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# 구간치 쇼케이적분에 의해 정의된 단조 구간치 집합함수의 구조적 성질에 관한 연구

# Structural characterizations of monotone interval-valued set functions defined by the interval-valued Choquet integral

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Abstract

We introduce nonnegative interval-valued set functions and nonnegative measurable interval-valued functions. Then the interval-valued Choquet integral determines a new nonnegative monotone interval-valued set function which is a generalized concept of monotone set function defined by Choquet integral in [17]. We also obtained absolutely continuity of them in [9]. In this paper, we investigate some characterizations of the monotone interval-valued set function defined by the interval-valued Choquet integral, and such as subadditivity, superadditivity, null-additivity, converse-null-additivity.

Key Words : monotone interval-valued set functions, interval-valued functions, fuzzy measures, Choquet integrals.

## 1. Preliminaries and Definitions

In a previous work [17] the authors investigated monotone set function defined by Choquet integral ([1,2,10,11,12]) instead of fuzzy integral ([13,14,15,16]). This construction is a useful method to form sound monotone set functions, including fuzzy measures, in various application areas, such as decision making, information theory, expected utility theory, and risk analysis.

Set-valued Choquet integrals was first introduced by Jang, Kil, Kim and Kwon([4]) and restudied by Zhang, Guo and Lia([19]) and that the theory about set-valued integrals has drawn much attention due to numerous applications in mathematics, economics, theory of control and many other fields. In the papers ([4,5,6,7,8,18,19]), they have been studied some properties of set-valued Choquet and interval-valued Choquet integrals. Recently, we obtained absolutely continuity of the monotone interval-valued set function defined by the interval-valued Choquet integral in [9]. In this paper, we investigate some characterizations of the monotone interval-valued set function defined by the interval-valued Choquet integral, and such as subadditivity, superadditivity, null-additivity, converse-null-additivity.

In order to construct a new useful tool defined by the interval-valued Choquet integral, we list the following definitions and basic properties.

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Let $X$ be a set and $(X, \Omega)$ a measurable space. A nonnegative set function $\mu$ is called a fuzzy measure if it is monotone and $\mu(\varnothing)=0$. A fuzzy measure $\mu$ is said to be lower semi-continuous if for every increasing sequence $A_n$ in $\Omega$, then $\mu(\cup_{n=1}^{\infty}A_n)=\lim_{n\to\infty}\mu(A_n)$. A fuzzy measure $\mu$ is said to be upper semi-continuous if for every decreasing sequence $A_n$ in $\Omega$ and $A_1<\infty$, then $\mu(\cap_{n=1}^{\infty}A_n)=\lim_{n\to\infty}\mu(A_n)$. If $\mu$ is both lower semi-continuous and upper semi-continuous, it is said to be continuous. A fuzzy measure $\mu$ is said to be finite if $\mu(X)$ is finite.

Definition 1.1 ([1,2,10,11,12]) (1) The Choquet integral of a measurable function $f$ with respect to a fuzzy measure $\mu$ on $A\in\Omega$ is defined by

$$(C)\int_A f d\mu=\int_0^{\infty}\mu(\{x|f(x)>r\}\cap A)\,dr$$

where the integrand on the right-hand side is an ordinary one.

(2) A measurable function $f$ is called $c$-integrable if the Choquet integral of $f$ can be defined and its value is finite.

Instead of $(C)\int_X f d\mu$, we will write $(C)\int f d\mu$.

Throughout this paper, $R^+$ will denote the interval $[0,\infty)$. The Choquet integral is a generalization of the Lebesgue integral, since they coincide when $\mu$ is a classical ($\sigma$-additive) measure.

Definition 1.2 ([1,2,10,11,12]) Let $f,g$ be nonnegative measurable functions. We say that $f$ and $g$ are comonotonic, in symbol $f \sim g$ if

$$f(x) < f(x') \rightarrow g(x) \leq g(x')$$

for all $x,x' \in X$.

Theorem 1.3 ([1,2,10,11,12]) Let $f,g$ be nonnegative measurable functions. Then the followings hold.
(1) $f \sim f$,
(2) $f \sim g \rightarrow g \sim f$,
(3) $f \sim a$ for all $a \in R^+$,
(4) $f \sim g$ and $g \sim h \rightarrow f \sim g+h$.

