

# Application of Field test to the Rate-dependent relation in Constitutive model

Youngsun Song<sup>1\*</sup>

## 변형률속도-의존 구성모델의 현장 시험 적용

송용선<sup>1\*</sup>

**Abstract** The rate-dependent constitutive model and the rate-independent model were analyzed for comparing with the piezocone penetration test and dissipation test. The mathematical expressions of the rate-dependent constitutive model were introduced, and the predictions using the both models were compared with the experimental results of in-situ field test. The rate-dependent model analysis gave better results than the rate-independent model analysis, that is appreciated since the process of the piezocone penetration and dissipation depends on the strain rate. Therefore, it is recommended the concept of the rate-dependent model for the prediction of soil behaviors using the field penetration test.

**Key words** : rate-dependent constitutive model, piezocone penetration test, strain rate

**요 약** 본 연구에서는 실제 현장에서 수행한 피에조콘관입 및 소산시험결과와 변형률 속도를 고려한 구성모델의 이론적 해석을 비교하였다. 이를 위하여 변형률 속도 의존적 구성모델의 수학적 유도과정을 전개하였다. 해석 결과 변형률속도를 무시한 구성모델보다 변형률속도-의존 구성모델을 활용한 지반의 거동 해석이 비의존적 구성모델인 경우의 해석 보다 현장의 시험결과와 잘 일치하므로 변형률속도-의존 구성모델의 적용이 바람직하다.

## 1. Introduction

Recently, modern constitutive models develops to estimate the behavior of cohesive soils considering the stress anisotropy and time-dependent effect. The stress anisotropy has been described in the form of the development of isotropic elastoplastic constitutive models[1][2][3].

Such time-dependent behaviors of cohesive soils as creep, stress relaxation, and strain rate effect might be properly simulated with the viscous theory, not with the elastoplastic theory alone. The general elastoviscous theory proposed by Perzyna(1966) has been, in many cases, adopted for the time-dependent constitutive

models since the theory has the merits of generality and practical usefulness[4][5][6][7]. The models based on Perzyna's theory, however, mainly focus on the description of the creep and stress relaxation other than the strain rate effect[2] [4][8][9][10].

In this study, the rate-independent constitutive model, utilizing the generalized Hooke's law and the modified Cam-Clay model, and the rate-dependent model, utilizing the Perzyna's generalized viscous theory, were mathematically formulated and the predictions from the both models were compared with the experimental results of the piezocone penetration and dissipation tests conducted in ○○ site. The concepts, the functions, and the mathematical formulations of the models are briefly presented in the following sections.

<sup>1</sup> Professor, Department of Civil and Environmental Engineering, Kongju University

\* Corresponding author: Youngsun Song(E-mail: ssong@kongju.ac.kr)

## II. Formulation of Rate-dependent Constitutive model

The strain rate can be decomposed into an elastic part and an inelastic part, assuming small deformation and rotations, the later consisting a delayed(viscoplastic) and an instantaneous(plastic) part[11][13]. Analytically these are expressed by

$$\dot{\epsilon}_{ij} = \dot{\epsilon}_{ij}^e + \dot{\epsilon}_{ij}^i = \dot{\epsilon}_{ij}^e + \dot{\epsilon}_{ij}^e + \dot{\epsilon}_{ij}^p + \dot{\epsilon}_{ij}^v \quad (1)$$

Concerning the each elastic, plastic, and viscoplastic response and using the general function of the state, that is defined by the generalized Hooke's law and the classical plasticity, the above expression can be reformed as

$$\dot{\epsilon}_{ij} = C_{ijkl} \dot{\sigma}_{kl} + \langle L \rangle R_{ij}^p + \langle \Phi \rangle R_{ij}^v \quad (2)$$

where  $C_{ijkl}$  represents the fourth order tensor of elastic compliance,  $L$  is the scalar loading index, and  $\Phi$  is the proper continuous scalar function of the overstress, that represents the viscoplastic response.

