Minimum Energy Cooperative Path Routing in All-Wireless Networks: NP-Completeness and Heuristic Algorithms

Fulu Li, Kui Wu, and Andrew Lippman

Abstract: **We study the routing problem in all-wireless networks based on cooperative transmissions. We model the minimum-energy cooperative path (MECP) problem and prove that this problem is NP-complete. We hence design an approximation algorithm called cooperative shortest path (CSP) algorithm that uses Dijkstra's algorithm as the basic building block and utilizes cooperative transmissions in the relaxation procedure. Compared with traditional non-cooperative shortest path algorithms, the CSP algorithm can achieve a higher energy saving and better balanced energy consumption among network nodes, especially when the network is in large scale. The nice features lead to a unique, scalable routing scheme that changes the high network density from the curse of congestion to the blessing of cooperative transmissions.**

Index Terms: **Cooperative transmissions, distributed algorithms, energy efficiency, wireless networks.**

I. INTRODUCTION

In this paper, we study the problem of minimum-energy cooperative path routing in all-wireless networks, which consist of numerous devices communicating via short-range ad hoc radio [1]. Cooperative routing approach allows multiple nodes along the path for cooperative transmission to the next hop as long as the combined signal at the receiver satisfies a given threshold signal-to-interference-plus-noise ratio (SINR). Such cooperative transmission is also termed as *transmit diversity*. A transmission is successful only if the received signal at the receiver is above a given threshold value of SINR, which is chosen to achieve a desired bit error rate (BER) for the given modulation scheme and data rate [2]. Traditional routing schemes select a route solely based on certain criteria such as the number of hops along the path, the cost of the path, and/or the message delay. In traditional routing schemes, network nodes generally compete for limited resources and a high network density is commonly deemed as the curse of congestion. The cooperative routing approach totally changes the philosophy behind traditional routing schemes. By combining route selection with cooperative transmissions, network nodes help each other and as the result, the more nodes a network has, the better performance the network can achieve.

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Among the many benefits of cooperative routing, we mainly focus on three prominent ones: Energy efficiency, scalability, and fairness. Energy is a scarce resource as most wireless nodes are powered by batteries. The lifetime of a wireless network totally depends on the energy consumption of each node. To increase network lifetime, power-efficient and power-aware protocols and techniques at different network layers have been employed to minimize the power consumption. Without causing confusion, we use energy and power interchangeably throughout the paper when the context is clear. As shown later in this paper, the cooperative shortest path algorithm can save about 30$\sim$50% power compared with non-cooperative shortest path algorithms, depending on the node density of the network. As more nodes are added in the network, higher power savings can be achieved, indicating that a dense network actually offers more opportunities for cooperative transmissions.

Virtually all wireless networks face scalability problems, since transmission with excessive power on one link often leads to severe interference to other links in the network. As discussed in [2], a wireless link is rather a *soft* concept in the sense that a *link* exists between two wireless nodes if the transmitting node transmits with sufficiently high power such that the SINR at the receiving node is above a given threshold. Moreover, besides interference and noise, wireless channel inherently has other impairment such as multi-path fading, attenuation, reflection, obstruction, etc. As such, our objective is to optimize the distribution of information by exploring transmit diversity and to minimize the wireless channel impairment effects via cooperative transmissions among nodes in the network. When network nodes change their role from competition to cooperation, the network prefers more nodes and becomes more scalable.

To become a scalable solution, the routing scheme must result in balanced energy consumption among network nodes. The concept of *fairness* here means that the distribution of information in the network is optimized in such a way that each node is treated fairly based on the pre-defined utility function for each node (we will provide the formal definition in Section VII). We demonstrate in this paper that cooperative routing can achieve better fairness compared with non-cooperative routing algorithms.

The results in this paper are built upon our previous work in [3] and [4]. The major contributions of this paper include:

- We formulate the minimum energy cooperative path (MECP) problem and prove that it is NP-complete.
- We develop a cooperative shortest path (CSP) algorithm for cooperative routing for all-wireless networks that uses Dijkstra's algorithm as the basic building block and utilizes co-

operative transmissions in the relaxation procedure.

- We perform a comprehensive performance study. The experimental results show that the CSP algorithm consistently outperforms existing algorithms like those in [5], regarding energy savings and fairness.

The rest of the paper is organized as follows. Related work is introduced in Section II. We give the description of the system model in Section III and the problem formulation in Section IV. We prove the NP-completeness of the minimum energy path problem in the context of cooperative routing in Section V and present a cooperative shortest path algorithm for cooperative routing in Section VI. The experimental results, which are presented in Section VII, demonstrate the better performance of our algorithm compared with other cooperative routing algorithms and non-cooperative shortest path algorithms. Distribution and implementation issues are discussed in Section VIII. We conclude the paper in Section IX.

II. RELATED WORK

The minimum energy cooperative path routing problem in wireless networks has been addressed in [5] and several heuristic algorithms were developed to approximate the minimum energy cooperative route based on a non-cooperative shortest-path algorithm. One of the presented algorithms in [5] is called cooperative along non-cooperative (CAN) shortest path algorithm. The basic idea is to run a non-cooperative shortest path algorithm to obtain the cooperative path. The computational complexity of the CAN algorithm is in the order of $O(N^2)$, where N is the number of nodes in the network. Our CSP algorithm is different from the CAN algorithm: The CSP algorithm proceeds as a cooperative version of Dijkstra's algorithm with a new relaxation procedure to reflect the cooperative transmission cost.

Another related work by Catovic *et al.* in [6] presents approaches to explore transmit diversity via user cooperation in next generation wireless multi-hop networks. The network model used in [6] greatly differs from that assumed in [5]. In essence, Catovic *et al.* assume that the m-finger RAKE receivers are used and each finger is in charge of the reception of the signal from a different transmitter. Khandani *et al.* use conventional receivers and assume that the channel parameters are estimated by the receiver, which returns the feedback to the transmitter. It is essentially a trade off between the use of complex receivers, e.g., the m-finger RAKE receivers, and the complexity to implement the feedback mechanism. In this paper, we present an abstract network model, based on which the basic framework of our algorithm should be equally applicable to other cooperative routing environment, e.g., different fading/attenuation models, different receiver types, etc.

Other researchers also address the power efficiency issue in wireless networks. Wieselthier *et al.* [7], Calgalj *et al.* [1], and Liang [8] presented approaches for the construction of minimum energy broadcast trees in wireless networks. The seminal work by Wieselthier *et al.* in [7] elucidates many fundamental aspects of energy-efficient routing in wireless networks. Rodoplu and Meng [9] proposed a novel distributed position-based network protocol to achieve the minimum-energy network topology. Wattenhofer *et al.* [10] presented an ingenious distributed topology control algorithm for power efficiency in wireless adhoc networks based on directional information. All the above work, however, does not consider cooperative transmissions.

Min and Chandrakasan address the energy consumption issues of wireless communication in [11]. In [12], Feeney and Nilson reveal that nodes usually spend most of their energy in communication in wireless ad hoc networks. Srinivas and Modiano [2] present algorithms for finding minimum energy disjoint paths in wireless ad hoc networks to achieve energy efficiency and reliability.

III. THE SYSTEM MODEL

A. The Network Model

We consider an all-wireless network consisting of N devices (nodes). These nodes are generally powered by batteries and are linked via short-range ad-hoc radio connections. We assume the use of omni-directional antennas and all nodes within communication range of a transmitting node can receive its transmission.