Theorem 1.4 ([1,2,10,11,12]) Let $f,g$ be nonnegative measurable functions. Then the followings hold.
(1) If $f \leq g$, then $(C)\int f d\mu \leq (C)\int g d\mu$.
(2) If $A \subset B$ and $A,B \in \Omega$, then

$$(C)\int_A f d\mu \leq (C)\int_B f d\mu.$$

(3) If $f \sim g$ and $a,b \in R^+$, then

$$(C)\int (af+bg)d\mu = a(C)\int f d\mu + b(C)\int g d\mu$$

(4) If $(f \vee g)(x) = f(x) \vee g(x)$ and $(f \wedge g)(x) = f(x) \wedge g(x)$ for all $x \in X$, then

$$(C)\int f \vee g d\mu \geqq (C)\int f d\mu \vee (C)\int g d\mu$$

and

$$(C)\int f \wedge g d\mu \geqq (C)\int f d\mu \wedge (C)\int g d\mu.$$

We denote $I(R^+)$ by

$$I(R^+) = \bar{a} = [a^-, a^+] \mid a^- \leq a^+, a^-, a^+ \in R^+.$$

For any $a \in R^+$, we define $a = [a,a]$. Obviously, $a \in I(R^+)$.

Definition 1.5 ([9]) If $\bar{a}, \bar{b} \in I(R^+)$, then we define
(1) $\bar{a} \wedge \bar{b} = [a^- \wedge b^-, a^+ \wedge b^+]$,
(2) $\bar{a} \vee \bar{b} = [a^- \vee b^-, a^+ \vee b^+]$,
(3) $\bar{a} \leq \bar{b}$ if and only if $a^- \leq b^-$ and $a^+ \leq b^+$,
(4) $\bar{a} < \bar{b}$ if and only if $\bar{a} \leq \bar{b}$ and $\bar{a} \neq \bar{b}$,
(5) $\bar{a} \subset \bar{b}$ if and only if $b^- \leq a^-$ and $a^+ \leq b^+$.

Theorem 1.6 ([9]) Let $\bar{a}, \bar{b} \in I(R^+)$. Then the followings hold.
(1) idempotent law: $\bar{a} \wedge \bar{a} = \bar{a}, \bar{a} \vee \bar{a} = \bar{a}$,
(2) commutative law: $\bar{a} \wedge \bar{b} = \bar{b} \wedge \bar{a}, \bar{a} \vee \bar{b} = \bar{b} \vee \bar{a}$,
(3) associative law: $(\bar{a} \wedge \bar{b}) \wedge \bar{c} = \bar{a} \wedge (\bar{b} \wedge \bar{c})$,
$(\bar{a} \vee \bar{b}) \vee \bar{c} = \bar{a} \vee (\bar{b} \vee \bar{c})$,
(4) absorption law: $\bar{a} \wedge (\bar{a} \vee \bar{b}) = \bar{a} \vee (\bar{a} \wedge \bar{b}) = \bar{a}$,
(5) distributive law: $\bar{a} \wedge (\bar{b} \vee \bar{c}) = (\bar{a} \wedge \bar{b}) \vee (\bar{a} \wedge \bar{c})$,
$\bar{a} \vee (\bar{b} \vee \bar{c}) = (\bar{a} \vee \bar{b}) \wedge (\bar{a} \vee \bar{c})$,

Clearly, we have the following theorem for multiplication and Hausdorff metric on $I(R^+)$.

Theorem 1.7 ([3]) (1)If we define

$$\bar{a} \cdot \bar{b} = \{x \cdot y \mid x \in \bar{a}, y \in \bar{b}\}$$

for $\bar{a}, \bar{b} \in I(R^+)$, then $\bar{a} \cdot \bar{b} = [a^- \cdot b^-, a^+ \cdot b^+]$.
(2) If $d_H : I(R^+) \times I(R^+) \rightarrow [0,\infty)$ is a Hausdorff metric, then $d_H(\bar{a},\bar{b}) = \max|a^- - b^-|, |a^+ - b^+|$.

Let $C(R^+)$ be the class of closed subsets of $R^+$. We denote a real-valued function $f: X \rightarrow R^+$, a closed set-valued function $\bar{f}: X \rightarrow C(R^+) \setminus \{\varnothing\}$.

