A Robust and Computationally Efficient Optimal Design Algorithm of Electromagnetic Devices Using Adaptive Response Surface Method

Yanli Zhang*, Hee Sung Yoon, Pan-Seok Shin***, and Chang Seop Koh†**

Abstract – This paper presents a robust and computationally efficient optimal design algorithm for electromagnetic devices by combining an adaptive response surface approximation of the objective function and (1+λ) evolution strategy. In the adaptive response surface approximation, the design space is successively reduced with the iteration, and Pareto-optimal sampling points are generated by using Latin hypercube design with the Max Distance and Min Distance criteria. The proposed algorithm is applied to an analytic example and TEAM problem 22, and its robustness and computational efficiency are investigated.

Keywords: Adaptive Response Surface Method, Latin Hypercube Design, Optimal Design, Pareto Optimization.

1. Introduction

Optimal design of electromagnetic devices often involves computationally expensive finite element analysis for performance evaluation through finding solutions of electromagnetic fields. Especially when a global optimization algorithm is adopted, the optimal design procedure requires huge computing time in regards to finite element analysis. Thus, in order to reduce the computational work related with finite element analysis during optimal design, response surface method (RSM) has been developed to approximate the objective function in the design space to a simple analytic expression, namely, response surface [1-5]. Once constructed, this response surface provides a rapid way of obtaining the objective function value at any point in the design space, and will be used in place of the finite element analysis. Hence, the RSM has been efficiently combined with a global optimization algorithm to determine an optimal design even when the objective function is multimodal or ill-conditioned [3].

As the number of sampling points increases, in general, the fitting ability of the RSM using multi-quadric radial basis function becomes better while numerical efficiency worsens. There have been researches, therefore, to guarantee both numerical accuracy and efficiency at the same time [2-4], [6-8]. These researches can be classified into two categories. One is improvement of the basis function [3-4], [7-8], and the other is adaptive insertion of new sampling points based on the error estimation of the response surface [6].

In this paper, an adaptive response surface method is developed by coupling successive reduction of the design space and adaptive insertion of new sampling points, combined with (1+λ) evolution strategy to give a numerically efficient and robust optimal design algorithm.

2. RSM with Multi-quadric Radial Basis Function

When RSM is used for the global interpolation of an objective function, multi-quadric radial basis function is one of the most attractive from the viewpoint of its smoothness and fitting ability with a limited number of sampling points in the design space. With the given sampling data, the response surface is constructed as follows [1]:

$$S(\mathbf{x}) = \sum_{i=1}^{N} \beta_i g(\mathbf{x} - \mathbf{x}_i) \tag{1-a}$$

$$g(\mathbf{x}) = \sqrt{\|\mathbf{x}\|^2 + h^2} \tag{1-b}$$

$$X = \{(\mathbf{x}_i, f_i), i = 1, 2, \cdots, N\} \tag{1-c}$$

where $\|\cdot\|$ is Euclidean distance, $\mathbf{x}$ is a design parameter vector, β_i and f_i are the coefficient and objective function value corresponding to the i-th

† Corresponding author, School of Electrical & Computer Engineering, Chungbuk National University, Chungbuk 361-763, Korea (kohcs@chungbuk.ac.kr).
* School of EE, Shenyang University of Technology, Liaoning 110023, China. She is currently a research associate with BK21 Project at Chungbuk National University, Korea (zhangyanli_sy@hotmail.com).
** School of ECE, Chungbuk National University, Cheongju, Chungbuk 361-763, Korea. (hsyoon@chungbuk.ac.kr)
*** Dept. of EE, Hongik University, Chungnam 361-763, Korea. (psshin@hongik.ac.kr).
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sampling point $\mathbf{x}_i$, respectively, X is the set of sampling data of size N, and h is the shape parameter. With a given shape parameter, the coefficients are determined, using point matching technique, as follows:

$$[\beta_i]=\left[g_{ij}\right]^{-1}[f_i] \tag{2}$$

where $g_{ij}=g(\mathbf{x}_i-\mathbf{x}_j)$ is called the interpolation matrix.

For the choice of a good value of the shape parameter, the interpolation error is defined using *Leave-One-Out* method, and an optimal value, which minimizes the estimated error, is found by applying $(1+\lambda)$ evolution strategy [1].