Following [5], we assume that the power level of a transmission can be chosen at each node within a given range of values, say $[0, P_{\max}]$. We assume that the channel parameters are estimated by the receiver, which also returns feedback to the transmitter. Each node can thus dynamically adjust its transmitted signal phase to possibly synchronize with other nodes. This mechanism can be realized by pre-compensation before transmitting based on the estimate of the phase and delay at each path as discussed by Tu and Pottie in [9]. This assumption is reasonable for slowly varying channels where the channel coherence time is much longer than the block transmission duration.

B. The Power Consumption Model

It is well known that the signal power attenuation in wireless communication is non-linear. We consider a commonly used wireless propagation model [1], [2], [5], [7]–[9] whereby the received signal power attenuates $d^{-\lambda}$, where d stands for the distance between the transmitting node antenna and the receiving node antenna and λ takes a value between 2 and 4, depending on the characteristics of the communication medium. Following [2], we assume that the required power to support a wireless link at a given data rate between node i and node j is given by

$$P_{i,j} = {d_{i,j}}^{\lambda} \tag{1}$$

where $d_{i,j}$ denotes the distance between node i and node j. We say node i can reach node j if and only if the transmitting power at node i is greater than or equal to ${d_{i,j}}^{\lambda}$. Notably, each node can add or remove links by adjusting its transmitting power levels, hence the network topology totally depends on the transmitting range of each node.

Regarding the transmission energy, we have the following definitions:

Definition 1: The power required for a transmitting node, say t, to directly reach a set of destination nodes, say $r_1, r_2, \cdots, r_m$, is determined by the maximum required power to reach any of them individually and other nodes essentially get the transmission for free. This is referred to as wireless broadcast advantage (WBA) by Wieselthier *et al.* in [7]. Let $d_{r_1,t}, d_{r_2,t}, \cdots, d_{r_m,t}$ stand for the distances from the transmitting node t to the destination nodes $r_1, r_2, \cdots, r_m$, respectively. The required power is determined by

$$P_{\text{broadcast}} = \max({d_{r_1,t}}^{\lambda}, {d_{r_2,t}}^{\lambda}, \cdots, {d_{r_m,t}}^{\lambda}). \quad (2)$$

Definition 2: Assume that nodes $t_1, t_2, \cdots, t_m$ cooperatively transmit information to a given destination node, say node r, where the received signal from each transmitting node is coherently combined. We assume that the cooperative transmission is successful if and only if the SINR of the coherently combined signal at the receiving node r is above a given threshold value. Let ${d_{t_1,r}}^{\lambda}, {d_{t_2,r}}^{\lambda}, \cdots, {d_{t_m,r}}^{\lambda}$ denote the required power for point-to-point transmission to the given destination node r from transmitting nodes $t_1, t_2, \cdots, t_m$, respectively. The discovery by Khandani *et al.* in [5] reveals that the total required power for this cooperative transmission is given by

$$P_{coop} = \frac{1}{\sum_{i=1}^{m} \frac{1}{{d_{t_i,r}}^{\lambda}}}. \quad (3)$$

We further derive that the required power for each of the transmitting node in this cooperative transmission is given by

$$P_{t_i} = \frac{{d_{t_i,r}}^{-\lambda}}{\sum_{j=1}^{m} {d_{t_j,r}}^{-\lambda}} \frac{1}{\sum_{j=1}^{m} \frac{1}{{d_{t_j,r}}^{\lambda}}} \quad (4)$$

where $1 \leq i \leq m$.

One observation from (3) is that if ${d_{t_1,r}}^{\lambda} = {d_{t_2,r}}^{\lambda} = \cdots = {d_{t_m,r}}^{\lambda}$, the total required power for this cooperative transmission is $\frac{1}{m}{d_{t_1,r}}^{\lambda}$, meaning that cooperative transmission can use as little as only a fraction of $\frac{1}{m}$ of the individual point-to-point transmission power if nodes cooperatively transmit to a given destination node.

IV. THE PROBLEM FORMULATION

Given an energy cost graph $G = (V, E)$, where $V = \{1, \cdots, N\}$ is the set of nodes and $E = \{< i,j >, i,j \in V\}$ is the set of links. For each link $< i,j >$, we assign a weight ${d_{i,j}}^{\lambda}$. Assume that the source and destination nodes are S and D, respectively. Also assume that the last L nodes along a path are allowed for cooperative transmission to the next hop, where $L < N$. Our goal is to find a $S - D$ path, $S \rightarrow t_1, t_2, \cdots, t_k \rightarrow D$ and a transmission schedule, which may consist of point-to-point transmissions, multicast and cooperative transmissions, such that the total required energy along this path is the minimum. That is,

$$\min \sum P_x \quad (5)$$

where $x \in \{S, t_1, t_2, \cdots, t_k, D\}$, P_x stands for the required power for node x.

An example of cooperative transmission along the path from node S to node D is given in Fig. 1, in which we allow the last two nodes along the path for cooperative transmission to the next hop, i.e., $L = 2$.

The corresponding path can be stated as $S \rightarrow A(S) \rightarrow B(A) \rightarrow D$. We use notation $A(B)$ to represent that node A and node B transmit cooperatively. The transmission procedure operates as follows: Node S first transmits to node A, then node S and node A cooperatively transmit to node B, then node A and node B cooperatively transmit to node D. The total energy cost for this path is give by

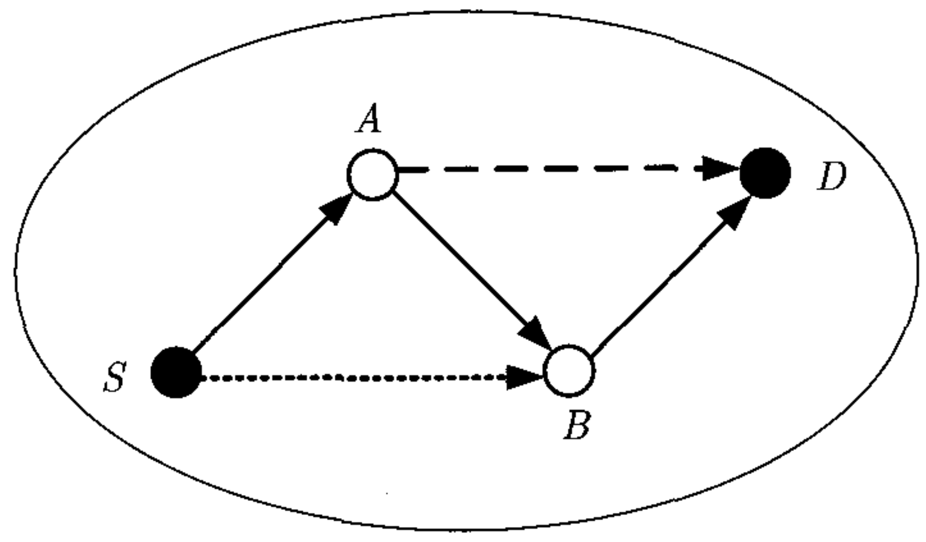

Fig. 1. An illustration of cooperative transmission with $L = 2$.

$$P_{S-D} = {d_{S,A}}^{\lambda} + \frac{1}{\frac{1}{{d_{S,B}}^{\lambda}} + \frac{1}{{d_{A,B}}^{\lambda}}} + \frac{1}{\frac{1}{{d_{A,D}}^{\lambda}} + \frac{1}{{d_{B,D}}^{\lambda}}}. \quad (6)$$

We need to emphasize that the minimum energy cooperative path could be a combination of multicast (one to many), cooperative transmissions (many to one), and point-to-point transmissions. Note that the case of many-to-many transmission is not a valid option as synchronizing transmissions for coherent receptions at multiple receivers is not feasible [5].