Definition 1.8 ([4,5]) A closed set-valued function $\bar{f}$ is said to be measurable if for each open set $O \subset R^+$,

$$\bar{f}^{-1}(O) = \{x \in X \mid \bar{f}(x) \cap O \neq \varnothing \in \Omega\}.$$

Definition 1.9 ([4,5,6,7,8,9,19]) (1) Let $A \in \Omega$. The Choquet integral of a closed set-valued $\bar{f}$ on $A$ is defined by

$$(C)\int_A \bar{f} d\mu = \left\{(C)\int_A f d\mu \mid f \in S(\bar{f})\right\}$$

where $S(\bar{f})$ is the family of measurable selections of $\bar{f}$.
(2) $\bar{f}$ is said to be $c$-integrable if

$$(C)\int \bar{f} d\mu \neq \varnothing.$$

(3) $\bar{f}$ is said to be Choquet integrably bounded if there is a $c$-integrable function $g$ such that

$$\|\bar{f}\| = \sup_{r \in \bar{f}(x)} |r| \leq g(x),$$

for all $x \in X$.

Instead of $(C)\int_X \bar{f} d\mu$, we write $(C)\int \bar{f} d\mu$. Obviously, it may be empty. We note that if $A,B \in C(X)$ (the class of closed subsets of $X$), then $A \leq B$ means $\infty A \leq \infty B$ and $\sup A \leq \sup B$.

Theorem 1.10 ([4,7,8,9]) (1) If a closed set-valued function $\bar{f}$ is $c$-integrable, then
(i) $A \leq B$ and $A,B \in C(X)$ $\Rightarrow$

$$(C)\int_A \bar{f} d\mu \leq (C)\int_B \bar{f} d\mu.$$

(ii) $A \subset B$ and $A,B \in C(X)$ $\Rightarrow$

$$(C)\int_A \bar{f} d\mu \subset (C)\int_B \bar{f} d\mu.$$

(2) If a fuzzy measure $\mu$ is continuous and a closed set-valued function $\bar{f}$ is Choquet integrably bounded,

then $(C)\int \bar{f}d\mu$ is a closed set.

(3) If a fuzzy measure $\mu$ is continuous and an interval-valued function $\bar{f}=[f^{-},f^{+}]$ is Choquet integrably bounded, then

$$(C)\int \bar{f}d\mu=[(C)\int f^{-}d\mu,(C)\int f^{+}d\mu].$$

## 2. Structural characteristics

In this section, we show that a monotone interval-valued set function defined by the interval-valued Choquet integral preserves some structural characteristics such as subadditivity, superadditivity, null-additivity, and converse-null-additivity.

We note that by using Theorem 1.10(1), for any Choquet integrably bounded closed set-valued function $\bar{f}$, the closed set-valued set function $\bar{\nu}_{\bar{f}}$ on $\Omega$defined by

$$\bar{\nu}_{\bar{f}}(A)=(C)\int_{A}\bar{f}d\mu, \quad \forall A\in\Omega \tag{2.1}$$

is also nonnegative monotone and vanishing at $\varnothing$. Then by Theorem 1.10(3), we also obtain that if $\bar{f}$ is a Choquet integrably bounded interval-valued function and a fuzzy measure $\mu$ is continuous, there exist two nonnegative monotone set functions $\nu_{f^{-}},\nu_{f^{+}}$ such that

$$\bar{\nu}_{\bar{f}}(A)=[\nu_{f^{-}}(A),,\nu_{f^{+}}(A)], \quad \forall\in\Omega \tag{2.2}$$

where $\nu_{f^{-}}(A)=(C)\int_{A}f^{-}d\mu$ and $\nu_{f^{+}}(A)=(C)\int_{A}f^{+}d\mu$.

Definition 2.1 ([17]) (1) A fuzzy measure $\mu$ is said to be subadditive if for any $A,B\in\Omega$,

$$\mu(A\cup B)\le\mu(A)+\mu(B).$$

(2) A fuzzy measure $\mu$ is said to be superadditive if for any $A,B\in\Omega$,

$$\mu(A\cup B)\ge\mu(A)+\mu(B).$$

(3) A nonnegative monotone interval-valued set function $\bar{\nu}$ is said to be subadditive if for any $A,B\in\Omega$,

$$\bar{\nu}(A\cup B)\le\bar{\nu}(A)+\bar{\nu}(B).$$

(4) A nonnegative monotone interval-valued set function $\bar{\nu}$ is said to be superadditive if for any $A,B\in\Omega$,

$$\bar{\nu}(A\cup B)\ge\bar{\nu}(A)+\bar{\nu}(B).$$

We note that a fuzzy measure $\mu$ is additive if and only if it is both subadditive and superadditive, and that a nonnegative monotone interval-valued set function $\bar{\nu}$ is additive if and only if it is both subadditive and superadditive.

Theorem 2.2 ([17]) Let $A\in\Omega$ and $\nu_{g}$ be defined in terms of a fuzzy measure $\mu$ and a real-valued function $g$ by

$$\nu_{g}(A)=\int_{A}gd\mu.$$

(1) If $g$ is measurable and $\mu$ is subadditive, then $\nu_{g}$ is subadditive.