The viscous behavior in eq. (1) is represented by eq. (3), based on the generalized viscous theory[4][9].

$$\dot{\epsilon}_{ij}^v = \langle \Phi \rangle \frac{\partial f}{\partial \sigma_{ij}} \quad (3)$$

where the overstress function  $\Phi$  plays an important role, influencing the plastic behavior, through the coupling effect, as well as the viscous behavior, in the strain rate dependent behavior of clays. Various forms of the overstress function have been proposed; however, the eqs. (4) and (5) has been widely and successfully used for geotechnical materials[7][8][10].

$$\Phi = \frac{1}{V} \exp(J_2/N I_1) (\Delta \hat{\sigma})^n \quad (4)$$

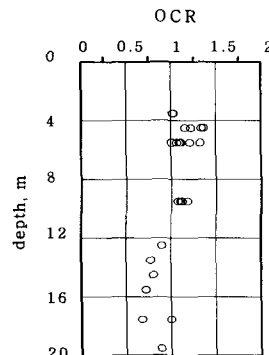
$$\hat{V} = V \frac{1}{1 + \left\langle \left( \frac{2}{3} e_{ij}^i e_{ij}^i \right)^{0.5} - \epsilon_m \right\rangle} \quad (5)$$

where  $J_2$  is the second invariant of the deviatoric stress tensor,  $I_1$  is the first invariant of the stress tensor,  $N$  is the slope of the critical state line in  $I_1$ - $J_2$

space.  $\Delta \hat{\sigma}$  is called normalized overstress and obtained through a closed form solution using the viscous nucleous parameter  $v$ ,  $e_{ij}^i$  is the inelastic deviatoric strain tensor and  $\left\{ (2/3) e_{ij}^i e_{ij}^i \right\}^{0.5}$  is the accumulated inelastic deviatoric strain tensor.  $n, V, \epsilon_m$  are the model parameters related to the viscous behavior.

## III. Application to Field tests

The piezocone penetration and dissipation tests have been conducted in ○○ site, in the vicinity of Pusan city. The site consists of sedimented fine soils near Nak-dong river and is currently undergoing large-scale industrial and residential area development. The soil profile of the test site is composed of sandy and silty soil layer of about 2.0 m thick on top and quite homogeneous clay layer to the depth of 25~30 m. The clay maybe unstable as natural water content is larger than liquid limit in all depth. According to laboratory test results, the clay was classified into CL(by USCS) and it has been also found that  $w_n$  is 54~70 %, LL is 42~57 %, PI is 20~27 %,  $G_s$  is 2.70~2.72,  $\gamma_t$  is 15.4 ~ 16.6 kN/m<sup>3</sup>,  $e_0$  is 1.6~1.9,  $c_v$  is  $8 \times 10^{-2} \sim 2 \times 10^{-1}$  cm<sup>2</sup>/min,  $k$  is  $2.0 \times 10^{-7} \sim 7.5 \times 10^{-8}$  cm/sec, the clay consists of illite (45.8~56.1 %), kaolinite (16.0~24.7 %), and smectite(1.8~10.0 %), and the undrained strength gradually increases with depth from 15 kPa to 40 kPa. Fig. 1 shows the OCR values with depth.



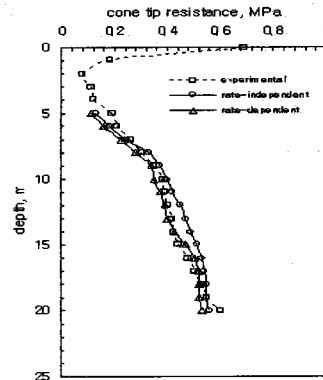
[Fig. 1] Variation of OCR[9]

The piezocone used in this study has 60° cone tip and 10 cm<sup>2</sup> base area with 150 cm<sup>2</sup> friction sleeve and filter element located behind the cone tip (above the cone base). This u2 type filter element is very effective for the nonhomogeneous area correction[9]. The cone penetrates ground using pushing rods connected to loading system. Cone tip resistance  $q_c$ , sleeve friction  $f_s$ , and pore water pressure  $u_2$  were measured. In addition to the penetration test, dissipation of generated pore water pressure with time were measured to evaluate the drainage properties of the soil. The penetration test was conducted at the standard rate of 2 cm/sec.

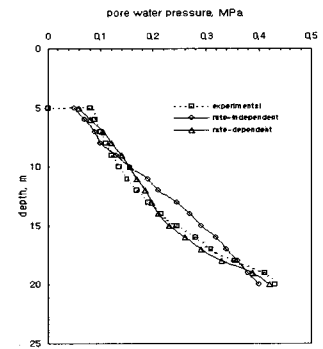
The piezocone penetration was simulated by imposing incremental nodal vertical displacements of the nodes representing the cone tip boundary until failure is achieved. The vertical displacement was applied at the rate of 2.0 cm/sec and the piezocone penetrometer was assumed to be infinitely stiff. No tensile stresses were allowed to develop along the centerline boundaries. The excess pore pressure was obtained assuming undrained condition during penetration.