Regarding the point-to-point transmissions in MECP, we have the following theorem.

Theorem 1: If $L \geq 2$, i.e., at least the last two nodes along the path are allowed for cooperative transmissions to the next hop, and if there are point-to-point transmissions in MECP, there must be only one point-to-point transmission and it must be the first-hop transmission.

Proof: We prove this theorem by contradiction. Assume that there is a point-to-point transmission, say from node t_i to node t_{i+1} in MECP, which is not the first-hop transmission. There must be at least one predecessor node, say node t_{i-1}, for node t_i in MECP. As L is equal to or greater than two, node t_{i-1} and node t_i are allowed for cooperative transmissions to node t_{i+1}. We also have

$$\frac{1}{\frac{1}{{d_{t_{i-1},t_{i+1}}}^{\lambda}} + \frac{1}{{d_{t_i,t_{i+1}}}^{\lambda}}} < {d_{t_i,t_{i+1}}}^{\lambda} \quad (7)$$

because $\frac{1}{{d_{t_{i-1},t_{i+1}}}^{\lambda}}$ is always greater than zero due to the fact that node t_{i-1} and node t_{i+1} are different nodes at different locations in the physical space. Therefore, a cooperative transmission from node t_{i-1} and node t_i to node t_{i+1} always leads to less energy consumption than a point-to-point transmission from node t_i to node t_{i+1} does. This contradicts the assumption that the point-to-point transmission from node t_i to node t_{i+1} is one of the transmissions along MECP that should have the minimum total energy consumption. This concludes the proof. □

V. COMPLEXITY ISSUES

The problem of finding MECP appears to be hard to solve [5]. To find a good solution, acquiring insights into the complexity of the MECP problem is of great importance. In the following we show that the minimum energy cooperative path problem is NP-complete.

The complexity theory originates from decision problems, i.e., the problems with either yes or no as an answer [1], [13]. Nevertheless, each optimization problem can be easily stated as a corresponding decision problem. A decision problem related to the MECP problem can be described as follows:

The MECP Problem in All-Wireless Networks
Instance: Assume that we have an energy cost graph $G = (V, E)$ with weights ${d_{i,j}}^{\lambda}$, where V is the set of nodes, E is the set of links, and ${d_{i,j}}^{\lambda}$ is the weight on the edge $< i, j > \in E$. Assume that $|V| = N$ and the last L nodes along the path are allowed for cooperative transmission to the next hop, where $L < N$. Let P_x stand for the required power for node x.
Question: Given a source-destination pair $S, D \in V$ and a constant $B \in R_+$, is there a $S - D$ path, $S \to t_1, t_2, \cdots, t_k \to D$ and a corresponding transmission schedule, such that $\sum P_x \leq B$ $(x \in \{S, t_1, t_2, \cdots, t_k, D\})$?

An example path that consists of point-to-point transmission, multicast, and cooperative transmissions is given in Fig. 2. The transmission procedure operates as follows: Node S first transmits to node t_1, then node t_1 multicasts to node t_2 and node t_3, then node t_2 and node t_3 cooperatively transmit to node t_4, then node t_4 multicasts to node t_5 and node t_6, then node t_5 and node t_6 cooperatively transmit to node D. The corresponding path can be stated as $S \to t_1 \to (t_2, t_3) \to t_4 \to (t_5, t_6) \to D$.

In the following, we prove that the MECP problem is NP-complete by showing that a special case of it is NP-complete. In order to obtain a special case of MECP, we specify the following restrictions to be placed on the instances of MECP. First, we only allow cooperative transmission to the destination node D and up to $N - 1$ nodes are allowed for this last-hop cooperative transmission. Further, we assume that the weight difference between link $< i, D >$ and link $< j, D >$, where $i, j \in V - \{S, D\}$, is negligible, e.g., ${d_{i,D}}^{\lambda} \approx {d_{j,D}}^{\lambda}$. This assumption is to simplify the calculation of the last-hop cooperative transmission cost and it is just for the convenience of analysis.

We call this special case of MECP as S-MECP. We prove NP-completeness of the S-MECP problem by reduction from the minimum energy broadcast (MEB) problem in wireless networks, which is known to be NP-complete [1], [8].

The MEB Problem in Wireless Networks
Instance: Assume that we have an energy cost graph $G = (V, E)$ with weights ${d_{i,j}}^{\lambda}$, where V is the set of nodes, E is the set of links, and ${d_{i,j}}^{\lambda}$ is the weight on the edge $< i, j > \in E$. Assume that $|V| = N$. let P_x stand for the required power for node x.
Question: Given a source node $S \in V$ and a constant $B \in R_+$, is there a subgraph of G, say $G' = (V', E')$, where $|V'| = |V| = N$ and G' is a tree rooted at node S, such that $\sum P_x \leq B$ $(x \in V)$?