(2) If $g$ is measurable and $\mu$ is superadditive, then $\nu_{g}$ is superadditive.

Theorem 2.3 If a fuzzy measure $\mu$ is continuous and subadditive, and if $\bar{f}$ is Choquet integrably bounded, then $\bar{\nu}_{\bar{f}}$ is subadditive.

Proof. Since a fuzzy measure $\mu$ is continuous and $\bar{f}$ is Choquet integrably bounded, by the equation (2.2),

$$\bar{\nu}_{\bar{f}}(A)=[\nu_{f^{-}}(A),\nu_{f^{+}}(A)], \quad \forall\in\Omega.$$

Since $\mu$ is subadditive, by Theorem 2.2(1), $\nu_{f^{-}}$ and $\nu_{f^{+}}$ are subadditive. Thus, if $A,B\in\Omega$, then

$$\nu_{f^{-}}(A\cup B)\le\nu_{f^{-}}(A)+\nu_{f^{-}}(B)$$

and

$$\nu_{f^{+}}(A\cup B)\le\nu_{f^{+}}(A)+\nu_{f^{+}}(B).$$

Therefore,

$$\begin{aligned}\bar{\nu}_{\bar{f}}(A\cup B)&=[\nu_{f^{-}}(A\cup B),\nu_{f^{+}}(A\cup B)]\\&\le[\nu_{f^{-}}(A)+\nu_{f^{-}}(B),\nu_{f^{+}}(A)+\nu_{f^{+}}(B)]\\&=[\nu_{f^{-}}(A),\nu_{f^{+}}(A)]+[\nu_{f^{-}}(B),\nu_{f^{+}}(B)]\\&=\bar{\nu}_{\bar{f}}(A)+\bar{\nu}_{\bar{f}}(B).\end{aligned}$$

Theorem 2.4 If a fuzzy measure $\mu$ is continuous and superadditive, and if $\bar{f}$ is Choquet integrably bounded, then $\bar{\nu}_{\bar{f}}$ is superadditive.

Proof. Since a fuzzy measure $\mu$ is continuous and $\bar{f}$ is Choquet integrably bounded, by the equation (2.2),

$$\bar{\nu}_{\bar{f}}(A)=[\nu_{f^{-}}(A),\nu_{f^{+}}(A)], \quad \forall\in\Omega.$$

Since $\mu$ is superadditive, by Theorem 2.2(2), $\nu_{f^{-}}$ and $\nu_{f^{+}}$ are superadditive. Thus, if $A,B\in\Omega$, then

$$\nu_{f^{-}}(A\cup B)\ge\nu_{f^{-}}(A)+\nu_{f^{-}}(B)$$

and

$$\nu_{f^{+}}(A\cup B)\ge\nu_{f^{+}}(A)+\nu_{f^{+}}(B).$$

Therefore,

$$\begin{aligned}\bar{\nu}_{\bar{f}}(A\cup B)&=[\nu_{f^{-}}(A\cup B),\nu_{f^{+}}(A\cup B)]\\&\ge[\nu_{f^{-}}(A)+\nu_{f^{-}}(B),\nu_{f^{+}}(A)+\nu_{f^{+}}(B)]\\&=[\nu_{f^{-}}(A),\nu_{f^{+}}(A)]+[\nu_{f^{-}}(B),\nu_{f^{+}}(B)]\\&=\bar{\nu}_{\bar{f}}(A)+\bar{\nu}_{\bar{f}}(B).\end{aligned}$$

Combining Theorems 2.3 and 2.4, we obtain the following theorem.

Theorem 2.5 If a fuzzy measure $\mu$ is continuous and additive, and if an interval-valued function $\overline{f}$ is Choquet integrably bounded, then $\overline{\nu}_{\overline{f}}$ is additive.

Remark 2.6 (1) We note that a fuzzy measure $\mu$ is both continuous and additive if and only if it is a classical(additive) measure. In this case, it may be not $\sigma$-additive.
(2) We can find an example that a fuzzy measure $\mu$ is continuous, but not a classical(additive) measure: if we put $m$ as a Lebesgue measure on $X$ and $\mu=m^2$, then clearly $\mu$ is a continuous fuzzy measure but not classical measure.

