

구간치 Intuitionistic Fuzzy Soft sets 관한 연구

Interval-Valued Intuitionistic Fuzzy Soft Sets

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요 약

본 논문은 구간치 intuitionistic fuzzy soft sets의 개념과 연산을 소개하며 기본적인 성질을 조사한다.

Abstract

We introduce the concept of interval-valued intuitionistic fuzzy soft sets, which is an extension of the interval-valued fuzzy soft set. We also introduce the concepts of operations for the interval-valued intuitionistic fuzzy soft sets and study basic some properties.

Key Words : interval-valued intuitionistic fuzzy soft sets, interval-valued fuzzy soft sets, null interval-valued intuitionistic fuzzy soft sets, absolute interval-valued intuitionistic fuzzy soft sets

1. Intorduction and Preliminaries

Molodtsov [7] introduced the concept of soft set to solve complicated problems in economic, engineering, environment because any mathematical tools can not successfully solve various types of uncertainties in these problems. In fact there are some theories: theory of fuzzy sets [9], theory of vague sets [3], theory of intuitionistic fuzzy sets [1], and theory of rough sets [8] which can be consider as mathematical tools for dealing with uncertainties. But all these theories cannot solve the complicated problems including various types of uncertainties. Soft set theory is a mathematical tool for dealing with uncertainties which is free from the difficulties of the above theories. In [5], Maji et al. introduced several operators for soft set theory and made a theoretical study of the "Soft Set Theory" in more detail. In [4], Maji et al. also introduced fuzzy soft sets, where the relevant values are represented by real values between zero and one.

In [6], Son introduced the concept of interval-value fuzzy soft sets based on [2].

In this paper, we introduce the concept of interval-valued intuitionistic fuzzy soft sets, which is an extension of interval-value fuzzy soft sets. Also we introduce the concepts of operations for interval-valued intuitionistic fuzzy soft set theory and modify some operations defined by interval-value fuzzy soft sets.

Let $D[0,1]$ be the set of all closed subintervals of the

interval $[0,1]$. Let X be a nonempty set. Atanassov and Gargov [2] introduced the concept of interval-valued intuitionistic fuzzy set (shortly *IVIF* set) in X as the following:

An interval-valued intuitionistic fuzzy set (shortly *IVIF* set) in X is an expression given by

$$A = \langle x, \mu_A(x), \nu_A(x) \rangle : x \in X$$

where $\mu_A: X \rightarrow D[0,1]$ and $\nu_A: X \rightarrow D[0,1]$ with the condition $0 \leq \sup_x \mu_A(x) + \sup_x \nu_A(x) \leq 1$.

The interval $\mu_A(x)$ and $\nu_A(x)$ denote, respectively, the degree of belongingness and degree of non-belongingness of the element x to the set A . Thus for each $x \in X$, $\mu_A(x)$ and $\nu_A(x)$ are closed intervals whose lower and upper end points are, respectively, denoted by $[\mu_A(x)]^L, [\mu_A(x)]^U$ and $[\nu_A(x)]^L, [\nu_A(x)]^U$. We will denote by $\mathcal{I}(X)$ the set of all the *IVIF* sets in X . The *IVIF* sets are extensions of the intuitionistic fuzzy sets (*IF* sets), as well as of the interval-valued fuzzy sets (*IVF* sets). In fact, the *IVIF* sets are essentially extensions of the ordinary *IF* sets. For any two intervals $[a, b]$ and $[c, d]$ with $b + d \leq 1$ belonging to $D[0,1]$, the *IVIF* set whose valued is the pair of intervals $([a, b], [c, d])$ for all $x \in X$, is denoted by $([a, b], [c, d])$.

Let $A, B \in \mathcal{I}(X)$. Subset relation is defined by

$$A \subset B \Leftrightarrow [\mu_A(x)]^L \leq [\mu_B(x)]^L, [\mu_A(x)]^U \leq [\mu_B(x)]^U, \\ \text{and} \\ [\nu_A(x)]^L \geq [\nu_B(x)]^L, [\nu_A(x)]^U \geq [\nu_B(x)]^U$$

for all $x \in X$.

The equality of $A, B \in \mathcal{I}(X)$ is defined by $A = B \Leftrightarrow A \subset B$ and $B \subset A$.

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The complement A^c of $A \in \mathcal{I}(X)$ is defined by

$$\mu_{A^c}(x) = \nu_A(x) \text{ and } \nu_{A^c}(x) = \mu_A(x)$$

i.e.,

$$A^c = \langle x, \nu_A(x), \mu_A(x) \rangle : x \in X,$$

where $A = \langle x, \mu_A(x), \nu_A(x) \rangle : x \in X$.

We denote $\tilde{0}$ and $\tilde{1}$ as follows:

$$\tilde{0} = ([0,0], [0,0]), \quad \tilde{1} = ([1,1], [1,1]).$$

2. Interval-valued intuitionistic fuzzy soft sets

Let U be an initial universe set and E be a set of parameters.