For the transformation from MEB to MECP, we first give

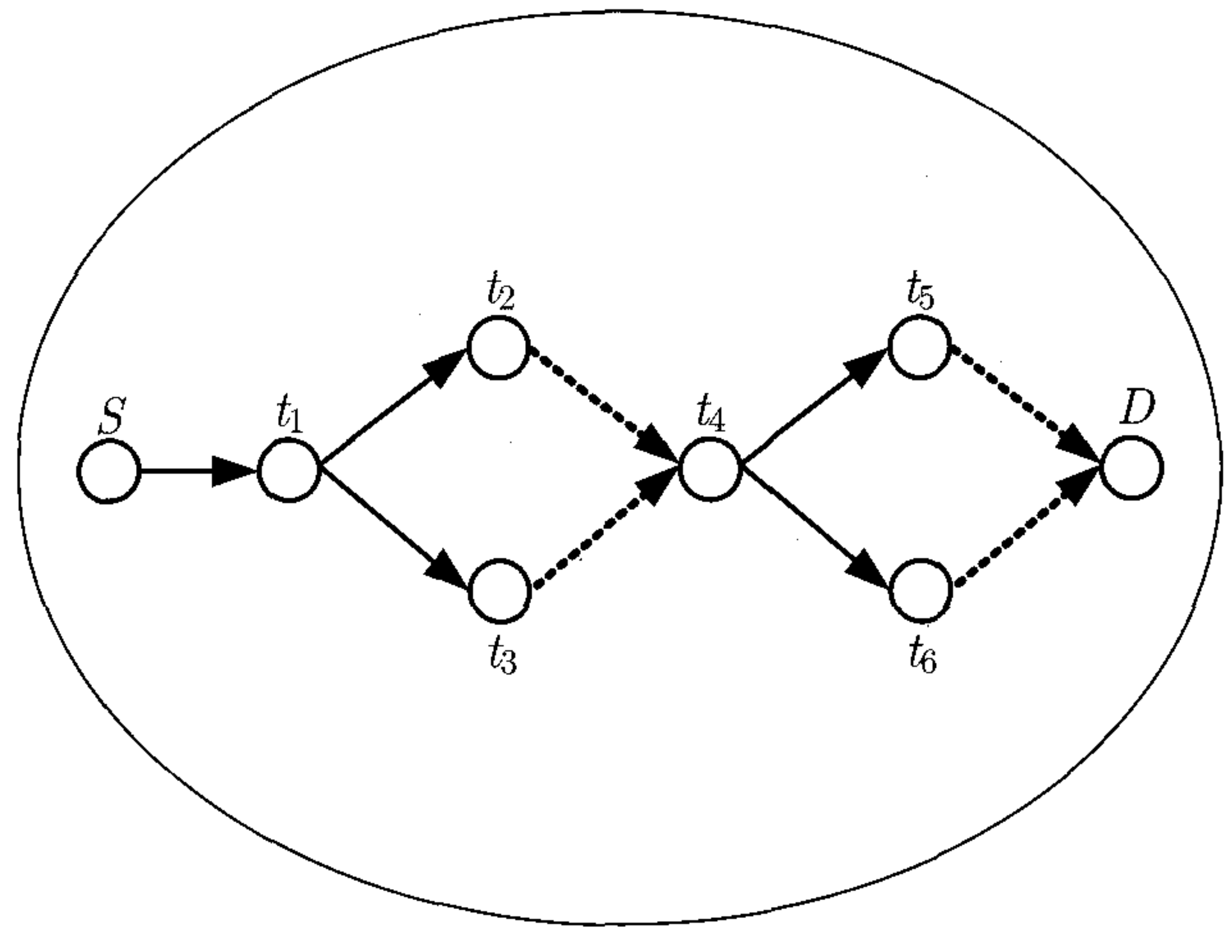

Fig. 2. An example path.

the following description of minimum energy i-node multicast problem.

Given a source node S and $N - 1$ potential destination nodes, we define that a minimum energy i-node multicast tree with source node S as a tree that is rooted at node S and reaching $i-1$ destination nodes among those $N - 1$ potential nodes with the least required power among all possible i-node multicast trees.

From the theorems in [1], [8], The MEB problem in wireless networks is NP-complete. We have the following corollary.

Corollary 1: Minimum energy i-node $(1 \leq i \leq N)$ multicast (MEiM) problem with one source node and $N - 1$ potential destination nodes is NP-complete.

Proof: It is easy to see that minimum energy i-node multicast problem belongs to the NP class since a nondeterministic algorithm needs only to guess a set of nodes, e.g., i nodes, and check in polynomial time whether there is a path from the source node to any of the remaining $i - 1$ destination nodes in a final solution, and whether the cost of the final solution is equal to or less than B. Since the MEB problem is a special case of the minimum energy i-node multicast (MEiM) problem when i is equal to N, and minimum energy i-node multicast problem is NP, the MEiM problem is NP-complete too. □

Let $T(S, i)$ denote the required power of a minimum energy i-node multicast tree with source node S. Regarding the series of $T(S, 1), T(S, 2), \cdots, T(S, i), T(S, i+1), \cdots, T(S, N)$, we have the following lemma.

Lemma 1: The series of $T(S, i)$ is monotonically increasing, i.e., $T(S, i) \leq T(S, i + 1)$, where $1 \leq i \leq N$.

Proof: We prove this lemma by contradiction. Assume that there is a minimum energy $(i + 1)$-node multicast tree and a minimum energy i-node multicast tree for the same given settings and $T(S, i + 1) < T(S, i)$ holds. For the minimum energy $(i + 1)$-node multicast tree, there must be at least one leaf node among the i destination nodes, the removal of which will not cause the connectivity changes of other nodes and the total cost of the remaining i-node multicast tree, say $T^*(S, i)$, is equal to or less than the original minimum energy $(i + 1)$-node multicast tree, i.e., $T(S, i + 1)$. Hence, $T^*(S, i) \leq T(S, i + 1)$. From the assumption, we also have $T(S, i + 1) < T(S, i)$. So, we have $T^*(S, i) < T(S, i)$. This contradicts the assertion that the original i-node multicast tree is the minimum energy i-node multicast tree. □

As discussed before, for S-MECP, we only allow cooperative transmission to the destination node D and up to $N-1$ nodes are allowed for this last-hop cooperative transmission. Let $C(i, D)$ stand for the required power for the last-hop cooperative transmission from i already-covered nodes including the source node S to the final destination node D. Recall that among the restrictions on the instance of MECP we assume that the link weight difference between link $< k, D >$ and link $< j, D >$, where $k, j \in V - \{S, D\}$, is negligible, i.e., $d_{k,D}{}^{\lambda} \approx d_{j,D}{}^{\lambda}$.

Lemma 2: The series of $C(1, D), C(2, D), \cdots, C(i, D), C(i+1, D), \cdots, C(N-1, D)$ is strictly decreasing, i.e., $C(i, D) > C(i+1, D)$, where $1 \leq i \leq N-2$.

Proof: According to the definition and the restriction assumptions, we have

$$C(1, D) = d_{S,D}{}^{\lambda},$$

$$C(2, D) = \frac{1}{\dfrac{1}{d_{S,D}{}^{\lambda}} + \dfrac{1}{d_{x,D}{}^{\lambda}}},$$

$$\vdots$$

$$C(i, D) = \frac{1}{\dfrac{1}{d_{S,D}{}^{\lambda}} + \dfrac{i-1}{d_{x,D}{}^{\lambda}}},$$

$$C(i+1, D) = \frac{1}{\dfrac{1}{d_{S,D}{}^{\lambda}} + \dfrac{i}{d_{x,D}{}^{\lambda}}},$$

$$\vdots$$

$$C(N-1, D) = \frac{1}{\dfrac{1}{d_{S,D}{}^{\lambda}} + \dfrac{N-2}{d_{x,D}{}^{\lambda}}}.$$

As $x \neq D$, we have $d_{x,D}{}^{\lambda} > 0$. Notably, all the link weights are non-negative. Without loss of generality, let us consider two consecutive elements $C(i, D)$ and $C(i+1, D)$. Based on the above analysis, we have $C(i, D) - C(i+1, D) = \frac{d_{S,D}{}^{2\lambda} \times d_{x,D}{}^{\lambda}}{(d_{x,D}{}^{\lambda} + (i-1) d_{S,D}{}^{\lambda})(d_{x,D}{}^{\lambda} + i \times d_{S,D}{}^{\lambda})} > 0$. So, the series of $C(i, D)$s is strictly decreasing. □

With Lemma 1, Lemma 2, and Corollary 1, we are ready to prove Theorem 2.

Theorem 2: The MECP problem in all-wireless networks is NP-complete.

Proof: We first show that S-MECP is NP-complete. As S-MECP is a special case of MECP, the theorem follows. To see that S-MECP is NP-complete, the proof first shows that S-MECP belongs to the NP class, and then shows that MEiM polynomially reduces to S-MECP.

It is easy to see that S-MECP problem belongs to the NP class since a nondeterministic algorithm needs only to guess a set of nodes and checks in polynomial time whether a given link schedule for the corresponding cooperative path from the source node S to the destination node D is feasible in a final solution, and whether the cost of the final solution is equal to or less than B.

