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Permanence of a Three-species Food Chain System with Impulsive Perturbations

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Abstract. We investigate a three-species food chain system with Lotka-Volterra functional response and impulsive perturbations. In [23], Zhang and Chen have studied the system. They have given conditions for extinction of lowest-level prey and top predator and considered the local stability of lower-level prey and top predator eradication periodic solution. However, they did not give a condition for permanence, which is one of important facts in population dynamics. In this paper, we establish the condition for permanence of the three-species food chain system with impulsive perturbations. In addition, we give some numerical examples.

1. Introduction

The mathematical study of predator-prey system in population dynamics has a long history starting with the work of Lotka and Volterra. The principles of Lotka-Volterra models, conservation of mass and decomposition of the rates of change in birth and death processes, have remained valid until today and many theoretical ecologists adhere to there principles. For the reason, we need to consider a Lotka-Volterra type food chain model, which can be described by the following differential equations:

$$\begin{cases} x'(t) = x(t)(a - bx(t) - cy(t)), \\ y'(t) = y(t)(-d_1 + c_1x(t) - e_1z(t)), \\ z'(t) = z(t)(-d_2 + e_2y(t)), \end{cases} \tag{1.1}$$

where $x(t), y(t), z(t)$ are the densities of the lowest-level prey, mid-level predator and top predator at time t, respectively, $a, b, c, d_1, c_1, e_1, d_2, e_2$ are positive constants.

On the other hand, there are number of factors in the environment to be considered in population models. One of important factors is impulsive perturbation such as fire, flood, etc, that are not suitable to be considered continually. These impulsive perturbations bring sudden changes to the system. For example, consider the interaction between crops and locusts in a local region. Once a year or

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once several years, a large number of locusts may invade into the region and cause damage to the crops together with the local locusts. Another important example is a biological control for pests (prey($x(t)$)). It is defined as the reduction of pest population by natural enemies(mid-level predator($y(t)$)) and typically involves an active human role. The key to successful pest control is to identify the pest and its natural enemy and release the beneficial insect at fixed times for pest control. It is natural to assume that these perturbations act instantaneously, that is, in the form of impulse. Thus, in this paper, we consider the following Lotka-Volterra type food chain model with periodic constant impulsive immigration of the middle predator.

$$
\left\{
\begin{array}{l}
\left.
\begin{array}{l}
x'(t) = x(t)(a - bx(t) - cy(t)),\\
y'(t) = y(t)(-d_1 + c_1 x(t) - e_1 z(t)),\\
z'(t) = z(t)(-d_2 + e_2 y(t)),
\end{array}
\right\} \; t \neq nT,\\
\left.
\begin{array}{l}
x(t^+) = x(t),\\
y(t^+) = y(t) + p,\\
z(t^+) = z(t),
\end{array}
\right\} \; t = nT,\\
(x(0^+), y(0^+)) = (x_0, y_0) = \mathbf{x}_0,
\end{array}
\right.
\tag{1.2}
$$

where T is the period of the impulsive immigration or stock of the predator and p is the size of immigration or stock of the predator. Such model is an impulsive differential equation whose theory and applications were greatly developed by the efforts of Bainov and Lakshmikantham et al. [2], [10] and, moreover, the theory of impulsive differential equations is being recognized to be not only richer than the corresponding theory of differential equations without impulses, but also represents a more natural framework for mathematical modeling of real world phenomena.

In recent years, many authors have studied predator-prey models with impulsive perturbations [11], [12], [13], [17], [18], [19], [25], [27]. Moreover, food chain models with sudden perturbations have been intensively researched, such as Holling-type [23], [24], Bedington-type [20], [21], [28], and Ivlev-type [22]. Especially, Zhang and Chen [23] have studied the model (1.2). They have gave conditions for extinction of lowest-level prey and top predator, considered the local stability of lower-level prey and top predator eradication periodic solution. However, they did not give a condition for permanence, which is one of important facts in population dynamics. The main purpose of this paper is to investigate the permanence of (1.2).

The organization of the paper is as follows. In the next section, we introduce some notations which are used in this paper. In section 3, we give a sufficient condition for the permanence of the system (1.2) by applying the comparison theorem. In section 4 we give some numerical examples. Finally, we have a conclusion in section 4.