In the numerical analysis, the total depth of penetration was divided into sub-layers and the steady value of cone tip resistance resulting from the finite element analysis of each layer was obtained, then the steady value of each layer was smoothly connected to each other as cone tip resistance profile. The same procedure was applied to pore water pressure profile. This technique was used to avoid tremendous numerical errors resulting from the numerical simulation of the deep and continuous penetration in one step, and successfully worked in this study but more accurate simulation corresponding to real penetration situation needs further research.

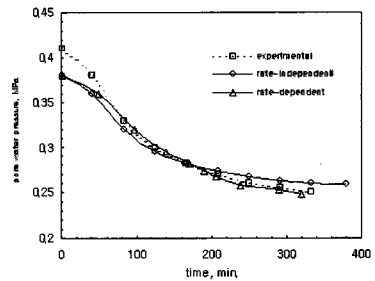
Figs. 2, 3, and 4 show the results of the finite element analysis and the experimental results of the piezocone penetration and dissipation tests conducted in ○○site. In Figs. 2 and 3, the numerical results were presented below 5 m depth since the top crust layer was excluded in the numerical simulation. The peak region like in Fig. 2 generally appears to initiate penetration even without the top crust.



[Fig. 2] Cone tip resistance profile



[Fig. 3] Pore water pressure profile



[Fig. 4] Dissipation curve at 19 m depth

As shown in Figs. 2, 3, and 4, the both model predictions showed good agreement with the experimental data. One reason is that the clay at the site is quite homogeneous. It has been proven during the trimming of samples. The other reason is the use of the Modified Cam-Clay model. It is well known that the Modified Cam-Clay model usually gives good prediction for normally consolidated clays.

The rate-dependent predictions give better results than the rate-independent predictions as expected, that is appreciated since the process of the piezocone penetration and dissipation depends on the testing speed, namely, strain rate. But it is not showed differentiated markedly in Figs. 2, 3, and 4 considering rate-dependent or rate-independent analysis because the constitutive models are inherently made of many parameters in the predictions. The OCR of the site, as shown in Fig. 1, appeared near unity in all depth and even less than unity for the depth below 12 m, which corresponds to the result of the previous research[12][13]. This can be explained by the fact that the clay is underconsolidated maybe due to the sudden rise of sea level following the Ice Age, or the fact that there are two distinct layers of the clays at the site, where although the mineralogy is the same, the microstructure varies, namely, the upper layer contains large aggregates with bridges in between, while the lower layer is composed of mostly single particles with little aggregation, and much less bridging[12][13]. This maybe the reason that the numerical results using the Modified Cam-Clay model, as shown in Fig. 2 to Fig. 4, showed little better-match with the experimental results in the upper layer since the microstructure in the upper layer shows more clayey characteristics.

In Fig. 3, the model predictions are a little bit larger than the experimental data, which maybe the sluggish behavior of pore water pressure due to incomplete saturation and/or clogging of the filter element.

When penetration stops for dissipation test in heavily overconsolidated clay, it is possible for negative excess pore water pressure measured behind cone tip ( $u_2$  type) to occur at the initial part of excess pore water pressure profile[4][11][12] but no negative values were measured experimentally and calculated numerically in this study, which is because the soil is not under heavily overconsolidated. For overconsolidated clay, it is also possible for pore water pressure measured behind cone tip ( $u_2$  type) to initially increase before dissipation[3][11] [13]. In Fig. 4, no obvious initial increase of pore water pressure was recorded, that confirms the clay is not under overconsolidated state. Actually, when penetration stops,

pore water pressure is influenced by many factors such as stress redistribution, stress relaxation, and viscous and dynamic effects[11]. More frequent reading of measurement and dissipation test at various depth, therefore, might have given interesting results, especially for the very initial stage of dissipation.

## IV. Conclusions

In this study, it is used the rate-independent and the rate-dependent constitutive models to predict soil behavior inferring the piezocone penetration and dissipation field test. The results from the model predictions have been compared with the experimental results of field test. The Modified Cam-Clay model was used to simulate the plastic behavior of the clay. The both model predictions showed good agreement with the experimental results all in cone tip resistance profile, pore water pressure profile, and dissipation behavior. The rate-dependent predictions gave better results than the rate-independent predictions as expected, that is appreciated since the process of the piezocone penetration and dissipation depends on the testing speed, namely, strain rate.

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**Youngsun Song**

[Regular Member]



- Feb. 1981: Dept. of Civil Engineering, Korea University(B.S.)
- Feb. 1983: Dept. of Civil Engineering, Korea University(M.S.)
- Aug. 1989: Dept. of Civil Engineering, Chung Nam University(Ph.D.)
- Currently Professor of Civil and Environmental Eng. Kongju University. since Mar. 1993.

<Research Area>

Rock Slope Stability, Site Investigation