# Outage Capacity Analysis for Cooperative DF and AF Relaying in Dissimilar Rayleigh Fading Channels

Suchitra Shrestha* *Associate Member*, KyungHi Chang* *Lifelong Member*

## ABSTRACT

Cooperative relaying permits one or more relay to transmit a signal from the source to the destination, thereby increasing network coverage and spectral efficiency. The performance of cooperative relaying is often measured as outage probability. However, appropriate measure for the channel quality is outage capacity. Although the outage probability for cooperative relaying protocol has been analyzed before, very little research has been addressed for the outage capacity. This paper is the first of its kind to derive a closed-form analytical solution of outage capacity using fixed decode and forward relaying and amplify and forward relaying in dissimilar Rayleigh fading channels, considering channel coefficients known to the receiver side. The analytical results show a tradeoff between the SNR and the number of relays for specific outage capacity. A comparison between decode and forward relaying and amplify and forward relaying shows that decode and forward relaying outperforms amplify and forward relaying for a large number of relays.

**Key Words :** Amplify and forward (AF) relaying, Cooperative relaying protocol, Decode and forward (DF) relaying, Outage capacity, Outage probability

## I. Introduction

Capacity is one of the most important quantities in communication systems. There are two main types of capacity of interest; ergodic capacity and outage capacity. Ergodic capacity, which represents the average capacity among all the channel states, is specifically chosen for the fast fading channels[1]. The outage capacity, on the other hand, which represents the maximum capacity over channel for prescribed outage probability, is relevant for slow fading channels. The notion of outage capacity is more practical in real applications, hence our main focus is on the outage capacity.

While developing future wireless systems such as IMT-advanced system, researchers are facing various challenges with regard to coverage and capacity. Cooperative relaying is one of the promising technology for solving these foreseen problems. There are various protocols of cooperatve relaying. Decode & forward (DF) and amplify & forward (AF) relaying are the most popular due to their simplicity. In the DF protocol, the source transmits a signal to the relays and the destination. The relay, which is able to decode the transmitted signal from the source, first decodes and retransmits the signal to the destination. However, in the AF relaying protocol, the relay node simply amplifies the received signal and forwards it directly to the destination.

The outage probability for independent and identically distributed (i.i.d.) channels and bounds for independent but non identical (i.ni.d.) channels using the DF relaying protocol has been derived in[2]. Later in[3], a closed-form expression of the outage probability for dissimilar Rayleigh fading

* 인하대학교 정보통신대학원 이동통신연구실 (khchang@inha.ac.kr)
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channels is presented for the same protocol. Similarly, a closed-form expression of the outage probability using AF relaying has been derived in[4].

Most of the previous works address the outage probability as a performance measure of cooperative relaying, the outage capacity is yet to be analyzed. In our novel approach, we derive a closed-form solution of the outage capacity using fixed DF and AF cooperative relaying in dissimilar Rayleigh fading channels. Furthermore, we present numerical results based on the closed-form solution of outage capacity, and discuss the performance of the DF and the AF in terms of the SNR and the number of relays.

The remaining part of the paper is outlined as follows. In section II, we describe the system model for the cooperative relaying under consideration. In section III, we derive the outage capacity using fixed DF and AF relaying protocol. Numerical results, based on closed-form solution of outage capacity, are discussed in section IV, and finally section V gives concluding remarks. For smooth flow of the paper, some of the proofs are appended in the Appendix.

## Ⅱ. System Model for Cooperative Relay

We consider a cooperative relay network as shown in fig. 1. The source node transmits a information to the destination node through a direct path and through m sets of relay nodes. In addition, we statistically modeled the channel gain

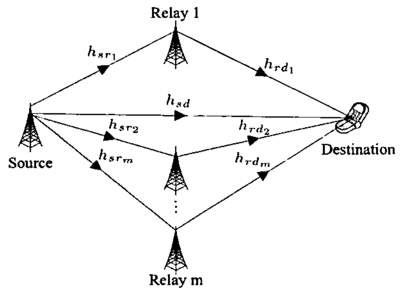



Fig. 1. Relay channel with three nodes: source, relay, and destination.

between the source node and the destination node $h_{sd}$, the source node and the relay nodes $h_{sr}$, and the relay nodes to the destination node $h_{rd}$, as zero mean circularly symmetric complex Gaussian random variables[5]. The magnitude square of channel gain from the source to the destination, the source to the relay, and the relay to the destination are as below, respectively.

$$\begin{aligned} x_0 &= |h_{sd}|^2 \\ \hat{x}_c &= |h_{sr}|^2 \\ x_c &= |h_{rd}|^2 \end{aligned} \tag{1}$$