2. Preliminaries

The three species food chain model (1.1) has four non-negative equilibrium:

(1) The system (1.1) has positive equilibrium: $E^* = (x^*, y^*, z^*)$ if and only if $ae_2c_1 - d_2cc_1 - d_1be_2 > 0$, where

$$x^* = \frac{ae_2 - d_2c}{be_2}, \quad y^* = \frac{d_2}{e_2}, \quad z^* = \frac{ae_2c_1 - d_2cc_1 - d_1be_2}{be_1e_2}.$$

(2) The system (1.1) has three equilibrium :

$$A(0,0,0), \quad B(\frac{d_1}{b},0,0), \quad C(\frac{d_1}{c_1}, \frac{ac_1 - d_1b}{cc_1}, 0), \quad (ac_1 - d_1b > 0).$$

The stability of equilibrium of the system (1.1) can be established as follows:

Lemma 2.1([23]).

(1) *If positive equilibrium E^* exists, then E^* is globally stable.*

(2) *If positive equilibrium E^* dose not exist and C exists, then C is globally stable.*

(3) *If positive equilibrium E^* and C do not exist, then B is globally stable.*

Now, we shall introduce a few notations and definitions together with a few auxiliary results relating to comparison theorem, which will be useful for our main results.

Let $\mathbb{R}_+ = [0, \infty)$ and $\mathbb{R}^3_+ = \{\mathbf{x} = (x(t), y(t), z(t)) \in \mathbb{R}^3 : x(t),\ y(t),\ z(t) \geq 0\}$. Denote $\mathbb{N}$ the set of all of nonnegative integers and $f = (f_1, f_2, f_3)^T$ the right hand of the first three equations in (1.2). Let $V : \mathbb{R}_+ \times \mathbb{R}^3_+ \to \mathbb{R}_+$, then V is said to be in a class V_0 if

(1) V is continuous on $(nT, (n+1)T] \times \mathbb{R}^3_+$, and $\lim_{\substack{(t,\mathbf{y}) \to (nT,\mathbf{x}) \\ t > nT}} V(t, \mathbf{x}) = V(nT^+, \mathbf{x})$ exists.

(2) V is a locally Lipschitzian in $\mathbf{x}$.

Definition 2.2. For $V \in V_0$, we define the upper right Dini derivative of V with respect to the impulsive differential system (1.2) at $(t, \mathbf{x}) \in (nT, (n+1)T] \times \mathbb{R}^3_+$ by

$$D^+V(t, \mathbf{x}) = \limsup_{h \to 0+} \frac{1}{h}[V(t + h, \mathbf{x} + hf(t, \mathbf{x})) - V(t, \mathbf{x})].$$

Remark 2.3.

(1) The solution of the system (1.2) is a piecewise continuous function $\mathbf{x} : \mathbb{R}_+ \to \mathbb{R}^3_+$, $\mathbf{x}(t)$ is continuous on $(nT, (n+1)T]$, $n \in \mathbb{N}$ and $\mathbf{x}(nT^+) = \lim_{t \to nT^+} \mathbf{x}(t)$ exists.

(2) The smoothness properties of f guarantee the global existence and uniqueness of solution of the system (1.2)(See [10] for the details).

We will use an important comparison theorem on an impulsive differential equation [10]. We suppose that $g : \mathbb{R}_+ \times \mathbb{R}_+ \to \mathbb{R}$ satisfies the following hypotheses.

(H) g is continuous on $(n\tau, (n+1)\tau] \times \mathbb{R}_+$ and the limit $\lim_{(t,y)\to(n\tau^+,x)} g(t,y) = g(n\tau^+, x)$ exists and is finite for $x \in \mathbb{R}_+$ and $n \in \mathbb{N}$.

