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Use of Markov Chain Monte Carlo in Estimating the Economy Model

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Abstract

This project follows the heterogeneous agent market segmented model of Landon-Lane and Occhino (2007) with using Korean data, M1 and GDP deflator from 1882:I to 2007:II. This paper estimates parameters with Monte Carlo Markov Chain. The fraction of traders, λ, in Korea is 15.64%. The quarterly preferences discount factor's, β, posterior mean is 0.9922. The posterior mean of the inverse of the elasticity of the labor supply to the real wage, φ, is 0.0316. The elasticity of the labor supply to the real wage has a very large value. By Hansen (1985) and Christiano and Eichenbaum (1992) and Cooley and Hansen (1989), models having large elasticity of the aggregate labor supply better match macroeconomic data.

Key words : Segmented Market, Dynamic Stochastic General Equilibrium, Markov Chain Monte Carlo

1. Introduction

Many economic literatures have assumed that all households participate in the financial market. Data in the real world, however, show that only a fraction of household participates in the financial market at any time, that is, most households change infrequently their financial asset such like stocks, bonds and money market bonds. So we can think the financial asset market is segmented.

Annette (2002)[15] found that a large fraction of households does not participate in the stock market at any given point in time. Of households with positive financial wealth 28.4 percent in 1984, 34.0 percent in 1989, and 44.1 percent in 1994 had participated in the stock market. Also this paper showed that the set of stock market participants changes over time and found that the portfolio shares in financial asset are very heterogeneous over the period. Heaton and Lucas (2000) [8] find that realistic differences in background risk from sources such as labor income, proprietary income, and real estate can make an effect on the portfolio allocation. If a household's income is very risky, the household is less likely to participate in the financial asset market. So what make the market segmented? Annette (2002)[15] considered three different costs of stock market participation, a fixed transactions cost, a proportional transactions cost, and a per period participation cost, and found that households with high wealth can participate more frequently than ones with low wealth. It supports that a fixed transaction cost leads to structural state dependence in the stock market participation decision. A transaction cost can be more easily accepted by wealthy households but can be an obstacle for poor households. Therefore we can set an economic market segmentation model that a fraction of households can only participate in the financial market at any given period from these empirical evidences.

This paper will follow the model and method of Landon-Lane and Occhino (2007)[11]. This paper is organized as follows. We will review the literature on the segmented market model in section 2. Section 3 and 4 will describe the model and the method respectively. Section 5 explains data and prior distribution of parameters. The estimation results are represented in section 6. Section 7 conclude.

2. Literature Review

Alvarez, Lucas, and Weber (2001)[1] showed that a policy of increasing short-term interest rates can reduce inflation with essentially quantity theoretic models of monetary equilibrium by using a segmented market model. Since the market segmentation is consistent with the existence of a liquidity effect, the market incom-

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pleteness played very important role in explaining the properties of Taylor rules, studied within a Keynesian framework, using a neoclassical framework on the quantity theory of money. They derived the money demand function without interest rates in the segmented asset market model, so inflation rate could be the sum of money growth rate and the rate of change in velocity. An open market operation bond purchase increasing money supply is to reduce interest rate, the liquidity effect, but if there is no market segmentation, there is no liquidity effect in the paper.

Khan and Thomas (2006)[10] found that households who participate in the financial market in which there are fixed transactions costs hold money balances in excess of current spending because of limitation of access to asset markets. Since households can choose to participate in the asset markets responds to economic shocks, the extent of market segmentation changes over time, endogenous market segmentation, in this paper. The price adjustment changes more slowly and the liquidity effect lasts more persistently in the endogenous than in the exogenous market segmentation. They found that the increasing persistence in the inflation response to endogenous changes in the distribution of households leads to persistent changes in interest rates.

Landon-Land and Occhino (2007)[11] estimates and evaluates a heterogeneous agents segmented market model with endogenous production. Their model is very different from the standard limited participation model in that the traders, participants in asset market, receive lump-sum monetary transfers from the government and firms borrow money form the traders to finance production. They found that the segmented market friction significantly gets better the statistical out-of-sample forecast performance of the model, and helps generate delayed and realistic impulse response functions to monetary policy shocks.