These square magnitude of channel gains are assumed to be exponentially distributed random variables with parameters of $\lambda_0$, $\hat{\lambda}_c$, and $\lambda_c$, respectively[5]. Probability distribution functions of $x_0$, $\hat{x}_c$, and $x_c$ are in order as shown below.

$$\begin{aligned} f(x_0;\lambda_0) &= \lambda_0 e^{-\lambda_0 x_0} \\ f(\hat{x}_c;\hat{\lambda}_c) &= \hat{\lambda}_c e^{-\hat{\lambda}_c \hat{x}_c} \\ f(x_c;\lambda_c) &= \lambda_c e^{-\lambda_c x_c} \end{aligned} \tag{2}$$

Furthermore, We consider time division multiple access (TDMA) arrangement with $m+1$ time slots to facilitate the orthogonal transmission. That is, in the first time slot, the source node broadcasts its information to the destination and the relay nodes. In the following m slots, the relay nodes transmit their information to the destination.

## Ⅲ. Outage Capacity Analysis for Cooperative Relay

In this section, we first find the maximum instantaneous capacity of the network shown in fig. 1. We, then derive the outage probability and, subsequently, closed-form solution of the outage capacity for the DF and AF cooperative relaying.

### 3.1 Direct Transmission

On the basis of information theory[6], we can express the mutual information between the source and the destination without relay as follows:

$$I_{SD} = \log_2(1 + SNR x_0) \tag{3}$$

where SNR is the transmit signal to noise ratio.

### 3.2 Decode and Forward Relay Transmission

As shown in[5], the mutual information between the source and the relay nodes $c = 1,2,\ldots,m$ is described by the following equation:

$$I_{SR} = \frac{1}{(m+1)} \cdot \log_2(1 + SNR \hat{x}_c) \tag{4}$$

where $m$ is the number of relays. If the mutual information between source and relay $I_{SR}$ is greater than some spectral efficiency $R$, then the $c-th$ relay is able to successfully decode the transmitted signal from the source and belongs to the decoding set $C$. The mutual information using the DF relaying[5] is given by

$$I_{DF} = \frac{1}{(m+1)} \cdot \log_2(1 + SNR x_0 + SNR x_c) \tag{5}$$

The capacity of the entire network using the DF relaying is minimum of two mutual information[7].

$$C(\gamma) = \min(I_{SR}, I_{DF}) \tag{6}$$

where $I_{SR}$ and $I_{DF}$ are given by equations (4) and (5), respectively.

The outage probability $P_{out}$ which can be defined as the probability that instantaneous capacity fall below outage capacity $C_{out}$[1], is expressed as follows:

$$P_{out} = \Pr[C(\gamma) < C_{out}] \tag{7}$$

where $C_{out}$ is given by equation (6). The outage event $C(\gamma) < C_{out}$ can be expressed as

$$\min[\hat{x}_c, x_0 + x_c] < \frac{2^{C_{out}(m+1)} - 1}{SNR} \tag{8}$$

Furthermore, let

$$w = \frac{2^{C_{out}(m+1)} - 1}{SNR} \tag{9}$$

Then, by using order statistics as in[8], we get

$$\Pr\left[\min(\hat{x}_c, x_0 + x_c) < w\right] = F(\hat{x}_c) + F(x_0 + x_c) - F(\hat{x}_c).F(x_0 + x_c) \tag{10}$$

Moreover, by using sum of exponential random variables, we get

$$\begin{aligned} &\Pr\left[\min(\hat{x}_c, x_0 + x_c) < w\right] \\ &= (1 - e^{-\hat{\lambda}_c w}) + \left(1 + \frac{\lambda_c e^{-\lambda_0 w}}{\lambda_c - \lambda_0} - \frac{\lambda_0 e^{-\lambda_c w}}{\lambda_c - \lambda_0}\right) \\ &- (1 - e^{-\hat{\lambda}_c w}).\left(1 + \frac{\lambda_c e^{-\lambda_0 w}}{\lambda_c - \lambda_0} - \frac{\lambda_0 e^{-\lambda_c w}}{\lambda_c - \lambda_0}\right) \end{aligned} \tag{11}$$

Rewriting equation (11), we can express the outage probability as

$$P_{out} = \Pr[\hat{x}_c < w] + P_r[\hat{x}_c > w].\Pr[(x_0 + x_c) < w] \tag{12}$$

