Multi-Objective Optimal Design of a Single Phase AC Solenoid Actuator Used for Maximum Holding Force and Minimum Eddy Current Loss

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Abstract – A new Pareto-optimal design algorithm, requiring least computational work, is proposed for a single phase AC solenoid actuator with multi-design-objectives: maximizing holding force and minimizing eddy current loss simultaneously. In the algorithm, the design space is successively reduced by a suitable factor, as iteration repeats, with the center of pseudo-optimal point. At each iteration, the objective functions are approximated to a simple second-order response surface with the CCD sampling points generated within the reduced design space, and Pareto-optimal solutions are obtained by applying (1+λ) evolution strategy with the fitness values of Pareto strength.

Keywords: Adaptive Response Surface Method, AC Solenoid Actuator, Multi-objective Optimal Design, Pareto-optimal Solution

1. Introduction

Solenoid actuators transform an electrical signal into mechanical movement, and provide linear motion over a short displacement. They have many advantages such as simple structure, reliability, durability, and low cost over linear or rotational motors. In addition, they can be easily real-time controlled using a microprocessor. For these reasons, the solenoid actuators have been widely used in devices such as fuel injectors, anti-lock braking systems, switches, and relay and electro-hydraulic automatic systems [1].

Especially, AC solenoid actuators are ever increasing their application area because of convenient power source, quicker response, and lower price. However, AC solenoid actuators also have drawbacks such as eddy current loss in the shading coil which results in rise in temperature, and force pulsation at the holding state which may induce chattering [2-4]. In their design stage, the shading coil should be optimally designed so that the eddy current loss is minimized and the holding force is maintained larger than a load force [3].

From the viewpoint of maximizing the holding force, the eddy current should be allowed as much as possible because the holding force is developed mainly by the eddy current in the shading coil. This will, however, result in incremental loss of the eddy current. The optimal design of a single phase AC solenoid, therefore, naturally becomes a multi-objective design problem. From the viewpoint of numerical implementation, on the other hand, the optimal design requires a huge computational work because the performance evaluation involves time-stepping finite element analysis. In case of a global optimization algorithm, which requires at least thousands of function calls, is employed, the optimal design itself seems impossible.

In this paper, a computationally efficient multi-objective optimal design algorithm is developed by adopting Pareto-optimization technique and (1+λ) evolution strategy coupled with adaptive response surface method. The developed algorithm is applied to an optimal design of a single phase AC solenoid actuator to achieve two conflicting design targets, namely, minimizing the eddy current loss in the shading coil and maximizing the holding force.

2. Design Objectives and Design Parameters

Fig. 1 shows the initial design of an axis-symmetric AC solenoid actuator, which is composed of a static core with a shading coil on its face, movable plunger, frame, and exciting winding. The detailed specifications are listed in Table 1.

The fundamental purpose of a shading coil in a single phase AC solenoid actuator is to mitigate the force

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pulsation between the plunger and core. Especially when the plunger is in contact with the core pole face, it is of importance to keep the plunger from hammering on the core pole face and prevent the undesirable effect known as *chattering* [2]. At the same time, the eddy current in the shading coil should be limited to avoid rise in temperature.

The optimal design targets are mathematically defined as follows:

$$\begin{aligned} &\text{Maximize} \quad \text{Holding Force, } F_{holding}, \\ &\text{Minimize} \quad \text{Eddy Current Loss, } W_{loss}, \\ &\text{Subject to} \quad F_{holding} > F_{load} \end{aligned} \tag{1}$$

where F_{load} represents a load force.

In this optimization, the computing time increases very rapidly as the number of design parameters increases because the sampling data are prepared through time-stepping finite element analysis. In this paper, therefore, the design parameters are selected based on theoretical analysis of the holding force and eddy current loss.

Fig. 2 (a) shows a typical magnetic circuit of a shading coil, where the magnetic flux Φ from the power current is, at the pole face, divided into Φ_1 and Φ_2, which links and does not link the shading coil, respectively. The vector diagram of the fluxes and shading coil current can be expressed in shown in Fig. 2 (b), where the angle α is the phase difference between Φ_1 and Φ_2. The magneto-motive forces have the following relationship [2]:

$$\Phi_1 \frac{R_1}{v_1} = \Phi_2' \frac{R_2}{v_2} - NI' \tag{2}$$

where the subscripts 1 and 2 denote the unshaded and shaded magnetic flux paths, respectively, and R and v are equivalent magnetic resistance and magnetic reluctivity, respectively [2].