Definition 2.1. A pair (F, A) is called an interval-valued intuitionistic fuzzy soft set over U if $A \subset E$ and F is a mapping of A into the set of interval-valued intuitionistic fuzzy sets of U .

Example 2.2. Let $U = \{d_1, d_2, d_3, d_4, d_5\}$ be the set of dresses and $E = \{e_1, e_2, e_3, e_4, e_5, e_6\}$ the set of six parameters, where, e_1 stands for 'expensive', e_2 stands for 'cheap', e_3 stands for 'beautiful', e_4 stands for 'elegant', e_5 stands for 'classical', e_6 stands for 'modern'.

Suppose

$$F(e_1) = \{ \langle d_1, [0.4, 0.7], [0.2, 0.3] \rangle, \\ \langle d_2, [1, 1], [0, 0] \rangle, \\ \langle d_3, [0.4, 0.5], [0.2, 0.3] \rangle, \\ \langle d_4, [0.3, 0.5], [0.3, 0.5] \rangle, \\ \langle d_5, [0.1, 0.2], [0.5, 0.7] \rangle \}.$$

$$F(e_2) = \{ \langle d_1, [0.4, 0.5], [0.3, 0.4] \rangle, \\ \langle d_2, [0.1, 0.2], [0.5, 0.8] \rangle, \\ \langle d_3, [0.4, 0.5], [0.2, 0.4] \rangle, \\ \langle d_4, [0.3, 0.6], [0.2, 0.3] \rangle, \\ \langle d_5, [1, 1], [0, 0] \rangle \}.$$

$$F(e_3) = \{ \langle d_1, [0.5, 0.7], [0.3, 0.3] \rangle, \\ \langle d_2, [0.5, 0.5], [0.3, 0.5] \rangle, \\ \langle d_3, [0.4, 0.5], [0.3, 0.4] \rangle, \\ \langle d_4, [1, 1], [0, 0] \rangle, \\ \langle d_5, [0.4, 0.6], [0.2, 0.4] \rangle \}.$$

$$F(e_4) = \{ \langle d_1, [0.3, 0.6], [0.1, 0.4] \rangle, \\ \langle d_2, [0.3, 0.5], [0.2, 0.4] \rangle, \\ \langle d_3, [0.4, 0.5], [0.3, 0.4] \rangle, \\ \langle d_4, [0.8, 0.9], [0.1, 0.1] \rangle, \\ \langle d_5, [0.2, 0.6], [0.1, 0.4] \rangle \}.$$

$$F(e_5) = \{ \langle d_1, [0.1, 0.4], [0.3, 0.6] \rangle,$$

$$\langle d_2, [0.2, 0.5], [0.2, 0.4] \rangle, \\ \langle d_3, [0.3, 0.6], [0.3, 0.4] \rangle, \\ \langle d_4, [0.8, 1], [0, 0] \rangle, \\ \langle d_5, [0.2, 0.5], [0.1, 0.3] \rangle \}.$$

$$F(e_6) = \{ \langle d_1, [0.4, 0.6], [0.3, 0.4] \rangle, \\ \langle d_2, [0.2, 0.3], [0.2, 0.6] \rangle, \\ \langle d_3, [0.3, 0.5], [0.4, 0.5] \rangle, \\ \langle d_4, [0.1, 0.3], [0.3, 0.7] \rangle, \\ \langle d_5, [0.2, 0.4], [0.4, 0.6] \rangle \}.$$

The interval-valued intuitionistic fuzzy soft sets (F, E) is a parameterized family $\{F(e_i) : i = 1, 2, 3, 4, 5, 6\}$ of all interval-valued intuitionistic fuzzy soft sets of the set U and gives us a collection of a approximate description of an object.

For example, $F(e_3)$ means "dresses(elegant)" whose functional value is the interval-valued intuitionistic fuzzy set:

$$\{ \langle d_1, [0.5, 0.7], [0.3, 0.3] \rangle, \\ \langle d_2, [0.5, 0.5], [0.3, 0.5] \rangle, \\ \langle d_3, [0.4, 0.5], [0.3, 0.4] \rangle, \\ \langle d_4, [1, 1], [0, 0] \rangle, \\ \langle d_5, [0.4, 0.6], [0.2, 0.4] \rangle \},$$

i.e., $F(e_3)$ means

$$\text{"elegant dresses"} = \{ \langle d_1, [0.5, 0.7], [0.3, 0.3] \rangle, \\ \langle d_2, [0.5, 0.5], [0.3, 0.5] \rangle, \\ \langle d_3, [0.4, 0.5], [0.3, 0.4] \rangle, \\ \langle d_4, [1, 1], [0, 0] \rangle, \\ \langle d_5, [0.4, 0.6], [0.2, 0.4] \rangle \}.$$