Mizrach and Occhino (2007)[13] estimate the impact of monetary policy to the dynamics of bond real returns by adopting a heterogeneous agents segmented market model. They found that the segmented markets model can reproduce the sign and the size of the impulse response of bond returns to monetary policy shocks.

Occhino (2007)[14] showed that the segmented market model can replicate results that a contractionary monetary policy shock is able to decrease the aggregate endowment for several quarters and lead real the real interest rate to remain persistently. In this paper, the nominal interest rate and the aggregate output are exogenously modeled and the real interest rate is an endogenous variable.

3. Model[1]

The economy is consisted with many households, many firms normalized to one, and a monetary authority and is a cash-in-advanced production economy. Firms issued one-period bonds and a monetary authority issued money. Firms input labor to produce a single non-durable consumption good. There are two uncertainties in the economy, monetary policy shocks and technology shocks. There are two types of households, traders who receive lump-sum transfers of money form the monetary authority, participate in the bond market, and purchase bonds and non-traders. The same type households are identical in all respects. The fraction of trader (non-trader) is denoted by $\lambda \in (1,0]$ ($\lambda^{*} \equiv 1\text{-}\lambda$).

Traders (Non-traders) hold the cash balances, $\alpha_t(\alpha_t^{*})$, at the beginning of period t. Bonds, labor and goods are traded in exchange of money and these three markets meet in sequence. Traders receive lump-sum transfers of money, τ_t, from the monetary authority. The nominal interest of bonds is $i_t > 0$. Firms issued bonds to finance the rent for labor. The wage rate is $w_t > 0$.

Aggregate production is

$$y_t \equiv h_t n_t \tag{1}$$

where $h_t > 0$ is the product of the stock of technology $n_t > 0$ and is labor demand. The price of the consumption good produced by firms is $p_t > 0$. The money from selling labor at period t cannot be used to purchase consumption goods in the same period. The money supply, the aggregate amount of dollars, $m_t \equiv p_t y_t$. The fact that the production technology is linear makes equilibrium profits be zero and the equilibrium does not depend on the firms' ownership.

The inflation rate is,

$$\pi_t \equiv \log(p_{t+1}) - \log(p_t) \tag{2}$$

and the money growth rate is $\mu_t \equiv \log(m_{t+1}) - \log(m_t)$.

[1]This paper will follow the model of Landon-Lane and Occhino (2007).

The lump-sum transfers of money are set to target the nominal interest rate. Taylor interest rate rule is used as the monetary policy rule:

$$\hat{i}_{t+1}=\rho_i\hat{i}_t+(1-\rho_i)[\kappa_\pi\hat{\pi}_t+\kappa_y\hat{y}_t]+\sigma_i\varepsilon_{i,t+1} \tag{3}$$

where $\hat{i}_t$, $\hat{\pi}_t$, and $\hat{y}_t$ are respectively the deviation of the interest rate, inflation rate and the percentage deviation of output from their non-stochastic steady state values.$\rho_i \in [0,1)$ is the conditional first-order autocorrelation of the interest rate, $\kappa_\pi \geq 0$ is the response coefficient to inflation deviations, is the response coefficient to output percentage deviations, $\sigma_i > 0$ is the volatility of the monetary policy shock, and $\varepsilon_{i,t+1}$ is the normalized monetary policy shock independently and identically distributed as standard normal.

The exogenous process of the stock of technology is following as

$$\hat{h}_{t+1}=\rho_h\hat{h}_t+\sigma_h\varepsilon_{h,t+1} \tag{4}$$

where $\hat{h}_t$ is the percentage deviation of the stock of technology from its non-stochastic steady state value, $\rho_h \in [0,1)$is the conditional first-order autocorrelation, $\sigma_h > 0$ is the volatility of the technology shock and $\varepsilon_{h,t+1}$ is the normalized technology shock independently and identically distributed as standard normal. The two uncertainties, monetary policy shocks and technology shocks, are independent.