3. Optimization Algorithm using Adaptive Response Surface Method

Once a response surface has been constructed, a minimum point can be easily found by applying $(1+\lambda)$ evolution strategy. The minimum point obtained at this stage, however, can not be considered as a true optimal point unless sufficient number of sampling data is involved in the construction of the response surface. This minimum point hereinafter will be referred to a *pseudo-optimal* point. In this paper, in order to find a global optimum point, an adaptive response surface method involving the minimum required number of sampling points is suggested.

3.1 Optimization Algorithm

The proposed optimization algorithm using adaptive response surface method can be summarized as follows:

Step 1 Define the initial design space, and generate uniformly distributed N_i Pareto-optimal sampling points in the whole design space by means of Latin hypercube design with Max Distance and Min Distance criteria.

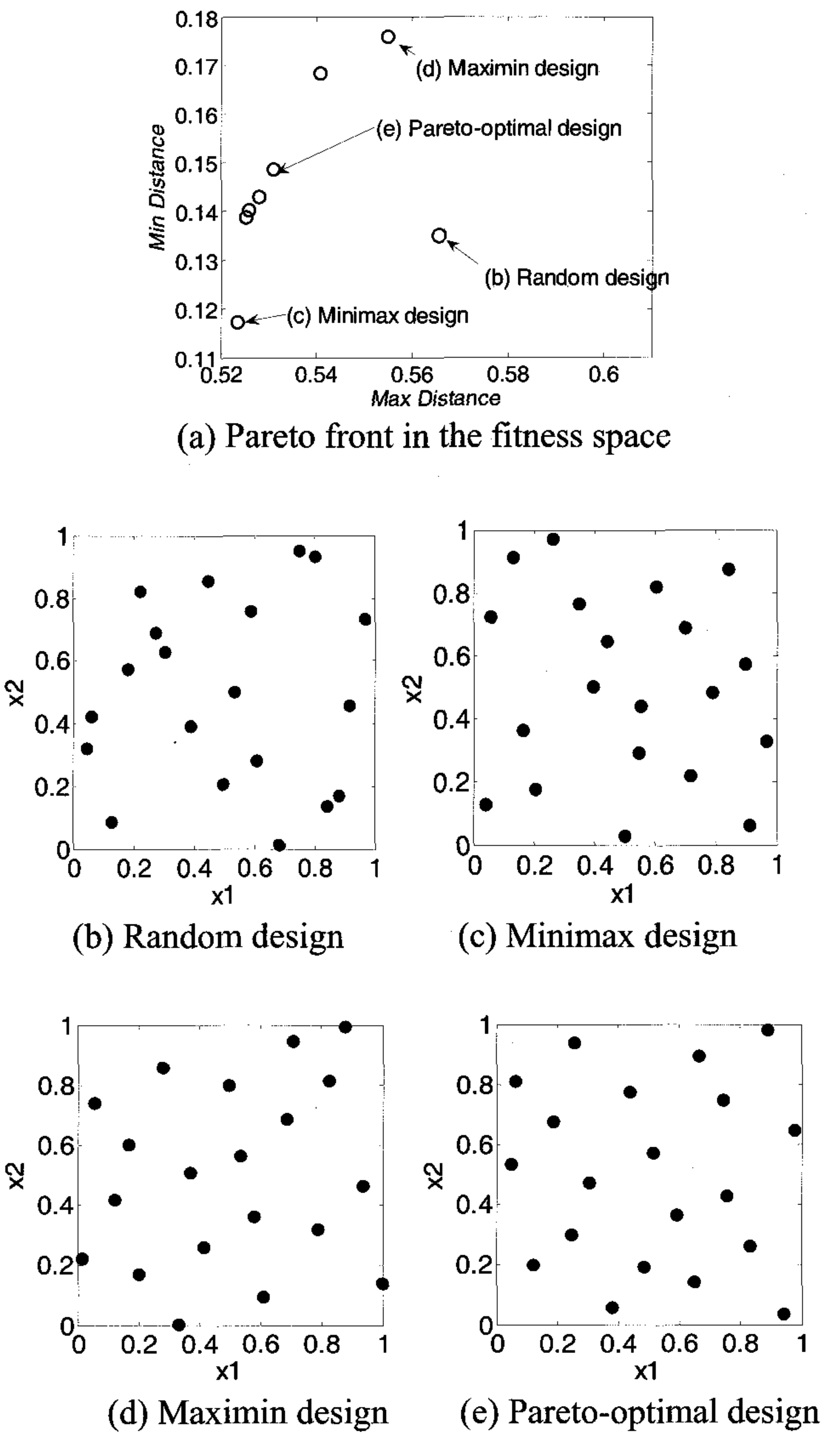
Step 2 Construct a response surface using multi-quadric radial basis function, and find a pseudo-optimal point by applying $(1+\lambda)$ evolution strategy.