For the approximation "elegant dresses =

$$\{ \langle d_1, [0.5, 0.7], [0.3, 0.3] \rangle, \\ \langle d_2, [0.5, 0.5], [0.3, 0.5] \rangle, \\ \langle d_3, [0.4, 0.5], [0.3, 0.4] \rangle, \\ \langle d_4, [1, 1], [0, 0] \rangle, \\ \langle d_5, [0.4, 0.6], [0.2, 0.4] \rangle \}.$$

we have

(i) the predicate name is 'elegant dresses', the predicate is denoted by p_3 ,

(ii) the approximate interval-valued intuitionistic fuzzy set (denoted by v_3) is

$$\{ \langle d_1, [0.5, 0.7], [0.3, 0.3] \rangle, \\ \langle d_2, [0.5, 0.5], [0.3, 0.5] \rangle, \\ \langle d_3, [0.4, 0.5], [0.3, 0.4] \rangle, \\ \langle d_4, [1, 1], [0, 0] \rangle, \\ \langle d_5, [0.4, 0.6], [0.2, 0.4] \rangle \}.$$

Thus an interval-valued intuitionistic fuzzy soft set

(F, E) can be viewed as a collection of interval-valued intuitionistic fuzzy approximations like as:

$$(F, E) = \{F(e_i) : i = 1, 2, 3, 4, 5, 6\} \\ = \{p_1=v_1, p_2=v_2, \dots, p_6=v_6\}.$$

Remark 2.3. The interval-valued intuitionistic fuzzy soft sets are extension of the interval-valued fuzzy soft sets.

Example 2.4. Let $U = \{n_1, n_2, n_3\}$ be the universe set and $E = \{e_1, e_2, e_3\}$ be the set of parameters. Consider an interval-valued fuzzy soft set (F, A) defined as the following:

$$F(e_1) = \{ \langle n_1, [0.5, 0.7] \rangle, \langle n_2, [0.4, 0.6] \rangle, \\ \langle n_3, [0.6, 0.7] \rangle \}.$$

$$F(e_2) = \{ \langle n_1, [0.4, 0.5] \rangle, \langle n_2, [0.1, 0.2] \rangle, \\ \langle n_3, [0.4, 0.5] \rangle \}.$$

$$F(e_3) = \{ \langle n_1, [0.5, 0.7] \rangle, \langle n_2, [0.5, 0.5] \rangle, \\ \langle n_3, [0.4, 0.5] \rangle \}.$$

Define $F: A \rightarrow \mathcal{I}(U)$ as the following:

$$F(e_1) = \{ \langle n_1, [0.5, 0.7], [0, 0] \rangle, \\ \langle n_2, [0.4, 0.6], [0, 0] \rangle, \\ \langle n_3, [0.6, 0.7], [0, 0] \rangle \}.$$

$$F(e_2) = \{ \langle n_1, [0.4, 0.5], [0, 0] \rangle, \\ \langle n_2, [0.1, 0.2], [0, 0] \rangle, \\ \langle n_3, [0.4, 0.5], [0, 0] \rangle \}.$$

$$F(e_3) = \{ \langle n_1, [0.5, 0.7], [0, 0] \rangle, \\ \langle n_2, [0.5, 0.5], [0, 0] \rangle, \\ \langle n_3, [0.4, 0.5], [0, 0] \rangle \}.$$

Then (F, A) is an interval-valued intuitionistic fuzzy soft set.

Definition 2.5. Let (F, A) and (G, B) be interval-valued intuitionistic fuzzy soft sets over a common universe set U . Then

- (a) (F, A) is a subset of (G, B) , denoted by $(F, A) \sqsubset (G, B)$, if
 - (i) $A \subset B$,
 - (ii) for any $e \in A$, $F(e)$ is an interval-valued intuitionistic fuzzy subset of $G(e)$.
- (b) (F, A) equals to (G, B) , denoted by $(F, A) = (G, B)$, if $(F, A) \sqsubset (G, B)$ and $(G, B) \sqsubset (F, A)$.

Example 2.6. Consider Example 2.2. Let $A = \{e_2, e_6\} \subset E$ and $B = \{e_1, e_2, e_6\} \subset E$; then $A \subset B$.

Let (F, A) and (G, B) be two interval-valued in-

tuitionistic fuzzy soft sets over a common universe set U such that

$$F(e_2) = \{ \langle d_1, [0.3, 0.5], [0.3, 0.5] \rangle, \\ \langle d_2, [0.1, 0.2], [0.6, 0.8] \rangle, \\ \langle d_3, [0.2, 0.4], [0.3, 0.6] \rangle, \\ \langle d_4, [0.3, 0.5], [0.3, 0.5] \rangle, \\ \langle d_5, [0.5, 0.7], [0.3, 0.3] \rangle \}.$$

$$F(e_6) = \{ \langle d_1, [0.2, 0.5], [0.3, 0.5] \rangle, \\ \langle d_2, [0.2, 0.2], [0.3, 0.7] \rangle, \\ \langle d_3, [0.3, 0.4], [0.5, 0.6] \rangle, \\ \langle d_4, [0, 0.2], [0.5, 0.7] \rangle, \\ \langle d_5, [0.2, 0.3], [0.5, 0.7] \rangle \}.$$