The outage probability for one relay, which is in the case for $m = 1$, can then be expressed as follows

$$\begin{aligned} P_{out_1} &= (1 - e^{-\hat{\lambda}_1 w}) + (1 - 1 + e^{-\hat{\lambda}_1 w}) \\ &\cdot \left[1 + \frac{1}{(\lambda_1 - \lambda_0)}(\lambda_1 e^{-\lambda_0 w} - \lambda_0 e^{-\lambda_1 w})\right] \\ &= 1 - \frac{\lambda_0 e^{-(\hat{\lambda}_1 + \lambda_1) w}}{(\lambda_1 - \lambda_0)} + \frac{\lambda_1 e^{-(\hat{\lambda}_1 + \lambda_0) w}}{(\lambda_1 - \lambda_0)} \end{aligned} \tag{13}$$

The terms $e^{-(\hat{\lambda}_1 + \lambda_1) w}$ and $e^{-(\hat{\lambda}_1 + \lambda_0) w}$ in equation (13) can be expanded as below.

$$\begin{aligned} e^{-(\hat{\lambda}_1 + \lambda_1) w} &= 1 - \frac{1}{1!}(\hat{\lambda}_1 + \lambda_1) w + \frac{1}{2!}(\hat{\lambda}_1 + \lambda_1)^2 w^2 + \ldots \\ &\approx 1 - (\hat{\lambda}_1 + \lambda_1) w \end{aligned} \tag{14}$$

Similarly,

$$e^{-(\hat{\lambda}_1 + \lambda_0) w} \approx 1 - (\hat{\lambda}_1 + \lambda_0) w \tag{15}$$

In the above series, we neglected the terms higher than the first order. In order to neglect higher terms, the value of w must be as small as possible, i.e., $w \ll 1$. Since $w$ is inversely proportional to SNR from equation (9), high value of SNR must be considered for the low value of $w$. The outage probability of the DF relaying with one relay node can be expressed as

$$P_{out_1} = \hat{\lambda}_1 w \tag{16}$$

From equation (16), outage capacity using one relay is expressed as

$$C_{out_1} = \frac{1}{2} \cdot \log_2 \left[ 1 + SNR \frac{P_{out_1}}{\hat{\lambda}_1} \right] \tag{17}$$

We can determine outage probability for $m$ relays in a similar manner. The outage probability of the DF relaying with $m$ relay nodes can be obtained as

$$P_{out} = \frac{\prod_{c=1}^{m} \hat{\lambda}_c w^m}{m!} \tag{18}$$

By substituting the values of $w$ in equation (9) to equation(18), the closed form solution of outage capacity for the DF relaying can be obtained as

$$C_{out} = \frac{1}{m+1} \cdot \log_2 \left[ 1 + SNR \left( \frac{m! P_{out}}{\prod_{c=1}^{m} \hat{\lambda}_c} \right)^{\frac{1}{m}} \right] \tag{19}$$

### 3.3 Amplify and Forward Relay Transmission

The mutual information between the source and the destination using AF relaying[5] is expressed as

$$I_{AF} = \frac{1}{(m+1)} \cdot \log_2 \left[ 1 + SNR x_0 + \frac{SNR x_c SNR \hat{x}_c}{(1 + SNR x_c + SNR \hat{x}_c)} \right] \tag{20}$$

From the definition of outage probability in (7), we get

$$P_{out} = \Pr \left[ (x_0 + \frac{1}{SNR} f(SNR x_c, SNR \hat{x}_c) < \frac{2^{C_{out}(m+1)} - 1}{SNR} \right] \tag{21}$$

For $m = 1$, we can utilize the CDF of sum, product and quotient of random variables, we get

$$\lim_{SNR \to \infty} \frac{1}{w^2} Pr \left[ x_0 + \frac{1}{SNR} f(SNR \hat{x}_1, SNR x_1) < w \right] = \frac{\lambda_0 (\hat{\lambda}_1 + \lambda_1)}{2} \tag{22}$$

$$P_{out_1} = w^2 \frac{\lambda_0 (\hat{\lambda}_1 + \lambda_1)}{2} \tag{23}$$

$$C_{out_1} = \frac{1}{2} \log_2 \left( 1 + \mathrm{SNR} \left[ \frac{2! \mathrm{P}_{out_1}}{\lambda_0 (\hat{\lambda}_1 + \lambda_1)} \right]^{1/2} \right) \tag{24}$$

We can find outage capacity for $m$ number of relays in a similar manner. That is, the closed-form solution of outage capacity for the AF relaying can be obtained as below.

$$C_{out} = \frac{1}{m+1} \cdot \log_2 \left( 1 + SNR \left[ \frac{(m+1)! P_{out}}{\lambda_0 \prod_{c=1}^{m} (\hat{\lambda}_c + \lambda_c)} \right]^{1/(m+1)} \right) \tag{25}$$

