Fuzzy Weakly r-Semicontinuous Mappings

Seok Jong Lee and Jin Tae Kim

Department of Mathematics, Chungbuk National University, Cheongju 361-763, Korea

Abstract

By generalizing the definition of fuzzy weakly semicontinuous mappings by B. S. Zhong, we introduce the concept of fuzzy weakly r-semicontinuous mappings in fuzzy topology of Chattopadhyay. Then the concept introduced by B. S. Zhong becomes a special case of our definition. Also, we show that fuzzy weakly r-semicontinuity and fuzzy weakly r-continuity are independent notions.

Key words : fuzzy weakly r-semicontinuous

1. Introduction

Chang [1] introduced fuzzy topological spaces and several other authors continued the investigation of such spaces. B. S. Zhong [2] introduced the concept of fuzzy weakly semicontinuous mappings in Chang's fuzzy topology. Chattopadhyay and his colleagues [3, 4] introduced another definition of fuzzy topology as a generalization of Chang's fuzzy topology. By generalizing the definition of fuzzy weakly semicontinuous mappings by B. S. Zhong, we introduce the concept of fuzzy weakly r-semicontinuous mappings in fuzzy topology of Chattopadhyay. Then the concept introduced by B. S. Zhong becomes a special case of our definition. Also, we show that fuzzy weakly r-semicontinuity and fuzzy weakly r-continuity are independent notions.

2. Preliminaries

We will denote the unit interval $[0,1]$ of the real line by I and $I_0 = (0,1]$. A member μ of I^X is called a *fuzzy set* in X. For any $\mu \in I^X$, μ^c denotes the complement $1-\mu$. By $\tilde{0}$ and $\tilde{1}$ we denote constant mappings on X with value 0 and 1, respectively. All other notations are standard notations of fuzzy set theory.

A *Chang's fuzzy topology* on X is a family T of fuzzy sets in X which satisfies the following properties:

(1) $\tilde{0}, \tilde{1} \in T$.

(2) If $\mu_1, \mu_2 \in T$ then $\mu_1 \wedge \mu_2 \in T$.

(3) If $\mu_i \in T$ for each i, then $\bigvee \mu_i \in T$.

Manuscript received Apr. 4, 2008; revised May. 7, 2008.

The pair (X, T) is called a *Chang's fuzzy topological space.*

A *fuzzy topology* on X is a mapping $\mathcal{T} : I^X \to I$ which satisfies the following properties:

(1) $\mathcal{T}(\tilde{0}) = \mathcal{T}(\tilde{1}) = 1$.

(2) $\mathcal{T}(\mu_1 \wedge \mu_2) \geq \mathcal{T}(\mu_1) \wedge \mathcal{T}(\mu_2)$.

(3) $\mathcal{T}(\bigvee \mu_i) \geq \bigwedge \mathcal{T}(\mu_i)$.

The pair $(X, \mathcal{T})$ is called a *fuzzy topological space.*

For each $\alpha \in (0,1]$, a *fuzzy point* x_α is a fuzzy set in X defined by

$$x_\alpha(y) = \begin{cases} \alpha & \text{if } y = x, \\ 0 & \text{if } y \neq x. \end{cases}$$

In this case, x and α are called the *support* and the *value* of x_α, respectively. A fuzzy point x_α is said to *belong* to a fuzzy set μ in X, denoted by $x_\alpha \in \mu$, if $\alpha \leq \mu(x)$.

Definition 2.1. ([5]) Let μ be a fuzzy set in a fuzzy topological space $(X, \mathcal{T})$ and $r \in I_0$. Then μ is said to be

(1) *fuzzy r-open* if $\mathcal{T}(\mu) \geq r$,

(2) *fuzzy r-closed* if $\mathcal{T}(\mu^c) \geq r$.

Definition 2.2. ([5, 6]) Let μ be a fuzzy set in a fuzzy topological space $(X, \mathcal{T})$ and $r \in I_0$. Then μ is said to be

(1) *fuzzy r-semiopen* if there is a fuzzy r-open set ρ in X such that $\rho \leq \mu \leq \mathrm{cl}(\rho, r)$,

(2) *fuzzy r-semiclosed* if there is a fuzzy r-closed set ρ in X such that $\mathrm{int}(\rho, r) \leq \mu \leq \rho$,

(3) *fuzzy r-regular open* if $\mathrm{int}(\mathrm{cl}(\mu, r), r) = \mu$,

(4) *fuzzy r-regular closed* if $\mathrm{cl}(\mathrm{int}(\mu, r), r) = \mu$.

