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Interval-Valued Fuzzy Soft sets 관한 연구

A Note on Interval-Valued Fuzzy Soft Sets

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요 약

본 논문은 부정확한 null interval-valued fuzzy soft sets 와 absolute interval-valued fuzzy soft sets 개념의 오류를 지적하였으며 interval-valued fuzzy soft sets를 위해 새롭게 정의된 개념을 소개하며 이를 이용한 기본적인 성질을 조사한다.

Abstract

In this paper, we show that the concepts of null interval-valued fuzzy and absolute interval-valued fuzzy soft sets are not reasonable. Thus we introduce the modified concepts for them and study some properties. Also we introduce an operation for the interval-valued fuzzy soft set theory and study basic properties.

Key Words : interval-valued fuzzy soft sets, null interval-valued fuzzy soft sets, absolute interval-valued fuzzy soft set

1. Intorduction and Preliminaries

In [4], Son introduced the concept of interval-value fuzzy soft sets based on [2, 3, 5]. In this paper, we show that the concepts of null interval-valued fuzzy soft set and absolute interval-valued fuzzy soft set are not reasonable. Thus we introduce the modified concepts for them and study some properties. Also we introduce an operation for the interval-valued fuzzy soft set theory and study basic properties.

Let $D[0,1]$ be the set of all closed subintervals of the interval $[0,1]$. Let X be a nonempty set. An interval-valued fuzzy set [1] in X as the following:

An interval-valued fuzzy set A on X is a map $A: X \rightarrow D[0,1]$. We will denote by $IVF(X)$ the set of all the interval-valued fuzzy sets. Let $A, B \in IVF(X)$.

$(A \cup B)(x) = [\max(\inf A(x), \inf B(x)), \max(\sup A(x), \sup B(x))]$, for all $x \in X$.

$(A \cap B)(x) = [\min(\inf A(x), \inf B(x)), \min(\sup A(x), \sup B(x))]$, for all $x \in X$.

$A \subset B \Leftrightarrow \inf A(x) \leq \inf B(x), \quad \sup A(x) \leq \sup B(x)$ for all $x \in X$.

The equality of $A, B \in IVF(X)$ is defined by $A = B \Leftrightarrow A \subset B$ and $B \subset A$.

The complement A^c of A is defined by

$A^c(x) = [1-\sup A(x), 1-\inf A(x)]$, for all $x \in X$.

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We denote $\tilde{0}$ and $\tilde{1}$ as follows: $\tilde{0} = [0,0]$, $\tilde{1} = [1,1]$.

Let U be an initial universe set and E be a set of parameters.

Definition 1.1 ([4]). A pair (F, A) is called an interval-valued fuzzy soft set over U if $A \subset E$ and F is a mapping of A into the set of interval-valued fuzzy sets of U.

Definition 1.2 ([4]). Let (F, A) and (G, B) be interval-valued fuzzy soft sets over a common universe set U. Then

(a) (F, A) is a subset of (G, B), denoted by $(F, A) \sqsubseteq (G, B)$, if

(i) $A \subset B$,

(ii) for any $e \in A$, $F(e)$ is an interval-valued fuzzy subset of $G(e)$.

(b) (F, A) equals to (G, B), denoted by $(F, A) = (G, B)$, if $(F, A) \sqsubseteq (G, B)$ and $(G, B) \sqsubseteq (F, A)$.

(c) The complement of (F, A), denoted by $(F, A)^c$, is defined by

$$(F, A)^c = (F^c, \neg A)$$

where $F^c: \neg A \rightarrow IVF(U)$ is a mapping given by $F^c(\alpha)=$ the interval-valued fuzzy complement of $F(\neg\alpha)$, for any $\alpha \in \neg A$.

(d) (F, A) is called the null interval-valued fuzzy soft set, denoted by Φ, if for any $e \in A$, $F(e) = \tilde{0}$.

(e) (F, A) is called the absolute interval-valued fuzzy soft set, denoted by $\tilde{A}$, if for any $e \in A$, $F(e) = \tilde{1}$.