Now, consider the following instance. Assume that we have an energy cost graph $G = (V, E)$ with weights $d_{i,j}{}^{\lambda}$, where V is the set of nodes, E is the set of links, and $d_{i,j}{}^{\lambda}$ is the weight on the edge $< i, j > \in E$. For a source-destination pair $S, D \in V$ and a constant $B \in R_{+}$, let us consider the instances of MEiM in which the destination nodes belong to $V' = V - \{S, D\}$. According to Lemma 1, the $T(S, i)$ series, which stands for the power of the minimum energy i-node multicast tree with source node S and destination nodes belong to V', is monotonically increasing. According to Lemma 2, the $C(i, D)$ series, which denotes the cooperative transmission cost from all the nodes in the minimum energy i-node multicast to the destination node D, is monotonically decreasing. The cost of the corresponding $S - D$ path, which satisfies the assumptions of S-MECP, is $P(S, i, D) = T(S, i) + C(i, D)$. Notably, the series of $P(S, i, D)$s may neither monotonically increase nor monotonically decrease. By evaluating the instances of $P(S, 0, D)$, $P(S, 1, D), \cdots$, $P(S, N-1, D)$, we can solve the instance of S-MECP. Clearly, this instance of S-MECP can be constructed in a polynomial time of $O(N)$ from MEiM instances. This completes the proof that S-MECP is NP-complete.

Since S-MECP is a special case of MECP problem, and MECP belongs to the NP class, which can be shown along the similar lines as for the S-MECP problem, MECP problem is NP-complete, too. □

VI. COOPERATIVE SHORTEST PATH ALGORITHM

In this section, we present a CSP algorithm that uses Dijkstra's algorithm as the basic building block and reflects the cooperative transmission properties in the relaxation procedure. The CSP algorithm takes as input an energy cost graph $G = (V, E)$ with weights $d_{i,j}{}^{\lambda}$ and a source-destination pair $S, D \in V$. We assume that the last L nodes along the path are allowed for cooperative transmission to the next hop. The CSP algorithm uses the basic structure of Dijkstra's algorithm and uses a modified relaxation procedure to reflect the cooperative transmission cost along the path. The presented approach (CSP algorithm) differs from those in [5] in the sense that we directly change the relaxation procedure of the Dijkstra's algorithm to adopt the cooperative transmission cost instead of calculating the non-cooperative shortest path first. All those heuristics presented in [5] have the common thread to calculate the non-cooperative shortest path first, either statically like the CAN algorithm (cooperation along the non-cooperative shortest path) or dynamically like the progressive cooperation heuristic (PC) algorithm. We refer interested readers to [5] for details. As shown in the next section, the CSP algorithm outperforms the existing algorithms with respect to both energy efficiency and fairness.

The new relaxation procedure for CSP is described in Fig. 3 and the rest of the CSP algorithm has the same structure as that of Dijkstra's algorithm. Notably, the algorithm maintains two labels for each node: $d[u]$ to represent the estimated total cost of the cooperative shortest path from the source node S to node u with respect to the cooperative transmission cost along the path and $\pi(u)$ to represent predecessors of node u along the cooperative shortest path. $\pi(u)$ only needs to keep as many as $L-1$ predecessors, e.g., the last $L-1$ predecessors along the cooper-

Relax (u,v)
if $d[v] > d[u] + coop(u,v)$ then
  $d[v] = d[u] + coop(u,v)$;
  set node u as node v's predecessor;
endif

$coop(u,v)$
// calculate the cooperative transmission cost from node u
// and its predecessors to node v
Assume $\text{Path}_u^* = \{S, t_1, \cdots, t_k, u\}$
if $(k+2) \le L$
  $\text{cost} = \dfrac{1}{\dfrac{1}{d_{S,v}^{\lambda}} + \sum_{i=1}^{k} \dfrac{1}{d_{t_i,v}^{\lambda}} + \dfrac{1}{d_{u,v}^{\lambda}}}$
elseif $(k+1) = L$
  $\text{cost} = \dfrac{1}{\sum_{i=1}^{k} \dfrac{1}{d_{t_i,v}^{\lambda}} + \dfrac{1}{d_{u,v}^{\lambda}}}$
  elseif $(k+1) > L$
    $\text{cost} = \dfrac{1}{\sum_{i=k-L+2}^{k} \dfrac{1}{d_{t_i,v}^{\lambda}} + \dfrac{1}{d_{u,v}^{\lambda}}}$
    endif
  endif
endif
return cost

Fig. 3. New relaxation procedure for CSP algorithm.

ative shortest path, which allows as many as L nodes (including node u) for cooperative transmission to another non-included node.

As described in Fig. 3, we define $coop(u, v)$ as the cooperative transmission cost from node u and its predecessors (at most $L-1$) to node v. Hence, the formulation of the problem depicted in (5) can be rewritten as

$$\min(coop(S,t_1) + \sum_{i=1}^{k-1} coop(t_i, t_{i+1}) + coop(t_k, D)) \quad (8)$$

where $\text{Path} = S \to t_1 \to t_2 \to \cdots \to t_k \to D$.

We omit the description of the rest of the CSP algorithm as it has the same structure as that of Dijkstra's algorithm, which can be found in virtually every algorithm book, e.g., the one by Cormen *et al.* [14].

The complexity of the presented cooperative shortest algorithm for cooperative routing is in the order of $O(N^2)$, where N is the number of nodes in the network. The CAN algorithm and the PC algorithm [5] have the computational complexities of $O(N^2)$ and $O(N^3)$, respectively. As verified by the experimental results in the following section, both of them have poor performance compared to the CSP algorithm.

A. *CSP with Additional Constraints*

Multi-hop cooperative transmission significantly reduces the total transmit energy along the path. However, delay may increase considerably when there are a large number of intermediate nodes along the path, each of which adds additional transmission delay, queuing delay, and processing delay. Hence, in addition to our first objective to minimize the total required energy of the path, we may need to add additional constraints to limit the number of hops.

Let H denote the number of hop counts permitted along a path. We want to find a path such that the total required energy for cooperative transmission along the path is the least and the number of hops of the path is equal to or less than H. We slightly modify the CSP algorithm to handle the constraint.

Notably, the algorithm maintains three labels for each node: $d[u]$ representing the estimated total cost of the cooperative shortest path from the source node S to node u with respect to the cooperative transmission cost along the path, $h[u]$ representing the number of hops of the current path from the source node S to node u, and $\pi(u)$ recording predecessors of node u along the cooperative shortest path. $\pi(u)$ only needs to keep as many as $L-1$ predecessors, which allow as many as L nodes including node u for cooperative transmission to the next hop.

The rest of the algorithm has the same structure as that of Dijkstra's algorithm. Hence, the computation complexity in the worst case is $O(N^2)$, which is the same as that of Dijkstra's algorithm.

VII. PERFORMANCE EVALUATION

In this section, we evaluate the performance of the CSP algorithm compared with the CAN algorithm and the uncooperative shortest path (USP) algorithm on three main aspects: (a) Mean normalized path power, (b) fairness, and (c) total consumed power of each node, over a substantially large number of random source-destination pairs.

Following [2] and [5], we simulate networks of a varying number of nodes, N, placed randomly within a 10×10 plane, in a variety of circumstances, e.g., with the power attenuation factor, $\lambda = 2, 3, 4$, and $L = 2, 3, 4$. We use $P_{\max} = 2 \times 10^{\lambda}$ for each node and this allows every node being able to reach every other node in one hop so long as it transmits at a sufficiently high power level. As discussed before, each node is able to adjust its transmitting power in the range of $[0, P_{\max}]$ to add or remove links in the energy cost graph.