Lemma 2.4([10]). *Suppose $V \in V_0$ and*

$$\begin{cases} D^+V(t,\mathbf{x}) \le g(t, V(t,\mathbf{x})), \ t \ne n\tau \\ V(t, \mathbf{x}(t^+)) \le \psi_n(V(t,\mathbf{x})), \ t = n\tau, \end{cases} \tag{2.1}$$

where $g : \mathbb{R}_+ \times \mathbb{R}_+ \to \mathbb{R}$ satisfies (H) and $\psi_n : \mathbb{R}_+ \to \mathbb{R}_+$ is non-decreasing for all $n \in \mathbb{N}$. Let $r(t)$ be the maximal solution for the impulsive Cauchy problem

$$\begin{cases} u'(t) = g(t, u(t)), \ t \ne n\tau, \\ u(t^+) = \psi_n(u(t)), \ t = n\tau, \\ u(0^+) = u_0, \end{cases} \tag{2.2}$$

defined on $[0,\infty)$. Then $V(0^+, \mathbf{x}_0) \le u_0$ implies that $V(t, \mathbf{x}(t)) \le r(t), t \ge 0$, where $\mathbf{x}(t)$ is any solution of (2.1).

Similar result can be obtained when all conditions of the inequalities in the Lemma 2.4 are reversed. Note that if we have some smoothness conditions of $g(t,x)$ to guarantee the existence and uniqueness of solutions for the Cauchy problem (2.2), then $r(t)$ is exactly the unique solution of (2.2).

The following lemma is obvious.

Lemma 2.5([23]). *Let $\mathbf{x}(t) = (x(t), y(t), z(t))$ be a solution of the system* (1.2). *Then we have*

(1) *If $\mathbf{x}(0^+) \ge 0$ then $\mathbf{x}x(t) \ge 0$ for all $t \ge 0$ and*

(2) *If $\mathbf{x}(0^+) > 0$ then $\mathbf{x}(t) > 0$ for all $t \ge 0$.*

Now, we give the basic properties of the following impulsive differential equation.

$$\begin{cases} y'(t) = -d_1 y(t), \ t \ne nT \\ y(t^+) = y(t) + p, \ t = nT, \\ y(0^+) = y_0. \end{cases} \tag{2.3}$$

Then we can easily obtain the following results.

Lemma 2.6.

(1) $y^*(t) = \dfrac{p\exp(-d_1(t-nT))}{1-\exp(-d_1T)}$, $t \in (nT,(n+1)T]$, $n \in \mathbb{N}$ *and* $y^*(0^+) = \dfrac{p}{1-\exp(-d_1T)}$ *is a positive periodic solution of* (2.3).

(2) $y(t) = \left(y(0^+) - \dfrac{p}{1-\exp(-d_1T)}\right)\exp(-d_1t) + y^*(t)$ *is the solution of* (2.3) *with* $y_0 \geq 0$, $t \in (nT,(n+1)T]$ *and* $n \in \mathbb{N}$.

(3) *All non-negative solutions* $y(t)$ *of* (2.3) *tend to* $y^*(t)$. *i.e.,* $|y(t)-y^*(t)| \to 0$ *as* $t \to \infty$.

It is from Lemma 2.6 that the general solution $y(t)$ of (2.3) can be synchronized with the positive periodic solution $y^*(t)$ of (2.3) for sufficiently large t and we can obtain the complete expression for the lowest-level prey and top predator free periodic solution of the system (1.2)

$$(0,y^*(t),0) = \left(0, \frac{q\exp(-d_1(t-nT))}{1-\exp(-d_1T)}, 0\right) \text{ for } t \in (nT,(n+1)T].$$

The stability of the periodic solution $(0,y^*(t),0)$ and the boundedness of the system (1.2) has been studied in [23]. Now we will mention their results as follows:

Theorem 2.7([23]). *Let* $(x(t),y(t),z(t))$ *be any solution of the system* (1.2). *Then* $(0,y^*(t),0)$ *is locally asymptotically stable provided* $\dfrac{ad_1T}{c} < p < \dfrac{d_1d_2T}{e_2}$.