Definition 1.3 ([4]). Let (F, A) and (G,B) be interval-valued fuzzy soft sets over a common universe set U. Then:

(a) The union (F, A) and (G,B) is an interval-valued fuzzy soft set $(H, A\cup B)$ defined by

$$H(e)=\begin{cases}F(e), & e\in A-B,\\ G(e), & e\in B-A,\\ F(e)\cup G(e), & e\in A\cap B.\end{cases}$$

and denoted by $(F, A)\sqcup(G,B)$.

(b) The intersection (F, A) and (G,B) is an interval-valued fuzzy soft set $(H, A\cap B)$ defined by $H(\alpha)=F(\alpha)$ or $G(\alpha)$ for $\alpha\in A\cap B$ and denoted by $(F, A)\sqcap(G,B)$.

Definition 1.4 ([3]). Let $E=\{e_1, e_2, \cdots, e_n\}$ be a set of parameters. The Not set of E denoted by $\neg E=\{\neg e_1, \neg e_2, \cdots, \neg e_n\}$ where $\neg e_i$ =not e_i for each $i\in\{1,\cdots, n\}$.

Theorem 1.5. ([3]). Let E be a set of parameters and $A\subset E$. Then the following hold:

(a) $\neg(\neg A)=A$.

(b) $\neg(A\cup B)=\neg A\cup\neg B$.

(c) $\neg(A\cap B)=\neg A\cap\neg B$.

2. Main results

Remark 2.1. In Proposition 3.10 [4], the statements (b) and (c) are incorrect as shown the following example.

Example 2.2. Let $U=\{h_1, h_2, h_3, h_4, h_5, h_6\}$ be the set of *wooden* houses and $E=\{e_1, e_2, e_3, e_4, e_5, e_6, e_7, e_8\}$ where, e_1 stands for 'very costly', e_2 stands for 'costly', e_3 stands for 'cheap', e_4 stands for 'beautiful', e_5 stands for 'in the green surrounding', e_6 stands for 'brick', e_7 stands for 'muddy', e_8 stands for 'stone'. Consider two interval-valued fuzzy soft sets (F, A) and (G,B) as in Example 3.6 [4]. For $A=\{$very costly, costly, cheap$\}$, $B=\{$beautiful, in the green surrounding, cheap$\}$,

$F(e_1)=\{h_1/[0.7, 0.7], h_2/[1,1], h_3/[0.8, 1], h_4/[1,1], h_5/[0.9,1], h_6/[0.3, 0.9]\}$,
$F(e_2)=\{h_1/[1,1], h_2/[1,1], h_3/[1,1], h_4/[1,1], h_5/[1,1], h_6/[0.4,1]\}$,
$F(e_3)=\{h_1/[0.4, 1], h_2/[0.2, 0.8], h_3/[0.5, 1], h_4/[0.4, 1], h_5/[0.3, 1], h_6/[1, 1]\}$,
$G(e_4)=\{h_1/[0.6, 0.9], h_2/[1,1], h_3/[1,1], h_4/[0.8, 0.9], h_5/[0.6,1], h_6/[0.8,0.8]\}$,
$G(e_5)=\{h_1/[0.5,0.7], h_2/[0.7,0.9], h_3/[0.8,0.9], h_4/[0.6, 0.9], h_5/[1,1], h_6/[1,1]\}$,
$G(e_3)=\{h_1/[0.4, 1], h_2/[0.2, 0.9], h_3/[0.5, 0.8], h_4/[0.4, 0.9], h_5/[0.3,1], h_6/[1,1]\}$.