The magnetic force between the core and plunger is expressed as follows [2]:

$$f \propto \left(\frac{\Phi_1^2}{S_1 v_1} + \frac{\Phi_2^2}{S_2 v_2} \right) - \left\{ \frac{\Phi_1^2}{S_1 v_1} \cos 2\omega t - \frac{\Phi_2^2}{S_2 v_2} \cos(2\omega t - 2\alpha) \right\} \tag{3}$$

where the first and second terms represent DC and AC components of the force, and S_1 and S_2 are equivalent areas of the unshaded and shaded pole faces, respectively.

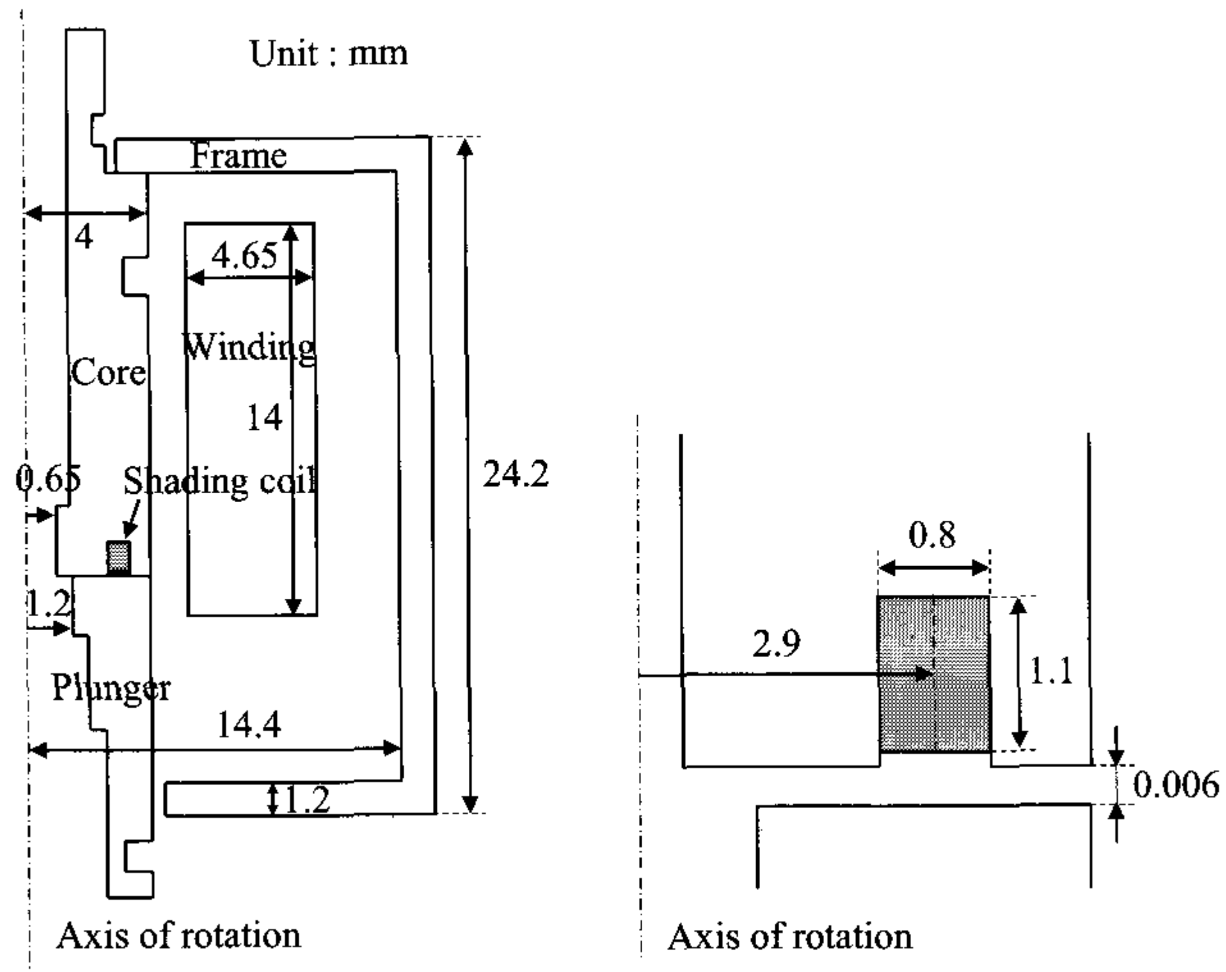
The eddy current loss in the shading coil can be expressed as follows:

$$W_{loss} = R_s \left(\frac{1}{R_s} \frac{d\Phi_2}{dt} \right)^2 \tag{4}$$

where R_s represents the resistance of the shading coil.