For the calculation of mean normalized path power, the results are averaged over 100 randomly-chosen source-destination pairs in randomly-generated networks in a variety of circumstances by varying N, λ, and L. For the performance comparison between the algorithms, we consider normalized path power. For example, suppose we have three approaches to generate cooperative routing paths, say approaches CAN, CSP, and USP, where USP stands for uncooperative shortest path algorithm. Let P_{CAN}, P_{CSP}, and P_{USP} stand for the required path power for the paths generated by approaches CAN, CSP, and USP, respectively, for the same source-destination pair in the same network topology. The normalized path power for each of these approaches is given by the following:

$$P'_{\text{CAN}} = \frac{P_{\text{CAN}}}{\min(P_{\text{CAN}}, P_{\text{CSP}}, P_{\text{USP}})}. \quad (9)$$

Similarly, we can define P'_{CSP} and P'_{USP}.

Let the R_i stand for the ratio of the number of transmission sessions, in which node i is either the source or the destination node, over the total number of transmission sessions that node i participates. We call R_i as node i's utility function. Clearly, a higher R_i value indicates more benefits node i can get from the cooperation with other nodes. Let STD be the standard deviation of R_is among all the nodes in the network, we have

$$\text{STD} = \sqrt{\frac{\sum_{i=1}^{N} (R_i - \sum_{j=1}^{N} \frac{R_i}{N})^2}{N}} \quad (10)$$

where N is the total number of nodes in the network.

The standard deviation of R_is describes how spread-out the values of R_is are. If the data all lies close to the mean, then the standard deviation will be small, while if the data spread out over a large range of values, STD will be large. As we consider R_i as node is utility function, clearly a smaller STD value shows better fairness among different nodes.

We begin with the evaluation of the mean normalized path power by CSP, CAN, and USP. As shown in Figs. 4 and 5, we first observe that CSP consistently outperforms CAN and USP in all circumstances. With more nodes added in the network, the gap between CSP and uncooperative shortest path (USP) approach slightly widens, ranging from 30~50% power savings. CSP outperforms CAN by a margin around 10% for the same settings. We also observe that as we allow more nodes along the path for cooperative transmission to the next hop, i.e., a larger value of L, both CAN and CSP achieve more power savings compared with the non-cooperative shortest path approach. This is due to the fact that a larger value of L offers more cooperative transmission opportunities, resulting in more power savings. Another observation is that the gap between CSP and CAN widens with a larger value of power attenuation factor (λ). This demonstrates that the energy saving of the CSP algorithm is higher than that of the CAN algorithm in a deep power attenuation environment.

We next explore the performance of CSP, CAN, and USP with respect to fairness defined in (10). As shown in Figs. 6 and 7, for $100,000$ random S-D pairs by CSP, CAN, and USP, an interesting observation is that as more nodes added in the network, the CSP algorithm achieves better fairness among nodes in terms of the cooperative transmission participation. In particular, when the number of nodes in the network exceeds 60, CSP outperforms CAN and USP in all circumstances.

Another observation is that CAN performs slightly worse than USP with respect to fairness. This is due to the fact that CAN uses the same path found by USP for cooperative transmission but it gives biased treatment for the source and destination. Notably, both source and destination nodes have less cooperative transmission participation opportunity than that of those in the middle. In particular, for the destination node, it has no participation to the cooperative transmission at all. Another general intuitive perception with respect to fairness is that nodes in the middle of a network always have more chances to relay packets for other nodes than those at the edge of a network do. Hence, it may be impossible for the STD to approach zero.

Lastly, we explore the total required power for each node over $100,000$ randomly-chosen source-destination pairs by CSP, CAN, and USP in a variety of circumstances, where $N = 100$. We first observe that some nodes may consume less power under one approach while some other nodes may consume less

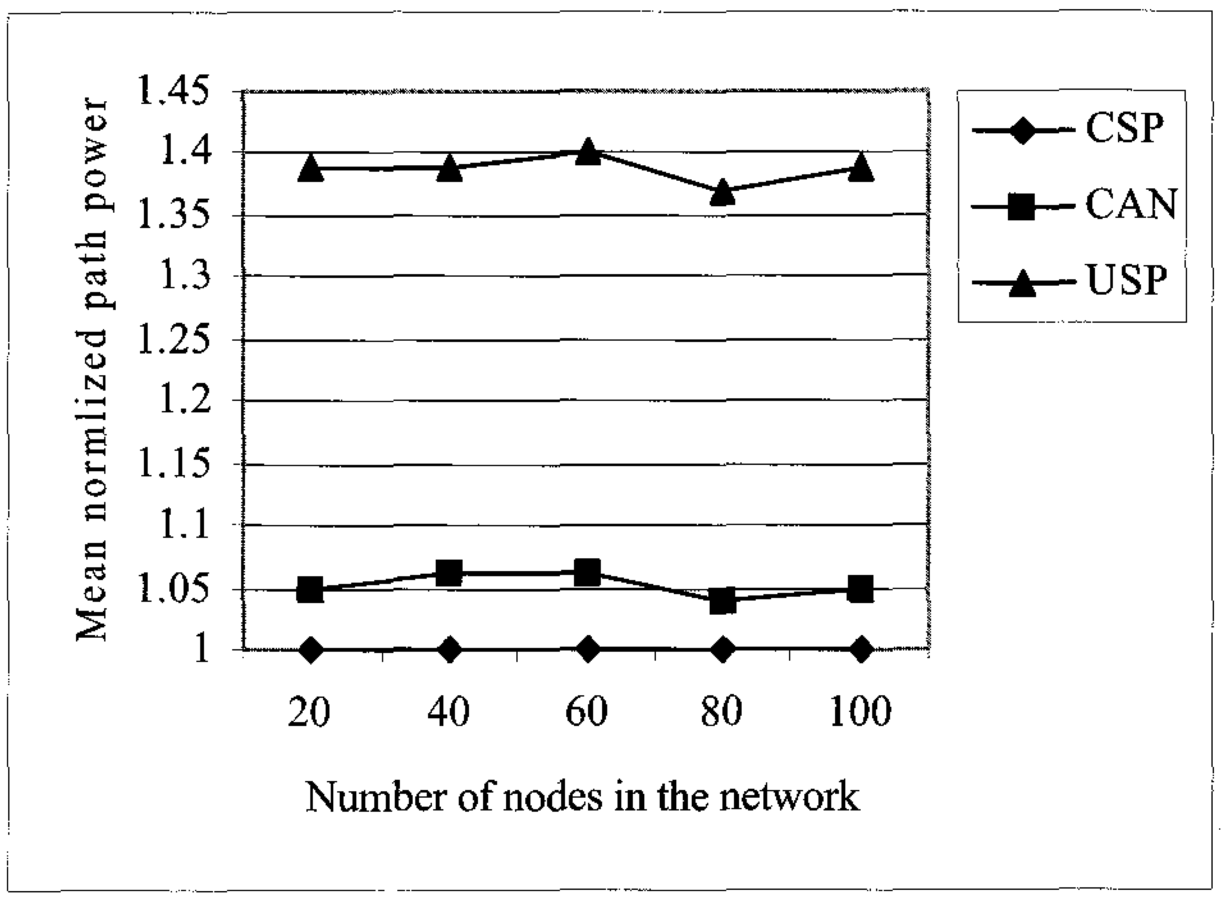

Fig. 4. Mean normalized path power with $\lambda = 2$ and $L = 2$.