Definition 2.7 (1) A fuzzy measure $\mu$ is said to be null-additive if $\mu(B\cup A)=\mu(B)$, whenever $A,B\in\Omega$ and $\mu(A)=0$.
(2) A fuzzy measure $\mu$ is said to be converse-null-additive if $\mu(B-A)=0$, whenever $A,B\in\Omega$, $A\subset B$ and $\mu(A)=\mu(B)$.
(3) A nonnegative monotone interval-valued set function $\overline{\nu}$ is said to be null-additive if $\overline{\nu}(B\cup A)=\overline{\nu}(B)$, whenever $A,B\in\Omega$ and $\overline{\nu}(A)=\overline{0}$.
(4) A nonnegative monotone interval-valued set function $\overline{\nu}$ is said to be converse-null-additive if $\overline{\nu}(B-A)=\overline{0}$, whenever $A,B\in\Omega$, $A\subset B$ and $\overline{\nu}(A)=\overline{\nu}(B)$.

We note that a fuzzy measure $\mu$ is null-additive if and only if it is both null-additive and converse-null-additive, and a nonnegative monotone interval-valued set function $\overline{\nu}$ is null-additive if and only if it is both null-additive and converse-null-additive.

Theorem 2.8 ([17]) Let $A\in\Omega$ and $\nu_g$ be defined in terms of a fuzzy measure $\mu$ and a real-valued function $g$ by

$$\nu_g(A)=(C)\int_A g d\mu.$$

(1) If $g$ is measurable and $\mu$ is null-additive, then $\nu_g$ is null-additive.
(2) If $g$ is measurable and $\mu$ is converse null-additive, then $\nu_g$ is converse null-additive.

Theorem 2.9 If a fuzzy measure $\mu$ is null-additive, and if a closed set-valued $\overline{f}$ is $c$-integrable, then $\overline{\nu}_{\overline{f}}$ is null-additive.
Proof. Let $A,B\in\Omega$ and $\overline{\nu}(A)=\overline{0}$. Then $\nu_f(A)=0$ for all $f\in S(\overline{f})$.
Theorem 2.8(1) implies that $\nu_f$ is null-additive, that is

$$\nu_f(B\cup A)=\nu_f(B),\ \forall B\in\Omega \text{ and } \forall f\in S(\overline{f}).$$

Thus

$$\begin{aligned}\overline{\nu}_{\overline{f}}(B\cup A)&=\{\nu_f(B\cup A)\mid f\in S(\overline{f})\}\\&=\{\nu_f(B)\mid f\in S(\overline{f})\}\\&=\overline{\nu}_{\overline{f}}(B).\end{aligned}$$

We also can prove the following theorem for preservation of converse-null-additive under some strong condition for a fuzzy measure $\mu$.

Theorem 2.10 If a fuzzy measure $\mu$ is converse-null-additive and continuous, and if an interval-valued function $\overline{f}$ is Choquet integrably bounded, then $\overline{\nu}_{\overline{f}}$ is converse-null-additive.
Proof. Since a fuzzy measure $\mu$ is continuous and $\overline{f}$ is Choquet integrably bounded, by the equation (2.2),

$$\overline{\nu}_{\overline{f}}(A)=[\nu_{f^-}(A),\nu_{f^+}(A)],\ \forall\in\Omega.$$

Now, we let $A,B\in\Omega$, $A\subset B$ and $\overline{\nu}_{\overline{f}}(A)=\overline{\nu}_{\overline{f}}(B)$. Then

$$[\nu_{f^-}(A),\nu_{f^+}(A)]=[\nu_{f^-}(B),\nu_{f^+}(B)]$$

and hence $\nu_{f^-}(A)=\nu_{f^-}(B)$ and $\nu_{f^+}(A)=\nu_{f^+}(B)$. By Theorem 2.8(2), $\nu_{f^-}$ and $\nu_{f^+}$ are converse-null-additive and hence $\nu_{f^-}(B-A)=0$ and $\nu_{f^+}(B-A)=0$. Thus

$$\begin{aligned}\overline{\nu}_{\overline{f}}(B-A)&=[\nu_{f^-}(B-A),\nu_{f^+}(B-A)]\\&=\overline{0}.\end{aligned}$$

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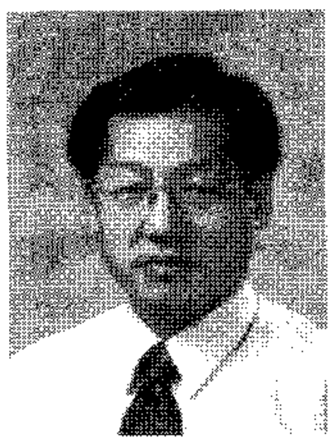

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