Step 3 Check the convergence of the pseudo-optimal points, and stop if converged.

Step 4 Reduce the design space by a suitable factor with the center of the current pseudo-optimal point.

Step 5 Generate additional N_a Pareto-optimal sampling points only within the reduced design space concentrating at the neighborhood of the current pseudo-optimal point as in *Step* 1, and go to *Step* 2.

In the algorithm, the iteration repeats until the *pseudo-optimal* points converge, and the converged *pseudo-optimal* point is considered as a true optimal point.



(a) Pareto front in the fitness space

(b) Random design (c) Minimax design

(d) Maximin design (e) Pareto-optimal design

Fig. 1. Comparison of the Latin hypercube sampling points of size n=20 with two design variables.

3.2 Initial Sampling Points using Latin Hypercube Design with Pareto-optimization

Latin hypercube design (LHD) is a type of *space-filling* experiment design strategy [9]. It corresponds to a $(n\times k)$ matrix where n and k are the numbers of sampling points and design variables, respectively. In order to distribute the sampling points, each variable is divided into n intervals with equal probability and each of the k columns is a random permutation of $\{1,2,\ldots,n\}$ which can be mapped onto the actual range of the variables.

Although sampling data can be randomly generated using LHD, it may have poor quality, as shown in Fig. 1(b), in the viewpoint of uniform *space-filling*.

3.2.1 Criteria for Sampling Points

The quality of a sampling data set is evaluated, in the viewpoint of *space-filling*, by introducing *Max distance* and *Min distance* criteria.

The *Max Distance* criterion is defined as follows [9]:

$$Max\ Distance = \underset{\mathbf{x}\in D}{Max}\left\{\underset{\mathbf{x}_i\in X}{Min}\ d\left(\mathbf{x},\mathbf{x}_i\right)\right\} \tag{3}$$

where D is design space, X is sampling data set, and $d(\cdot,\cdot)$ is Euclidean distance between any two points in D. This criterion physically means possible maximum distance from an arbitrary point in the design space to the nearest sampling point.

The *Min Distance* criterion, of which the physical meaning is minimum distance between any two sampling points, is defined as in [9]:

$$Min\ Distance = \underset{\mathbf{x}_i,\mathbf{x}_j\in X}{Min}\ d\left(\mathbf{x}_i,\mathbf{x}_j\right) \tag{4}$$

A set of sampling points minimizing the *Max Distance* criterion is called *Minimax design,* and it is expected to cover all the design space with minimum distance as shown in Fig. 1(c). In this design, however, some sampling points may be located very closely to each other. On the contrary, a set of sampling points, obtained by maximizing the *Min Distance* criterion and known as *Maximin design*, locates some sampling points unexpectedly on the boundary of the design space as shown in Fig. 1(d).

3.2.2 Pareto-optimal Sampling Points

In the viewpoint of *space-filling* quality, a set of sampling points, which minimizes the *Max Distance* criterion and maximizes the *Min Distance* criterion, simultaneously is considered as the best one. However, the two goals, minimizing the *Max Distance* criterion and maximizing the *Min Distance* criterion, conflict with each other, and selecting a good sampling data arrives at a multi-objective optimization problem.

In this paper, a Pareto-optimal set of sampling data is obtained by using (1+λ) evolution strategy where the optimization objective is "Minimize *Max Distance* and Maximize *Min Distance"*.

Fig. 1(a) shows a Pareto front in the two criteria space from which an acceptable set of sampling data is selected. Fig. 1(e) shows a distribution of finally selected sampling data, where the design space is almost uniformly covered by the sampling points.

3.3 Reduction of Design Space and Adaptive Inserting of New Sampling Points

Once a *pseudo-optimal* point is found at the initial iteration by scanning the entire design space, a true optimal point is assumed to exist not too far from the *pseudo-optimal* point. With this assumption, the region of interest may be reduced, i.e., the design space is reduced by a suitable factor (for example 0.618) for more precise investigation without missing a true optimal point. Furthermore, by confining our interest to only the reduced design space, a more accurate response surface can be efficiently obtained by inserting only a few sampling points.