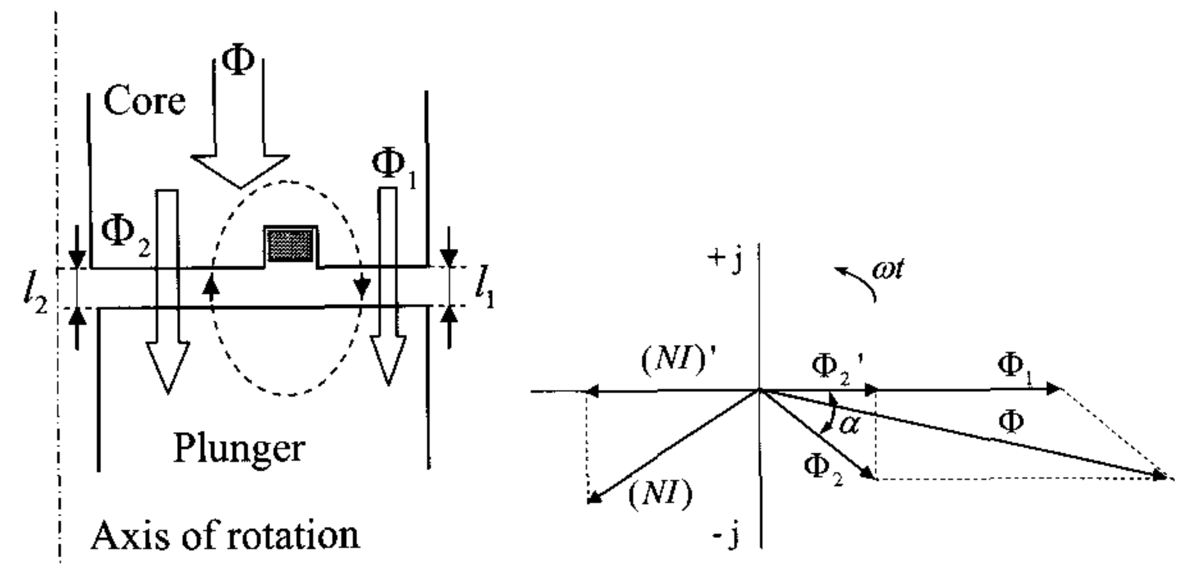
It is clear from (3) and (4) that the holding force and the eddy current loss can be optimized by changing Φ_1, Φ_2, and α, which are controlled by the radial position, width, and thickness of the shading coil. Equation (3) implies that the holding force also depends on, by means of v_1 and v_2, the air-gap profile.

Based on these investigations, the three design parameters are finally selected as indicated in Fig. 3, where the air-gap profile is ignored for the purpose of manufacturability.



(a) AC solenoid actuator (b) shading coil

Fig. 1. Initial design of a single phase AC solenoid actuator.



(a) magnetic circuit (b) vector diagram

Fig. 2. Magnetic circuit and vector diagram of shading coil.

Table 1. Specifications of an AC Solenoid Actuator

Items		Specifications
Operating voltage, Frequency		$220 \pm 30\%$(V) , 60Hz
Minimum stroke		0.006(mm)
Winding turns, Resistance		12600, 4910 (Ω)
Load force		3.8283 (N)
Materials	Core, Plunger	QMR5L
	Frame	Pure iron
	Shading coil	Copper (Bulk type)

3. Multi-Objective Optimization Algorithm using Adaptive Response Surface

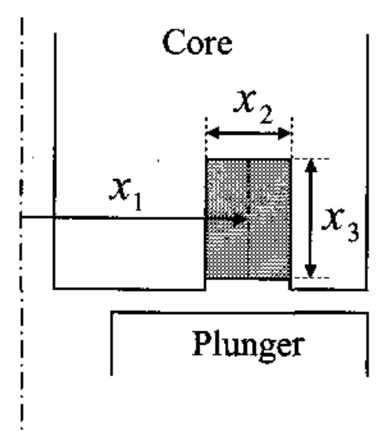
3.1 Second-order Response Surface with Central Composite Design

With given N_s sampling points, the second-order response surface is constructed as follows [5]:

$$f(\mathbf{x}) = \beta_0 + \sum_{i=1}^{K} \beta_i x_i + \sum_{i=1}^{K} \beta_{ii} x_{ii}^2 + \sum_{i=1}^{K} \sum_{i<j=2}^{K} \beta_{ij} x_i x_j \quad (5)$$