Theorem 2.3. ([5]) Let μ be a fuzzy set in a fuzzy topological space $(X, \mathcal{T})$ and $r \in I_0$. Then the following statements are equivalent:

(1) μ is a fuzzy r-semiopen set.

(2) μ^c is a fuzzy r-semiclosed set.

(3) $\mathrm{cl}(\mathrm{int}(\mu, r), r) \geq \mu$.

(4) $\mathrm{int}(\mathrm{cl}(\mu^c, r), r) \leq \mu^c$.

Definition 2.4. ([5]) Let $(X, \mathcal{T})$ be a fuzzy topological space. For each $r \in I_0$ and for each $\mu \in I^X$, the *fuzzy r-semiclosure* is defined by

$$\mathrm{scl}(\mu, r) = \bigwedge\{\rho \in I^X \mid \mu \leq \rho,\ \rho \text{ is fuzzy } r\text{-semiclosed}\},$$

and the *fuzzy r-semiinterior* is defined by

$$\mathrm{sint}(\mu, r) = \bigvee\{\rho \in I^X \mid \mu \geq \rho,\ \rho \text{ is fuzzy } r\text{-semiopen}\}.$$

Obviously $\mathrm{scl}(\mu, r)$ is the smallest fuzzy r-semiclosed set which contains μ and $\mathrm{sint}(\mu, r)$ is the greatest fuzzy r-semiopen set which is contained in μ. Also, $\mathrm{scl}(\mu, r) = \mu$ for any fuzzy r-semiclosed set μ and $\mathrm{sint}(\mu, r) = \mu$ for any fuzzy r-semiopen set μ. Moreover, we have

$$\mathrm{int}(\mu, r) \leq \mathrm{sint}(\mu, r) \leq \mu \leq \mathrm{scl}(\mu, r) \leq \mathrm{cl}(\mu, r).$$

Also, we have the following results :

(1) $\mathrm{scl}(\tilde{0}, r) = \tilde{0}$, $\mathrm{scl}(\tilde{1}, r) = \tilde{1}$, $\mathrm{sint}(\tilde{0}, r) = \tilde{0}$, $\mathrm{sint}(\tilde{1}, r) = \tilde{1}$.

(2) $\mathrm{scl}(\mu, r) \geq \mu$, $\mathrm{sint}(\mu, r) \leq \mu$.

(3) $\mathrm{scl}(\mu \vee \rho, r) \geq \mathrm{scl}(\mu, r) \vee \mathrm{scl}(\rho, r)$, $\mathrm{sint}(\mu \wedge \rho, r) \leq \mathrm{sint}(\mu, r) \wedge \mathrm{sint}(\rho, r)$.

(4) $\mathrm{scl}(\mathrm{scl}(\mu, r), r) = \mathrm{scl}(\mu, r)$, $\mathrm{sint}(\mathrm{sint}(\mu, r), r) = \mathrm{sint}(\mu, r)$.

Theorem 2.5. ([7]) For a fuzzy set μ in a fuzzy topological space X and $r \in I_0$, we have :

(1) $\mathrm{sint}(\mu, r)^c = \mathrm{scl}(\mu^c, r)$.

(2) $\mathrm{scl}(\mu, r)^c = \mathrm{sint}(\mu^c, r)$.

Definition 2.6. ([7, 5, 6]) Let $f : (X, \mathcal{T}) \to (Y, \mathcal{U})$ be a mapping from a fuzzy topological space X to a fuzzy topological space Y and $r \in I_0$. Then f is called

(1) a *fuzzy r-continuous* mapping if $f^{-1}(\mu)$ is a fuzzy r-open set in X for each fuzzy r-open set μ in Y,

(2) a *fuzzy r-semicontinuous* mapping if $f^{-1}(\mu)$ is a fuzzy r-semiopen set in X for each fuzzy r-open set μ in Y, or equivalently, $f^{-1}(\mu)$ is a fuzzy r-semiclosed set in X for each fuzzy r-closed set μ in Y,

(3) a *fuzzy almost r-continuous* mapping if $f^{-1}(\mu)$ is a fuzzy r-open set in X for each fuzzy r-regular open set μ in Y,

(4) a *fuzzy weakly r-continuous* mapping if $f^{-1}(\mu) \leq \mathrm{int}(f^{-1}(\mathrm{cl}(\mu, r)), r)$ for each fuzzy r-open set μ in Y.