In this stage, the additional sampling points are generated by means of LHD using Gaussian random numbers, so that they are concentrated near the *pseudo-optimal* point, i.e., by letting the mean value correspond to the *pseudo-optimal* point, and by controlling the standard deviation, the degree of closeness to the *pseudo-optimal* point is controlled. The Pareto optimization is also adopted to select a good set of additional sampling points. During the Pareto optimization, the distance, $d(\cdot,\cdot)$, is rescaled and computed as follows:

$$d\left(\mathbf{x}_i,\mathbf{x}_j\right) = d\left(\mathbf{x}_i,\mathbf{x}_j\right)\cdot d\left((\mathbf{x}_i+\mathbf{x}_j)/2,\mathbf{x}_p\right) \tag{5}$$

where $\mathbf{x}_p$ is the *pseudo-optimal* point.

It should be noted that the reducing factor has strong influence on the robustness and convergence rate of the suggested optimization algorithm, namely, a small value of the reducing factor may increase the robustness, but will decrease the convergence rate. Several tests with analytic functions show a factor of 0.85 or 0.618 giving a good convergence without falling into local minima.

Fig. 2 presents an example of the additional insertion of 16 sampling points with a reducing factor of 0.85 to the old sampling points shown in Fig. 1(e).

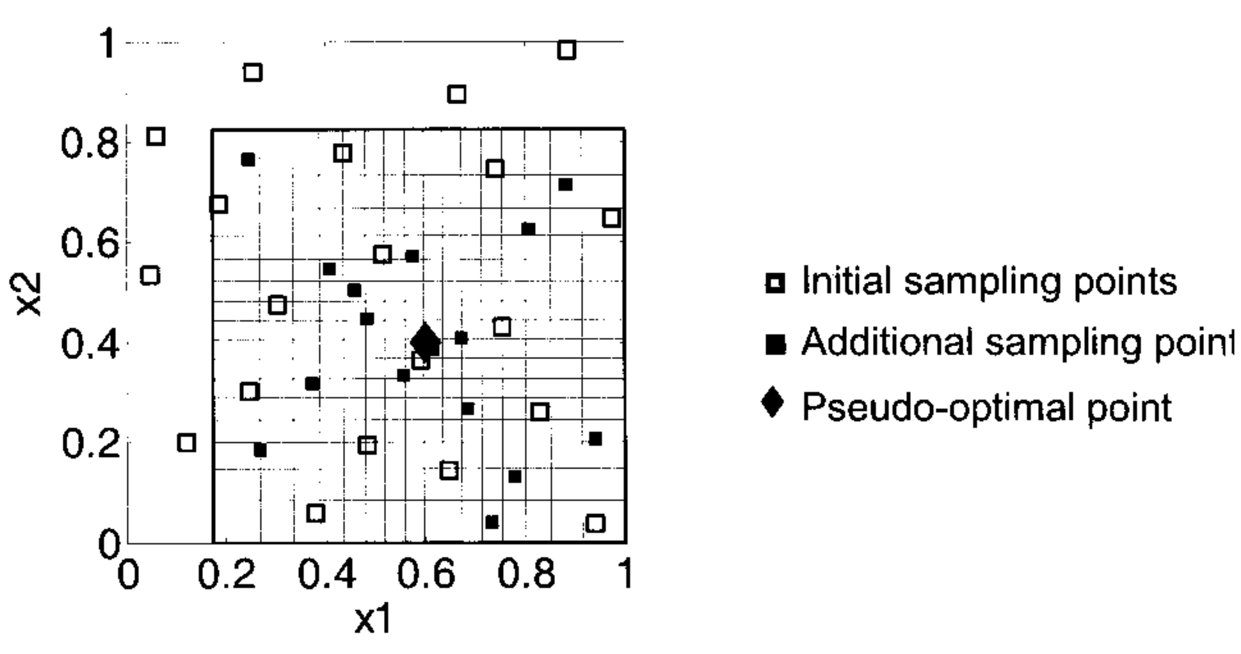


Fig. 2. The insertion of the sampling points using LHD based on Gaussian distribution.

4. Numerical Experiments

4.1 Analytic Function with Two Design Variables

In order to illustrate graphically the proposed procedure, an analytic function with two design variables is taken as an example. The objective function to be minimized in the range of $-3.0 \le x_1, x_2 \le 3.0$ is given as [4]:

$$F(\mathbf{x}) = 3(1-x_1)^2 e^{[-x_1^2-(x_2+1)^2]} - 10\left(\frac{x_1}{5} - x_1^3 - x_2^5\right)e^{[-x_1^2-x_2^2]} - \frac{1}{3}e^{[-(x_1+1)^2-x_2^2]} \quad (6)$$

This analytic function has 3 local maxima, 1 local minimum, and 1 global minimum point at (0.2282, -1.6256) with corresponding objective function value -6.5511.