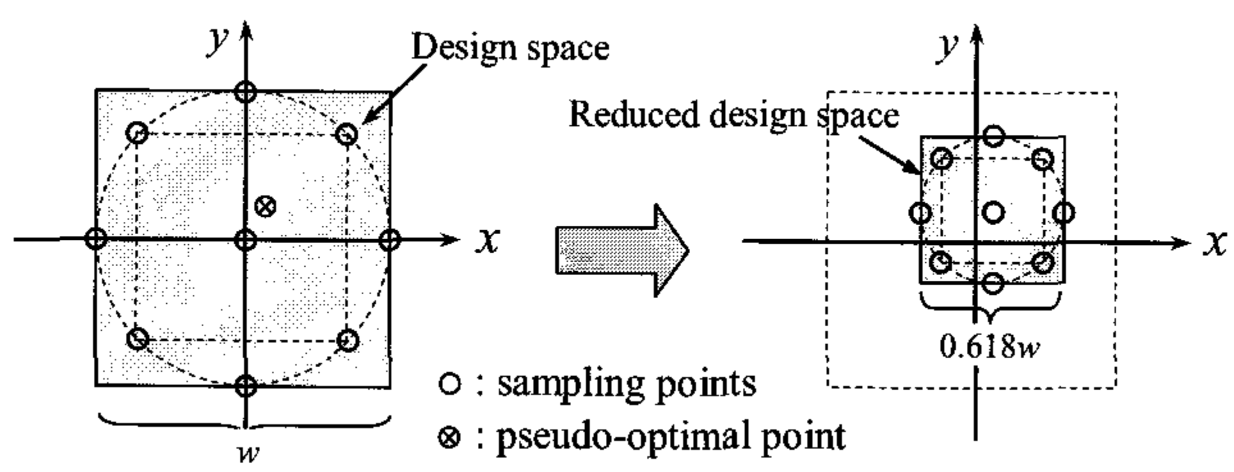
where $\mathbf{x} = (x_1, x_2, \cdots, x_K)$ with K being the number of design parameters. The coefficients are numerically obtained by applying point matching technique for the N_s sampling points [5].

In order to decide the locations of the sampling points, in this paper, the central composite design (CCD) is utilized. In this method, the number of sampling points is defined as ($N_s=2^K+2K+1$) [5]. Fig. 4(a) shows an example of distribution of the sampling points in a two dimensional design space.

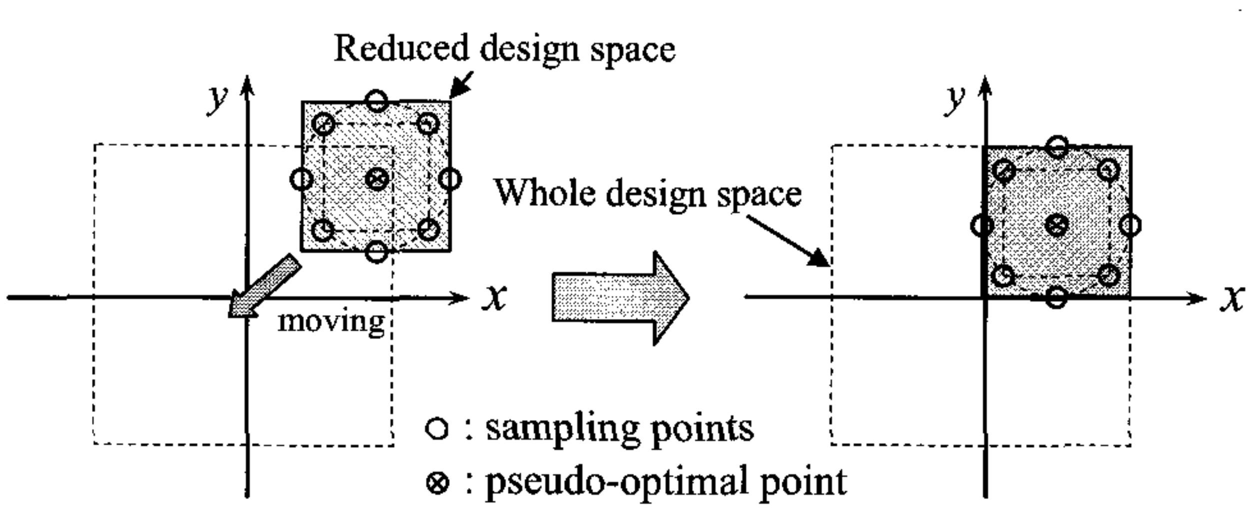


Design parameters	Range (mm) Lower	Upper
x_1	1.55	3.1
x_2	0.20	1.4
x_3	0.20	2.3

Fig. 3. Design parameters and their ranges.