$$G(e_1) = \{ \langle d_1, [0.4, 0.7], [0.2, 0.3] \rangle, \\ \langle d_2, [1, 1], [0, 0] \rangle, \\ \langle d_3, [0.4, 0.5], [0.2, 0.3] \rangle, \\ \langle d_4, [0.3, 0.5], [0.3, 0.5] \rangle, \\ \langle d_5, [0.1, 0.2], [0.5, 0.7] \rangle \}.$$

$$G(e_2) = \{ \langle d_1, [0.4, 0.5], [0.3, 0.4] \rangle, \\ \langle d_2, [0.1, 0.2], [0.5, 0.8] \rangle, \\ \langle d_3, [0.4, 0.5], [0.2, 0.4] \rangle, \\ \langle d_4, [0.3, 0.6], [0.2, 0.3] \rangle, \\ \langle d_5, [1, 1], [0, 0] \rangle \}.$$

$$G(e_6) = \{ \langle d_1, [0.4, 0.6], [0.3, 0.4] \rangle, \\ \langle d_2, [0.2, 0.3], [0.2, 0.6] \rangle, \\ \langle d_3, [0.3, 0.5], [0.4, 0.5] \rangle, \\ \langle d_4, [0.1, 0.3], [0.3, 0.7] \rangle, \\ \langle d_5, [0.2, 0.4], [0.4, 0.6] \rangle \}.$$

Thus $(F, A) \sqsubset (G, B)$.

Definition 2.7. Let (F, A) and (G, B) be interval-valued intuitionistic fuzzy soft sets over a common universe set U . Then (F, A) is an absolute subset of (G, B) , denoted by $(F, A) \sqsubset\sqsubset (G, B)$, if

- (i) $A \subset B$,
- (ii) for any $e \in A$, $F(e) = G(e)$.

Theorem 2.8. Let (F, A) and (G, B) be interval-valued intuitionistic fuzzy soft sets over a common universe set U . Then:

- (a) If $(F, A) \sqsubset\sqsubset (G, B)$, then $(F, A) \sqsubset (G, B)$.
- (b) If $(F, A) \sqsubset\sqsubset (G, B)$ and $(G, B) \sqsubset\sqsubset (F, A)$, then $(F, A) = (G, B)$.

Proof. Obvious.

Theorem 2.9. ([5]). Let $E = \{e_1, e_2, \dots, e_n\}$ be a set of parameters. The Not set of E denoted by

$\neg E = \neg e_1, \neg e_2, \dots, \neg e_n$ where $\neg e_i = \text{not } e_i$ for each $i = 1, \dots, n$.

Example 2.10. In Example 2.2,

$\neg E = \{\text{not expensive; not cheap; not beautiful; not elegant; not classical; not modern}\}$.

Theorem 2.11. ([5]). Let E be a set of parameters and $A \subset E$. Then the following hold:

- (a) $\neg(\neg A) = A$.
- (b) $\neg(A \cup B) = \neg A \cup \neg B$.
- (c) $\neg(A \cap B) = \neg A \cap \neg B$.

Definition 2.12. Let (F, A) be interval-valued intuitionistic fuzzy soft set over a common universe set U . The complement of (F, A) , denoted by $(F, A)^c$, is defined by

$$(F, A)^c = (F^c, \neg A)$$

where $F^c : \neg A \rightarrow \mathcal{I}(U)$ is a mapping given by $F^c(\alpha) =$ the interval-valued intuitionistic fuzzy complement of $F(\neg\alpha)$, for any $\alpha \in \neg A$.

Example 2.13. In Example 2.6,

$$F(\neg e_2) = \{ \langle d_1, [0.3, 0.5], [0.3, 0.5] \rangle, \langle d_2, [0.6, 0.8], [0.1, 0.2] \rangle, \langle d_3, [0.3, 0.6], [0.2, 0.4] \rangle, \langle d_4, [0.3, 0.5], [0.3, 0.5] \rangle, \langle d_5, [0.3, 0.3], [0.5, 0.7] \rangle \}.$$

$$F(\neg e_6) = \{ \langle d_1, [0.3, 0.5], [0.2, 0.5] \rangle, \langle d_2, [0.3, 0.7], [0.2, 0.2] \rangle, \langle d_3, [0.5, 0.6], [0.3, 0.4] \rangle, \langle d_4, [0.5, 0.7], [0, 0.2] \rangle, \langle d_5, [0.5, 0.7], [0.2, 0.3] \rangle \}.$$

$$G(\neg e_1) = \{ \langle d_1, [0.2, 0.3], [0.4, 0.7] \rangle, \langle d_2, [0, 0], [1, 1] \rangle, \langle d_3, [0.2, 0.3], [0.4, 0.5] \rangle, \langle d_4, [0.3, 0.5], [0.3, 0.5] \rangle, \langle d_5, [0.5, 0.7], [0.1, 0.2] \rangle \}.$$