Fig. 3 shows the variation of the response surfaces and distributions of the sampling points as the new sampling points are inserted at each iteration. At the initial iteration, 25 sampling points are generated in the whole design space using Pareto-optimal LHD, and a response surface is constructed to give a *pseudo-optimal* point of (0.5035,-1.400) as presented in Fig. 3(a). At the second and third iterations, as indicated in Fig. 3(b) and Fig. 3(c), the design space is successively reduced by a factor of 0.85, and 16 and 11 additional sampling points are inserted concentrating at the neighborhood of the previous *pseudo-optimal* points, respectively. After the fourth iteration with additional 6 sampling points, a converged pseudo-optimal point (0.2278, -1.6254) and corresponding objective function value -6.5513 are obtained.

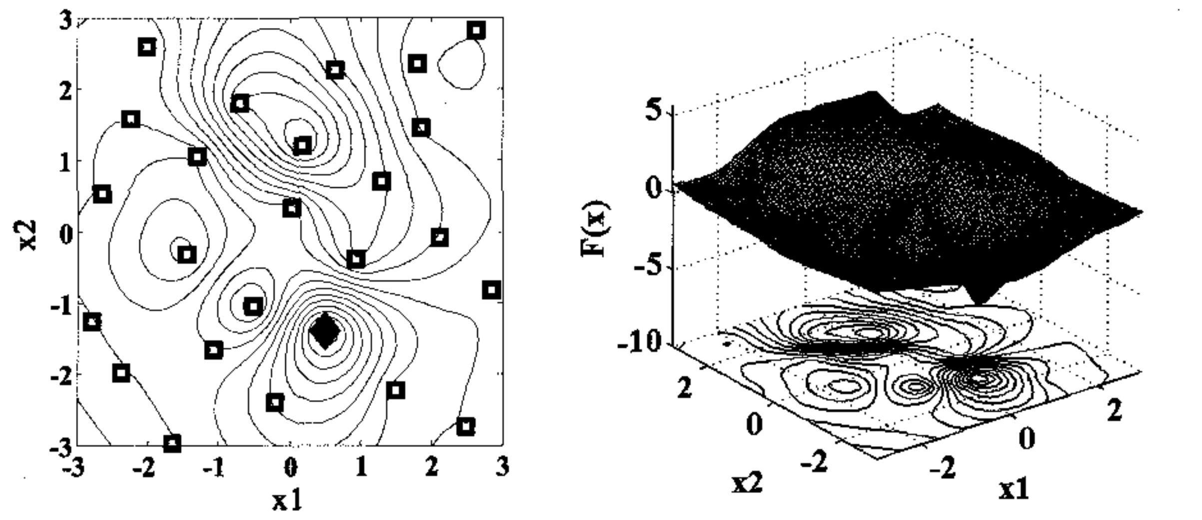
Fig. 4 shows the convergence of the objective function value and the *pseudo-optimal* points during the iterations. It can be seen that the *pseudo-optimal* point converges robustly to the true global optimal point with the iterations.

4.2 Electromagnetic Application

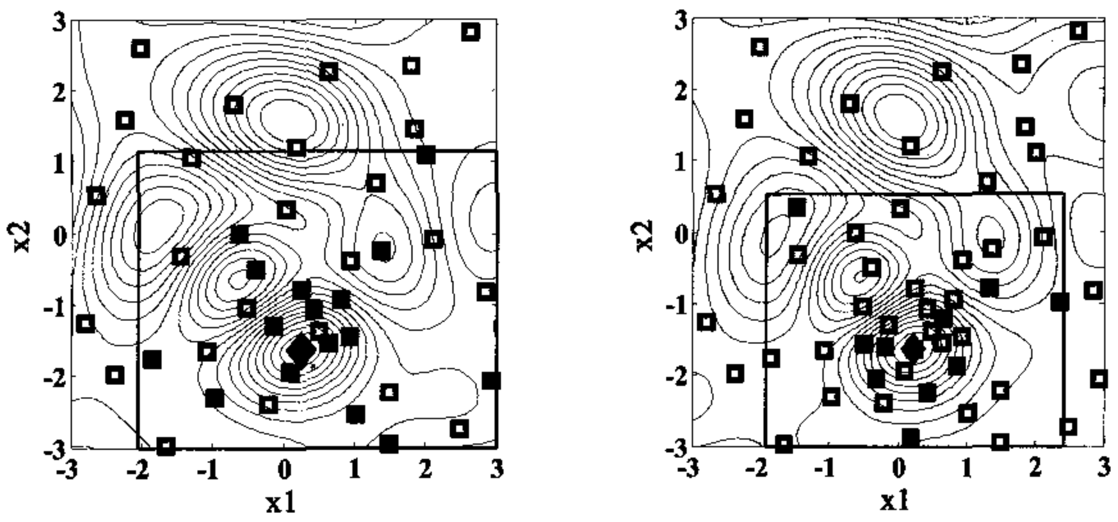
Fig. 5(a) gives a superconducting magnetic energy storage device (SMES) model, taken from TEAM Problem 22 [10], which consists of two concentric coils with current densities J_1 and J_2, respectively, which are in opposite directions. The design target is determining the optimum design parameters that minimize the following objective function:

$$OF = \frac{B_{stray}^2}{B_{norm}^2} + \frac{|Energy - E_{ref}|}{E_{ref}} \quad (7\text{-}1)$$

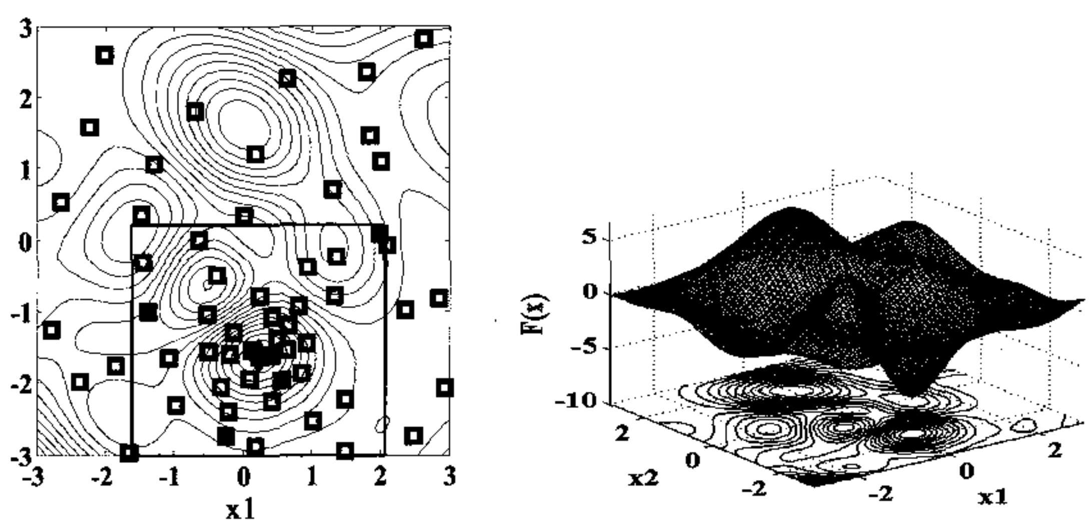
$$B_{stray}^2 = \frac{1}{22}\sum_{i=1}^{22} B_{stray,i}^2, \; E_{ref} = 180\text{MJ}, B_{norm} = 3\times10^{-3}\text{T} \quad (7\text{-}2)$$