Theorem 2.8([23]). *There exists a constant* $M > 0$ *such that* $x(t) \leq M$, $y(t) \leq M$ *and* $z(t) \leq M$ *for each solution* $(x(t),y(t),z(t))$ *of the system* (1.2) *with all* t *large enough.*

For the system (1.2), there exist the following two subsystems. If the top-predator is absent i.e., $z(t) = 0$, then the system (1.2) can be reduced

$$\text{(2.4)} \qquad \left\{\begin{array}{l} \left.\begin{array}{l} x'(t) = x(t)(a - bx(t) - cy(t)), \\ y'(t) = y(t)(-d_1 + c_1x(t)), \end{array}\right\} t \neq nT, \\ \left.\begin{array}{l} x(t^+) = x(t), \\ y(t^+) = y(t) + p. \end{array}\right\} t = nT, \end{array}\right.$$

If the prey is extinct, then the system (1.2) can be reduced

$$\text{(2.5)} \qquad \left\{\begin{array}{l} \left.\begin{array}{l} y'(t) = y(t)(-d_1 - e_1z(t)), \\ z'(t) = z(t)(-d_2 + e_2y(t)), \end{array}\right\} t \neq nT, \\ \left.\begin{array}{l} y(t^+) = y(t) + p, \\ z(t^+) = z(t). \end{array}\right\} t = nT, \end{array}\right.$$

Definition 2.9. The system (1.2) is permanent if there exist $M \geq m > 0$ such that, for any solution $(x(t), y(t), z(t))$ of the system (1.2) with $x(0^+), y(0^+) > 0$,

$m \leq \lim_{t\to\infty} \inf x(t) \leq \lim_{t\to\infty} \sup x(t) \leq M$, $m \leq \lim_{t\to\infty} \inf y(t) \leq \lim_{t\to\infty} \sup y(t) \leq M$ and $m \leq \lim_{t\to\infty} \inf z(t) \leq \lim_{t\to\infty} \sup z(t) \leq M$.

Especially, the reference [12] have given a condition for permanence of the subsystem (2.4).

Theorem 2.10([12]). *If* $p < \frac{ad_1T}{c}$*, then the subsystem* (2.4) *is permanent.*

3. Main results

Theorem 3.1. *If* $p > \frac{d_1d_2T}{e_2}$*, then the subsystem* (2.5) *is permanent.*

Proof. Let $(y(t), z(t))$ be a solution of the subsystem (2.5) with $y(0) > 0, z(0) > 0$. From Theorem 2.8, we may assume that $y(t) \leq M$ and $z(t) \leq \frac{M}{e_1}$. Then $y'(t) \geq -(d_1 + M)y(t)$. From Lemmas 2.4 and 2.6, we have $y(t) \geq u^*(t) - \epsilon$ for $\epsilon > 0$, where $u^*(t) = \frac{p\exp(-(d_1+M)(t-nT))}{1-\exp(-(d_1+M))}$ for $t \in (nT, (n+1)T]$. Thus, we obtain that $y(t) \geq \frac{p(\exp(-(d_1+M)T)}{1-\exp(-(d_1+M))} - \epsilon \equiv m_0$ for sufficiently large t. Therefore, we only need to find $m_2 > 0$ such that $z(t) \geq m_2$ for large enough t.

We will do this in the following two steps.

(Step1) Since $p > \frac{d_1d_2T}{e_2}$, we can choose $m_1 > 0$, $\epsilon_1 > 0$ small enough such that $R = \exp(\frac{e_2p - d_1d_2T - m_1e_1d_2T}{d_1 + e_1m_1} - e_2\epsilon_1) > 1$. In this step we will show that $z(t_1) \geq m_1$ for some $t_1 > 0$. Suppose not. i.e., there exists $t_1 > 0$ such that $z(t) < m_1$ for $t > 0$. Consider the following system.

$$
\begin{cases} \left.\begin{aligned} v'(t) &= -(d_1 + e_1m_1)v(t), \\ w'(t) &= (-d_2 + e_2v(t))w(t), \end{aligned}\right\} & t \neq nT, \\ \left.\begin{aligned} v(t^+) &= v(t) + p, \\ w(t+) &= w(t). \end{aligned}\right\} & t = nT, \end{cases} \tag{3.1}
$$

Then, by Lemmas 2.4 and 2.6, we can obtain $z(t) \geq w(t)$ and a periodic solution $v^*(t)$ of the subsystem (3.1), where

$$v^*(t) = \frac{p\exp(-(d_1+e_1m_1)(t-nT))}{1-\exp(-(d_1+e_1m_1)T)}, \quad t \in (nT, (n+1)T], \quad n \in \mathbb{N}.$$