$$G(\neg e_2) = \{ \langle d_1, [0.3, 0.4], [0.4, 0.5] \rangle, \langle d_2, [0.5, 0.8], [0.1, 0.2] \rangle, \langle d_3, [0.2, 0.4], [0.4, 0.5] \rangle, \langle d_4, [0.2, 0.3], [0.3, 0.6] \rangle, \langle d_5, [0, 0], [1, 1] \rangle \}.$$

$$G(\neg e_6) = \{ \langle d_1, [0.3, 0.4], [0.4, 0.6] \rangle, \langle d_2, [0.2, 0.6], [0.2, 0.3] \rangle, \langle d_3, [0.4, 0.5], [0.3, 0.5] \rangle, \langle d_4, [0.3, 0.7], [0.1, 0.3] \rangle, \}.$$

$$\langle d_5, [0.4, 0.6], [0.2, 0.4] \rangle \}.$$

Definition 2.14. Let U be an initial universe set and E be a set of parameters.

- (a) (F, A) is called the relative null interval-valued intuitionistic fuzzy soft set on A , denoted by Φ_A , if for any $e \in A$, $F(e) = \tilde{0}$.
- (b) (F, A) is called the absolute interval-valued intuitionistic fuzzy soft set, denoted by \tilde{A} , if for any $e \in A$, $F(e) = \tilde{1}$.

Theorem 2.15. Let (F, A) be interval-valued intuitionistic fuzzy soft set over a common universe set U .

- (a) $((F, A)^c)^c = (F, A)$.
- (b) $\tilde{A}^c = \Phi_A$ and $\Phi_A^c = \tilde{A}$.

Proof. Obvious.

Definition 2.16. Let (F, A) and (G, B) be interval-valued fuzzy soft sets over a common universe set U . Then:

- (a) $(F, A) \wedge (G, B)$ is an interval-valued intuitionistic fuzzy soft set defined by $(F, A) \wedge (G, B) = (H, A \times B)$, where $H(\alpha, \beta) = F(\alpha) \cap G(\beta)$ for any $\alpha \in A$ and $\beta \in B$, where \cap is the intersection operation of interval-values intuitionistic fuzzy sets.
- (b) $(F, A) \vee (G, B)$ is an interval-valued intuitionistic fuzzy soft set defined by $(F, A) \vee (G, B) = (K, A \times B)$, where $K(\alpha, \beta) = F(\alpha) \cup G(\beta)$ for any $\alpha \in A$ and $\beta \in B$, where \cup is the intersection operation of interval-values intuitionistic fuzzy sets.

Theorem 2.17. Let (F, A) and (G, B) be interval-valued intuitionistic fuzzy soft sets over a common universe set U . Then

- (a) $(F, A) \wedge (G, B) = (G, B) \wedge (F, A)$.
- (b) $(F, A) \vee (G, B) = (G, B) \vee (F, A)$.

Proof. Let $(F, A) \wedge (G, B) = (H, A \times B)$ and $(G, B) \wedge (F, A) = (K, B \times A)$. For $\alpha \in A$ and $\beta \in B$, $(\alpha, \beta) \in A \times B$, $(\beta, \alpha) \in B \times A$.

$$\begin{aligned} H(\alpha, \beta) &= F(\alpha) \cap G(\beta) \\ &= G(\beta) \cap F(\alpha) \\ &= K(\beta, \alpha). \end{aligned}$$

- (b) Similar to (a).

Theorem 2.18. Let (F, A) and (G, B) be interval-valued intuitionistic fuzzy soft sets over a common universe set U . Then

- (a) $((F, A) \wedge (G, B))^c = (F, A)^c \vee (G, B)^c$.
- (b) $((F, A) \vee (G, B))^c = (F, A)^c \wedge (G, B)^c$.

Proof. (a) Let $(F, A) \wedge (G, B) = (H, A \times B)$, where H is a mapping defined as the following: For any $\alpha \in A$ and $\beta \in B$,

$$\begin{aligned}
 H(\alpha, \beta) &= F(\alpha) \cap G(\beta) \\
 &= \{ \langle x, [\mu_{F(\alpha)}(x)]^L \wedge [\mu_{G(\beta)}(x)]^L, \\
 &\quad [\mu_{F(\alpha)}(x)]^U \wedge [\mu_{G(\beta)}(x)]^U, \\
 &\quad [\nu_{F(\alpha)}(x)]^L \vee [\nu_{G(\beta)}(x)]^L, \\
 &\quad [\nu_{F(\alpha)}(x)]^U \vee [\nu_{G(\beta)}(x)]^U \rangle : x \in U \}.
 \end{aligned}$$