## Ⅳ. Numerical Results

### 4.1 Decode and Forward Relay Transmission

We have so far analysed a closed-form solution of outage capacity for both DF and AF relaying. In this section, we analyze the outage capacity of the DF relay transmission in terms of the outage probability, the SNR, and the number of relays. Fig.2 reveals the outage capacity for constant outage probability of 10 %. At SNR of 15 dB, the highest outage capacity is achieved by the system with two relays. However, as the SNR increases to 25 dB, the system with one relay has the maximum outage capacity. i.e., in DF

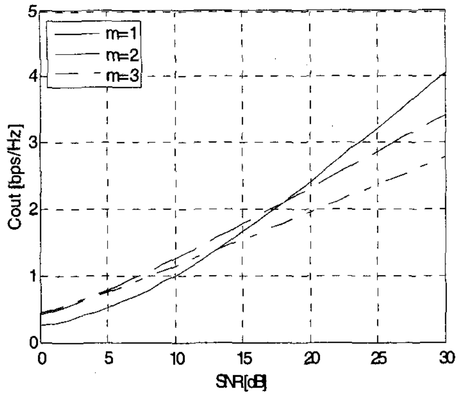
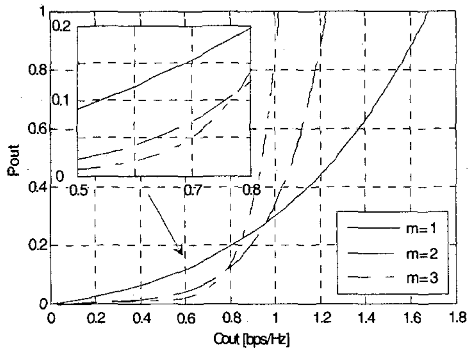
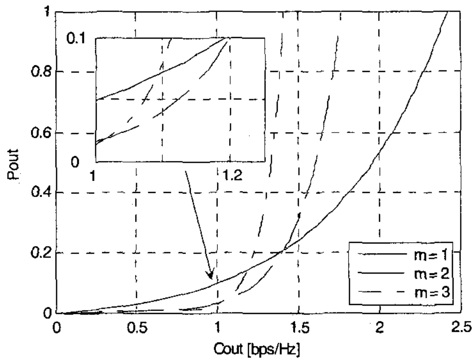



Fig. 2. Outage capacity for different values of m for $P_{out}=0.1$ in DF relay transmission.




(a)




(b)

Fig. 3. Outage capacity vs. outage probability for different values of m in DF relay transmission. (a) $SNR=5\,dB$. (b) $SNR=10\,dB$.

relaying, the number of relay can be decreased under a high SNR as expected.

Fig. 3.(a) and 3.(b) show the outage capacity with fixed SNR of 5 dB and 10 dB, respectively. The figures show that, for specific outage capacity the outage probability decreases with increase in number of relays. Nonetheless, after the tradeoff point, an increase in number of relay does not mean a decrease in outage probability. This is due to the exploitation of the time division channel allocation, which allows less time for the source to transmit signal as number of relays increases, which in turn increases the possibility of outage condition. From these figures we also depict that the tradeoff point depends upon the SNR value.

### 4.2 Amplify and Forward Relay Transmission

In this section, we analyze the outage capacity of AF relay transmission. Fig. 4 depicts the outage capacity for constant probability of 10 %. The maximum outage capacity is achieved by the system with one relay for both SNR of 15 dB as well as SNR of 25 dB. That is, in contrast to DF relaying, the performance tradeoff due to the relationship between SNR and the number of relays does not exist in AF relaying, except at very low SNR, around 0 to 1.5 dB.

Fig. 5.(a) and 5.(b) represent the scenario with fixed SNR of 5 dB and 10 dB, respectively. As in DF relaying, these figures show that outage capacity depends upon the number of relays and the value of SNR. We can decrease the outage probability by increasing the number of relays but up to tradeoff point. After tradeoff point, an