Now consider another two interval-valued fuzzy soft sets (L, S) and $(M, \neg S)$. For $S=\{$brick, muddy, stone$\}$, $\neg S=\{$not brick, not muddy, not stone$\}$

$L(e_6)=\{h_1/[0,0], h_2/[0,0], h_3/[0,0], h_4/[0,0], h_5/[0,0], h_6/[0,0]\}$,
$L(e_7)=\{h_1/[0,0], h_2/[0,0], h_3/[0,0], h_4/[0,0], h_5/[0,0], h_6/[0,0]\}$,
$L(e_8)=\{h_1/[0,0], h_2/[0,0], h_3/[0,0], h_4/[0,0], h_5/[0,0], h_6/[0,0]\}$,
$M(\neg e_6)=\{h_1/[1, 1], h_2/[1,1], h_3/[1,1], h_4/[1, 1], h_5/[1, 1], h_6/[1, 1]\}$,
$M(\neg e_7)=\{h_1/[1, 1], h_2/[1,1], h_3/[1,1], h_4/[1, 1], h_5/[1, 1], h_6/[1, 1]\}$,
$M(\neg e_8)=\{h_1/[1, 1], h_2/[1,1], h_3/[1,1], h_4/[1, 1], h_5/[1, 1], h_6/[1, 1]\}$.

Then (L, S) is the null interval-valued fuzzy soft set Φ and $(M, \neg S)$ the absolute interval-valued fuzzy soft set $\tilde{A}$.

(a) Let $(F, A)\sqcup(L,S)=(F, A)\sqcup\Phi=(H, A\cup S)$, where

$$H(e)=\begin{cases}F(e), & e\in A-S,\\ L(e), & e\in S-A,\\ F(e)\cup L(e), & e\in A\cap S.\end{cases}$$

Then for $e\in A\cup S=\{e_1, e_2, e_3, e_4, e_5, e_6, e_7, e_8\}$,

$H(e_1)=F(e_1)$, $H(e_2)=F(e_2)$, $H(e_3)=F(e_3)$, $H(e_6)=F(e_6)$, $H(e_7)=F(e_7)$, $H(e_8)=F(e_8)$.
i.e.,

$(H, A\cup S)$={very costly=$\{h_1/[0.7, 0.7], h_2/[1,1], h_3/[0.8,1], h_4/[1,1], h_5/[0.9,1], h_6/[0.3,0.9]$,
costly=$\{h_1/[1,1], h_2/[1,1], h_3/[1,1], h_4/[1,1], h_5/[1,1], h_6/[0.4,1]\}$,
cheap=$\{h_1/[0.4, 1], h_2/[0.2,0.8], h_3/[0.5,1], h_4/[0.4,1], h_5/[0.3,1], h_6/[1,1]\}$,
brick=$\{h_1/[0,0], h_2/[0,0], h_3/[0,0], h_4/[0,0], h_5/[0,0], h_6/[0,0]\}$,
muddy=$\{h_1/[0,0], h_2/[0,0], h_3/[0,0], h_4/[0,0], h_5/[0,0], h_6/[0,0]\}$,
stone=$\{h_1/[0,0], h_2/[0,0], h_3/[0,0], h_4/[0,0], h_5/[0,0], h_6/[0,0]\}$.

Therefore, $(F, A)\sqcup(L,S)=(F, A)\sqcup\Phi$ is neither the null interval-valued fuzzy soft set nor (F, A).

Similarly, we can say $(F, A)\sqcup(M,\neg S)=(F, A)\sqcup\tilde{A}$ is neither the absolute interval-valued fuzzy soft set nor (F, A). And since $(L, S)\sqcap(M,\neg S)=(L, S)\sqcap\tilde{A}$ and

$S \cap \neg S = \varnothing$, $(L, S) \sqcap \tilde{A} \neq \tilde{A}$.

Now we introduce the modified concepts of null interval-valued fuzzy soft set and absolute interval- valued fuzzy soft set for the interval-valued fuzzy soft set theory. And we redefine the intersection of two interval-valued fuzzy soft sets and study basic properties.

Definition 2.3. Let U be an initial universe set, E a universe set of parameters and $A, B \subset E$.

(a) (F, A) is called the relative-null interval-valued fuzzy soft set on A, denoted by Φ_A, if for any $e \in A$, $F(e) = \varnothing$.