Now for each $(\neg\alpha, \neg\beta) \in \neg(A \times B)$,

$$\begin{aligned}
 H^c(\neg\alpha, \neg\beta) &= H(\alpha, \beta)^c \\
 &= ((F(\alpha) \cap G(\beta))^c \\
 &= \{ \langle x, [[\nu_{F(\alpha)}(x)]^L \vee [\nu_{G(\beta)}(x)]^L], \\
 &\quad [[\nu_{F(\alpha)}(x)]^U \vee [\nu_{G(\beta)}(x)]^U], \\
 &\quad [[\mu_{F(\alpha)}(x)]^L \wedge [\mu_{G(\beta)}(x)]^L], \\
 &\quad [[\mu_{F(\alpha)}(x)]^U \wedge [\mu_{G(\beta)}(x)]^U] \rangle : x \in U \} \\
 &= \{ \langle x, [[\nu_{F(\alpha)}(x)]^L, [\nu_{F(\alpha)}(x)]^U], \\
 &\quad [[\mu_{F(\alpha)}(x)]^L, [\mu_{F(\alpha)}(x)]^U] \rangle : x \in U \\
 &\cup \{ \langle x, [[\nu_{G(\beta)}(x)]^L, [\nu_{G(\beta)}(x)]^U], \\
 &\quad [[\mu_{G(\beta)}(x)]^L, [\mu_{G(\beta)}(x)]^U] \rangle : x \in U \\
 &= F(\alpha)^c \cup G(\beta)^c \\
 &= (F^c, \neg\alpha) \cup (G^c, \neg\beta).
 \end{aligned}$$

Thus since $(H^c, \neg(A \times B)) = ((F, A) \wedge (G, B))^c$ and $(F^c, \neg A) \vee (G^c, \neg B) = (F, A)^c \vee (G, B)^c$, so $((F, A) \wedge (G, B))^c = (F, A)^c \vee (G, B)^c$.

(b) It is similar to (a)

Definition 2.19. Let (F, A) and (G, B) be interval-valued intuitionistic fuzzy soft sets over a common universe set U . Then

(a) The union (F, A) and (G, B) is an interval-valued intuitionistic fuzzy soft set $(H, A \cup B)$ defined by

$$H(e) = \begin{cases} F(e), & e \in A - B, \\ G(e), & e \in B - A, \\ F(e) \cup G(e), & e \in A \cap B. \end{cases}$$

and denoted by $(F, A) \sqcup (G, B)$.

(b) The intersection (F, A) and (G, B) is an interval-valued intuitionistic fuzzy soft set $(H, A \cap B)$ defined by $H(\alpha) = F(\alpha) \cap G(\alpha)$ for $\alpha \in A \cap B$ and denoted by $(F, A) \cap (G, B)$.

Example 2.20. Consider two (F, A) and (G, B) be interval-valued intuitionistic fuzzy soft sets over a common universe set U as in Example 2.6.

Let $(F, A) \sqcup (G, B) = (H, A \cup B)$, where

$$H(e) = \begin{cases} G(e), & e \in B - A, \\ F(e) \cup G(e), & e \in A \cap B. \end{cases}$$

Then

$$\begin{aligned}
 H(e_1) = G(e_1) &= \{ \langle d_1, [0.4, 0.7], [0.2, 0.3] \rangle, \\
 &\quad \langle d_2, [1, 1], [0, 0] \rangle \},
 \end{aligned}$$

$$\begin{aligned}
 &\langle d_3, [0.4, 0.5], [0.2, 0.3] \rangle, \\
 &\langle d_4, [0.3, 0.5], [0.3, 0.5] \rangle, \\
 &\langle d_5, [0.1, 0.2], [0.5, 0.7] \rangle \}.
 \end{aligned}$$

$$\begin{aligned}
 H(e_2) &= F(e_2) \cup G(e_2) \\
 &= \{ \langle d_1, [0.4, 0.5], [0.3, 0.4] \rangle, \\
 &\quad \langle d_2, [0.1, 0.2], [0.5, 0.8] \rangle, \\
 &\quad \langle d_3, [0.4, 0.5], [0.2, 0.4] \rangle, \\
 &\quad \langle d_4, [0.3, 0.6], [0.2, 0.3] \rangle, \\
 &\quad \langle d_5, [1, 1], [0, 0] \rangle \}.
 \end{aligned}$$

$$\begin{aligned}
 H(e_6) &= F(e_6) \cup G(e_6) \\
 &= \{ \langle d_1, [0.4, 0.6], [0.3, 0.4] \rangle, \\
 &\quad \langle d_2, [0.2, 0.3], [0.2, 0.6] \rangle, \\
 &\quad \langle d_3, [0.3, 0.5], [0.4, 0.5] \rangle, \\
 &\quad \langle d_4, [0.1, 0.3], [0.3, 0.7] \rangle, \\
 &\quad \langle d_5, [0.2, 0.4], [0.4, 0.6] \rangle \}.
 \end{aligned}$$

Let $(F, A) \cap (G, B) = (H, A \cap B)$, where $H(e) = F(e) \cap G(e)$, $\alpha \in A \cap B$.