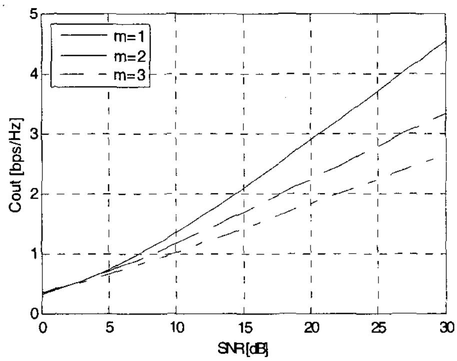
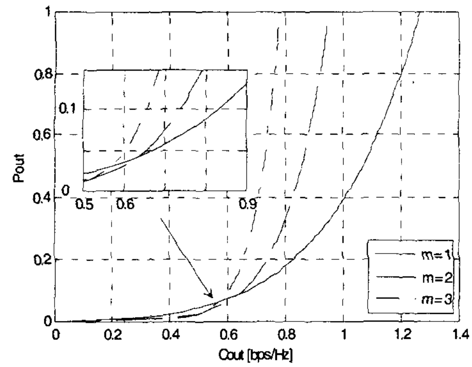
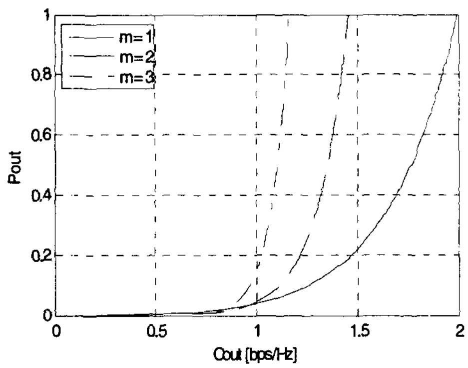



Fig. 4. SNR vs. outage capacity for different values of m for $P_{out}=0.1$ in AF relay transmission.




(a)




(b)

Fig. 5. Outage capacity vs. outage probability for different values of m in AF relay transmission.
(a) $SNR = 5\ dB$. (b) $SNR = 10\ dB$.

increase in the number of relays does not mean a decrease in outage probability as in the case of DF relaying. In addition, it is also observed that tradeoff outage capacity of the AF relaying is smaller than that of the DF relaying.

### 4.3 DF vs. AF Relay Transmissions

This section compares the performance of DF relaying with that of AF relaying. Fig. 6 shows that AF relaying outperforms DF relaying in the aspect of outage probability, for the system with relay less than three, at low outage capacity. However, as the outage capacity increases, DF relaying outperforms AF relaying. Furthermore, we also observe that DF relaying outperforms AF relaying for the system with more than three relays. i.e., DF relaying is more suitable for large the number of relays as compared to AF relaying. This is due to the fact that effect of noise amplification in AF relaying becomes more pronounced as the number of relay is increased. However, in DF relaying, the noise-free decoding by deploying more number of DF relays expands the coverage and enhance the capacity.

Fig. 7 shows that AF relaying outperforms DF relaying for low value of outage capacity, howevever, DF relaying gets better with increasing outage capacity. In addition, from fig. 7 it is also clear that tradeoff outage capacity increases with the SNR values. These phenomena are consistent with the value of *m*.

Fig. 8 shows that for a fixed value of SNR and the number of relays, DF relaying outperforms AF relaying for the outage capacity of 70 %. However, for low value of outage probability, such as 10%, since AF relaying is so reliable that DF relaying transmission costs more

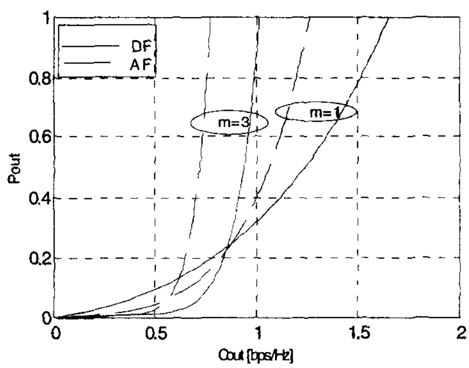



Fig. 6. Comparison of DF and AF relay transmission for different m at 5 dB SNR.

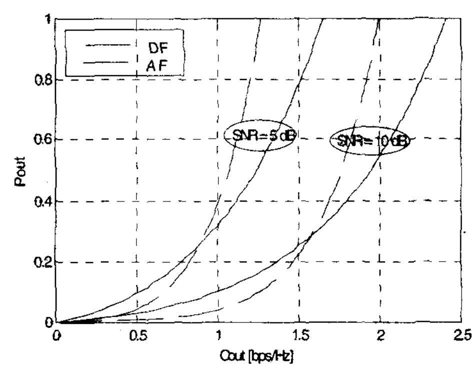



Fig. 7. Outage capacity vs. outage probability of DF and AF at different values of SNR.