(5) a *fuzzy r-irresolute* mapping if $f^{-1}(\mu)$ is a fuzzy r-semiopen set in X for each fuzzy r-semiopen set μ in Y.

3. Fuzzy weakly r-semicontinuous mappings

We define the notion of fuzzy weakly r-semicontinuous mappings, and investigate some of their properties.

Definition 3.1. Let $f : (X, \mathcal{T}) \to (Y, \mathcal{U})$ be a mapping from a fuzzy topological space X to a fuzzy topological space Y and $r \in I_0$. Then f is called a *fuzzy weakly r-semicontinuous* mapping if $f^{-1}(\mu) \leq \mathrm{sint}(f^{-1}(\mathrm{scl}(\mu, r)), r)$ for each fuzzy r-open set μ in Y.

Remark 3.2. It is obvious that a fuzzy r-semicontinuous mapping is also a fuzzy weakly r-semicontinuous mapping for each $r \in I_0$. But the converse does not hold as in the following example.

Example 3.3. Let $X = \{x, y, z\}$ and μ_1 and μ_2 be fuzzy sets in X defined as

$$\mu_1(x) = \frac{1}{3},\ \mu_1(y) = \frac{1}{3},\ \mu_1(z) = \frac{1}{2};$$

and

$$\mu_2(x) = \frac{1}{2},\ \mu_2(y) = \frac{1}{2},\ \mu_2(z) = \frac{1}{2}.$$

Define $\mathcal{T}_1 : I^X \to I$ and $\mathcal{T}_2 : I^X \to I$ by

$$\mathcal{T}_1(\mu) = \begin{cases} 1 & \text{if } \mu = \tilde{0}, \tilde{1}, \\ \frac{1}{2} & \text{if } \mu = \mu_2, \\ 0 & \text{otherwise;} \end{cases}$$

and

$$\mathcal{T}_2(\mu) = \begin{cases} 1 & \text{if } \mu = \tilde{0}, \tilde{1}, \\ \frac{1}{2} & \text{if } \mu = \mu_1,\ \mu_2, \\ 0 & \text{otherwise.} \end{cases}$$

Then $\mathcal{T}_1$ and $\mathcal{T}_2$ are fuzzy topologies on X. Consider the mapping $f : (X, \mathcal{T}_1) \to (X, \mathcal{T}_2)$ defined by $f(x) = x$ for each $x \in X$. Note that

$$f^{-1}(\tilde{0}) = \tilde{0} \leq \mathrm{sint}(f^{-1}(\mathrm{scl}(\tilde{0}, \frac{1}{2})), \frac{1}{2}) = \tilde{0},$$

$$f^{-1}(\tilde{1}) = \tilde{1} \leq \text{sint}(f^{-1}(\text{scl}(\tilde{1}, \frac{1}{2})), \frac{1}{2}) = \tilde{1},$$

$$\begin{aligned} f^{-1}(\mu_1) &= \mu_1 \leq \text{sint}(f^{-1}(\text{scl}(\mu_1, \frac{1}{2})), \frac{1}{2}) \\ &= \text{sint}(f^{-1}(\mu_2), \frac{1}{2}) = \mu_2, \end{aligned}$$

and

$$\begin{aligned} f^{-1}(\mu_2) &= \mu_2 \leq \text{sint}(f^{-1}(\text{scl}(\mu_2, \frac{1}{2})), \frac{1}{2}) \\ &= \text{sint}(f^{-1}(\mu_2), \frac{1}{2}) = \mu_2. \end{aligned}$$

Thus f is fuzzy weakly $\frac{1}{2}$-semicontinuous mapping. But $f^{-1}(\mu_1) = \mu_1$ is not fuzzy $\frac{1}{2}$-semiopen in $(X, \mathcal{T}_1)$ and hence f is not a fuzzy $\frac{1}{2}$-semicontinuous mapping.

Theorem 3.4. Let $f : (X, \mathcal{T}) \to (Y, \mathcal{U})$ be a fuzzy almost r-continuous mapping. Then f is also a fuzzy weakly r-semicontinuous mapping.