(a) i-th itaretion (b) (i+1)-th iteration



(c) reduced design space when a *pseudo-optimal* point exists at a corner

Fig. 4. Reduced design space with sampling points.

3.2 Optimization Strategy

Once a response surface is constructed, a minimum point, in Pareto-optimal sense for the multi-objective functions, can be found by using (1+λ) evolution strategy. The minimum point obtained at this stage, however, cannot be considered as a true optimal point. This is because the objective function is approximated to a simple second-order function in the whole design space. This minimum point hereinafter will be referred to a *pseudo-optimal* point. In order to find a true optimal point, we need an adaptive response surface method involving the minimum required number of sampling data.

3.2.1 Optimization Algorithm using Adaptive Response Surface Method

The proposed optimization algorithm can be summarized as follows:

Step 1 Define the initial design space.
Step 2 In the design space, generate N_s sampling data using CCD and time-stepping finite element analysis.
Step 3 Construct a second-order response surface, and find a *pseudo-optimal* point for the multi-objective functions by using (1+λ) evolution strategy.
Step 4 Check the convergence of the *pseudo-optimal* points. Stop if converged.
Step 5 Reduce the design space by a suitable factor with the center of the current *pseudo-optimal* point, and go to *Step* 2.

In the algorithm, the iteration repeats until the *pseudo-optimal* points converge, and the converged *pseudo-optimal* point is considered as a true optimal point. Fig. 4 shows an example, with a reducing factor of 0.618, of reduced design space with the distribution of the sampling points in a two dimensional design space. In case that the *pseudo-optimal* point locates near the boundary of the design space, the reduced design space is moved, as shown in Fig. 4(c), so that it may belong to the initial design space.

It should be noted that the performance of the algorithm depends on the reducing factor of the design space. For example, a smaller reducing factor is expected to give faster convergence with higher possibility of missing the global optimum point, while a bigger one, on the contrary, more robust optimization with slower convergence.

3.2.2 Multi-Objective Optimization using (1+λ) Evolution Strategy

In the optimization using (1+λ) evolution strategy, a major difference of a multi-objective optimization from a typical single objective optimization is how to assign a fitness value to each of the design parameter vectors. In this paper, fitness value based on Pareto strength is introduced.

The suggested multi-objective optimization algorithm using (1+λ) evolution strategy is summarized briefly as follows:

Step 1 Generate λ children vectors, and create empty set P of size N_p for Pareto-optimal solutions.
Step 2 Evaluate objective function values for all children vectors, and copy non-dominated children vectors, if any, to set P.
Step 3 Reduce set P by removing covered solutions from P.
Step 4 Calculate the fitness values for all children vectors and solutions in P using Pareto strength.
Step 5 Select a parent vector, and go to *Step* 1.

where $\mathbf{x}_m$ *covers* $\mathbf{x}_n$ if $f_i(\mathbf{x}_m) \geq f_i(\mathbf{x}_n), i = 1,2,\cdots,N$. In maximizing a problem with N as the number of objective functions, f, and $\mathbf{x}_i$ is *non-dominated* when no member in set P *covers* $\mathbf{x}_i$ [6],[7]. The calculation of the fitness values is accomplished as follows [6]:

i) when a design parameter vector $\mathbf{x}_i$ belongs to set P, i.e., if $\mathbf{x}_i$ is one of the Pareto-optimal solutions,

$$Fitness(\mathbf{x}_i) = \frac{n}{N_{PO}+1} \tag{6}$$

where N_{PO} is the number of members in set P, and n is the number of children *covered* by $\mathbf{x}_i$ among the λ.

ii) when a design parameter vector $\mathbf{x}_i$ does not belong to set P,

$$Fitness(\mathbf{x}_i) = 1 + \sum_{j=1}^{C_j} Fitness(\mathbf{x}_j) \tag{7}$$

where C_j is the number of members in P which *covers* $\mathbf{x}_i$.

4. Numerical Optimization Results

During the optimization, the holding force and eddy current loss are computed by using time-stepping finite element method. In axis-symmetric plane, the field governing equation and circuit equations are, ignoring the conductivities of frame, plunger and core, given as follows, and they are solved by using the Maxwell 2D Program:

$$\nabla \times \frac{1}{\mu} \nabla \times A = J + \sigma \frac{\partial A}{\partial t} \tag{8-a}$$