(a) initial 25 sampling points and corresponding response surface

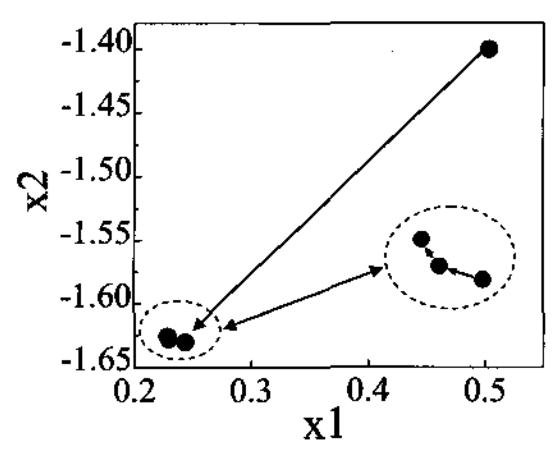


(b) 2-nd iteration (c) 3-rd iteration

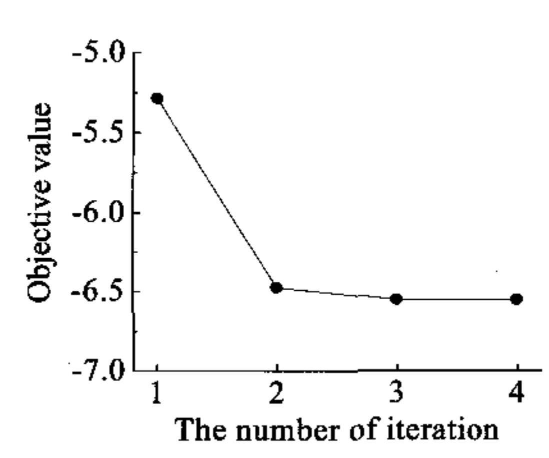


(d) final 58 sampling points and corresponding response surface

Fig. 3. Response surfaces and movement of the pseudo-optimal point (The symbols have the same meaning as in Fig. 2.).



(a) optimal points



(b) objective function

Fig. 4. Convergence of the suggested optimization strategy.

where the stray field B_{stray} is evaluated at 22 equidistant points along lines *a* and *b*, as shown in Fig. 5(a), and Energy is the stored magnetic energy. The three design variables are associated with the external coil (R_2, h_2, d_2), and the ranges and fixed values are listed in Table I.

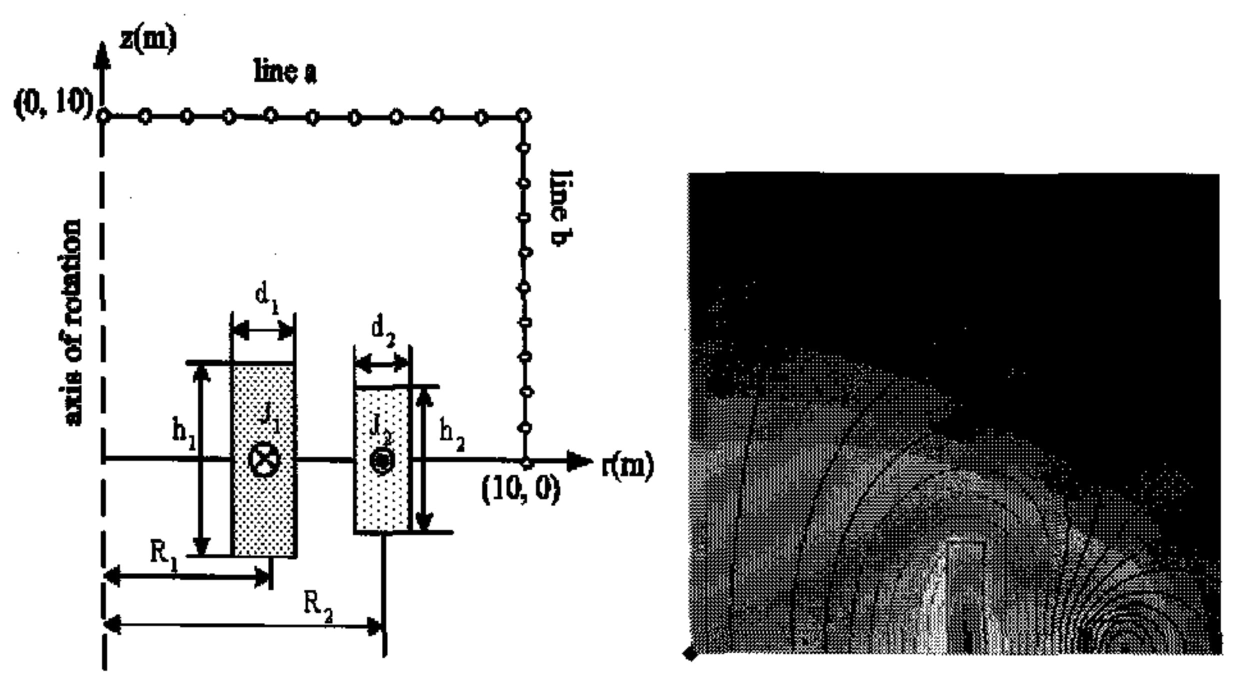
The magnetic field analysis of the SMES system was done by using a finite element code using a triangular element of second order (around 14,800 nodes and 7,400 elements). An artificial boundary was placed relatively far, at 20m from the coils' center.

During the optimization procedure, the initial 80 Pareto-optimal sampling points were distributed in the whole design space, and followed by the successive additional insertion of 51, 32, and 17 sampling points into the reduced design space with a factor of 0.618 to the previous region. The field plot of optimal configuration obtained by the above optimization strategy is shown in Fig. 5(b).