Then

$$\begin{aligned}
 H(e_2) &= F(e_2) \cap G(e_2) \\
 &= \{ \langle d_1, [0.3, 0.5], [0.3, 0.5] \rangle, \\
 &\quad \langle d_2, [0.1, 0.2], [0.6, 0.8] \rangle, \\
 &\quad \langle d_3, [0.2, 0.4], [0.3, 0.6] \rangle, \\
 &\quad \langle d_4, [0.3, 0.5], [0.3, 0.5] \rangle, \\
 &\quad \langle d_5, [0.5, 0.7], [0.3, 0.3] \rangle \},
 \end{aligned}$$

$$\begin{aligned}
 H(e_6) &= F(e_6) \cap G(e_6) \\
 &= \{ \langle d_1, [0.2, 0.5], [0.3, 0.5] \rangle, \\
 &\quad \langle d_2, [0.2, 0.2], [0.3, 0.7] \rangle, \\
 &\quad \langle d_3, [0.3, 0.4], [0.5, 0.6] \rangle, \\
 &\quad \langle d_4, [0, 0.2], [0.5, 0.7] \rangle, \\
 &\quad \langle d_5, [0.2, 0.3], [0.5, 0.7] \rangle \}.
 \end{aligned}$$

Theorem 2.21. Let (F, A) and (G, B) be interval-valued intuitionistic fuzzy soft sets over a common universe set U . Then

- $(F, A) \sqcup (F, A) = (F, A)$.
- $(F, A) \cap (F, A) = (F, A)$.
- $(F, A) \cap \Phi_A = \Phi_A$, where Φ_A is the null interval-valued intuitionistic fuzzy soft set on A .
- If $B \subset A$, then $(F, A) \sqcup \Phi_B = (F, A)$, where Φ_B is the null interval-valued intuitionistic fuzzy soft set on B .
- $(F, A) \sqcup \tilde{A} = \tilde{A}$ and $(F, A) \cap \tilde{A} = (F, A)$, where \tilde{A} is the absolute interval-valued intuitionistic fuzzy soft set.

Proof. From Definition 2.19, it is obvious.

Remark 2.22. In Theorem 2.21, the equality of the part (d) is not always true as shown the following example.

Example 2.23. As in Example 2.6, consider $A = \{e_2, e_6\} \subset E$ and $B = \{e_1, e_2, e_6\} \subset E$, then $A \subset B$. Now $(F, A) \sqcup \Phi_B = (H, A \cup B)$, where H is a mapping from $A \cup B$ to $\mathcal{I}(U)$ defined as the following:

$$H(e) = \begin{cases} \langle x, [0,0], [1,1] \rangle, & e \in B - A, \\ F(e), & e \in A \cap B. \end{cases}$$

Since $(F, A) \sqcup \Phi_B = (H, A \cup B)$ is an interval-valued intuitionistic fuzzy soft set defined by the mapping $H: B \rightarrow \mathcal{I}(U)$ and (F, A) is an interval-valued intuitionistic fuzzy soft set defined by the mapping $F: A \rightarrow \mathcal{I}(U)$ and $A \neq B$, we get $(F, A) \sqcup \Phi_B \neq (F, A)$.

Theorem 2.24. Let (F, A) and (G, B) be interval-valued intuitionistic fuzzy soft sets over a common universe set U . Then

$$((F, A) \sqcup (G, B))^c \sqsubset (F, A)^c \sqcup (G, B)^c.$$

Proof. Let $(F, A) \sqcup (G, B) = (H, A \cup B)$, where

$$H(e) = \begin{cases} F(e), & e \in A - B, \\ G(e), & e \in B - A, \\ F(e) \cup G(e), & e \in A \cap B. \end{cases}$$

Then $((F, A) \sqcup (G, B))^c = (H, A \cup B)^c = (H^c, \neg A \cup \neg B)$.

Since $H^c(\neg e) = H(e)^c$ for $\neg e \in \neg A \cup \neg B$, by Definition 2.19

$$\begin{aligned} H^c(\neg e) &= H(e)^c \\ &= \begin{cases} F(e)^c, & \neg e \in \neg A - \neg B, \\ G(e)^c, & \neg e \in \neg B - \neg A, \\ (F(e) \cup G(e))^c, & \neg e \in \neg A \cap \neg B. \end{cases} \\ &= \begin{cases} F^c(\neg e), & \neg e \in \neg A - \neg B, \\ G^c(\neg e), & \neg e \in \neg B - \neg A, \\ F^c(\neg e) \cap G^c(\neg e), & \neg e \in \neg A \cap \neg B. \end{cases} \end{aligned}$$

And

$$\begin{aligned} (F, A)^c \sqcup (G, B)^c &= (F^c, \neg A) \sqcup (G^c, \neg B) \\ &= \begin{cases} F^c(\neg e), & \neg e \in \neg A - \neg B, \\ G^c(\neg e), & \neg e \in \neg B - \neg A, \\ F^c(\neg e) \cup G^c(\neg e), & \neg e \in \neg A \cap \neg B. \end{cases} \end{aligned}$$

Hence $((F, A) \sqcup (G, B))^c \sqsubset (F, A)^c \sqcup (G, B)^c$.