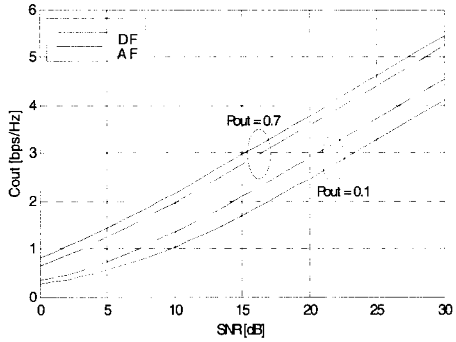



Fig. 8. SNR vs. Outage capacity of DF and AF at different values of outage probability for $m=1$.

latency due to the operation of decode and forward. That is why AF relaying shows more outage capacity than DF relaying at 10 % of outage probability.

## V. Conclusions

In this paper, we present an a closed-form solution of the outage capacity for dissimilar Rayleigh fading channels using DF and AF relaying. Outage capacity versus outage probability is analysed for different values of SNR with a variable number of relays. For both DF and AF relaying, we observe that the outage probability decreases with increase in the number of relays, for specific outage capacity, but after tradeoff point, an increase in the number of relays does not mean a decrease in the outage probability, due to the exploitation of time division channel allocation, which increase the possibility of outage condition. Moreover, it is also illustrated that the tradeoff outage capacity depends on the value of SNR. For large number of relays, it is worthy to use DF relaying as compared to AF relaying, because more number of relays mean more noise amplification in AF relaying, but coverage expansion and capacity enhancement are expected due to the noise-free decoding in DF relaying. In this paper, the proposed analysis is based on the high SNR assumption, which is the necessary condition to obtain the closed-form analytical result. However, there may be some performance gap in the low SNR range due to the above assumption. Further research might be recommended to relax the high SNR assumption.

## APPENDIX A
## CDF of Sum of Random variables$(x_0+x_c)$.

Let $x_0$ and $x_c$ be the exponential random variable with parameters $\lambda_0$ and $\lambda_c$ respectively. Let $w$ be the sum of two random variables $x_0$ and $x_c$, i.e.,

$$w=x_0+x_c \tag{A-1}$$

The PDF of these two random variables is the convolution of the two individual PDFs. For $\lambda_0 \neq \lambda_c$,

$$\begin{aligned} p_{w(w)} &= \int_{-\infty}^{\infty} p_{x_0}p_{x_c}(w-x_0)dx_0 \\ &= \int_0^w \lambda_0 e^{-\lambda_0 x_0}\lambda_{x_c} e^{-\lambda_c(w-x_0)}dx_0 \\ &= \lambda_0\lambda_c e^{-\lambda_c w}\int_0^w e^{(\lambda_c-\lambda_0)x_0}dx_0 \\ &= \frac{\lambda_0\lambda_c}{(\lambda_c-\lambda_0)}[e^{-\lambda_0 w}-e^{-\lambda_c w}] \end{aligned} \tag{A-2}$$

Now, taking integration of PDF (A-2), CDF for $\lambda_0 \neq \lambda_c$ becomes

$$\begin{aligned} &\Pr[(x_0+x_c)<w] \\ &=1+\frac{\lambda_0}{(\lambda_c-\lambda_0)}e^{-\lambda_c w}-\frac{\lambda_c}{(\lambda_c-\lambda_0)}e^{-\lambda_0 w} \end{aligned} \tag{A-3}$$

## APPENDIX B
## CDF of Random variable $\left(\frac{\hat{x_c}x_c}{\hat{x_c}+x_{c+1}}\right)$

For any positive value of $\delta$, let $r_\delta=f(\frac{\hat{x_c}}{\delta},\frac{x_c}{\delta})$ where $\hat{x_c}$ and $x_c$ are independent exponential random variables with parameters $\hat{\lambda_c}$ and $\lambda_c$, respectively. Let $h(\delta)>0$ be continuous with $h(\delta)\rightarrow 0$, as $\delta\rightarrow 0$. Then $\Pr[r_\delta<\delta]$ satisfies

$$\lim_{\delta\rightarrow 0}\frac{1}{h(\delta)}Pr[r_\delta<h(\delta)]=\hat{\lambda_c}+\lambda_c \tag{B-1}$$