Proof. Let μ be a fuzzy r-open set in Y. Using Theorem 4.2 in [6], we have

$$\begin{aligned} f^{-1}(\mu) &\leq \text{int}(f^{-1}(\text{int}(\text{cl}(\mu, r), r)), r) \\ &\leq \text{sint}(f^{-1}(\text{int}(\text{cl}(\mu, r), r)), r) \\ &\leq \text{sint}(f^{-1}(\text{int}(\text{cl}(\text{scl}(\mu, r), r), r)), r). \end{aligned}$$

Since $\text{scl}(\mu, r)$ is a fuzzy r-semiclosed set in Y, by Theorem 2.3,

$$\begin{aligned} f^{-1}(\mu) &\leq \text{sint}(f^{-1}(\text{int}(\text{cl}(\text{scl}(\mu, r), r)), r) \\ &\leq \text{sint}(f^{-1}(\text{scl}(\mu, r)), r). \end{aligned}$$

Hence f is a fuzzy weakly r-semicontinuous mapping. □

Remark 3.5. The following example shows that the converse of Theorem 3.4 need not be true.

Example 3.6. Let $X = I$ and μ_1 and μ_2 be fuzzy sets in X defined as

$$\mu_1(x) = \begin{cases} 0 & \text{if } 0 \leq x \leq \frac{1}{2}, \\ 2x - 1 & \text{if } \frac{1}{2} \leq x \leq 1; \end{cases}$$

and

$$\mu_2(x) = \begin{cases} 1 & \text{if } 0 \leq x \leq \frac{1}{4}, \\ -4x + 2 & \text{if } \frac{1}{4} \leq x \leq \frac{1}{2}, \\ 0 & \text{if } \frac{1}{2} \leq x \leq 1. \end{cases}$$

Define $\mathcal{T} : I^X \to I$ by

$$\mathcal{T}(\mu) = \begin{cases} 1 & \text{if } \mu = \tilde{0}, \tilde{1}, \\ \frac{1}{2} & \text{if } \mu = \mu_1,\ \mu_2,\ \mu_1 \vee \mu_2, \\ 0 & \text{otherwise.} \end{cases}$$

Then $\mathcal{T}$ is a fuzzy topology on X. Let $f : (X, \mathcal{T}) \to (X, \mathcal{T})$ be defined by $f(x) = \frac{1}{2}x$. It is easy to see that $f^{-1}(\tilde{0}) = \tilde{0}$, $f^{-1}(\tilde{1}) = \tilde{1}, f^{-1}(\mu_1) = \tilde{0}$ and $f^{-1}(\mu_2) = f^{-1}(\mu_1 \vee \mu_2) = \mu_1^c$. Since $\text{cl}(\mu_2, \frac{1}{2}) = \mu_1^c$, μ_1^c is a fuzzy $\frac{1}{2}$-semiopen set and thus f is a fuzzy $\frac{1}{2}$-semicontinuous mapping. Hence f is a fuzzy weakly $\frac{1}{2}$-semicontinuous mapping. Note that $\text{int}(\text{cl}(\mu_2, \frac{1}{2}), \frac{1}{2}) = \mu_2$. Thus μ_2 is a fuzzy $\frac{1}{2}$-regular open set in Y. But $f^{-1}(\mu_2) = \mu_1^c$ is not fuzzy $\frac{1}{2}$-open. Hence f is not a fuzzy almost $\frac{1}{2}$-continuous mapping.

Remark 3.7. The following examples show that a fuzzy weakly r-semicontinuous mapping need not be fuzzy weakly r-continuous vice versa.

Example 3.8. A fuzzy weakly r-semicontinuous mapping need not be a fuzzy weakly r-continuous mapping.

Let $X = \{x, y, z\}$ and μ_1, μ_2 and μ_3 be fuzzy sets in X defined as

$$\mu_1(x) = \frac{3}{10},\ \mu_1(y) = \frac{1}{10},\ \mu_1(z) = \frac{1}{10};$$

$$\mu_2(x) = \frac{1}{2},\ \mu_2(y) = \frac{1}{2},\ \mu_2(z) = \frac{1}{2};$$

and

$$\mu_3(x) = \frac{1}{5},\ \mu_3(y) = \frac{1}{10},\ \mu_3(z) = 0.$$

Define $\mathcal{T}_1 : I^X \to I$ and $\mathcal{T}_2 : I^X \to I$ by

$$\mathcal{T}_1(\mu) = \begin{cases} 1 & \text{if } \mu = \tilde{0}, \tilde{1}, \\ \frac{1}{2} & \text{if } \mu = \mu_3, \\ 0 & \text{otherwise;} \end{cases}$$

and

$$\mathcal{T}_2(\mu) = \begin{cases} 1 & \text{if } \mu = \tilde{0}, \tilde{1}, \\ \frac{1}{2} & \text{if } \mu = \mu_1,\ \mu_2, \\ 0 & \text{otherwise.} \end{cases}$$