Remark 2.25. In Theorem 2.24, the converse may not be true as the following example.

Example 2.26. Consider two (F, A) and (G, B) be interval-valued intuitionistic fuzzy soft sets over a common universe set U as in Example 2.6. Let $(F, A) \sqcup (G, B) = (H, A \cup B)$, where

$$H(e) = \begin{cases} G(e), & e \in B - A, \\ F(e) \cup G(e), & e \in A \cap B. \end{cases}$$

Then from Example 2.20, it is obtained the following:

$$\begin{aligned} H(\neg e_1) &= \{ \langle d_1, [0.2, 0.3], [0.4, 0.7] \rangle, \\ &\quad \langle d_2, [0, 0], [1, 1] \rangle, \\ &\quad \langle d_3, [0.2, 0.3], [0.4, 0.5] \rangle, \\ &\quad \langle d_4, [0.3, 0.5], [0.3, 0.5] \rangle, \\ &\quad \langle d_5, [0.5, 0.7], [0.1, 0.2] \rangle \}. \end{aligned}$$

$$\begin{aligned} H(\neg e_2) &= \{ \langle d_1, [0.3, 0.4], [0.4, 0.5] \rangle, \\ &\quad \langle d_2, [0.5, 0.8], [0.1, 0.2] \rangle, \\ &\quad \langle d_3, [0.2, 0.4], [0.4, 0.5] \rangle, \\ &\quad \langle d_4, [0.2, 0.3], [0.3, 0.6] \rangle, \\ &\quad \langle d_5, [0, 0], [1, 1] \rangle \}. \end{aligned}$$

$$\begin{aligned} H(\neg e_6) &= \{ \langle d_1, [0.3, 0.4], [0.4, 0.6] \rangle, \\ &\quad \langle d_2, [0.2, 0.6], [0.2, 0.3] \rangle, \\ &\quad \langle d_3, [0.4, 0.5], [0.3, 0.5] \rangle, \\ &\quad \langle d_4, [0.3, 0.7], [0.1, 0.3] \rangle, \\ &\quad \langle d_5, [0.4, 0.6], [0.2, 0.4] \rangle \}. \end{aligned}$$

From Definition 2.19,

$$\begin{aligned} (F, A)^c \sqcup (G, B)^c &= (F^c, \neg A) \sqcup (G^c, \neg B) \\ &= (K, \neg A \cup \neg B) \\ &= K(\neg e) \\ &= \begin{cases} G^c(\neg e), & \neg e \in \neg B - \neg A, \\ F^c(\neg e) \cup G^c(\neg e), & \neg e \in \neg A \cap \neg B. \end{cases} \end{aligned}$$

Hence $((F, A) \sqcup (G, B))^c \sqsupseteq (F, A)^c \sqcup (G, B)^c$.

$$\begin{aligned} K(\neg e_1) &= \{ \langle d_1, [0.2, 0.3], [0.4, 0.7] \rangle, \\ &\quad \langle d_2, [0, 0], [1, 1] \rangle, \\ &\quad \langle d_3, [0.2, 0.3], [0.4, 0.5] \rangle, \\ &\quad \langle d_4, [0.3, 0.5], [0.3, 0.5] \rangle, \\ &\quad \langle d_5, [0.5, 0.7], [0.1, 0.2] \rangle \}. \end{aligned}$$

$$\begin{aligned} K(\neg e_2) &= F^c(\neg e_2) \cup G^c(\neg e_2) \\ &= \{ \langle d_1, [0.3, 0.5], [0.3, 0.5] \rangle, \\ &\quad \langle d_2, [0.6, 0.8], [0.1, 0.2] \rangle, \\ &\quad \langle d_3, [0.3, 0.6], [0.2, 0.4] \rangle, \\ &\quad \langle d_4, [0.3, 0.5], [0.3, 0.5] \rangle, \\ &\quad \langle d_5, [0.3, 0.3], [0.5, 0.7] \rangle \}. \end{aligned}$$

$$\begin{aligned} K(\neg e_6) &= F^c(\neg e_6) \cup G^c(\neg e_6) \\ &= \{ \langle d_1, [0.3, 0.5], [0.2, 0.5] \rangle, \\ &\quad \langle d_2, [0.3, 0.7], [0.2, 0.2] \rangle, \\ &\quad \langle d_3, [0.5, 0.6], [0.3, 0.4] \rangle, \\ &\quad \langle d_4, [0.5, 0.7], [0, 0.2] \rangle, \\ &\quad \langle d_5, [0.5, 0.7], [0.2, 0.3] \rangle \}. \end{aligned}$$

Hence $((F, A) \sqcup (G, B))^c \neq (F, A)^c \sqcup (G, B)^c$.

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