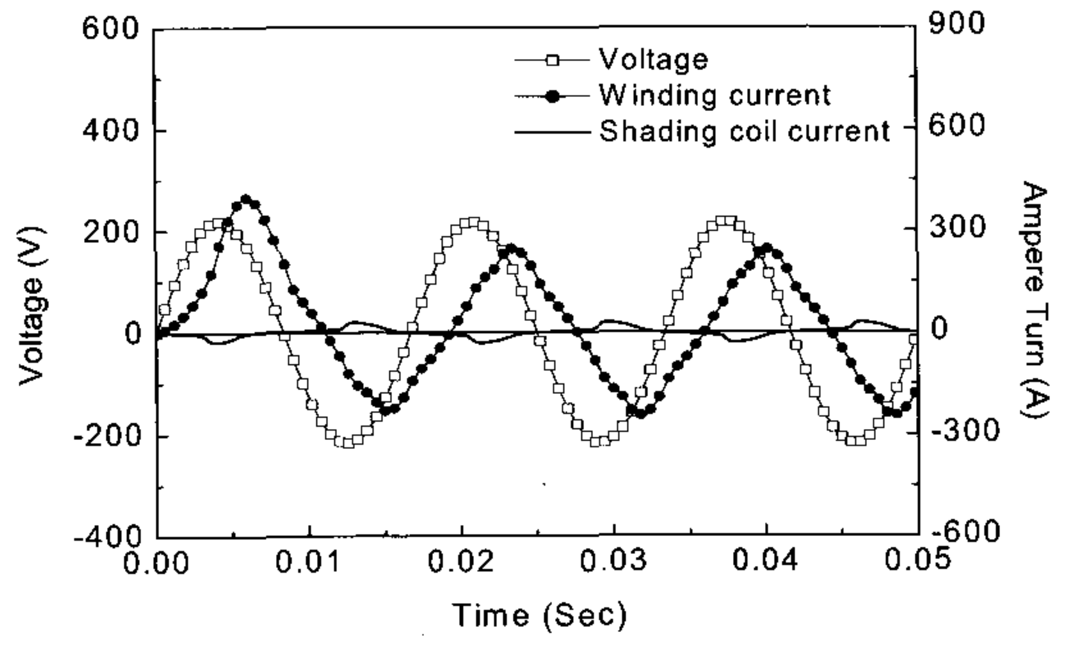


Fig. 5. Waveform of power and eddy currents in initial design.

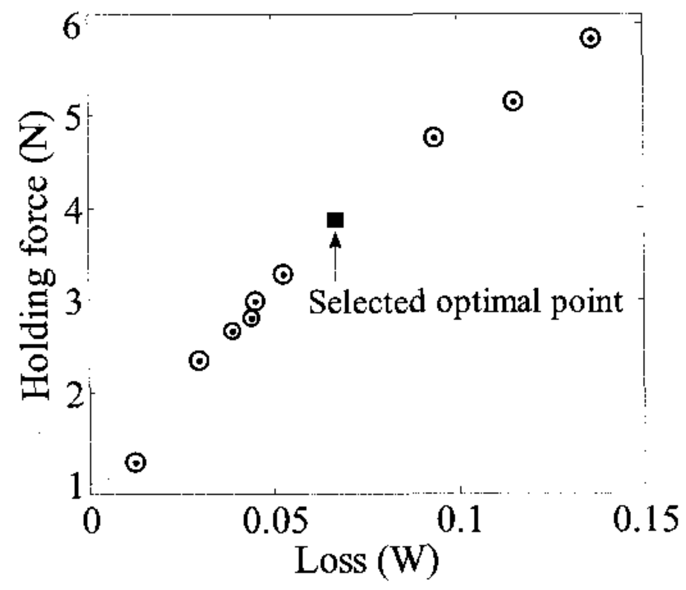


Fig. 6. Distribution of the Pareto-optimal solutions.

$$V(t) = Ri(t) + \frac{d\lambda}{dt} \tag{8-b}$$

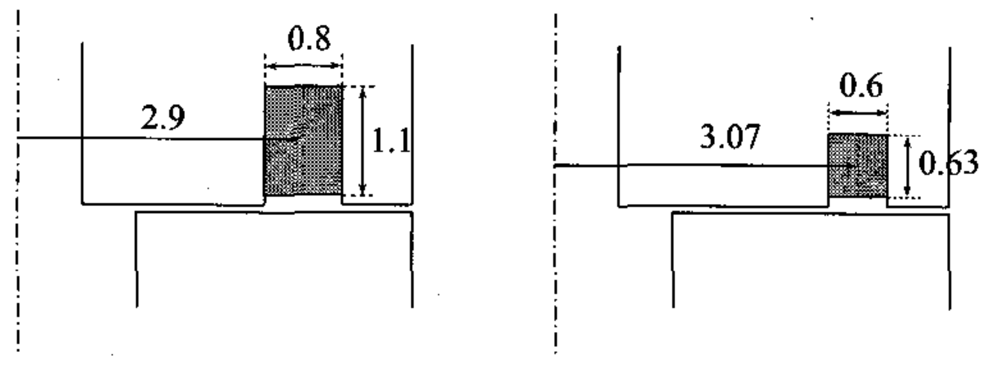
Fig. 5 shows the variations of the power and eddy currents with an initial design (x_1, x_2, x_3) =(2.9,0.8,1.1), where the steady state solutions are obtained after 3 cycles. From the analysis results, the holding force $F_{holding}$ (it corresponds to the minimum force) is computed by using Maxwell's stress tensor, and the eddy current loss W_{loss} is computed as:

$$W_{loss} = \sum_{e=1}^{N_e} \int_e \sigma E^2 dv \quad [W] \tag{9}$$

where N_e is total number of elements belonging to the shading coil.

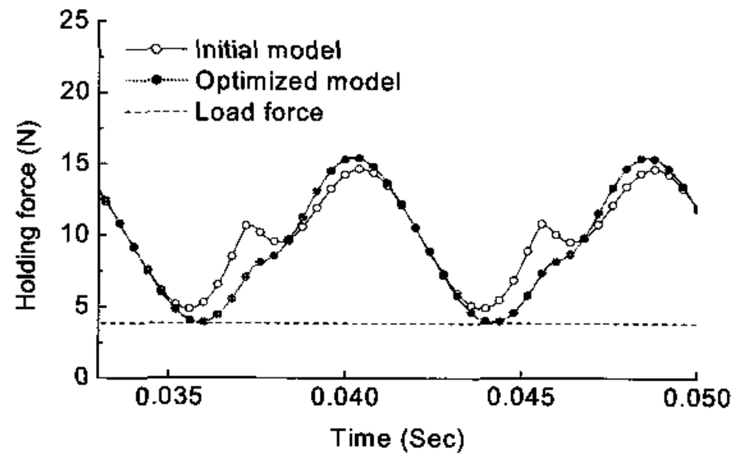
The optimal design is obtained after 4 iterations, namely with only 60 times of time-stepping finite element analysis. As compared with the other adaptive response surface method in [8], this proves that the suggested optimization algorithm is computationally very efficient. Fig. 6 shows a distribution of 10 Pareto-optimal solutions in objective function space. Although any of them can be taken as an optimal solution, (x_1,x_2,x_3)=(3.07,0.6,0.63) marked by a rectangle, is taken as the optimal solution because it gives less eddy current loss maintaining the holding force larger than the load force.