Each trader's optimal problem is as following:

$$\underset{\{b_c, c_t>0, l_t>0, a_{t+1}>0\}_{t=0}^{\infty}}{Max} E_0\left[\sum_{t=0}^{\infty}\beta^t\left(\frac{c_t^{1-\alpha}}{1-\alpha}-\phi\frac{l_t^{1+\varphi}}{1+\varphi}\right)\right]$$

subject to: $b_t+p_tc_t=a_t+\tau_t \quad a_{t+1}=w_tl_t+(1+i_t)b_t$ (5)

where c_t is consumption demand l_t, is labor supply, b_t is bond demand, and a_{t+1} is next period cash balances, and the trader's initial cash balances $a_0>0$ in period zero is given. $\beta \in (0,1)$is the preference discount factor $a>0(\neq 1)$, is the relative risk aversion, $\phi>0$, and $\varphi>0$ is the inverse of the elasticity of the labor supply to the real wage. The first-order conditions are as following:

$$v_t^1+v_t^2(1+i_t)=0 \quad \beta^t c_t^{-\alpha}-v_t^1p_t=0$$
$$-\beta^t\phi l_t^{\varphi}+v_t^2w_t=0 \quad E_t[v_{t+1}^1]=v_t^2 \tag{6}$$

where v_t^1 and v_t^2 are the Lagrange multipliers associated with the two constraints (5).

Non-trader's optimal problem is as following:

$$\underset{\{c_t^*>0, l_t^*>0, a_{t+1}^*>0\}_{t=0}^{\infty}}{Max} E_0\left[\sum_{t=0}^{\infty}\beta^t\left(\frac{c_t^{*1-a}}{1-\alpha}-\phi\frac{l_t^{*1+\varphi}}{1+\varphi}\right)\right]$$

subject to: $p_tc_t^*=a_{t+1}^*=w_tl_t^*$ (7)

where $a_0^*>0$ is given. The first condition is as following:

$$\beta^t c_t^{*-\alpha}-v_t^{1*}p_t=0 \quad -\beta^t\phi l_t^{*\varphi}+v_t^{2*}w_t=0$$
$$E_t[v_{t+1}^{1*}]=v_t^{2*} \tag{8}$$

Firms' optimal problem is as following:

$$\underset{\{n_t>0, d_t\}}{Max}\{h_tn_t-w_tn_t-i_td_t\}$$

subject to: $w_tn_t=d_t$ (9)

The equilibrium zero-profit condition is:

$$h_tp_t-w_t-i_tw_t=0 \tag{10}$$

The equilibrium conditions for the bond market, labor market and goods market are:

$$\lambda b_t=d_t \quad \lambda l_t+\lambda^* l_t^*=n_t \quad \lambda c_t+\lambda^* c_t^*=h_tn_t \tag{11}$$

A set of contingent sequences satisfying the definitions (1) and (2), the processes (3) and (4), the agents' constraints and necessary conditions, (5)-(10), and the equilibrium conditions (11) is an equilibrium.

4. Methodology

In this section, we will compute the non-stochastic steady state, and log-linearize the system around it. Let s_t (8×1) be the state vector consisting of $\hat{i}_t$, $\hat{h}_t$, $\hat{a}_t$, their lags and the lags of two Lagrange multipliers, one for traders and one for non-traders. Given these exogenous disturbance process the log-linearized Dynamic Stochastic General Equilibrium (DSGE) model can be expressed as a linear rational expectations (LRE) model solved by the methods described in Sims (2002). Then the LRE system will have the following solution by

Sims' method:

$$s_t = \Phi_1 s_{t-1} + \Phi_\varepsilon \varepsilon_t \quad (12a)$$

where $\varepsilon_t \equiv [\varepsilon_{i,t}, \varepsilon_{h,t}] \sim iidN(0,Q)$. From the equation (12a), we can express the log-linearized model as state-space model:

$$x_t = Bs_t \text{ ; measurement equation.} \quad (12b)$$

where x_t is an vector of observable variables, $[\hat{\mu}_t, \hat{\pi}_t]'$, where $\hat{\mu}_t$, and $\hat{\pi}_t$, are the deviations of the money growth rate and the inflation rate from steady state. From equations (12a) and (12b), we can derive a likelihood function of the linearized DSGE model and in the process obtaining the likelihood function the Kalman Filter was used.