Proof:

$$\begin{aligned}\Pr[r_\delta<\delta] &= \Pr\left[\frac{\hat{x}_c x_c}{\hat{x}_c+x_c+\delta}\le h(\delta)\right]\\ &= \Pr\left[\frac{1}{\hat{x}_c}+\frac{1}{x_c}+\frac{\delta}{\hat{x}_c x_c}\ge\frac{1}{h(\delta)}\right]\\ &= \Pr\left[\max\left(\frac{1}{\hat{x}_c},\frac{1}{x_c}\right)\ge\frac{1}{h(\delta)}\right]\\ &= 1-\Pr\left[\max\left(\frac{1}{\hat{x}_c},\frac{1}{x_c}\right)\le\frac{1}{h(\delta)}\right]\\ &= 1-\Pr\left[\frac{1}{\hat{x}_c}\le\frac{1}{h(\delta)}\right].\Pr\left[\frac{1}{x_c}\le\frac{1}{h(\delta)}\right]\\ &= 1-\Pr\left[\hat{x}_c\ge h(\delta)\right].\Pr\left[x_c\ge h(\delta)\right]\\ &= 1-\exp[-\hat{\lambda}_c h(\delta)].\exp[-\lambda_c h(\delta)]\\ &= 1-\exp[-(\hat{\lambda}_c+\lambda_c)h(\delta)] \end{aligned} \tag{B-2}$$

Expansion of exponential term leads to

$$\begin{aligned}&\exp[-(\hat{\lambda}_c+\lambda_c)h(\delta)]\\ &\quad=1-(\hat{\lambda}_c+\lambda_c)h(\delta)+(\hat{\lambda}_c+\lambda_c)^2h^2(\delta)-\ldots\\ &\quad\approx 1-(\hat{\lambda}_c+\lambda_c)h(\delta)\end{aligned} \tag{B-3}$$

By substituting the values of (B-3) into (B-2)

$$\Pr[r_\delta<\delta]=(\hat{\lambda}_c+\lambda_c)h(\delta) \tag{B-4}$$

Thereby, taking limits on both sides,

$$\lim_{\delta\to 0}\frac{1}{h(\delta)}Pr[r_\delta<\delta]=(\hat{\lambda}_c+\lambda_c) \tag{B-5}$$

## APPENDIX C
## CDF of Random variable $\left(x_0+\frac{\hat{x}_c x_c}{\hat{x}_c+x_c+1}\right)$

Let $x_0$, $\hat{x}_c$ and $x_c$ be the independent exponential random variables with parameters $\lambda_0$, $\hat{\lambda}_c$, and $\lambda_c$, respectively. Let $g(\epsilon)>0$ be continuous with $g(\epsilon)\to 0$, as $\epsilon\to 0$. Then

$$\begin{aligned}&\lim_{\varepsilon\to 0}\frac{1}{g^2(\varepsilon)}Pr\left[x_0+\varepsilon f\left(\frac{\hat{x}_c}{\varepsilon},\frac{x_c}{\varepsilon}\right)<g(\varepsilon)\right]\\ &\quad=\frac{\lambda_0(\hat{\lambda}_c+\lambda_c)}{2}\end{aligned} \tag{C-1}$$

Proof:

$$\begin{aligned}&\Pr\left[x_0+\varepsilon f\left(\frac{\hat{x}_c}{\varepsilon},\frac{x_c}{\varepsilon}\right)<g(\varepsilon)\right]\\ &=\Pr[x_0+r_\varepsilon<g(\varepsilon)]\\ &=\int_0^{g(\varepsilon)}\Pr[r_\varepsilon<g(\varepsilon)-x_0].p_{x_0}x_0dx_0\end{aligned} \tag{C-2}$$

Let

$$\begin{aligned}x_0' &= g(\epsilon)x_0'\\ dx_0' &= g(\epsilon)dx_0'\end{aligned} \tag{C-3}$$

By changing limits,

$$\begin{aligned}x_0'=0\ &;x_0=0\\ =1&;x_0=g(\epsilon)\end{aligned} \tag{C-4}$$

Thus we get,

$$\begin{aligned}&\Pr\left[x_0+\varepsilon f\left(\frac{\hat{x}_c}{\varepsilon},\frac{x_c}{\varepsilon}\right)<g(\varepsilon)\right]\\ &=\int_0^1\Pr[r_\varepsilon<g(\varepsilon)(1-x_0')].p_{x_0}(g(\varepsilon)x_0')g(\varepsilon)dx_0'\\ &=\lambda_0g^2(\varepsilon)\\ &\quad.\int_0^1\frac{\Pr[r_\varepsilon<g(\varepsilon)(1-x_0')](1-x_0)'e^{-\lambda_0g(\varepsilon)x_0'}}{g(\varepsilon)(1-x_0)'}dx_0'\end{aligned} \tag{C-5}$$