Then $\mathcal{T}_1$ and $\mathcal{T}_2$ are fuzzy topologies on X. Consider the mapping $f : (X, \mathcal{T}_1) \to (X, \mathcal{T}_2)$ defined by $f(x) = x$ for each $x \in X$. Then

$$f^{-1}(\tilde{0}) = \tilde{0} \leq \text{sint}(f^{-1}(\text{scl}(\tilde{0}, \frac{1}{2})), \frac{1}{2}) = \tilde{0},$$

$$f^{-1}(\tilde{1}) = \tilde{1} \leq \text{sint}(f^{-1}(\text{scl}(\tilde{1}, \frac{1}{2})), \frac{1}{2}) = \tilde{1},$$

$$f^{-1}(\mu_1) = \mu_1 \leq \text{sint}(f^{-1}(\text{scl}(\mu_1, \frac{1}{2})), \frac{1}{2}) = \mu_2,$$

and

$$f^{-1}(\mu_2) = \mu_2 \leq \text{sint}(f^{-1}(\text{scl}(\mu_2, \frac{1}{2})), \frac{1}{2}) = \mu_2.$$

Hence f is a fuzzy weakly $\frac{1}{2}$-semicontinuous mapping. Also, it is easy to see that

$$f^{-1}(\mu_1) = \mu_1 \not\leq \text{int}(f^{-1}(\text{cl}(\mu_1, \frac{1}{2})), \frac{1}{2}) = \mu_3.$$

Thus f is not a fuzzy weakly $\frac{1}{2}$-continuous mapping.

Example 3.9. A fuzzy weakly r-continuous mapping need not be a fuzzy weakly r-semicontinuous mapping.

Let $X = \{x, y, z\}$ and μ_1 and μ_2 be fuzzy sets in X defined as

$$\mu_1(x) = \frac{1}{5},\ \mu_1(y) = \frac{1}{5},\ \mu_1(z) = \frac{3}{10};$$

and

$$\mu_2(x) = \frac{7}{10},\ \mu_2(y) = \frac{4}{5},\ \mu_2(z) = \frac{1}{2}.$$

Define $\mathcal{T}_1 : I^X \to I$ and $\mathcal{T}_2 : I^X \to I$ by

$$\mathcal{T}_1(\mu) = \begin{cases} 1 & \text{if } \mu = \tilde{0}, \tilde{1}, \\ \frac{1}{2} & \text{if } \mu = \mu_2, \\ 0 & \text{otherwise}; \end{cases}$$

and

$$\mathcal{T}_2(\mu) = \begin{cases} 1 & \text{if } \mu = \tilde{0}, \tilde{1}, \\ \frac{1}{2} & \text{if } \mu = \mu_1, \\ 0 & \text{otherwise}. \end{cases}$$

Then $\mathcal{T}_1$ and $\mathcal{T}_2$ be fuzzy topologies on X. Consider the mapping $f : (X, \mathcal{T}_1) \to (X, \mathcal{T}_2)$ defined by $f(x) = x$ for each $x \in X$. Then

$$f^{-1}(\tilde{0}) = \tilde{0} \leq \text{int}(f^{-1}(\text{cl}(\tilde{0}, \frac{1}{2})), \frac{1}{2}) = \tilde{0},$$

$$f^{-1}(\tilde{1}) = \tilde{1} \leq \text{int}(f^{-1}(\text{cl}(\tilde{1}, \frac{1}{2})), \frac{1}{2}) = \tilde{1},$$