To estimate the structural parameters of the model, Bayesian likelihood method is used. When some parameters of the model are poorly identified, Bayesian methods are useful. Posterior distributions

$$p(\vartheta|X_t,\text{M}) \propto p(\vartheta|\text{M})\, p(X_T|\vartheta,\text{M}) \quad (13)$$

where ϑ parameter vector, $X_T = \{x_t\}_{t=1}^{T}$ and M is likelihood function of model. Posterior distribution can tell us about our model and about our result illustrate our beliefs given the data. To analyze the Bayesian inference, we have to calculate a marginal distribution associated with ϑ. Markov Chain Monte Carlo (MCMC) methods is used to simulate N serially correlated draws from $p(\vartheta|X_T,\text{M})$.

5. Data and Prior Distributions

In this paper, seasonally adjusted quarterly data of M1 and the GDP deflator for the period 1981:IV-2007:II are used[2]. In this paper a high-pass infinite impulse response (IIR) filter filtering out the low frequency components of the data (cycles greater than ten years) is used to convert the data into deviations. Then we can use the high frequency information of the data in the estimation of the models.

The prior distributions are reported in (Table 1)[3].

The key structural parameter, λ, is to a mean of 0.35 and a 95% inter-quantile range (IQR) of [0.104, 0.652]. The quarterly preference discount factor, β, is set to a mean of 0.99 and a IQR of [0.980, 0.997]. Since the real interest rate is equal to the preference discount rate in the non-stochastic steady state, the range of annualized real interest rates from 1.2% to 8% and the mean of real interest rate is 4% annually. The inverse of the elasticity of the labor supply to the real wage, φ, is set to a mean of 1 and a 95% IQR of [0.122, 2.795]. The common annualized growth rate of all nominal variables in the non-stochastic steady state, γ, is set to a mean of 2 and a 95% of IQR [1.294, 2.856].

[2]Data can be obtained from website of Bank of Korea.

[3]This distribution is from Landon-Lane and Occhino (2007). For the details, refer to Landon-Lane and Occhino (2007).

6. Results

The 20,000 draws is produced from posterior distribution by Hastings-Metropolis chain and the first 1,000

Table 1. Prior Distribution for Structural Parameters

Parameter	Distribution	Mean	95% IQR
γ	Beta	0.35	[0.104 0.652]
β	Beta	0.99	[0.980 0.997]
α	Gamma	2.00	[0.544 4.372]
φ	Gamma	1.00	[0.122 2.795]
γ	Gamma	2.00	[1.294 2.856]
ρ_i	Beta	0.80	[0.717 0.872]
κ_π	Gamma on [1,∞]	2.00	[1.675 2.387]
κ_y	Gamma	0.50	[0.357 0.666]
ρ_h	Beta	0.90	[0.739 0.987]
σ_i	Gamma	2.00	[0.544 4.372]
σ_h	Gamma	2.00	[0.544 4.372]

Table 2. Posterior Moments (Quarterly data 1982:I– 2007:II)

Parameter	Distribution	Mean	95% IQR
γ	Beta	0.1564	[0.0312 0.1648]
β	Beta	0.9922	[0.9830 0.9978]
φ	Gamma	0.0316	[0.0064 0.0336]
γ	Gamma	1.9318	[1.1982 2.7755]
ρ_i	Beta	0.8334	[0.8060 0.8984]
κ_π	Gamma on [1,∞]	2.5032	[2.1517 2.7179]
κ_y	Gamma	0.2879	[0.1918 0.3484]
ρ_h	Beta	0.9941	[0.9888 1.0000]
σ_i	Gamma	3.6123	[1.6638 3.9646]
σ_h	Gamma	7.2810	[6.1234 7.6795]