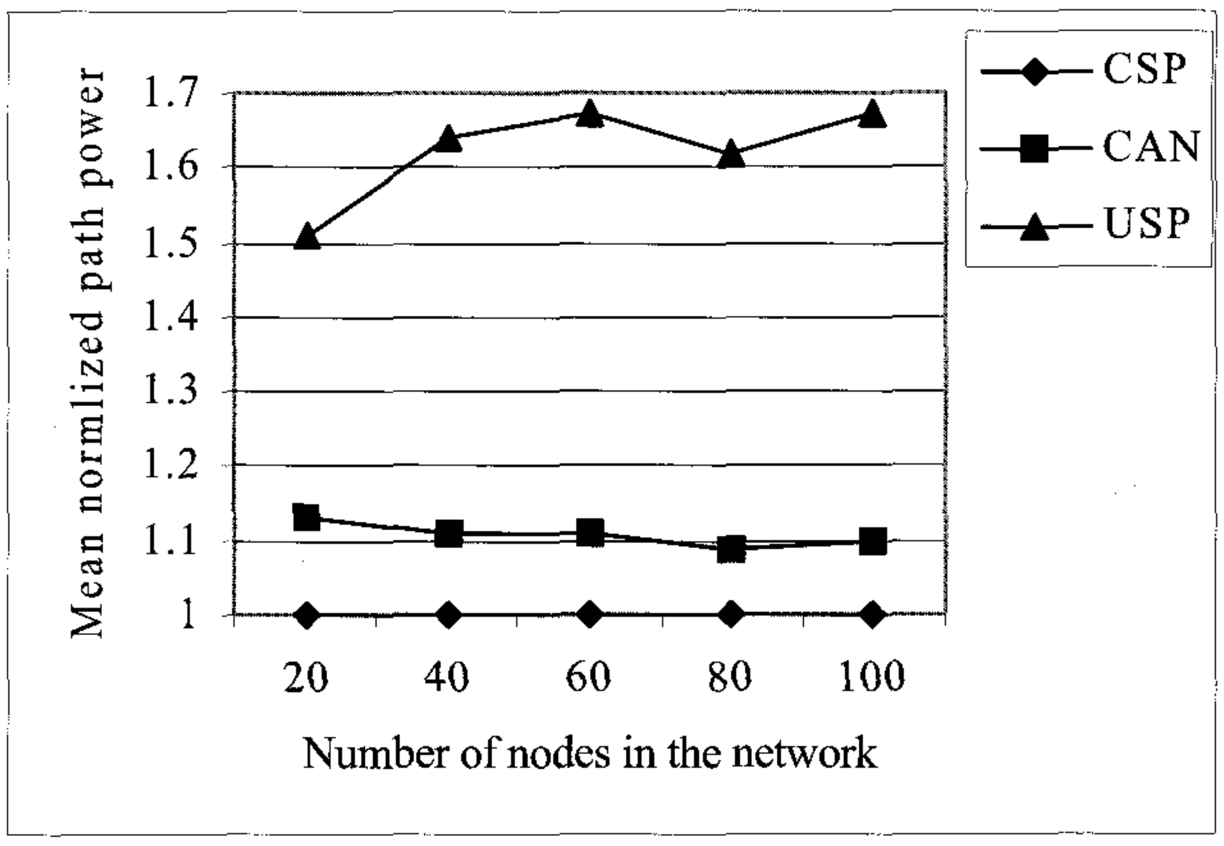

Fig. 5. Mean normalized path power with $\lambda = 2$ and $L = 4$.

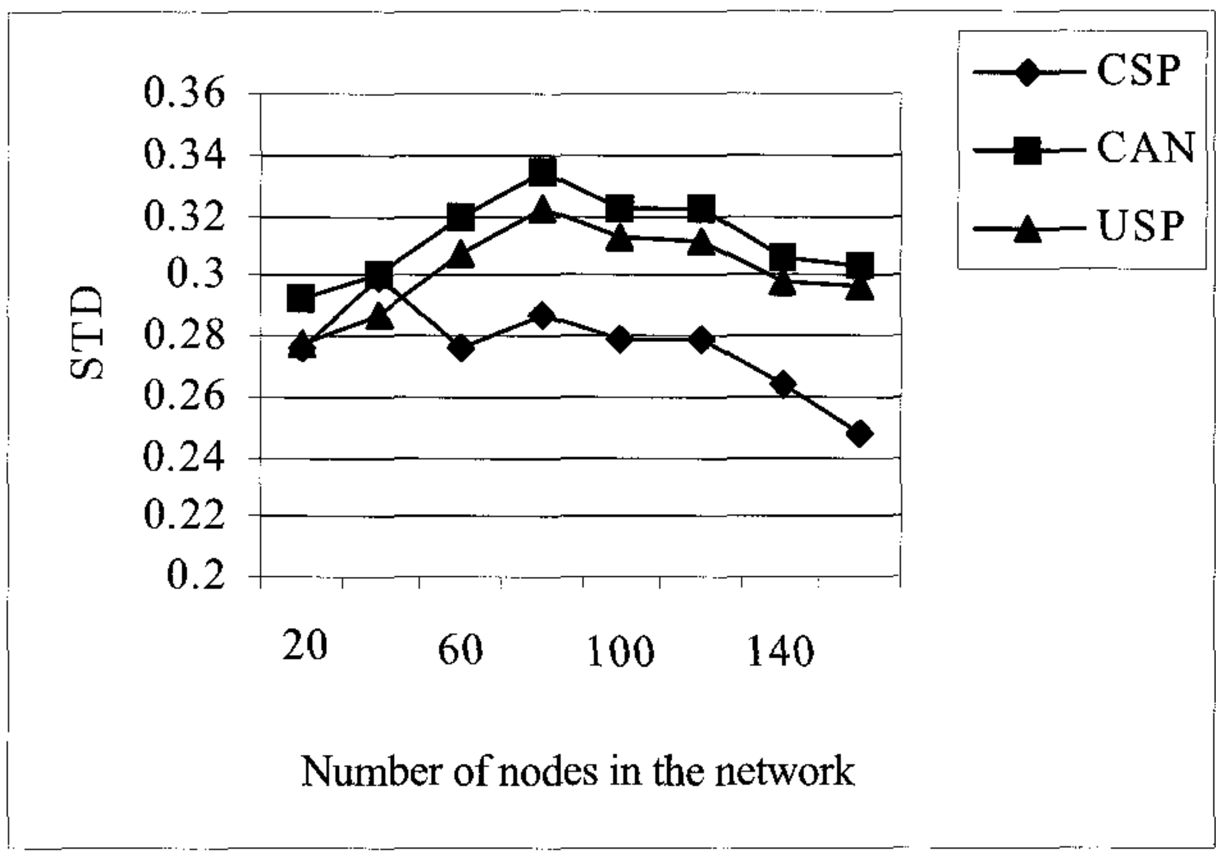

Fig. 6. STD among each node's utility function with $\lambda = 2$ and $L = 2$.