Since $|v(t) - v^*(t)| \to 0$ as $t \to \infty$, we have $v(t) \geq v^*(t) - \epsilon_1$. Thus

$$w'(t) \geq (-d_2 + e_2(v^*(t) - \epsilon_1))w(t). \tag{3.2}$$

Integrating (3.2) on $(nT, (n+1)T]$, we get

$$w((n+1)T) \geq w(nT^+) \exp\Big(\int_{nT}^{(n+1)T} -d_2 + e_2(v^*(t) - \epsilon_1)dt\Big) = w(nT)R$$

Therefore $z((n+k)T) \geq w((n+k)T) \geq w(nT)R^k \to \infty$ as $k \to \infty$ which is a contradiction to the boundedness of $z(t)$.

(Step 2) Without loss of generality, we may let $z(t_1) = m_1$. If $z(t) \geq m_1$ for all $t > t_1$, then the subsystem (2.5) is permanent. If not, we may let $t_2 = \inf\limits_{t>t_1}\{z(t) < m_1\}$. Then $z(t) \geq m_1$ for $t_1 \leq t \leq t_2$ and, by continuity of $z(t)$, we have $z(t_2) = m_1$ and $t_1 < t_2$. There exist a $t'(> t_2)$ such that $z(t') \geq m_1$ by step 1. Set $t_3 = \inf_{t>t_2}\{z(t) \geq m_1\}$. Then $z(t) < m_1$ for $t_2 < t < t_3$ and $z(t_3) = m_1$. We can continue this process by using step 1. If the process is stopped in finite times, we complete the proof. Otherwise, there exists an interval's sequence $[t_{2k}, t_{2k+1}], k \in \mathbb{N}$, which has the following property : $z(t) < m_1, t \in (t_{2k}, t_{2k+1})$, $t_{2k-1} < t_{2k} \leq t_{2k+1}$ and $z(t_n) = m_1$, where $k, n \in \mathbb{N}$. Let $T_0 = \sup\{t_{2k+1} - t_{2k} | k \in \mathbb{N}\}$. If $T_0 = \infty$, then we can take a subsequence $\{t_{2k_i}\}$ satisfying $t_{2k_i+1} - t_{2k_i} \to \infty$ as $k_i \to \infty$. As in the proof of the first step, this will lead to a contradiction to the boundedness of $z(t)$. Then we obtain $T_0 < \infty$. Note that

$$\begin{aligned} z(t) &= z(t_{2k}) \exp\Big(\int_{t_{2k}}^{t} -d_2 + e_2(v^*(s) - \epsilon_1)ds\Big) \\ &\geq m_1 \exp(-d_2 T_0) \equiv m_2, t \in (t_{2k}, \quad t_{2k+1}], \quad k \in \mathbb{N}. \end{aligned}$$

Thus we obtain that $\liminf\limits_{t\to\infty} z(t) \geq m_2$. Therefore we complete the proof. □

Theorem 3.2. *If* $\dfrac{d_1 d_2 T}{e_2} < p < \dfrac{a d_1 T}{c}$, *then the system* (1.2) *is permanent.*

Proof. Consider two subsystem of the system (1.2) as follows:

$$\left\{ \begin{array}{l} \left.\begin{array}{l} x_1'(t) = x_1(t)(a - bx_1(t) - cy_1(t)), \\ y_1'(t) = y_1(t)(-d_1 + c_1 x_1(t)), \end{array}\right\} t \neq nT, \\ \left.\begin{array}{l} x_1(t^+) = x_1(t), \\ y_1(t^+) = y_1(t) + p. \end{array}\right\} t = nT, \end{array} \right. \tag{3.3}$$

and

$$(3.4)\qquad \begin{cases} \left.\begin{aligned} y_2'(t) &= y_2(t)(-d_1 - e_1 z_2(t)), \\ z_2'(t) &= z_2(t)(-d_2 + e_2 y_2(t)), \end{aligned}\right\} & t \neq nT, \\ \left.\begin{aligned} y_2(t^+) &= y_2(t) + p, \\ z_2(t^+) &= z_2(t). \end{aligned}\right\} & t = nT, \end{cases}$$