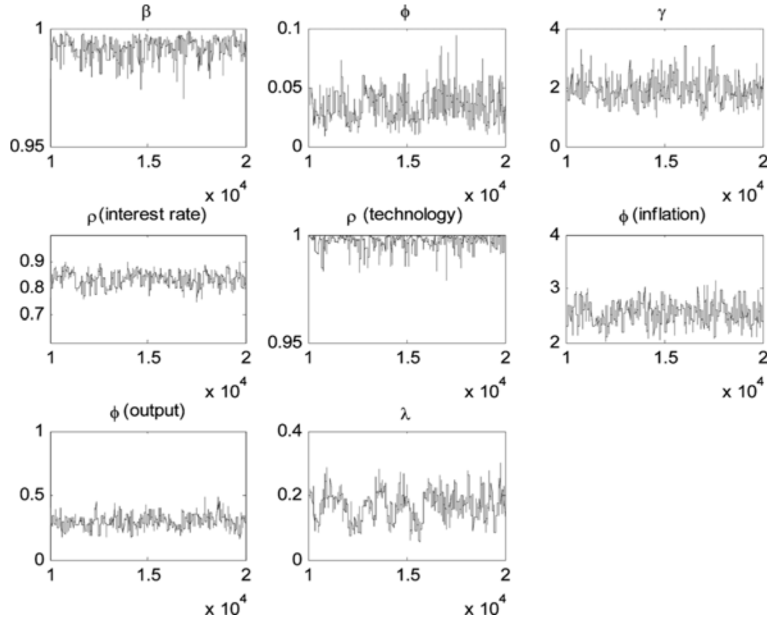

Fig. 1. Plot of Parameters by H-M RWHM.

draws is burn.

(Fig. 1) and (Table 2) make a summary on information of the posterior distribution of parameters for the period 1982:I-2007:II. The acceptance rate is 0.30. The fraction of traders, λ, in Korea is 15.64%. The 95% IQR is from 3.12% to 16.48%. The fraction of traders in USA is about 21% by Landon-Lane and Occhino (2007)[11]. From this result, we can confirm that the financial market in Korea is less developed than in USA. The quarterly preferences discount factor's, β, posterior mean is 0.9922. This numerical value is equal to the value of the quarterly preferences discount factor in USA by Landon-Lane and Occhino (2007). The posterior mean of the inverse of the elasticity of the labor supply to the real wage, φ, is 0.0316. The elasticity of the labor supply to the real wage has a very large value. By Hansen (1985)[7] and Christiano and Eichenbaum (1992)[3] and Cooley and Hansen (1989)[5], models having large elasticity of the aggregate labor supply better match macroeconomic data.

In (Table 3), convergence diagnostics for chains are reported. The convergence statistic (GR) defined in Gelman and Rubin (1992)[6] is used for convergence diagnostics in this paper. The GR statistic is calculated using the first and second half of the chain respectively. The GR statistic is the ratio of the within chain variance and the between partial chain variance of our estimates. If a value of the GR statistic close to 1, we can refer that the chain has converged. From (Table 3), we can show that the values of GR statistics of all parameters close to 1.

7. Conclusion

This project follows the heterogeneous agent market segmented model of Landon-Lane and Occhino (2007) [11]. In the model of Landon-Lane and Occhino, there are very important three assumptions. First, only traders take delivery of lump-sum transfers of money from the monetary authority that is introduced to model the liquidity effect of monetary injections. Second, firms borrow money by selling bonds to finance in producing goods. From this assumption, the model can produce the real effects of monetary injections. Third, traders and non-traders belong to separate households. So, monetary policy can have distributional and more persistent and delayed effects. In the model, households cannot purchase consumption with the money earned by selling labor in the same period.

The fraction of traders, λ, in Korea is 15.64%. The quarterly preferences discount factor's, β, posterior mean is 0.9922. The posterior mean of the inverse of the elasticity of the labor supply to the real wage, φ, is 0.0316. The elasticity of the labor supply to the real wage has a very large value.

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