(b) (F, B) is called the absolute interval-valued fuzzy soft set on B, denoted by $\tilde{B}$, if for any $e \in B$, $F(e) = U$.

Theorem 2.4. Let (F, A) and (G, B) be interval- valued fuzzy soft sets over a common universe set U. Then

(a) $(F, A) \sqcap \Phi_A = \Phi_A$, where Φ_A is the relative- null interval-valued fuzzy soft set on A.

(b) If $B \subset A$, then $(F, A) \sqcup \Phi_B = (F, A)$, where Φ_B is the relative-null interval-valued fuzzy soft set on B.

(c) $(F, A) \sqcup \tilde{A} = \tilde{A}$ and $(F, A) \sqcap \tilde{A} = (F, A)$, where $\tilde{A}$ is the absolute interval-valued fuzzy soft set on A.

Proof. It is obvious.

Remark 2.5. In the above Theorem, the equality of the part (b) is not always true as shown the following example.

Example 2.6. As in Example 2.2, consider $A = \{e_6, e_7\}$, $B = \{e_6, e_7, e_7\} \subset E$, then $A \subset B$.

Let

$N(e_6) = \{h_1/[0,0],\ h_2/[0,0],\ h_3/[0,0],\ h_4/[0,0],\ h_5/[0,0],\ h_6/[0,0]\}$.

$N(e_7) = \{h_1/[0,0],\ h_2/[0,0],\ h_3/[0,0],\ h_4/[0,0],\ h_5/[0,0],\ h_6/[0,0]\}$.

Then $(N, A) \sqcup \Phi_B = (H, A \cup B)$, where H is a mapping from $A \cup B$ to $IVF(U)$ defined as the following:

$$H(e) = \begin{cases} \Phi_B(e), & e \in B - A, \\ N(e), & e \in A \cap B. \end{cases}$$

Since $(N, A) \sqcup \Phi_B = (H, A \cup B)$ is an interval-valued fuzzy soft set defined by $H: A \cup B \rightarrow IVF(U)$ and (N, A) is an interval-valued fuzzy soft set defined by the mapping $N: A \rightarrow IVF(U)$ and $A \neq B$, we get $(N, A) \sqcup \Phi_B \neq (N, A)$.

Theorem 2.7. Let (F, A) and (G, B) be interval- valued fuzzy soft sets over a common universe set U. Then

$$((F, A) \sqcup (G, B))^c \sqsubseteq (F, A)^c \sqcup (G, B)^c.$$

Proof. Let $(F, A) \sqcup (G, B) = (H, A \cup B)$, where

$$H(e) = \begin{cases} F(e), & e \in A - B, \\ G(e), & e \in B - A, \\ F(e) \cup G(e), & e \in A \cap B. \end{cases}$$

Then $((F, A) \sqcup (G, B))^c = (H, A \cup B)^c = (H^c, \neg A \cup \neg B)$. Since $H^c(\neg e) = H(e)^c$, for $\neg e \in \neg A \cup \neg B$,

$$H^c(\neg e) = H(e)^c = \begin{cases} F(e)^c, & \neg e \in \neg A - \neg B, \\ G(e)^c, & \neg e \in \neg B - \neg A, \\ (F(e) \cup G(e))^c, & \neg e \in \neg A \cap \neg B. \end{cases} = \begin{cases} F^c(\neg e), & \neg e \in \neg A - \neg B, \\ G^c(\neg e), & \neg e \in \neg B - \neg A, \\ F^c(\neg e) \cap G^c(\neg e), & \neg e \in \neg A \cap \neg B. \end{cases}$$

And

$$(F, A)^c \sqcup (G, B)^c = (F^c, \neg A) \sqcup (G^c, \neg B) = \begin{cases} F^c(\neg e), & \neg e \in \neg A - \neg B, \\ G^c(\neg e), & \neg e \in \neg B - \neg A, \\ F^c(\neg e) \cup G^c(\neg e), & \neg e \in \neg A \cap \neg B. \end{cases}$$

Hence $((F, A) \sqcup (G, B))^c \sqsubseteq (F, A)^c \sqcup (G, B)^c$.

