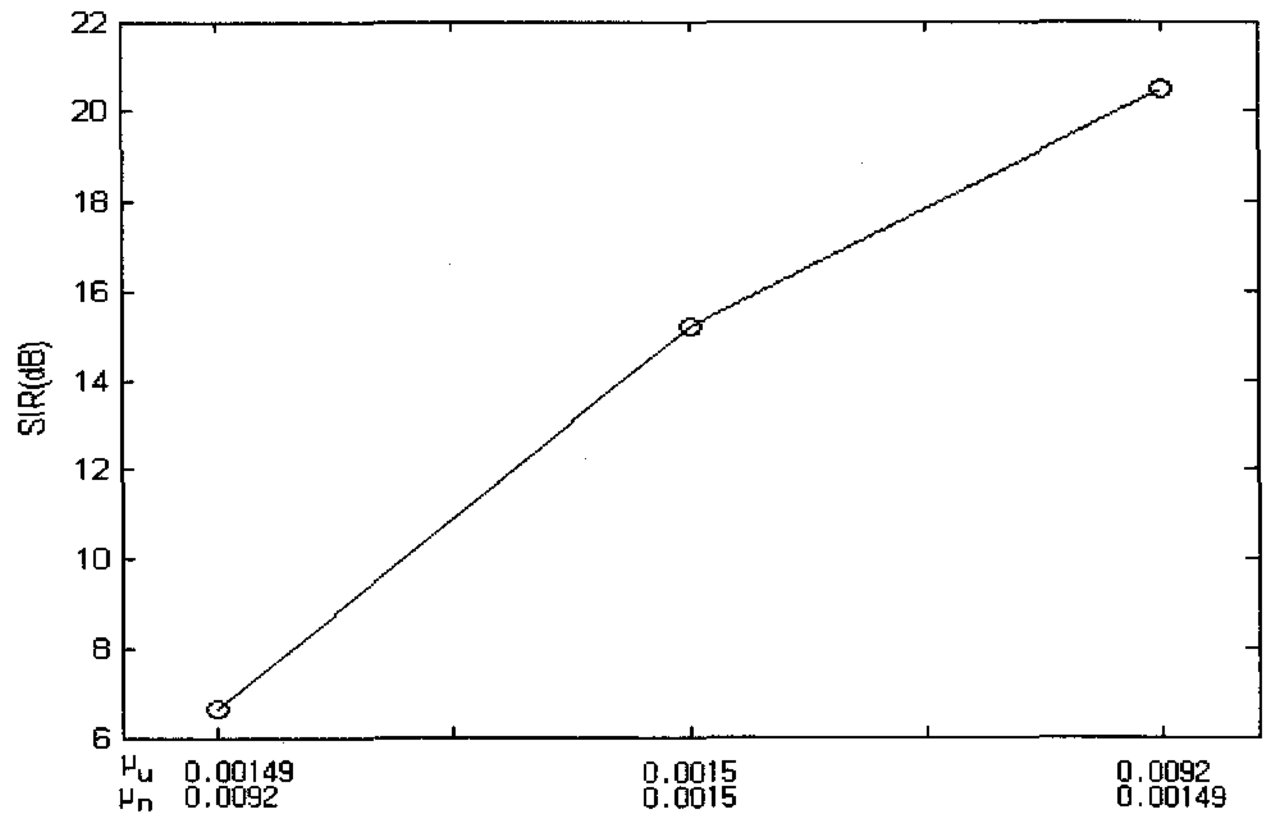

Fig. 13. Variation of SIR with respect to the values of μ_u and μ_n.

It is shown in Figs. 9 and 10 that the alternate mainbeam nulling approach performs better than the spatial smoothing approach. It is observed in Figs. 11 and 12 that in the proposed method, a null is formed around a region of four interferences while at other directions (27.9° 43.4°) appropriate nulls are not formed. But the gain at 43.4°of the proposed method is lower than that of the spatial smoothing approach.

The array performance in terms of the signal-to-interference ratio(SIR) is examined in the array output with respect to the values of μ_u and μ_n for the case of one interference. The output signal to interference ratio is expressed by

$$SIR=\frac{E[|\widehat{w}_o^H d_k|^2]}{E[|\widehat{w}_o^H i_k|^2]} \tag{13}$$

where $\widehat{w}_o$ is the steady-state weight vector obtained by (10), d_k is the desired signal vector, and i_k is the sum of the interference signal vectors. The variation of SIR with respect to the values of μ_u and μ_n is shown in Fig. 13. It is observed that the SIR performance improves as the values of μ_u and μ_n get closer to the optimal values. It is to be noted that the optimal values for μ_u and μ_n are 0.0092 and 0.00149 respectively.

V. Conclusions

The linearly constrained adaptive array processor suffers from signal cancellation in the presence of incident interferences. If the interferences are coherent, the desired signal is totally cancelled in the array output. To prevent the signal cancellation phenomenon, we proposed the alternate mainbeam nulling approach in the linearly constrained LMS algorithm. The two adaptive algorithms for the unit gain and null constraints are implemented alternately to reduce the signal interaction between the desired signal and the interferences. It is shown that the performance of the proposed method depends on the values of convergence parameters for the two algorithms. It is observed that the output SIR improves as the convergence parameters approach the optimal values. It is shown that the proposed method performs better than the spatial smoothing approach.

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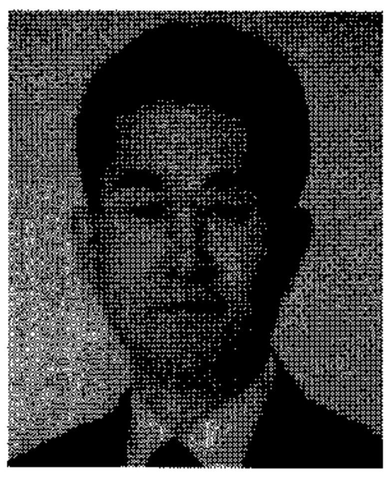

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