Finally, taking limits on both sides results in

$$\begin{aligned}&\lim_{\varepsilon\to 0}\frac{1}{g^2(\varepsilon)}Pr\left[x_0+\varepsilon.f\left(\frac{\hat{x}_c}{\varepsilon},\frac{x_c}{\varepsilon}\right)<g(\varepsilon)\right]\\ &=\int_0^1\frac{\lambda_0\Pr[r_\varepsilon<g(\varepsilon)(1-x_0')](1-x_0')e^{-\lambda_0g(\varepsilon)x_0'}}{g^2(\varepsilon)g(\varepsilon)(1-x_0')}dx_0'\\ &=\lambda_0(\hat{\lambda}_c+\lambda_c)\int_0^1(1-x_0')dx_0'\\ &=\frac{\lambda_0(\hat{\lambda}_c+\lambda_c)}{2}\end{aligned} \tag{C-6}$$

## APPENDIX D
## Derivation of Outage Capacity Using AF Relaying For m Number of Relays.

PDF of $P_{out_1}$ from (23) becomes

$$\begin{aligned}pdf_1 &= \frac{d}{dg(\epsilon)}g^2(\varepsilon)\frac{\lambda_0(\hat{\lambda}_1+\lambda_1)}{2}\\ &= g(\varepsilon)\lambda_0(\hat{\lambda}_1+\lambda_1)\end{aligned} \tag{D-1}$$

Now, for $m=2$ ,

$$u=g(\epsilon)\lambda_0(\hat{\lambda}_1+\lambda_1)$$
$$v=\hat{\lambda}_2+\lambda_2 \tag{D-2}$$

Let

$$y=u+v \tag{D-3}$$

Then, by using the sum of two random variable as in Appendix A, CDF for $m=2$ becomes

$$P_{out_2}=\lambda_0(\hat{\lambda}_1+\lambda_1)(\hat{\lambda}_2+\lambda_2)\frac{g^3(\varepsilon)}{2.3} \tag{D-4}$$

From (D-4), outage capacity for $m=2$ results in as below.

$$C_{out_2}=\frac{1}{3}\cdot\log_2\left(1+SNR\left[\frac{3!P_{out_2}}{\lambda_0(\hat{\lambda}_1+\lambda_1)(\hat{\lambda}_2+\lambda_2)}\right]^{1/3}\right) \tag{D-5}$$

Similarly, for $m=3$,

$$u=\lambda_u(\hat{\lambda}_1+\lambda_1)(\hat{\lambda}_2+\lambda_2)\frac{g^2(\varepsilon)}{2}$$
$$v=(\hat{\lambda}_3+\lambda_3) \tag{D-6}$$

$$P_{out_3}=\lambda_0(\hat{\lambda}_1+\lambda_1)(\hat{\lambda}_2+\lambda_2)(\hat{\lambda}_3+\lambda_3)\frac{g^4(\varepsilon)}{2.3.4} \tag{D-7}$$

From (D-7), outage capacity for $m=3$ can be obtained as

$$C_{out_3}=$$
$$\frac{1}{4}\cdot\log_2\left(1+SNR\left[\frac{4!P_{out_3}}{\lambda_0(\hat{\lambda}_1+\lambda_1)(\hat{\lambda}_2+\lambda_2)(\hat{\lambda}_3+\lambda_3)}\right]^{1/4}\right) \tag{D-8}$$

Finally, the generalized solution of the outage capacity using AF relaying is solved as

$$C_{out}=\frac{1}{m+1}\cdot\log_2\left(1+SNR\left[\frac{(m+1)!P_{out}}{\lambda_0\prod_{c=1}^{m}(\hat{\lambda}_c+\lambda_c)}\right]^{1/(m+1)}\right) \tag{D-9}$$

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**스레스타** (Suchitra Shrestha) 준회원

2005년1월 Institute of Engineering, T.U., Nepal, BE, Electrical Engineering (공학사)

2006년 9월~현재 인하대학교 정보 통신대학원 (공학석사)

<관심분야>Cooperative Relaying System, Information Theory, Radio Resource Management, MIMO System.

**장 경 희** (KyungHi Chang) 종신회원

1985년 2월 연세대학교 전자공학과 (공학사)

1987년 2월 연세대학교 전자공학과 (공학석사)

1992년 8월 Texas A&M Univ., EE Dept. (Ph.D.)

1989년~1990년 삼성종합기술원 주임연구원

1992년~2003년 한국전자통신연구원, 이동통신연구소 무선전송방식연구팀장 (책임연구원)

2003년~현재 인하대학교 정보통신대학원 부교수

<관심분야> 4세대 이동통신 및 3GPP LTE 무선전송방식, WMAN 및 DMB 시스템 무선전송기술, Cognitive Radio, Cross-layer Design, Cooperative Relaying System, RFID / USN Systems.