It follows from Lemma 2.4 that $y_1(t) \geq y(t)$, $y_2(t) \leq y(t)$, $x_1(t) \leq x(t)$, and $z_2(t) \leq z(t)$. If $p < \dfrac{ad_1T}{c}$, by Theorem 2.10 , the subsystem (3.3) is permanence. Thus we can take $T_1 > 0$ and $m_1 > 0$ such that $x(t) \geq m_1$ for $t \geq T_1$. Further, if $\dfrac{d_1d_2T}{e_2} < p$, by Theorem 3.1, the subsystem (3.4) is also permanent. Therefore, there exists $T_2 > 0$ and $m_2, m_3 > 0$ such that $y(t) \geq m_2$ and $z(t) \geq m_3$ for $t \geq T_2$. The proof is complete. $\square$

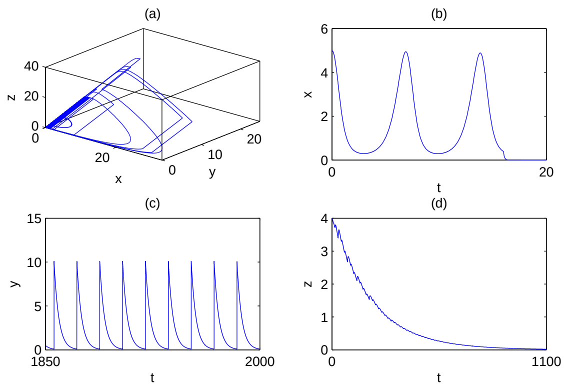

Figure 1: Phase portrait of the system (1.2) with $a = 2$, $b = 0.0002$, $c = 1$, $d_1 = 0.3$, $c_1 = 0.3$, $e_1 = 0.05$, $d_2 = 0.01$, $e_2 = 0.0025$ and an initial point $(x(0), y(0), z(0)) = (5, 2, 4)$ when $p = 10$. (a)-(c) time series of x, y and z.

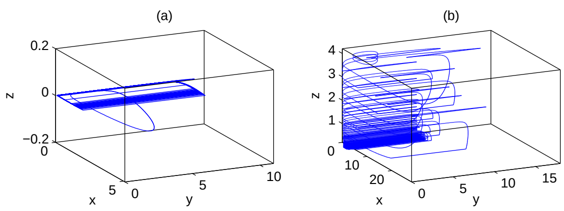

Figure 2: Phase portrait of the system (1.2) with $a = 2, b = 0.0002, c = 1, d_1 = 0.3, c_1 = 0.3, e_1 = 0.05, d_2 = 0.01, e_2 = 0.0025$ when $p = 9$. (a) $(x(0), y(0), z(0)) = (5, 2, 0)$, (b) $(x(0), y(0), z(0)) = (5, 2, 4)$.

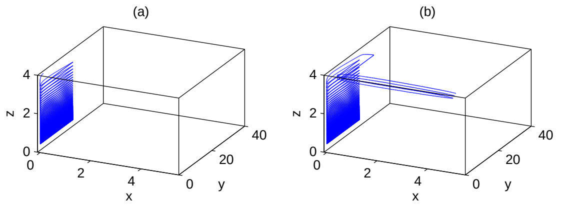

Figure 3: Phase portrait of the system (1.2) with $a = 2, b = 0.0002, c = 1, d_1 = 0.3, c_1 = 0.3, e_1 = 0.05, d_2 = 0.01, e_2 = 0.0025$ when $p = 20$. (a) $(x(0), y(0), z(0)) = (0, 2, 4)$, (b) $(x(0), y(0), z(0)) = (5, 2, 4)$.

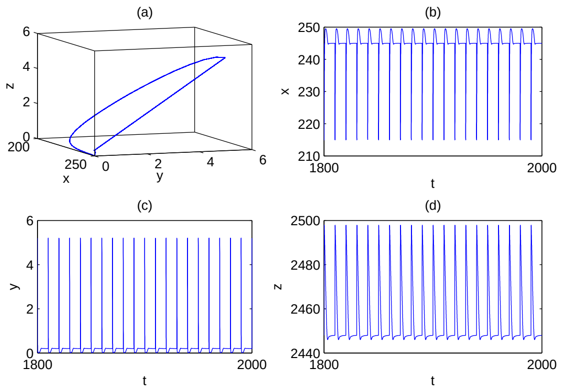

Figure 4: Phase portrait of the system (1.2) with $a = 5.0, b = 0.02, c = 0.5, c_1 = 0.5, d_1 = 0.1, d_2 = 0.01, e_1 = 0.05, e_2 = 0.05$ and $T = 10$. and an initial point $(x(0), y(0), z(0)) = (5, 2, 4)$ when $p = 7$.