Remark 2.8. In Theorem 2.7, the equality is not always true as the following example.

Example 2.9. Consider two interval-valued fuzzy soft sets (F, A) and (G, B) be interval- valued fuzzy soft sets over a common universe set U as in Example 2.2. Let $(N, A) \sqcup (G, B) = (H, A \cup B)$, where

$$H(e) = \begin{cases} G(e), & e \in B - A, \\ F(e), & e \in A - B, \\ F(e) \cup G(e), & e \in A \cap B. \end{cases}$$

Then

$H(e_1) = F(e_1) = \{h_1/[0.7,\ 0.7],\ h_2/[1,1],\ h_3/[0.8,1],\ h_4/[1,1],\ h_5/[0.9,1],\ h_6/[0.3,0.9]\}$,

$H(e_2) = F(e_2) = \{h_1/[1,1],\ h_2/[1,1],\ h_3/[1,1],\ h_4/[1,1],\ h_5/[1,1],\ h_6/[0.4,1]\}$,

$H(e_4) = F(e_4) = \{h_1/[0.6,\ 0.9],\ h_2/[1,1],\ h_3/[1,1],\ h_4/[0.8,0.9],\ h_5/[0.6,1],\ h_6/[0.8,0.8]\}$,

$H(e_5) = F(e_5) = \{h_1/[0.5,\ 0.7],\ h_2/[0.7,0.9],\ h_3/[0.8,0.9],\ h_4/[0.6,0.9],\ h_5/[1,1],\ h_6/[1,1]\}$,

$H(e_3) = F(e_3) \cup G(e_3) = \{h_1/[0.4,1], h_2/[0.2,0.9], h_3/[0.5,1],\ h_4/[0.4,1],\ h_5/[0.3,1],\ h_6/[1,1]\}$.

Thus for $\neg e_3 \in \neg A \cap \neg B$,

$H^c(\neg e_3) = \{h_1/[0,\ 0.6],\ h_2/[0.1,0.8],\ h_3/[0,0.5],\ h_4/[0,0.6],\ h_5/[0,0.7],\ h_6/[0,0]\}$.

Let $(F^c, \neg A) \sqcup (G^c, \neg B) = (K, \neg A \cup \neg B)$, where

$$K(\neg e)=\begin{cases} G^c(\neg e), & \neg e\in\neg B-\neg A,\\ F^c(\neg e), & \neg e\in\neg A-\neg B,\\ F^c(\neg e)\cup G^c(\neg e), & \neg e\in\neg A\cap\neg B.\end{cases}$$

Then for $\neg e_3\in\neg A\cap\neg B$,

$$\begin{aligned}K(\neg e_3)&=F^c(\neg e_3)\cup G^c(\neg e_3)\\ &=\{h_1/[0,0.6],\ h_2/[0.2,0.8],\ h_3/[0.2,0.5],\\ &\quad h_4/[0.1,0.6],\ h_5/[0,0.7],\ h_6/[0,0]\},\end{aligned}$$

where

$$\begin{aligned}F^c(\neg e_3)&=\{h_1/[0,\ 0.6],\ h_2/[0.2,0.8],\ h_3/[0,0.5],\\ &\quad h_4/[0,0.6],\ h_5/[0,0.7],\ h_6/[0,0]\},\\ G^c(\neg e_3)&=\{h_1/[0,\ 0.6],\ h_2/[0.1,0.8],\ h_3/[0.2,0.5],\\ &\quad h_4/[0.1,0.6],\ h_5/[0,0.7],\ h_6/[0,0]\}.\end{aligned}$$

Thus $K(\neg e_3)\neq H^c(\neg e_3)$ for $\neg e_3\in\neg A\cap\neg B$.
Consequently, $((F,A)\sqcup(G,B))^c \neq (F,A)^c\sqcup(G,B)^c$.

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