Table II compares the performance of the proposed method with those of the other reported solutions. The best solution for this problem is reported in [10] as listed in Table II. From the table, the proposed algorithm requires only 180 finite element analysis with a good global optimum solution, while the other algorithms require at least more than two times of analysis.

5. Conclusion

A global optimization algorithm has been presented by combining an adaptive response surface method employing successive reduction of design space, adaptive inserting of sampling points, and (1+λ) evolution strategy. The numerical results indicate that the proposed optimization strategy is very robust and computationally efficient with fewer sampling points and higher computation efficiency.



(a)design variables (b) flux line

Fig. 5. SMES model from TEAM problem 22, where the flux line is from the optimal design.

Table 1. Variable Ranges and Values Used

Variable [Unit]	R_1 [m]	$h_1/2$ [m]	d_1 [m]	R_2 [m]	$h_2/2$ [m]	d_2 [m]	J_1 [MA/m^2]	J_2 [MA/m^2]
Min	-	-	-	2.6	0.204	0.1	-	-
Max	-	-	-	3.4	1.1	0.4	-	-
Value	2.0	0.8	0.27	-	-	-	22.5	-22.5

Table 2. Comparison of Different Algorithms

Algorithm	R_2	$h_2/2$	d_2	OF	No. of FEM computations
Proposed	3.093	0.239	0.391	0.0895	180
RSM [8]	3.09	0.242	0.389	0.082	495
GA [11]	3.05	0.246	0.400	0.122	2400
TEAM [10]	3.08	0.239	0.394	0.088	-

Acknowledgements

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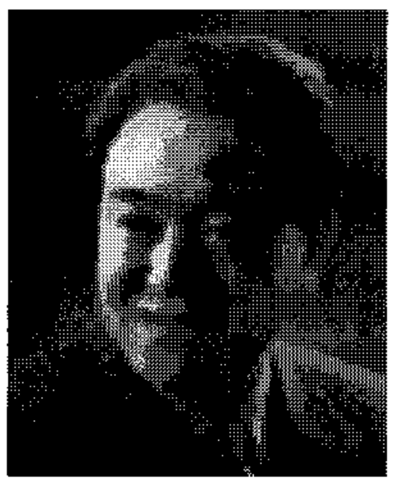
Yanli Zhang
She received her B.S., M.S., and Ph.D. degrees in Electrical Engineering from Shenyang University of Technology, Shenyang, China, in 1998, 2001, and 2006, respectively. Her research interests include numerical analysis and optimal design of electrical machines. She can be contacted at zhangyanli_sy@hotmail.com.

Hee Sung Yoon
He received his B.S. and M.S. degrees in the Department of Electrical and Electronics Engineering, Chungbuk National University in 2005 and 2007, respectively. He is currently working towards his Ph.D. degree in Electric Machines. His research interests include numerical analysis of electromagnetic fields and shape optimal design of electromagnetic devices. He can be contacted at hsyoon@chungbuk.ac.kr, (telephone) +82-43-274-2426, and (fax) +82-43-274-2426.

Pan-Seok Shin
He received his B.S. degree in Electrical Engineering from Seoul National University in 1977, and his M.S. and Ph.D. degrees in Electric Power Engineering from the Rensselaer Polytechnic Institute in 1986 and 1989, respectively. During 1980-1993, he worked at KERI as a Researcher for the projects of magnetically levitated vehicles, various motors, and linear actuators. In 1993, he joined the Electrical Engineering Department of Hong-Ik University as a Professor. His teaching and research interests are in the analysis of electromagnetic fields, electric machinery, special actuators, and optimization techniques. He can be contacted at psshin@hongik.ac.kr.



Chang Seop Koh
He received his B.S., M.S., and Ph.D. degrees in Electrical Engineering from Seoul National University, Seoul, Korea, in 1982, 1986, and 1992, respectively. He was a Visiting Professor at the Department of Electrical and Computer Engineering, Florida International University, Florida, USA, and at the Department of Electrical and Computer Engineering, Texas A&M University, Texas, USA, from May 1993 to April 1994, and from February 2003 to January 2004, respectively. He was also a Senior Researcher at the Central Research Institute of Samsung Electro- Mechanics Co., Ltd., from May 1994 to August 1996. He has been a Professor with the School of Electrical and Computer Engineering, Chungbuk National University, Korea, since 1996. His research interests include electric machine design, and numerical analysis of electric machines using the finite element and boundary element methods. He can be contacted at kohcs@chungbuk.ac.kr, (telephone) +82-43-261-2426, and (fax) +82-43-274-2426.