and

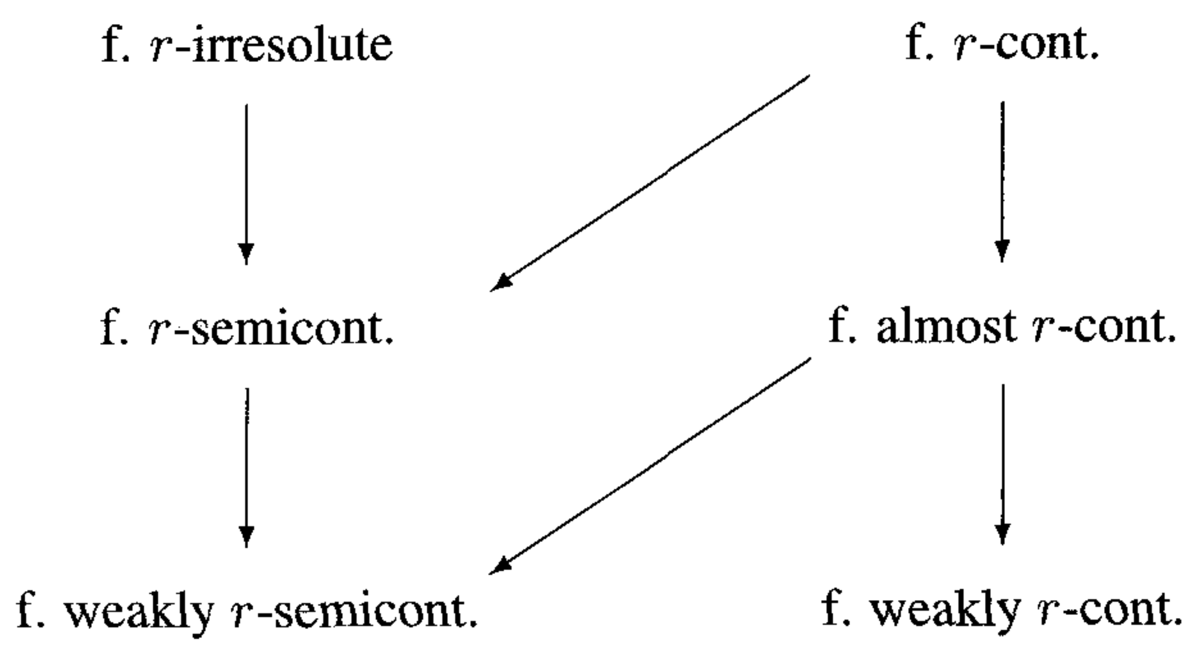
$$f^{-1}(\mu_1) = \mu_1 \leq \text{int}(f^{-1}(\text{cl}(\mu_1, \frac{1}{2})), \frac{1}{2}) = \mu_2.$$

Hence f is a fuzzy weakly $\frac{1}{2}$-continuous mapping. On the other hand, it is easy to see that

$$f^{-1}(\mu_1) = \mu_1 \not\leq \text{sint}(f^{-1}(\text{scl}(\mu_1, \frac{1}{2})), \frac{1}{2}) = \tilde{0}.$$

Thus f is not a fuzzy weakly $\frac{1}{2}$-semicontinuous mapping.

Remark 3.10. From the above results one may easily verify the following implications. And none of the undrawn implications holds.



Theorem 3.11. Let $f : (X, \mathcal{T}) \to (Y, \mathcal{U})$ be a mapping from a fuzzy topological space X to a fuzzy topological space Y and $r \in I_0$. Then the following statements are equivalent:

(1) f is a fuzzy weakly r-semicontinuous mapping.

(2) $\text{scl}(f^{-1}(\text{sint}(\mu, r)), r) \leq f^{-1}(\mu)$ for each fuzzy r-closed set μ in Y.

(3) $f^{-1}(\text{int}(\rho, r)) \leq \text{sint}(f^{-1}(\text{scl}(\rho, r)), r)$ for each fuzzy set ρ in Y.

(4) $\text{scl}(f^{-1}(\text{sint}(\rho, r)), r) \leq f^{-1}(\text{cl}(\rho, r))$ for each fuzzy set ρ in Y.