For the initial and optimized designs, Fig. 7 compares

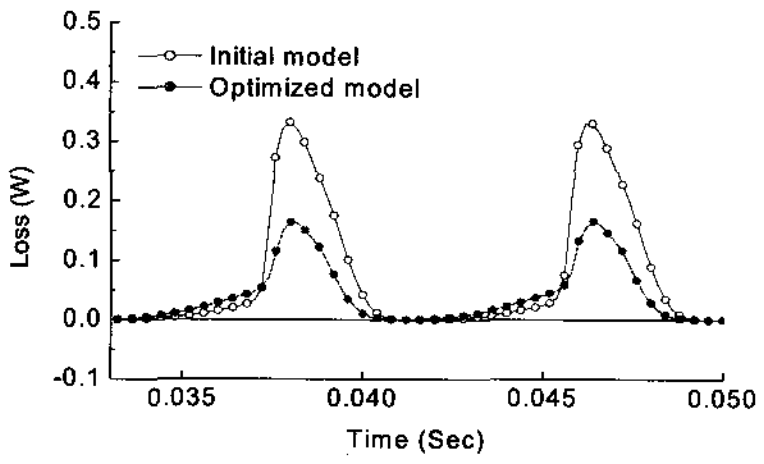


(a) initial model (b) optimized model

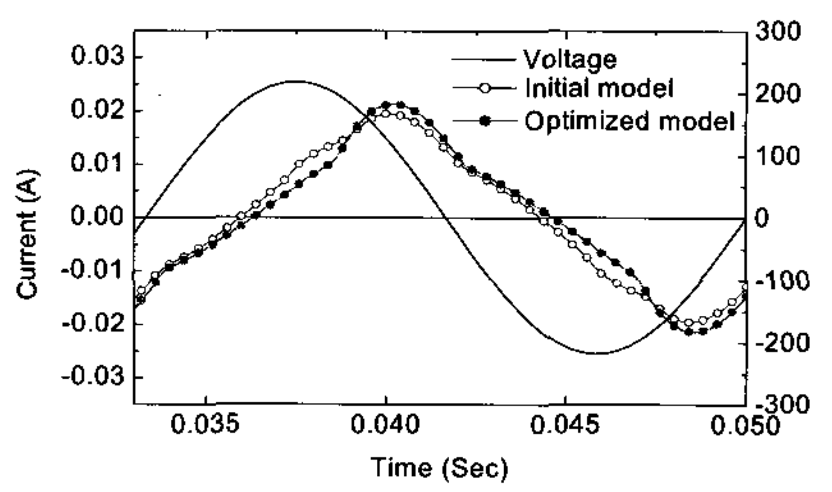
Fig. 7. Comparison of optimized shape with initial one.



(a) holding force

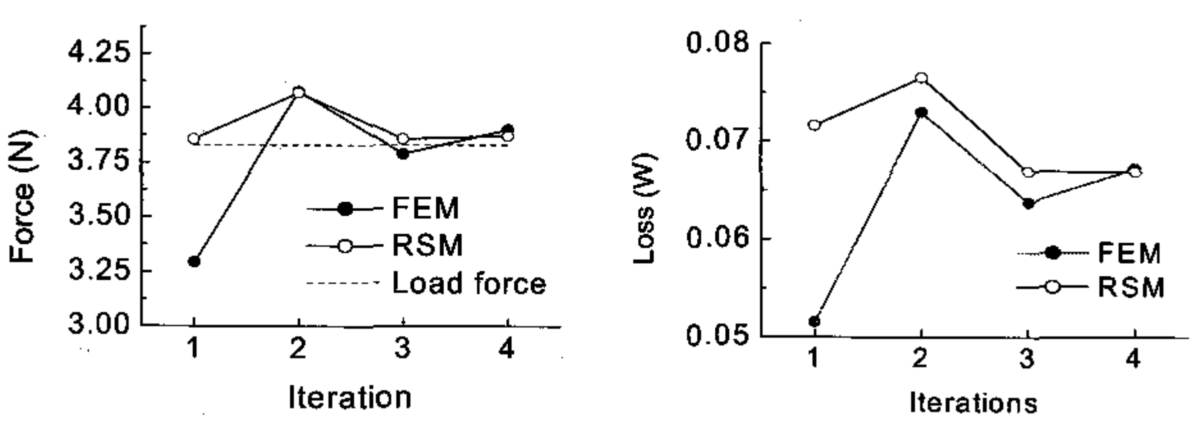


(b) eddy current loss



(c) input power

Fig. 8. Comparison of the performances between the optimized and initial models.



(a) holding force (b) eddy current loss

Fig. 9. Convergence of the performances.

the shapes of the core including the shading coil. Fig. 8 reveals the comparison of the performances, holding force, eddy current loss, and input power. The holding force is somewhat lowered while satisfying the constraint and the eddy current loss is greatly reduced. Since the holding force is still maintained higher than the load force, the optimal design is considered to be acceptable. However, in case that more holding force is preferred, another solution from the Pareto-optimal solutions may be selected. In order to compare the input power of the optimized design with that of the initial one, Fig. 8(c) shows the input voltage and current waveforms. It can be seen that the optimal design does not change the current waveform too drastically. The optimized design reduced the average input power from 1.0814(W) at initial design to 0.8806(W). Fig. 9 presents the convergence of the holding force and eddy current loss. It can be seen that the suggested optimization algorithm gives a very stable convergence.

5. Conclusion

A computationally efficient multi-objective optimization algorithm is developed and applied to an optimal design of an AC solenoid actuator with the design objectives of maximizing the holding force and minimizing the eddy current loss. During the optimization process, the design space is successively reduced by a factor of 0.618, and the objective functions are approximated by using second-order response surface with CCD sampling points. Through a numerical application, the suggested algorithm is concluded to give not only fast but robust convergence with less computational work.

Acknowledgements

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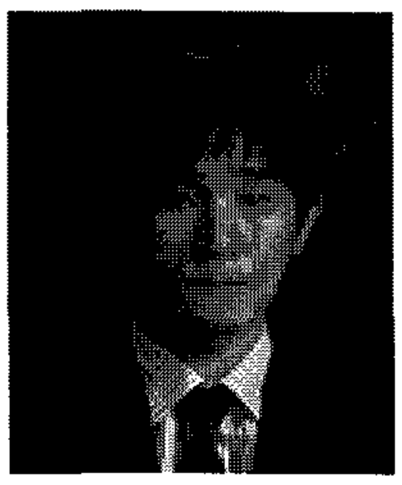

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