4. Numerical examples

In this section we consider the following three cases.

(1) $a = 2$, $b = 0.0002$, $c = 1$, $d_1 = 0.3$, $c_1 = 0.3$, $e_1 = 0.05$, $d_2 = 0.01$, $e_2 = 0.0025$ and $T = 16$.

(2) $a = 5.0$, $b = 0.02$, $c = 0.5$, $c_1 = 0.5$, $d_1 = 0.1$, $d_2 = 0.01$, $e_1 = 0.05$, $e_2 = 0.05$ and $T = 10$.

(3) $a = 4.0$, $b = 0.01$, $c = 1.0$, $c_1 = 0.7$, $d_1 = 0.3$, $d_2 = 0.4$, $e_1 = 1.0$, $e_2 = 0.5$ and $T = 5$.

For (1), by Lemma 2.1, we know the system (1.1) does not have positive equilibrium, but exists a globally stable equilibrium point $C = (1, 1.9980, 0)$. From Theorem 2.7, we know that the lowest-level prey and top predator free periodic solution $(0, y^*(t), 0)$ of the system (1.2) is locally stable if $9.6 < p < 19.2$.(see Figure 1). When $p < 9.6$ and $z(0) = 0$, we know from Theorem 2.10 that the prey and mid-predator can coexist.(see Figure 2(a)). Actually, Figure 2(b) illustrates the three species can also coexist. i.e. the system may be permanent even if $p < 9.6$. On the contrary, when $p > 19.2$ and $x(0) = 0$, we know from Theorem 3.1 that the mid-predator and top predator can coexist.(see Figure 3(a)). But, Figure 3(b) shows that the system (1.2) may not be permanent even if we take $x(0) > 0$. For

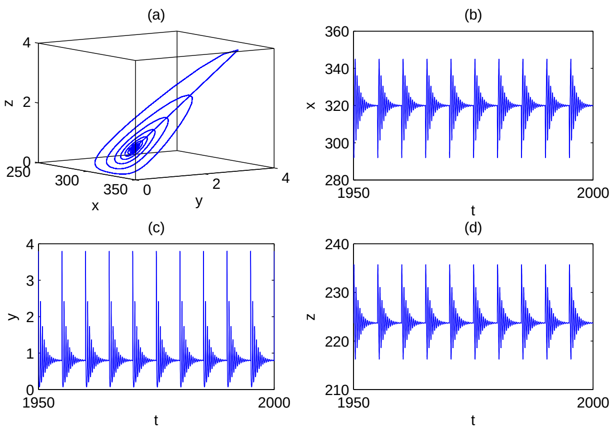

Figure 5: Phase portrait of the system (1.2) with $a = 4.0, b = 0.01, c = 1.0, c_1 = 0.7, d_1 = 0.3, d_2 = 0.4, e_1 = 1.0, e_2 = 0.5$ and an initial point $(x(0), y(0), z(0)) = (5, 2, 4)$ when $p = 3$.

(2), by Lemma 2.1, we know the system (1.1) has a globally asymptotically stable positive equilibrium point $E^* = (13750, 1.25, 82494)$. Moreover, from Theorem 3.2, we see the system (1.2) is permanent when $0.2 < p < 10$(see Figure 4).

For (3), by Lemma 2.1, we know the system (1.1) has a globally asymptotically stable positive equilibrium point $E^* = (320, 0.8, 223.8)$. It follows from 3.2 that the system (1.2) is permanent if $1.2 < p < 6$(see Figure 5).

5. Conclusion

In this paper, we have studied a three species food chain system with Lokta-Volterra functional response and impulsive perturbations. We have found the condition for permanence of this system and the subsystem of having no prey by using the comparison theorem. In addition, we have given numerical examples. These have shown that the condition found in this paper may be not optimal. We will discuss about this fact in the next paper.

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