Proof. (1) $\Rightarrow$ (2) Let f be fuzzy weakly r-semicontinuous and μ a fuzzy r-closed set in Y. Since μ^c is a fuzzy r-open set in Y,

$$\begin{aligned} (f^{-1}(\mu))^c &= f^{-1}(\mu^c) \\ &\leq \text{sint}(f^{-1}(\text{scl}(\mu^c, r)), r) \\ &= \text{scl}(f^{-1}(\text{sint}(\mu, r)), r)^c. \end{aligned}$$

Hence we have $\text{scl}(f^{-1}(\text{sint}(\mu, r)), r) \leq f^{-1}(\mu)$.
(2) $\Rightarrow$ (1) Let μ be a fuzzy r-open set in Y. Then μ^c is a fuzzy r-closed set in Y. By (2),

$$\begin{aligned} (f^{-1}(\mu))^c &= f^{-1}(\mu^c) \geq \text{scl}(f^{-1}(\text{sint}(\mu^c, r)), r) \\ &= \text{sint}(f^{-1}(\text{scl}(\mu, r)), r)^c. \end{aligned}$$

Thus we have $f^{-1}(\mu) \leq \text{sint}(f^{-1}(\text{scl}(\mu, r)), r)$. Hence f is a fuzzy weakly r-semicontinuous mapping.
(1) $\Rightarrow$ (3) Let ρ be a fuzzy set in Y. Then $\text{int}(\rho, r)$ is a fuzzy r-open set in Y. Since f is fuzzy weakly r-semicontinuous,

$$\begin{aligned} f^{-1}(\text{int}(\rho, r)) &\leq \text{sint}(f^{-1}(\text{scl}(\text{int}(\rho, r), r)), r) \\ &\leq \text{sint}(f^{-1}(\text{scl}(\rho, r)), r). \end{aligned}$$

(3) $\Rightarrow$ (1) It is obvious.
(2) $\Rightarrow$ (4) Let ρ be a fuzzy set in Y. Then $\text{cl}(\rho, r)$ is a fuzzy r-closed set in Y. By (2),

$$\begin{aligned} \text{scl}(f^{-1}(\text{sint}(\rho, r)), r) &\leq \text{scl}(f^{-1}(\text{sint}(\text{cl}(\rho, r), r)), r) \\ &\leq f^{-1}(\text{cl}(\rho, r)). \end{aligned}$$

(4) $\Rightarrow$ (2) It is obvious. □

Definition 3.12. Let $f : (X, \mathcal{T}) \to (Y, \mathcal{U})$ be a mapping from a fuzzy topological space X to a fuzzy topological space Y and $r \in I_0$. Then f is said to be *fuzzy weakly r-semicontinuous at a fuzzy point* x_α in X if for each fuzzy r-open set μ in Y and $f(x_\alpha) \leq \mu$, there exists a fuzzy r-semiopen set ρ in X such that $x_\alpha \in \rho$ and $f(\rho) \leq \mathrm{scl}(\mu, r)$.

Theorem 3.13. Let $f : (X, \mathcal{T}) \to (Y, \mathcal{U})$ be a mapping from a fuzzy topological space X to a fuzzy topological space Y and $r \in I_0$. Then f is fuzzy weakly r-semicontinuous if and only if f is fuzzy weakly r-semicontinuous for each fuzzy point x_α in X.

Proof. Let f be a fuzzy weakly r-semicontinuous mapping, x_α a fuzzy point in X and μ a fuzzy r-open set in Y such that $f(x_\alpha) \leq \mu$. Then $x_\alpha \leq f^{-1}(\mu) \leq \mathrm{sint}(f^{-1}(\mathrm{scl}(\mu, r)), r)$. Let $\rho = \mathrm{sint}(f^{-1}(\mathrm{scl}(\mu, r)), r)$. Then ρ is a fuzzy r-semiopen set in X and

$$\begin{aligned} f(\rho) &= f(\mathrm{sint}(f^{-1}(\mathrm{scl}(\mu, r)), r)) \\ &\leq f(f^{-1}(\mathrm{scl}(\mu, r))) \\ &\leq \mathrm{scl}(\mu, r). \end{aligned}$$