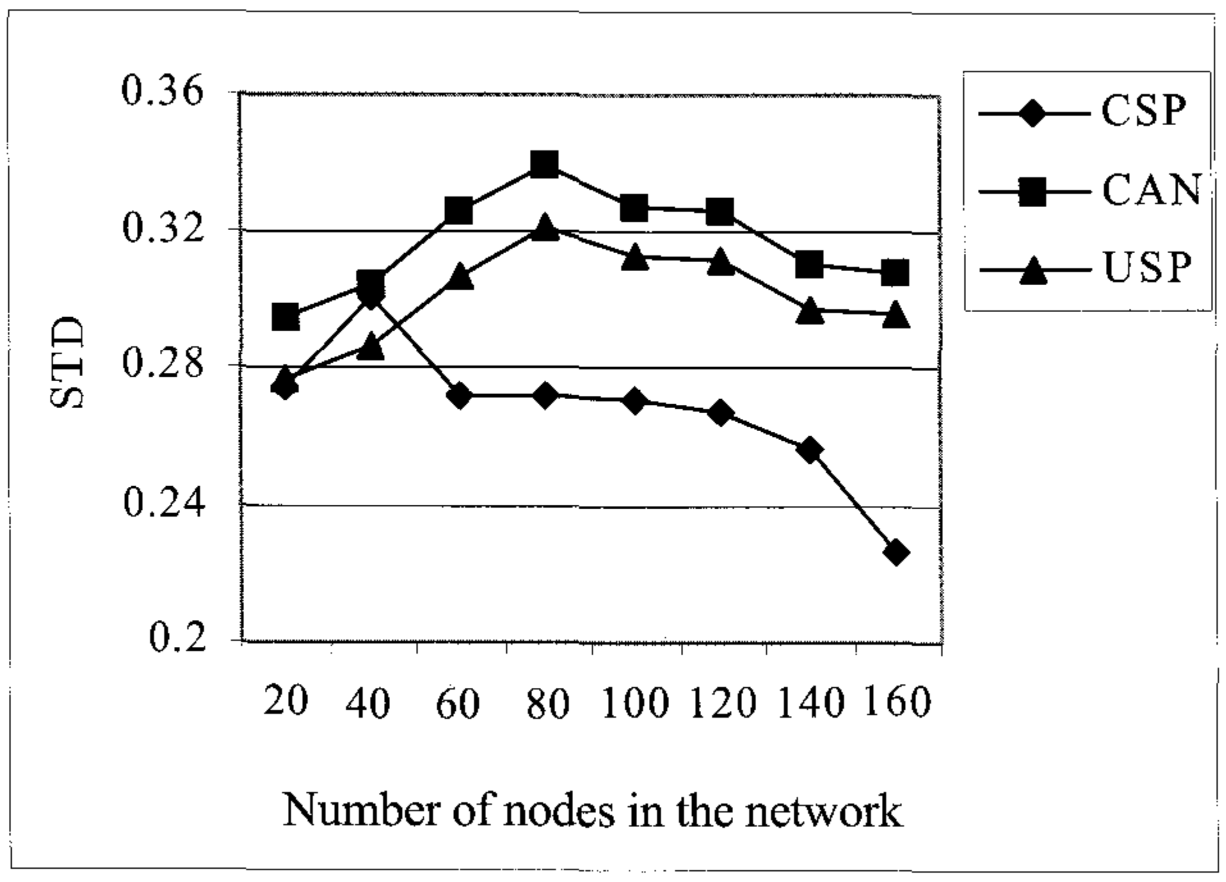

Fig. 7. STD among each node's utility function with $\lambda = 2$ and $L = 4$.

power under other approaches. Mostly, the non-cooperative approach consumes much more power in most cases and the CSP scheme requires the least power among CSP, CAN, and USP. In our empirical studies, the ratios among total required power of all nodes by CSP, CAN, and USP are $1.0 : 1.08 : 1.48$ with $L = 3$ and $\lambda = 3$, while the ratios among total required power of all nodes by CSP, CAN, and USP are $1.0 : 1.10 : 1.55$ with $L = 4$ and $\lambda = 3$ as a larger value of L offers more cooperative transmission opportunities.

VIII. DISTRIBUTION AND IMPLEMENTATION ISSUES

In this section, we give a brief discussion on the distribution and implementation issues of the presented algorithm. In the situations where the global information about the network topology is not immediately available to all the nodes in the network and/or where the network topology may be changing frequently, a distributed implementation of the algorithm is much more desirable. Fortunately, the CSP algorithm presented in this paper lends itself to such a distributed implementation as there are already efficient implementations of the traditional shortest path algorithms, e.g., the distributed Bellman-Ford algorithm. The additional difficulty is that each node needs to maintain as many as $L-1$ predecessors along the current best path from the source node to reflect the cooperative transmission cost in the new relaxation procedure. Another issue is to deal with the situation in which links or nodes fail or nodes move as the algorithm is running. A distributed asynchronous shortest path algorithm can deal with those difficulties more efficiently.

In the following, we give a brief description of the distributed version of the CSP algorithm. Let d_i be the cooperative shortest distance from node i to the destination node D. The update equation is given by

$$d_i = \min(coop(i,j) + d_j) \tag{11}$$

where $1 \leq j \leq N$, $coop(i,j)$ stands for the cooperative transmission cost from node i to node j (see detailed description in Fig. 3). Each node i regularly updates the values of d_i using the update (11) and each node maintains the values of $coop(i,j)$ to its neighbors as well as the values of d_j received from its neighbors. If no changes occur in the network, the algorithm will converge to cooperative shortest paths in no more than N steps, where N denotes the number of nodes in the network.

Even with the advent of commercial tuner receiver for phase coherence or the use of RAKE receivers, the coordination of transmissions from multiple transmitters to one receiver simultaneously to explore transmit diversity in a large wireless network is still a challenge. Collaborative media access control (MAC) protocols and adaptive scheduling algorithms have to be developed for the realization of cooperative routing in wireless networks. We leave this as one of our future research directions.

IX. CONCLUSION AND FUTURE DIRECTIONS

In this paper, we study MECP routing in all-wireless networks. The cooperative routing scheme combines network layer route selection, and the physical layer transmit diversity via cooperative transmission. Such a cross-layer design approach may be beneficial for wireless networks to minimize the inherent impairments of wireless channels such as interference, multi-path fading, attenuation, etc.

One major contribution of this paper is the proof of the NP-completeness of the MECP problem in all-wireless networks. To the best of our knowledge, it is the first time that the NP-completeness proof of the MECP problem has been presented. We also present a CSP algorithm to approximate the MECP in all-wireless networks. The extensive experimental results show that our presented approach consistently outperforms other existing schemes. We also find that as more nodes added in the network, the gap between the presented approach and non-cooperative shortest path algorithm widens and the fairness among nodes with respect to the transmission cooperation participation is greatly improved. All these findings indicate that the presented approach tends to make the network more scalable and more efficient with respect to both the energy conservation and fairness.

Finally, the algorithm presented in this paper can be equally applicable to other cooperative routing environments, e.g., other fading or attenuation models, etc. We will further explore efficient ways for the distributed implementation of the CSP algorithm as well as collaborative MAC protocols and adaptive scheduling algorithms as our future directions.

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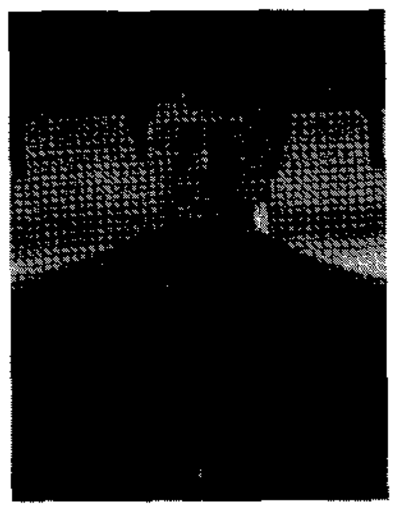

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