Hence f is a fuzzy weakly r-semicontinuous mapping for each fuzzy point x_α in X.
Conversely, let μ be a fuzzy r-open set in Y and x_α a fuzzy point in $f^{-1}(\mu)$. Then $f(x_\alpha) \leq \mu$. By hypothesis, there exists a fuzzy r-semiopen set ρ_{x_α} in X such that $x_\alpha \in \rho_{x_\alpha}$ and $f(\rho_{x_\alpha}) \leq \mathrm{scl}(\mu, r)$. Hence $x_\alpha \leq \rho_{x_\alpha} \leq f^{-1}(f(\rho_{x_\alpha})) \leq f^{-1}(\mathrm{scl}(\mu, r))$. So, $x_\alpha \leq \rho_{x_\alpha} = \mathrm{sint}(\rho_{x_\alpha}, r) \leq \mathrm{sint}(f^{-1}(\mathrm{scl}(\mu, r)), r)$. Thus

$$\begin{aligned} f^{-1}(\mu) &= \bigvee\{x_\alpha \mid x_\alpha \in f^{-1}(\mu)\} \\ &\leq \bigvee\{\rho_{x_\alpha} \mid x_\alpha \in f^{-1}(\mu)\} \\ &\leq \mathrm{sint}(f^{-1}(\mathrm{scl}(\mu, r)), r). \end{aligned}$$

Therefore f is a fuzzy weakly r-semicontinuous mapping. □

Let $(X, \mathcal{T})$ be a fuzzy topological space. For an r-cut $\mathcal{T}_r = \{\mu \in I^X \mid \mathcal{T}(\mu) \geq r\}$, it is obvious that $(X, \mathcal{T}_r)$ is a Chang's fuzzy topological space for all $r \in I_0$.

Let (X, T) be a Chang's fuzzy topological space and $r \in I_0$. Recall [3] that a fuzzy topology $T^r : I^X \to I$ is defined by

$$T^r(\mu) = \begin{cases} 1 & \text{if } \mu = \tilde{0}, \tilde{1}, \\ r & \text{if } \mu = T - \{\tilde{0}, \tilde{1}\}, \\ 0 & \text{otherwise.} \end{cases}$$

Theorem 3.14. Let $f : (X, \mathcal{T}) \to (Y, \mathcal{U})$ be a mapping from a fuzzy topological space X to a fuzzy topological space Y and $r \in I_0$. Then f is fuzzy weakly r-semicontinuous if and only if $f : (X, \mathcal{T}_r) \to (Y, \mathcal{U}_r)$ is fuzzy weakly semicontinuous.

Proof. Straightforward. □

Theorem 3.15. Let $f : (X, T) \to (Y, U)$ be a mapping from a Chang's fuzzy topological space X to a Chang's fuzzy topological space Y and $r \in I_0$. Then f is fuzzy weakly semicontinuous if and only if $f : (X, T^r) \to (Y, U^r)$ is fuzzy weakly r-semicontinuous.

Proof. Straightforward. □

Remark 3.16. By the above two theorems, we know that the concept of a fuzzy weakly r-semicontinuous mapping is a generalization of the concept of a fuzzy weakly semicontinuous mapping.

Acknowledgement. This work was supported by the research grant of the Chungbuk National University in 2007.

References

[1] C. L. Chang, "Fuzzy topological spaces," *J. Math. Anal. Appl.*, vol. 24, pp. 182–190, 1968.

[2] S. Z. Bai, "Fuzzy weak semicontinuity," *Fuzzy Sets and Systems*, vol. 47, no. 1, pp. 93–98, 1992.

[3] K. C. Chattopadhyay, R. N. Hazra, and S. K. Samanta, "Gradation of openness : Fuzzy topology," *Fuzzy Sets and Systems*, vol. 49, pp. 237–242, 1992.

[4] R. N. Hazra, S. K. Samanta, and K. C. Chattopadhyay, "Fuzzy topology redefined," *Fuzzy Sets and Systems*, vol. 45, no. 1, pp. 79–82, 1992.

[5] S. J. Lee and E. P. Lee, "Fuzzy r-continuous and fuzzy r-semicontinuous maps," *Int. J. Math. Math. Sci.*, vol. 27, no. 1, pp. 53–63, 2001.

[6] S. J. Lee and E. P. Lee, "Fuzzy r-regular open sets and fuzzy almost r-continuous maps," *Bull. Korean Math. Soc.*, vol. 39, no. 3, pp. 441–453, 2002.

[7] E. P. Lee, *Various kinds of continuity in fuzzy topological spaces*. PhD thesis, Chungbuk National University, 1998.

Seok Jong Lee
Professor of Chungbuk National University
Research Area: Fuzzy mathematics, Fuzzy topology, General topology
E-mail : sjl@chungbuk.ac.kr

Jin Tae Kim
Research Area: Fuzzy mathematics, Fuzzy topology, General topology
E-mail : kjtmath@hanmail.net